Electrical Power and Energy Systems 44 (2013) 52–59

Contents lists available at SciVerse ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A renewal-process-based component outage model considering the effects of aging and maintenance

Guoqiang Ji, Wenchuan Wu *, Boming Zhang, Hongbin Sun

State Key Lab of Power Systems, Dept. of Electrical Engineering, Tsinghua University, Beijing, China

ARTICLE INFO

Article history:
Received 28 March 2012
Received in revised form 15 June 2012
Accepted 5 July 2012
Available online 9 August 2012

Keywords:
Aging failure
Staircase function
Renewal process
Outage model
Availability

ABSTRACT

The development of accurate component outage models for reliability assessment in power systems is a fundamental problem. Traditional outage models with constant failure rate cannot reflect the impact of changes in operating conditions and repairs after a failure. In this paper, a staircase function is used to approximate the aging failure rate curve, and a renewal-process-based model is introduced to calculate time-varying failure probabilities. The proposed time-varying outage model is able to reflect the effects of component aging and repair activities on the failure rate. Application to an actual transformer shows that given the aging failure rate curve, the new model can evaluate the life probability distribution and steady-state availability of a component as accurate as the simulation method. Compared to the traditional constant-rate model, the proposed model is more accurate and practical.

1. Introduction

The two-state Markov model is the most widely used outage model in power system reliability analysis. With this type of model, the failure rate is obtained from long-term statistical records, and is always constant [1–3]. However, these constant-rate models cannot reflect the impact of changes in operating conditions or the effect of aging, and may lead to inaccurate or erroneous results [4].

Component outages can be divided into two categories: incident failure and aging failure. The aging failure rate is actually time varying [5–7]. Generally speaking, there are two factors that may alter the aging failure rate. One is component aging, which increases the aging failure rate [8]. The other is repair activities, which always reduce the aging failure rate [9]. For short time-scale problems (such as operational risk assessment over the next few hours), the effect of component aging can be ignored. On the other hand, for long time-scale problems (such as maintenance scheduling in a power system), the effect of component aging is significant and should be considered [10]. Neither of these factors can be incorporated into traditional models with a constant aging failure rate. Hence, a more accurate outage model should be developed for reliability assessment and maintenance scheduling in power systems.

Markov-based outage model is widely used in the reliability analysis. In [11], a Markov model is proposed to demonstrate the performance of power transformers and some analyses and sensitivity studies are given. A multi-state Markov model is proposed in [12] and a method for evaluation of transition rates is given. The model is used in reliability indices calculation and some results can be obtained. Ref. [13] proposes new state diagrams for maintenance models based on classical Markov model. Although the Markov models can be solved analytically, the calculation may be very complicated when there are too many states in the model. In [14], a method for reliability assessment in distribution systems is proposed considering the random failure and random repair time. The time-varying failure rates of components in power distribution system are given in [15]. The failure modes are classified into two categories and the increasing aging failure rate is described using exponential distribution and Weibull distribution. The effect of adverse weather and component aging on failure rate has been discussed in [16]. But the proposed model in this paper assumes that repair of the component cannot affect the failure rate, which disagrees with the reality. To incorporate the effects of component aging and repair activities, component outage models with time-varying failure rates have been developed in [17,18]. The models employ Monte Carlo simulations to capture the effects of changing failure rates on system reliability, but such simulations are fairly time consuming. While considering the time-varying factors for components, these models also bring large calculation amount and make reliability analysis in power system complicated. Therefore, a more precise model with less calculation burden should be proposed.

In this paper, a new component outage model is proposed, having the following characteristics: (1) the model can reflect the effects of component aging and repair activities on failure rates;

* Corresponding author.
E-mail address: wuwench@tsinghua.edu.cn (W. Wu).

