Electrical Power and Energy Systems 51 (2013) 89–97

Contents lists available at SciVerse ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Solvability and solutions for bus-type extended load flow

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ARTICLE INFO

Article history:
Received 24 October 2012
Received in revised form 5 February 2013
Accepted 22 February 2013
Available online 29 March 2013

Keywords:
Power systems
Load flow
Bus type

ABSTRACT

In traditional load flow calculation, only three types of buses, *PQ*, *PV*, and $V\theta$, are generally specified. To accommodate the integration of new kinds of power equipment and flexible load flow control facilities, extra bus types are needed in load flow model. The load flow model incorporated with all possible bus types is named as bus-type extended load flow (BELF) for short in this paper. Both Newton–Raphson and decoupled BELF solutions are developed. An important problem for BELF is its solvability, which is carefully studied and topology-based criteria are proposed to ascertain BELF's solvability. Numerical tests are carried out to verify the convergence of the BELF solution and the correctness of the proposed solvability criteria.

1. Introduction

Load flow calculation is a basic function in power system analysis. It is widely used in security analysis, optimal power flow, and some other network analysis applications. Although the study on load flow calculation algorithms have a long history [1–4], some techniques pertaining to special load flows, such as ill-conditioned load flows [5,6] and distribution network load flows [7–10], are still being actively researched. It is noticed that more and more new types of electrical equipment, such as distributed generators (DGs) and flexible AC transmission system (FACTS) components are being installed in power systems. It is improper to model all these new types of electrical equipment as traditional *PQ* or *PV* buses in load flow calculation. On the other hand, introduce extra bus types can make the load flow model to be more confirmed to the performance of new control facilities such as bus voltage control, external network merging and voltage stability analysis.

There are some published papers on modeling new types of equipment by introducing extra bus types, including DGs [11,12], static volt-ampere reactive (VAR) compensators [13], unified power flow controllers [14–16], and other FACTS equipment [17,18]. A current-based modeling technique of the FACTS devices in load flow calculation is proposed in paper [19–21]. On the other hand, for load flow control, there are also several researches include extra bus types to achieve their respective control objective, instead of solving an optimization problem. For example, Refs. [22,23] introduce *PQV* and *P* buses, thus an effective method for bus voltage control has been developed, *PQV* and *P* buses are also introduced for static voltage stability analysis in [24].

Although load flow calculations including extra bus types have been studied for specified purposes, there are still some important problems to be solved for this bus type extended load flow (BELF). This first one is how to formulate a BELF with various bus types and its solution. The second one is how to evaluate the solvability of BELF, because improper combination of bus types may lead to its insolvability.

The main work of this paper is to provide a systematic study on BELF. An important contribution of this paper is analysis the solvability problem for BELF, and very efficient topology-based solvability criteria are given. The content of this paper is organized as follows: The BELF model is presented and all the possible bus types are given in Section 2. Newton–Raphson and fast decoupled solutions are briefly introduced in Section 3. The solvability problem is discussed and topology-based solvability criteria are given in Section 4. In Section 5, extensive numerical tests have been done to verify the convergence of BELF and the solvability criteria.

2. Model

Load flow calculation is based on bus voltage equation, thus for each bus, four electric variables are generally considered: *P*, *Q*, *V*, and θ. Each variable among them may be known or unknown, thus there are 16 possible bus types under all conditions. All the 16 bus types are named according to their known variables, and are listed in Table 1. The traditional bus types are highlighted in grey.

The selection of bus types depends on practical needs. Traditional, *PQ* buses are used to model loads or generators without voltage control ability, and *PV* buses are used to model generators with voltage control ability. By the development of flexible load flow control and new types of electric equipment, more and more extra bus types will be introduced in the load flow model. In fact,

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http://dx.doi.org/10.1016/j.ijepes.2013.02.013

Table 1
All possible bus types.

P	Q	V	Θ	Bus Type	P	Q	V	θ	Bus Type
√	-	-	-	P	-	-	-	-	0
√	-	-	√	Pθ	-	-	-	√	θ
√	-	√	-	PV	-	-	√	-	V
√	-	√	√	PVθ	-	-	√	√	Vθ
√	√	-	-	PQ	-	√	-	-	Q
√	√	-	√	PQθ	-	√	-	√	Qθ
√	√	√	-	PQV	-	√	√	-	QV
√	√	√	√	PQVθ	-	√	√	√	QVθ

"√" = known, "–" = unknown.

many of them have been used, such as PQV buses, P buses, $PQV\theta$ buses and 0 buses.

The load flow equations considering all possible bus types can be formulated as:

$$\begin{aligned}
&\Delta P_i = P_i^{sp} - V_i\sum_{j\in i} V_i(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) = 0 \quad && i\in N_P\\
&\Delta Q_i = Q_i^{sp} - V_i\sum_{j\in i} V_i(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) = 0 \quad && i\in N_Q\\
&\Delta V_i = V_i^{sp} - V_i = 0 && i\in N_V\\
&\Delta\theta_i = \theta_i^{sp} - \theta_i = 0 && i\in N_\theta
\end{aligned} \tag{1}$$

where N_P, N_Q, N_V, and N_θ are the bus sets, whose respective P, Q, V, and θ are specified. The superscript sp indicates the specified value. In polar coordinates, V and θ are normally used as state variables, and the latter two equations are not necessary. Thus, load flow equations can be written as:

$$\begin{aligned}
&\Delta P_i = P_i^{sp} - V_i\sum_{j\in i} V_i(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) = 0 \quad i\in N_P\\
&\Delta Q_i = Q_i^{sp} - V_i\sum_{j\in i} V_i(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) = 0 \quad i\in N_Q
\end{aligned} \tag{2}$$

The existence of each bus type depends on whether its P and Q are specified. In BELF N_P includes all the eight bus types with P specified (P, $P\theta$, PV, $PV\theta$, PQ, $PQ\theta$, PQV and $PQV\theta$) as listed in Table 1 while N_P only include the PQ and PV buses in the conventional load flow. The connotation for N_Q is similar.

3. Solution

3.1. Newton–Raphson solution

BELF can be viewed as an extension of ordinary load flow, and hence its solution can be derived directly from the solution of ordinary load flow, while the extra bus types should be considered properly. The correction equation of Newton–Raphson method is:

$$\boldsymbol{J}\begin{bmatrix}\Delta\boldsymbol{\theta}\\ \Delta\boldsymbol{V}\end{bmatrix} = \begin{bmatrix}\Delta\boldsymbol{P}\\ \Delta\boldsymbol{Q}\end{bmatrix} \tag{3}$$

where $\boldsymbol{J}$ is the Jacobian matrix used in the Newton method (which should be a square matrix). It should be noticed that the following necessary condition should be satisfied to make the load flow problem solvable.

Condition 1. $n_P + n_Q + n_V + n_\theta = 2N$, where n_P, n_Q, n_V, and n_θ are the numbers of buses whose P, Q, V, and θ are specified, and N is the total number of buses.

Condition 1 means that the number of known variables in a load flow problem should be equal to that of unknown variables.

The Newton–Raphson correction Eq. (3) can be further expressed as:

$$\begin{bmatrix}\boldsymbol{J}_{P\theta} & \boldsymbol{J}_{PV}\\ \boldsymbol{J}_{Q\theta} & \boldsymbol{J}_{QV}\end{bmatrix}\begin{bmatrix}\Delta\boldsymbol{\theta}_{\overline{N}_\theta}\\ \Delta\boldsymbol{V}_{\overline{N}_V}\end{bmatrix} = \begin{bmatrix}\Delta\boldsymbol{P}_{N_P}\\ \Delta\boldsymbol{Q}_{N_Q}\end{bmatrix} \tag{4}$$

where

$$\boldsymbol{J}_{P\theta} = \frac{\partial\boldsymbol{P}}{\partial\boldsymbol{\theta}},\quad \boldsymbol{J}_{PV} = \frac{\partial\boldsymbol{P}}{\partial\boldsymbol{V}},\quad \boldsymbol{J}_{Q\theta} = \frac{\partial\boldsymbol{Q}}{\partial\boldsymbol{\theta}},\quad \boldsymbol{J}_{QV} = \frac{\partial\boldsymbol{Q}}{\partial\boldsymbol{V}} \tag{5}$$

$\Delta\boldsymbol{P}_{N_P}$ is the vector of active power unbalance for the buses in N_P, $\Delta\boldsymbol{Q}_{N_Q}$ is the vector of reactive power unbalance for the buses in N_Q. $\Delta\boldsymbol{\theta}_{\overline{N}_\theta}$ is the vector of bus voltage angle correction for the buses beside N_θ, and $\Delta\boldsymbol{V}_{\overline{N}_V}$ is the vector of bus voltage amplitude correction for the buses besides N_V. In (5), $\boldsymbol{J}_{P\theta}$ is a $n_P\times(n-n_\theta)$ matrix:

$$\boldsymbol{J}_{P\theta,(i,j)} = \frac{\partial\Delta\boldsymbol{P}_i}{\partial\boldsymbol{\theta}_j} = \begin{cases} -V_i^2B_{ii} + V_i\sum_{j\in i}V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) & i=j\\ V_iV_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) & i\neq j\end{cases} \quad i\in N_P, j\notin N_\theta \tag{6}$$

Table 2
Four bus types in P-sub iteration.

Bus type indexes	P	θ	Examples
1	√	–	P, PQ, PV, PQV
2	–	–	0, Q, V, QV
3	√	√	$P\theta$, $PQ\theta$, $PV\theta$, $PQV\theta$
4	–	√	θ, $Q\theta$, $V\theta$, $QV\theta$

The expression of $\boldsymbol{J}_{PV}$, $\boldsymbol{J}_{P\theta}$ and $\boldsymbol{J}_{QV}$ are similar with $\boldsymbol{J}_{P\theta}$. Using the conventional Newton–Raphson iteration procedure and the correct equation described in (4), load flow problem can be solved.

3.2. Fast decoupled solution

The correction equations of fast decoupled solutions are:

$$\begin{aligned} -\boldsymbol{B}'\Delta\boldsymbol{\theta} &= \Delta\boldsymbol{P}/\boldsymbol{V} \\ -\boldsymbol{B}''\Delta\boldsymbol{V} &= \Delta\boldsymbol{Q}/\boldsymbol{V} \end{aligned} \tag{7}$$

where the matrices $\boldsymbol{B}'$ and $\boldsymbol{B}''$ are the constant coefficient matrices in the P-sub iteration and Q-sub iteration, respectively. Similarly, following necessary condition should be satisfied for solvability.Condition 2 $n_P + n_\theta = N$, $n_Q + n_V = N$.

The P-sub iteration is used to demonstrate the proposed method. Since the P-sub iteration involves only P and θ, all buses can be divided into four types, according to their P and θ are known or not. These bus types are listed in Table 2.