0142-0615/$ - see front matter
http://dx.doi.org/10.1016/j.ijepes.2012.07.035

Nomenclature

$F(t)$	probability cumulative distribution function for component life
$f(t)$	probability density function for component life
$H(t)$	probability cumulative distribution function for the first life period of a component
$h(t)$	probability density function for the first life period of a component
$G(t)$	probability cumulative distribution function for repair time
$g(t)$	probability density function for repair time
$Q(t)$	probability cumulative distribution function for the renewal period
$q(t)$	probability density function for the renewal period
$W(t)$	probability cumulative distribution function for the first renewal period
$w(t)$	probability density function for the first renewal period
$A(t)$	availability of a component in the renewal process model
$A_{delayed}(t)$	availability of a component in the delayed renewal process model
λ	failure rate for a component
μ	repair rate for a component
$*$	convolution
X	component life
Y	repair time for a component
Z	renewal period for a component
L	Laplace transform
L^{-1}	inverse Laplace transform

(2) the duration of a component outage and that of its subsequent maintenance are formulated as random variables; (3) the calculations are relatively simple.

This paper is organized as follows. A staircase function is introduced in Section 2 to approximate the aging failure rate curve. This staircase function is used to construct a renewal-process-based component outage model in Section 3. The proposed model is extended to a more general form in Section 4. Application to an actual transformer is used to demonstrate the feasibility and advantages of the new model in Section 5. Conclusions are stated in Section 6.

2. Description of the component aging failure rate

In general, the variation of the component aging failure rate can be represented as a bathtub curve [19,20], as shown in Fig. 1. The curve of Fig. 1 can be divided into three intervals. In interval t_1, the component is in the wear-in period, and the aging failure rate decreases. In interval t_2, the aging failure rate is approximately constant. In interval t_3, the component reaches the wear-out period, and the aging failure rate increases. The wear-in and wear-out periods are always simulated by Weibull or exponential functions, which leads to a model too complicated to yield an analytic solution [21,22].

Staircases functions can be used to approximate the aging failure rate while avoiding this type of model complexity, as shown in Fig. 2. The accuracy of this approximation can be infinitely refined by increasing the number of steps, and random process theory can be used to derive an analytic solution, since the failure rate is constant on each step.

3. Component outage model based on a renewal process

In this section, a component outage model with time-varying aging failure rates is developed, in which a staircase function is used to approximate the aging failure rate curve. For the sake of clarity, the model is first formulated in terms of a two-step staircase function, and then generalized in Section 4.

3.1. Two-step staircase function for approximating the aging failure rate

A staircase function with two steps is used to approximate the aging failure rate, as shown Fig. 3. The aging failure rate of the component during the initial working period has a constant value λ_0. At time t_0, the aging failure rate changes to λ_1. The change from λ_0 to λ_1 at time t_0 is the embodiment of component aging. The time λ_0 is the cumulative time point.

There are two outage cases that can occur during a component's lifetime. In the first case, an outage occurs before t_0; in the second, an outage occurs after t_0. In both cases, we assume that the aging failure rate returns to λ_0 after maintenance.

It should be noted that the outage time t_{outage} is random, and is called the random outage time point.

3.2. Introduction to renewal processes

A renewal process is a generalization of a Poisson process, in which the times between successive events are independent and identically distributed with an arbitrary distribution [23]. A renewal process is ideal for describing the component outage process, as illustrated in Fig. 4. Suppose we have a single component that is working at some initial time. When an outage occurs, the component must be repaired, and returns to the working state when the repairs are completed. Let the random variable X_i denote the component life for the ith time, and let the random variable Y_i represent the repair duration for the ith time. Then, the ith renewal period can be expressed as $Z_i = X_i + Y_i$. When the component re-

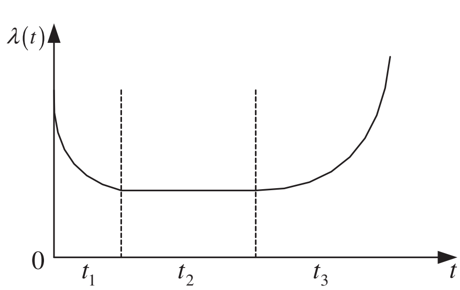

Fig. 1. Bathtub curve for component aging failure rate.

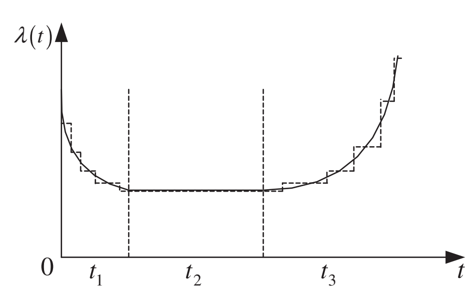

Fig. 2. Staircase curve for approximating a bathtub curve.