By reordering all the buses according to the bus types of Table 2, the P–θ correction equation including all types of buses can be rewritten as:

$$-\begin{bmatrix} \boldsymbol{B}'_{11} & \boldsymbol{B}'_{12} & \boldsymbol{B}'_{13} & \boldsymbol{B}'_{14} \\ \boldsymbol{B}'_{21} & \boldsymbol{B}'_{22} & \boldsymbol{B}'_{23} & \boldsymbol{B}'_{24} \\ \boldsymbol{B}'_{31} & \boldsymbol{B}'_{32} & \boldsymbol{B}'_{33} & \boldsymbol{B}'_{34} \\ \boldsymbol{B}'_{41} & \boldsymbol{B}'_{42} & \boldsymbol{B}'_{43} & \boldsymbol{B}'_{44} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_1 \\ \Delta\boldsymbol{\theta}_2 \\ \Delta\boldsymbol{\theta}_3 \\ \Delta\boldsymbol{\theta}_4 \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_1/\boldsymbol{V}_1 \\ \Delta\boldsymbol{P}_2/\boldsymbol{V}_2 \\ \Delta\boldsymbol{P}_3/\boldsymbol{V}_3 \\ \Delta\boldsymbol{P}_4/\boldsymbol{V}_4 \end{bmatrix} \tag{8}$$

The subscripts represent the bus type indices of Table 2.

For type 2 and type 4 buses, their $\boldsymbol{P}$ are unknown, thus $\Delta\boldsymbol{P}_2$ and $\Delta\boldsymbol{P}_4$ are unavailable. The corresponding $\boldsymbol{P}$-equations should be removed. For type 3 and 4 buses, their $\boldsymbol{\theta}$ are specified, thus $\Delta\boldsymbol{\theta}_3$ and $\Delta\boldsymbol{\theta}_4$ need not be calculated and can be removed. So (8) can be simplified to:

$$-\begin{bmatrix} \boldsymbol{B}'_{11} & \boldsymbol{B}'_{12} \\ \boldsymbol{B}'_{31} & \boldsymbol{B}'_{32} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_1 \\ \Delta\boldsymbol{\theta}_2 \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_1/\boldsymbol{V}_1 \\ \Delta\boldsymbol{P}_3/\boldsymbol{V}_3 \end{bmatrix} \tag{9}$$

Similarly, the four bus types in the Q-sub iteration are given in Table 3, and the Q–V correction equation is as follows:

$$-\begin{bmatrix} \boldsymbol{B}11'' & \boldsymbol{B}''_{12} \\ \boldsymbol{B}''_{31} & \boldsymbol{B}''_{32} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{V}_1 \\ \Delta\boldsymbol{V}_2 \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{Q}_1/\boldsymbol{V}_1 \\ \Delta\boldsymbol{Q}_3/\boldsymbol{V}_3 \end{bmatrix} \tag{10}$$

4. Solvability

For traditional load flow, it has been proved solvable under normal operation conditions. However, the form of load flow equations in BELF has changed and its solvability cannot be guaranteed any more since the including of extra bus types. Fig. 1 can be used to illustrate the solvability problem in BELF.

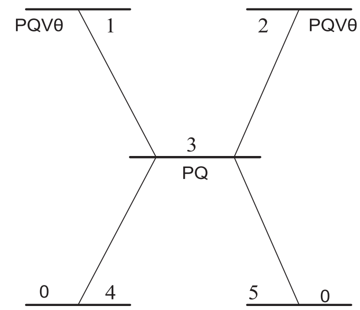


Fig. 1. An example for solvability.

In Fig. 1, buses 1 and 2 are $PQV\theta$ buses, bus 3 is a PQ bus, and buses 4 and 5 are 0 buses (without any specified quantities). It can be easily found that this load flow problem satisfies necessary Condition 1 and Condition 2. One finds that it is impossible to calculate the load flow in branches 3–4 and 3–5 because there are no specified quantities at buses 4 and 5. Hence, the load flow problem for the network of Fig. 1 is unsolvable.

Instead of checking the rank of the high dimension coefficient matrix, convenient criteria which directly based on topology are given in the following part of this paper.

Criterion 1. The number of type 2 buses is equal to the number of type 3 buses, and there exists at least one path connecting each type 2 bus to a type 3 bus. If all these paths are disjoint and without type 4 buses, the P-sub iteration of Eq. (9) will be solvable.

P-sub iteration of Eq. (9) is used to illustrate and prove this solvable criterion. In P-sub iteration, P and θ on type 2 bus are both specified, and on type 3 bus are both unknown. The unknown variables should be equal to specified variables in load flow model, thus the number of type 2 buses should be equal to type 3 buses. An induction proof for this criterion is as follows.

The P-sub iteration of Eq. (9) is solvable if the coefficient matrix in Eq. (9) has full rank. By eliminating the $\boldsymbol{B}'_{31}$ sub matrix in Eq. (9), the correction equation becomes:

$$-\begin{bmatrix} \boldsymbol{B}'_{11} & \boldsymbol{B}'_{12} \\ \boldsymbol{0} & \widetilde{\boldsymbol{B}}'_{32} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_1 \\ \Delta\boldsymbol{\theta}_2 \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_1/\boldsymbol{V}_1 \\ \Delta\widetilde{\boldsymbol{P}}_3 \end{bmatrix} \tag{11}$$

where

$$\begin{aligned} \widetilde{\boldsymbol{B}}'_{32} &= \boldsymbol{B}'_{32} - \boldsymbol{B}'_{31}\boldsymbol{B}'^{-1}_{11}\boldsymbol{B}'_{12}, \\ \Delta\widetilde{\boldsymbol{P}}_3 &= \Delta\boldsymbol{P}_3/\boldsymbol{V}_3 - \boldsymbol{B}'_{31}\boldsymbol{B}'^{-1}_{11}\Delta\boldsymbol{P}_1/\boldsymbol{V}_1 \end{aligned} \tag{12}$$

$\boldsymbol{B}'_{11}$ is the coefficient matrix of the P–θ correction equation for ordinary load flow, which is well known to be non-singular. So the P-sub iteration is solvable only if the $\widetilde{\boldsymbol{B}}'_{32}$ sub matrix has full rank. Assume that the number of type 2 buses is equal to type 3 buses, and denote this number by m:

(1) Suppose that $m = 1$. If there is generally no direct physical branch connecting a type 2 bus with a type 3 bus, B'_{32} in (12) will equal to zero. The topological implication of Gaussian elimination is the buses connected with a certain bus will be directly connected by an injection branch after this bus is eliminated [25]. Assume that Criterion 1 is satisfied, it means that there will exist such an injection branch between the types 2 and 3 buses after all the type 1 buses on the path have been eliminated. Thus $-B'_{31}\boldsymbol{B}^{-1'}_{11}B'_{12}\neq 0$. Therefore the scalar $\widetilde{\boldsymbol{B}}'_{32}\neq 0$, and the P-sub iteration will be solvable.
(2) Suppose that $m = k$. Then, $\boldsymbol{B}'_{32}$ and $\widetilde{\boldsymbol{B}}'_{32}$ in Eq. (11) are k-dimensional. When Criterion 1 is satisfied, $\widetilde{\boldsymbol{B}}'_{32}$ has full rank, and the P-sub iteration is solvable.
(3) We now verify the case $m = k + 1$. When $m = k + 1$, the P–θ correction Eq. (9) can be expanded as follows:

Table 3
Four bus types in Q-sub iteration.

Bus type indexes	Q	V	Examples
1	√	–	Q, PQ, $Q\theta$, $PQ\theta$
2	–	–	0, P, θ, $P\theta$
3	√	√	QV, PQV, $QV\theta$, $PQV\theta$
4	–	√	V, $V\theta$, PV, $PV\theta$

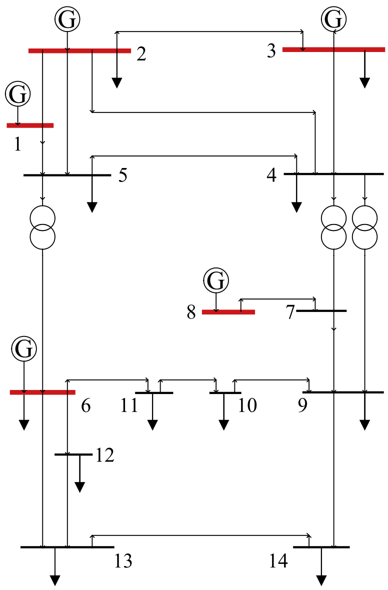


Fig. 2. IEEE 14-bus system.

$$-\begin{bmatrix} \boldsymbol{B}'_{11} & \boldsymbol{B}'_{12} & \boldsymbol{B}'_{12'} \\ \boldsymbol{B}'_{31} & \boldsymbol{B}'_{32} & \boldsymbol{B}'_{32'} \\ \boldsymbol{B}'_{3'1} & \boldsymbol{B}'_{3'2} & B'_{3'2'} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_1 \\ \Delta\boldsymbol{\theta}_2 \\ \Delta\theta_{2'} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_1/\boldsymbol{V}_1 \\ \Delta\boldsymbol{P}_3/\boldsymbol{V}_3 \\ \Delta P_{3'}/V_{3'} \end{bmatrix} \tag{13}$$

The subscripts 2′ and 3′ represent the additional types 2 and 3 buses beyond the case $m = k$ of step 2. By eliminating the type 3 buses (k buses) and type 3′ bus (1 bus) from Eq. (13), the following equation can be obtained:

$$-\begin{bmatrix} \boldsymbol{B}'_{11} & \boldsymbol{B}'_{12} & \boldsymbol{B}'_{12'} \\ \boldsymbol{0} & \widetilde{\boldsymbol{B}}'_{32} & \widetilde{\boldsymbol{B}}'_{32'} \\ \boldsymbol{0} & \boldsymbol{0} & \widetilde{B}'_{3'2'} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_1 \\ \Delta\boldsymbol{\theta}_2 \\ \Delta\theta_{2'} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_1/\boldsymbol{V}_1 \\ \Delta\widetilde{\boldsymbol{P}}_3 \\ \Delta\widetilde{P}_{3'} \end{bmatrix} \tag{14}$$

where

$$\widetilde{\boldsymbol{B}}'_{32} = \boldsymbol{B}'_{32} - \boldsymbol{B}'_{31}\boldsymbol{B}'^{-1}_{11}\boldsymbol{B}'_{12}$$

$$\widetilde{B}'_{3'2'} = B'_{3'2'} - \boldsymbol{B}'_{3'1}\boldsymbol{B}'^{-1}_{11}\boldsymbol{B}'_{12'} - \widetilde{\boldsymbol{B}}'_{3'2}\widetilde{\boldsymbol{B}}'^{-1}_{32}\widetilde{\boldsymbol{B}}'_{32'} \tag{15}$$

In step 2, it is assumed that $\widetilde{\boldsymbol{B}}'_{32}$ has full rank. Hence, to determine whether the coefficient matrix of (14) is singular, it suffices to check whether or not $\widetilde{B}'_{3'2'} \neq 0$.

If $B'_{3'2'}$ in Eq. (13) is equal to zero. The following two cases must be considered:

(a) If there originally exists a path between buses 2′ and 3′ that satisfies Criterion 1, then $-\boldsymbol{B}'_{3'1}\boldsymbol{B}'^{-1}_{11}\boldsymbol{B}'_{12'} \neq 0$. Hence $\widetilde{B}'_{3'2'} \neq 0$.
(b) After eliminating all type 1 buses, if there exists a branch between the type 2′ bus and a type 3 bus, $\widetilde{\boldsymbol{B}}'_{32'} \neq 0$. At the same time, if there exists another branch between the type 3′ bus and a type 2 bus, $\widetilde{\boldsymbol{B}}'_{3'2} \neq 0$. Since Criterion 1 is satisfied, $\widetilde{\boldsymbol{B}}'_{32'}$ and $\widetilde{\boldsymbol{B}}'_{3'2}$ should both be nonzero. Since $\widetilde{\boldsymbol{B}}'_{32}$ has full rank, $-\widetilde{\boldsymbol{B}}'_{3'2}\widetilde{\boldsymbol{B}}'^{-1}_{32}\widetilde{\boldsymbol{B}}'_{32'} \neq 0$, and again $\widetilde{B}'_{3'2'} \neq 0$.