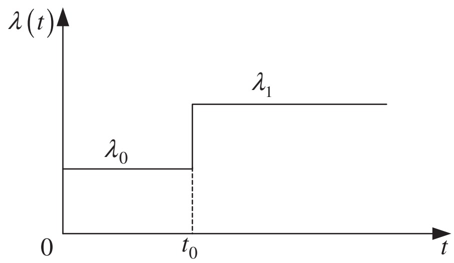

Fig. 3. Two-step staircase function for approximating the aging failure rate.

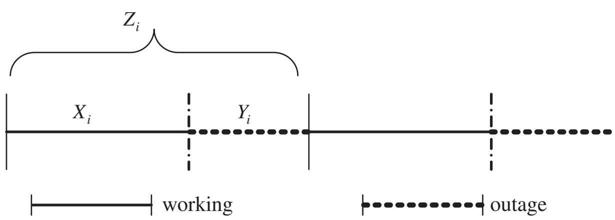

Fig. 4. Renewal for the component outage process.

turns to the working state after repair, we can say that a renewal has taken place. Different renewal periods are independent and identically distributed, based on the hypothesis that the state of the component after repair is the same as the initial state.

3.3. Derivation of the life probability distribution function for the component renewal process outage model

We propose a random process model based on two states, with a time-varying failure rate $\lambda(t)$ to describe the component outage process. The state space is illustrated in Fig. 5. The failure rate $\lambda(t)$ is formulated as a staircase function with two steps, and the repair rate μ is constant.

Aside from the effect of aging, operating conditions such as bad weather, may also alter a component's failure rate. Here, the proposed model is used for maintenance scheduling, so only component aging is considered in this paper. Other outage causes can easily be included in the model by extending the state space to three or more states, and the Semi-Markov process can be used in the derivation of model in these cases [24].

The random process model illustrated in Fig. 5 can be formulated as a renewal process. Therefore, the component outage model can be defined as a component renewal process outage model, based on a time-varying failure rate with a staircase function. If the aging failure rate is constant, the proposed model reduces to a traditional two-state Markov model.

The following two cases are considered to calculate the probability distribution function for the component life X_i.

(1) An outage occurs before time t_0.

The probability of this case is $P_1 = 1 - e^{-\lambda_0 t_0}$. The component life follows a clipped exponential distribution. The component life probability density function is given by

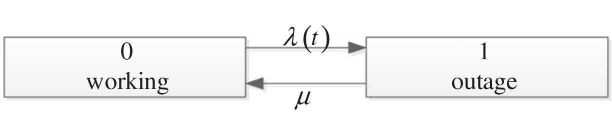

Fig. 5. State space of the random process model with a time-varying failure rate.

$$f_1(t) = \begin{cases} \frac{\lambda_0 e^{-\lambda_0 t}}{1-e^{-\lambda_0 t_0}}, & 0 \leqslant t < t_0 \\ 0, & t \geqslant t_0 \end{cases} \quad (1)$$

The component life probability distribution function can be written as

$$F_1(t) = \begin{cases} \frac{1-e^{-\lambda_0 t}}{1-e^{-\lambda_0 t_0}}, & 0 \leqslant t < t_0 \\ 1, & t \geqslant t_0 \end{cases} \quad (2)$$

(2) An outage occurs after time t_0.

The probability of this case is $P_2 = e^{-\lambda_0 t_0}$. The component life follows a delayed exponential distribution. The component life probability density function can be expressed as

$$f_2(t) = \begin{cases} 0, & 0 \leqslant t < t_0 \\ \lambda_1 e^{-\lambda_1 (t-t_0)}, & t \geqslant t_0 \end{cases} \quad (3)$$

The component life probability distribution function is

$$F_2(t) = \begin{cases} 0, & 0 \leqslant t < t_0 \\ 1 - e^{-\lambda_1 (t-t_0)}, & t \geqslant t_0 \end{cases} \quad (4)$$