Thus, the P-sub iteration will be solvable, and Criterion 1 is sufficient.

The necessity of Criterion 1 can be proved in a similar way. And Criterion 1 is a necessary and sufficient condition.

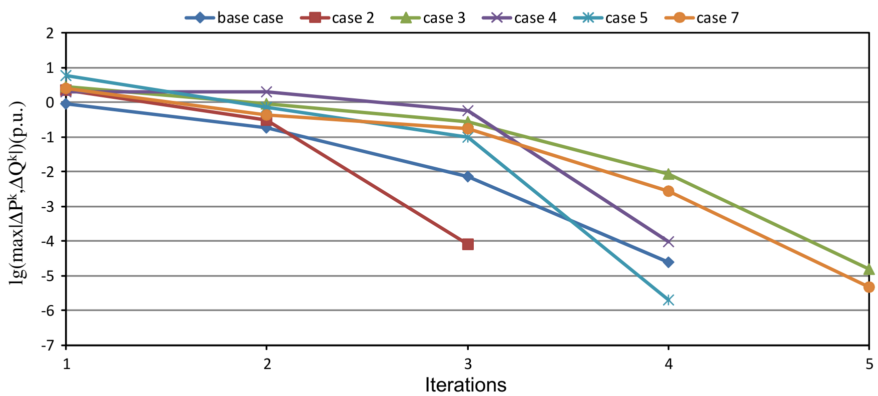
Similarly, for Q-sub iteration, all buses can be classified into the four types listed in Table 3, and the criteria for the solvability of the Q-sub iteration is as follows:

Criteria 2. The number of type 2 buses is equal to the number of type 3 buses, and there exists at least one path connecting each type 2 bus to a type 3 bus. If all these paths are disjoint and without type 4 buses, the Q-sub iteration of Eq. (11) will be solvable.

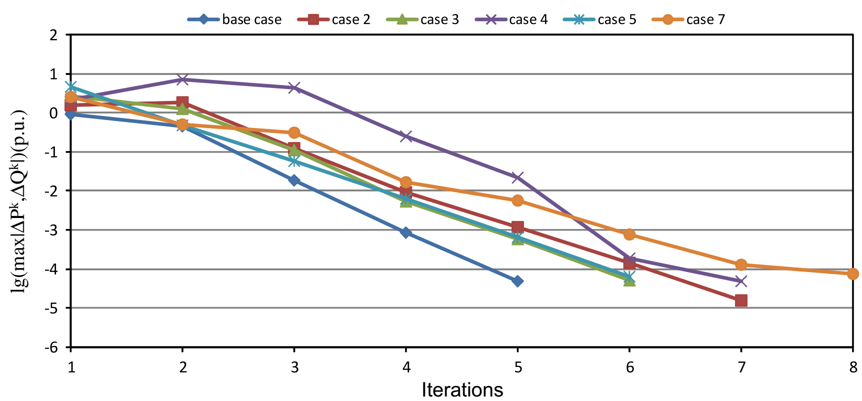
The analysis above is based on fast decoupled solution. In fact, it is well known that the coefficient matrix in fast decoupled load flow solution is an approximation of the Jacobian matrix used in Newton–Raphson solution, so these solvability criteria can also be used in Newton–Raphson solution approximately.

Table 4
Test results of IEEE 14-bus system for solvability criterion.

Case indexes	Bus type adjustment	Iteration no. (NR/FDLF)	Solvability by criterion	Effective paths for Criterion 1 (P-sub iteration)	Effective paths for Criterion 2 (Q-sub iteration)	Criterion correctness
1	13, 14: PQ bus → $PQV\theta$ buses 2, 3: PV bus → 0 buses	–	Unsolvable	–		Correct
2	13, 14: PQ bus → $PQV\theta$ buses 6, 8: PV bus → 0 buses	3/7	Solvable	8 → 7 → 9 → 14 6 → 13	8 → 7 → 9 → 14 6 → 13	Correct
3	5: PQ bus → Q bus 13: PQ bus → $PQ\theta$ bus 8: PV bus → P bus 4: PQ bus → PQV bus	5/6	Solvable	5 → 6 → 13	8 → 7 → 4	Correct
4	2: PV bus → P bus 11: PQ bus → $PQV\theta$ bus 3: PV bus → V bus	4/7	Solvable	2 → 4 → 7 → 9 → 11	3 → 4 → 7 → 9 → 11	Correct
5	1: $V\theta$ bus → $P\theta$ bus 2: PV bus → QV bus 8: PV bus → QV bus 7: PQ bus → $P\theta$ bus	4/6	Solvable	2 → 1 8 → 7	1 → 2 7 → 8	Correct
6	1: $V\theta$ bus → $P\theta$ bus 10: PQ bus → QV bus 8: PV bus → QV bus 9: PQ bus → $P\theta$ bus	–	Unsolvable	–		Correct
7	1: $V\theta$ bus → $P\theta$ bus 2: PV bus → QV bus 3: PV bus → P bus 4: PQ bus → PQV bus 5: PQ bus → PQV bus 6: PV bus → P bus 7: PQ bus → PQV bus 8: PV bus → P bus 9: PQ bus → P bus 10: PQ bus → P bus 11: PQ bus → PQV bus 12: PQ bus → PQV bus 13: PQ bus → P bus 14: PQ bus → PQV bus	5/8	Solvable	2 → 1	1 → 2 3 → 4 6 → 5 8 → 7 9 → 14 10 → 11 13 → 12	Correct



(a) Newton-Raphson method



(b) FDLF method

Fig. 3. Convergence curves for the solvable cases in the IEEE 14-bus system.

Generally speaking, not all BELF problems can be solvable. However, in most practical applications, such as modeling new kinds of power equipment or load flow control, unconventional bus types are usually concentrated in terms of electrical distance, which makes it easy to satisfy Criteria 1 and 2.

5. Numerical tests

To verify the solvability criteria introduced in this paper, numerical tests were conducted on IEEE 14-bus, 30-bus and 118-bus systems. The convergence thresholds for all load flow calculations were set as 0.0001 p.u.

5.1. IEEE 14-bus system

The IEEE 14-bus system is shown in Fig. 2. The original bus types in the IEEE 14-bus system are as follows: bus 1 is a $V\theta$ bus, buses 2, 3, 6, and 8 are *PV* buses, and the remaining buses are *PQ* buses. The $V\theta$ and *PV* buses are denoted by generator icons in Fig. 2.

In what follows, two different cases are designed and studied in detail to verify the criteria for solvability.

Case 1: buses 2 and 3 are changed to 0-type buses, and buses 13 and 14 are changed to $PQV\theta$ buses. According to Criteria 1 and 2, two disjoint paths between the $PQV\theta$ buses and 0 buses are required to guarantee solvability. To satisfy Criterion 1, these paths must exclude $V\theta$ buses (type 4 buses in the *P*-sub iteration), and to satisfy Criterion 2, they must exclude *PV* buses and $V\theta$ buses (type 4 buses in the *Q*-sub iteration). Since Criteria 1 and 2 should be both satisfied, the two disjoint paths between the $PQV\theta$ buses and 0 buses should not include any *PV* buses or $V\theta$ buses. As

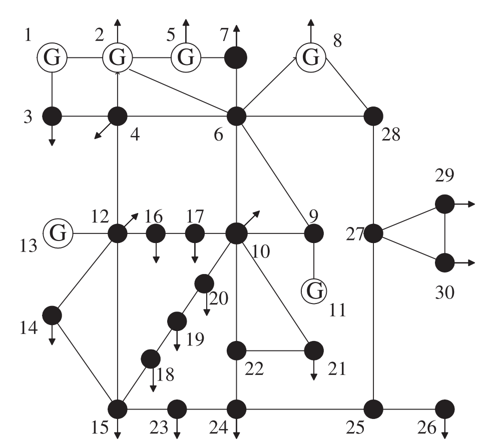


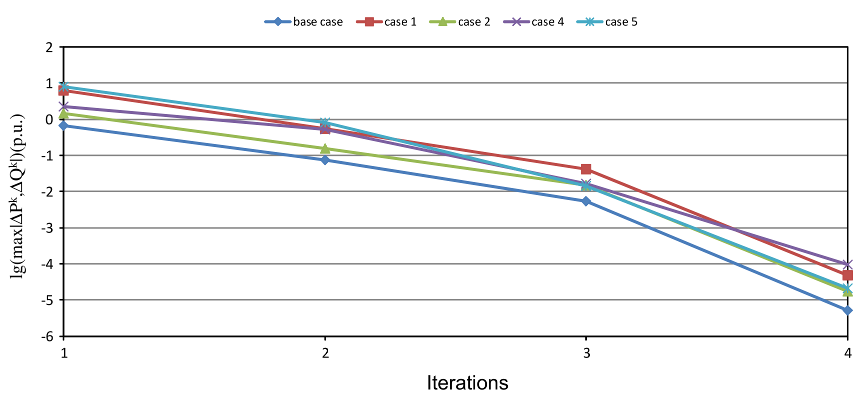
Fig. 4. IEEE 30-bus system.

Fig. 2 indicates, any path connecting a $PQV\theta$ bus (13 or 14) and a 0 bus (2 or 3) must contain bus 6 or 9. Since bus 6 is a *PV* bus, there is no pair of disjoint paths between $PQV\theta$ buses and 0 buses that excludes *PV* buses. Hence, this problem is unsolvable according to the proposed criteria.

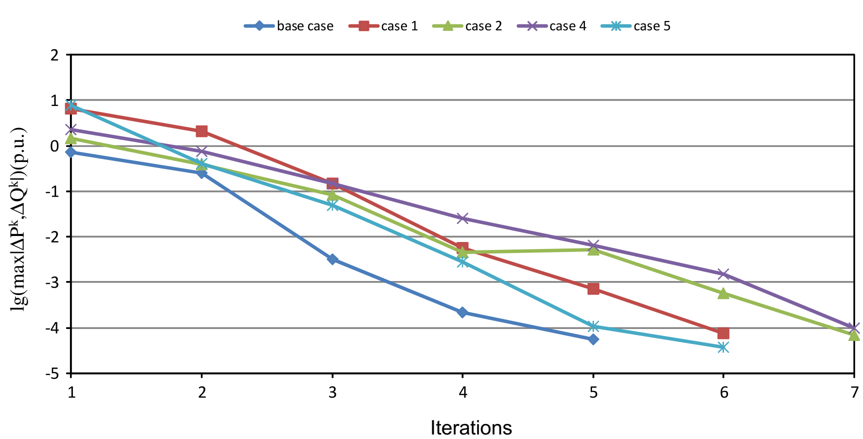
To verify this conclusion, the matrix $\widetilde{\boldsymbol{B}}''_{32} = \boldsymbol{B}''_{32} - \boldsymbol{B}''_{31}\boldsymbol{B}''^{-1}_{11}\boldsymbol{B}''_{12}$ in the *Q*-sub iteration is calculated from actual data. As with the matrix $\widetilde{\boldsymbol{B}}'_{32}$ defined in (11), the *Q*-sub iteration will be solvable only if the matrix $\widetilde{\boldsymbol{B}}''_{32}$ has full rank. By direct calculation, the $\widetilde{\boldsymbol{B}}''_{32}$ matrix in this case is:

Table 5
Test results of IEEE 30-bus system for solvability criterion.

Case indexes	Bus type adjustment	Iteration no. (NR/FDLF)	Solvability by criterion	Effective paths for Criterion 1 (*P*-sub iteration)	Effective paths for Criterion 2 (*Q*-sub iteration)	Criterion correctness
1	15: *PQ* bus → *PQV* buses 10: *PQ* bus → *PQV*θ bus 13: *PV* bus → *P* bus 8 *PV* bus → 0 bus	4/6	Solvable	8 → 6 → 10	8 → 6 → 10 13 → 12 → 15	Correct
2	1: *V*θ → θ bus 5: *PV* bus → *P*θ bus 8: *PV* bus → *V* bus 12: *PQ* bus → *PQV* bus 30: *PQ* bus → *PQV* bus	4/7	Solvable	8 → 6 → 7 → 5	1 → 3 → 4 → 12 5 → 7 → 6 → 28 → 27 → 30	Correct
3	1: *V*θ → θ bus 5: *PV* bus → *P*θ bus 8: *PV* bus → *V* bus 29: *PQ* bus → *PQV* bus 30: *PQ* bus → *PQV* bus	–	Unsolvable	–	–	Correct
4	1: *V*θ → θ bus 10: *PQ* bus → *PQV* bus 9: *PQ* bus → *QV*θ bus 13: *PV* bus → *P* bus	4/7	Solvable	–	1 → 3 → 4 → 6 → 9 13 → 12 → 16 → 17 → 10	Correct
5	1: Vθ → *V* bus 3: *PQ* bus → *Q* bus 6: *PQ* bus → *PQ*θ bus 7: *PQ* bus → *PQ*θ bus	4/6	Solvable	1 → 2 → 5 → 7 3 → 4 → 6	–	Correct
6	1: *V*θ → θ bus 3: *PQ* bus → *P* bus 6: *PQ* bus → *PQV* bus 7: *PQ* bus → *PQV* bus	–	Unsolvable	–	–	Correct



(a) Newton-Raphson method

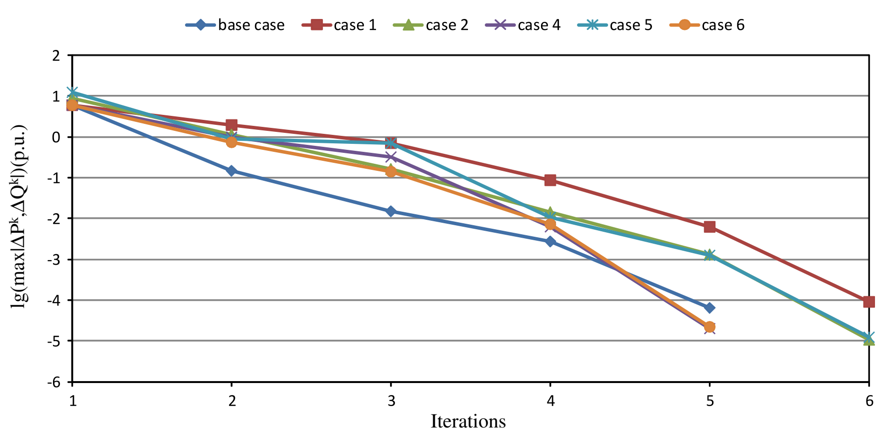


(b) FDLF method

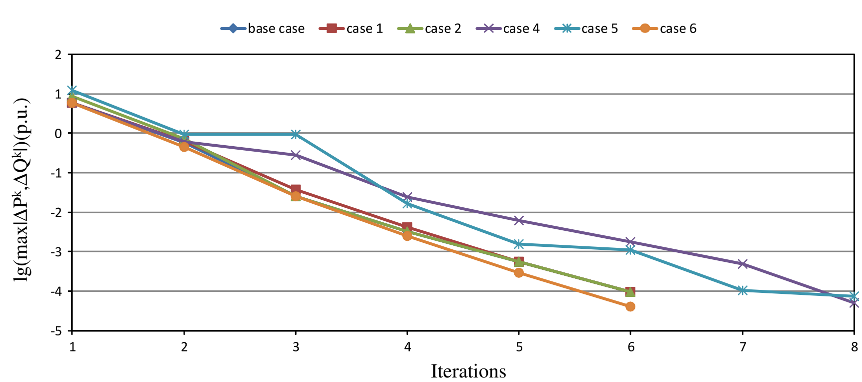
Fig. 5. Convergence curves for the solvable BELF cases in the IEEE 30-bus system.

Table 6
Test results of IEEE 118-bus system for solvability criterion.

Case indexes	Bus type adjustment	Iteration No.	Solvability by criterion	Effective paths for Criterion 1 (*P*-sub iteration)	Effective paths for Criterion 2 (*Q*-sub iteration)	Criterion correctness
1	33: *PQ* bus → *PQV* bus 34: *PV* bus → *P* bus 93: *PQ* bus → *PQVθ* bus 96 *PV* bus → 0 bus 59: *PV* bus → *PVθ* bus 60: *PQ* bus → *Q* bus	6/6	Solvable	96 → 94 → 93 60 → 59	34 → 37 → 33 96 → 94 → 93	Correct
2	2: *PQ* bus → *Q* bus 6: *PV* bus → *PVθ* bus	6/6	Solvable	2 → 12 → 7 → 6	–	Correct
3	2: *PQ* bus → *P* bus 6: *PV* bus → *PQV* bus	–	Unsolvable	–	–	Correct
4	8: *PV* → *P* bus 39: *PQ* bus → *PQV* bus 19: *PV* bus → 0 bus 23: *PQ* bus → *PQVθ* bus 58: *PV* bus → *QV* bus 53: PQ bus → *Pθ* bus	5/9	Solvable	19 → 20 → 21 → 22 → 23 58 → 51 → 52 → 53	8 → 30 → 38 → 37 → 39 19 → 20 → 21 → 22 → 23 53 → 52 → 51 → 58	Correct
5	69: *Vθ* → *Pθ* bus 49: *PV* bus → *QV* bus 77: *PV* bus → *PQV* bus 85: *PV* bus → *P* bus 31: *PV* bus → *QVθ* bus 28: *PQ* bus → *P* bus	6/8	Solvable	49 → 69	69 → 49 28 → 29 → 31 85 → 84 → 83 → 82 → 77 28 → 29 → 31	Correct
6	69: *Vθ* → *PQθ* bus 27: *PV* bus → *V* bus 87: *PV* bus → *PVθ* bus 112: *PV* bus → *V* bus	5/6	Solvable	27 → 32 → 23 → 24 → 70 → 69 112 → 110 → 103 → 100 → 92 → 89 → 85 → 86 → 87	–	Correct



(a) Newton-Raphson method



(b) FDLF method

Fig. 6. Convergence curves for the solvable cases in the IEEE 118-bus system.

$$\widetilde{\boldsymbol{B}}''_{32} = \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \left[\begin{array}{cc} 0 & 0 \\ 0.4030 & 0.2470 \end{array}\right] \end{array} \begin{array}{c} \\ 13 \\ 14 \end{array} \tag{16}$$

$\widetilde{\boldsymbol{B}}''_{32}$ is obviously singular, thus this problem is unsolvable. This conclusion can also be verified by the practical load flow calculation.

Case 2: buses 6 and 8 are changed to 0-type buses, and buses 13 and 14 are changed to $PQV\theta$ buses. As in the previous case, the problem will be solvable if there exist two disjoint paths between $PQV\theta$ buses and 0 buses that exclude PV buses and $V\theta$ buses. In this case, the paths $8 \to 7 \to 9 \to 14$ and $6 \to 13$ meet this requirement, and hence it can be concluded that this problem is solvable according to the proposed criteria.

In this case, the matrix $\widetilde{\boldsymbol{B}}''_{32}$ is:

$$\widetilde{\boldsymbol{B}}''_{32} = \begin{array}{c} \begin{array}{cc} 6 & 8 \end{array} \\ \left[\begin{array}{cc} 7.4204 & 0 \\ 0.6311 & 0.8264 \end{array}\right] \end{array} \begin{array}{c} \\ 13 \\ 14 \end{array} \tag{17}$$

which is obviously non-singular, and this problem is solvable. This conclusion can also be verified via a load flow calculation.

More cases and results are listed in Table 4. The second column (bus type adjustment) compares the bus type modifications with the original configuration. The iteration numbers of Newton–Raphson solution and fast decoupled solution are given in the column 'Iteration No. (NR/FDLF)', NR means Newton–Raphson solution and FDLF means fast decoupled load flow solution. 'Solvability by criteria' refers to solvability judged by Criteria 1 and 2. The paths satisfying Criteria 1 and 2 are listed in the columns labeled 'Effective paths for Criteria 1' and 'Effective paths for Criteria 2', respectively, starting from type 2 buses and ending at type 3 buses. The last column indicates whether or not the solvability conclusions drawn from Criteria 1 and 2 are consistent with practical calculations. For the solvable cases, the convergence curves for Newton–Raphson solutions are plotted in Fig. 3a and the convergence curves for fast decoupled solutions are plotted in Fig. 3b.

As Table 4 shows, the proposed solvability criteria are correct for all cases. Furthermore, Fig. 3 implies that the proposed solutions converge robustly in all solvable cases. In the last case listed in Table 4, none of the buses are ordinary bus types, but the iteration still converges robustly.

5.2. IEEE 30-bus system

Further tests were conducted on the IEEE 30-bus system shown in Fig. 4, and the test results are listed in Table 5.

For the solvable cases, the convergence curves are plotted in Fig. 5.

According to the results in Table 5, the solvability criteria proposed in this paper are reliable. Fig. 5 indicates that the convergence is robust for the solvable cases.

5.3. IEEE 118-bus system

The IEEE 118-bus system is also used to test the convergence and the correctness of the solvability criteria. The results are listed in Table 6.

For the solvable cases, the convergence curves are plotted in Fig. 6.

It can be concluded that the convergence is robust and the solvability criteria are effective for IEEE 118-bus systems.

6. Conclusion

Due to the development of new electric equipment and load flow control technique, new bus types besides PQ, PV and $V\theta$ are needed to be considered in load flow model. This paper provides a systematically study on the bus-type extended load flow problem. Considering there are four variables for each bus: P, Q, V, θ, there 16 bus types all together. Newton–Raphson and fast decoupled solution techniques for the BELF are introduced briefly. Solvability is a new problem raised by the addition of unconventional bus types, which has been discussed in detail. According to the load flow calculation characteristics, simple topology-based solvability criteria are given. Extensive numerical tests have been conducted and the results show that the proposed topology-based solvability criteria are reliable and convenient. The solution and solvability criterion for BELF can be used in many real applications such as the load flow modeling for bus voltage control, network merging and FACTS devices integration.

Acknowledgements

This work was supported in part by the National Key Basic Research Program of China (2013CB228203), National Science Fund for Distinguished Young Scholars (51025725) and New century excellent talents in university (NCET-11-0281).

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