The component life probability density and distribution functions for a combination of the two cases can be obtained from the total probability formula. The component life probability density function is

$$f(t) = f_1(t)P_1 + f_2(t)P_2 = \begin{cases} \lambda_0 e^{-\lambda_0 t}, & 0 \leqslant t < t_0 \\ \lambda_1 e^{-\lambda_1 t + (\lambda_1 - \lambda_0) t_0}, & t \geqslant t_0 \end{cases} \quad (5)$$

The component life probability distribution function can be written

$$F(t) = F_1(t)P_1 + F_2(t)P_2 = \begin{cases} 1 - e^{-\lambda_0 t}, & 0 \leqslant t < t_0 \\ 1 - e^{-\lambda_1 t + (\lambda_1 - \lambda_0) t_0}, & t \geqslant t_0 \end{cases} \quad (6)$$

The component life distribution curves for a two-step aging failure rate and a constant aging failure rate are shown in Fig. 6. The blue squares curves represent a two-step aging failure rate, and the green circles curves represent a constant aging failure rate. From Fig. 6 it is shown that the component life probability density function for a two-step aging failure rate changes suddenly at time t_0. This is caused by the large aging failure increment at time t_0. The life probability density curve becomes smoother as the number of steps of the staircase function increases.

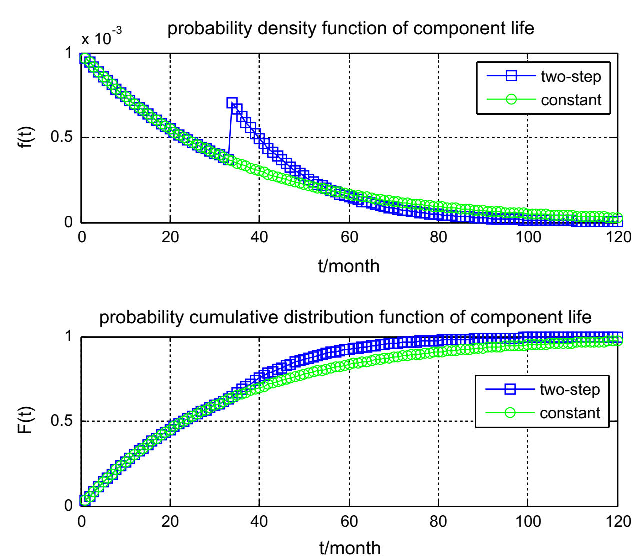

Fig. 6. Component life distribution curves for a two-step aging failure rate and a constant aging failure rate.

3.4. Derivation of the renewal period probability distribution function for the component renewal process outage model

The probability distribution function of the ith life X_i has been obtained above, and the probability distribution function of the ith repair duration Y_i follows the exponential distribution as the repair rate μ is assumed to be constant. The probability distribution function of the renewal period Z_i can be calculated from the convolution of the distributions of X_i and Y_i.

The probability density function of Z_i is given by

$$q(t) = f(t)^* g(t) = \begin{cases} \frac{\mu\lambda_0}{\mu-\lambda_0}[e^{-\lambda_0 t} - e^{-\mu t}], & 0 \leqslant t < t_0 \\ -\frac{\mu\lambda_0}{\mu-\lambda_0} e^{-\mu t} + \left[\frac{\mu\lambda_0}{\mu-\lambda_0} - \frac{\mu\lambda_1}{\mu-\lambda_1}\right] e^{-\mu(t-t_0)-\lambda_0 t_0} \\ +\frac{\mu\lambda_1}{\mu-\lambda_1} e^{-\lambda_1(t-t_0)-\lambda_0 t_0}, & t \geqslant t_0 \end{cases} \tag{7}$$

where $g(t)$ is the probability density function of the repair duration Y_i and $*$ denotes convolution.

The probability distribution function of Z_i is

$$Q(t) = F(t)^* G(t) = \begin{cases} 1 - \frac{\mu}{\mu-\lambda_0} e^{-\lambda_0 t} + \frac{\lambda_0}{\mu-\lambda_0} e^{-\mu t}, & 0 \leqslant t < t_0 \\ 1 + \left(\frac{\lambda_1}{\mu-\lambda_1} - \frac{\lambda_0}{\mu-\lambda_0}\right) e^{-\mu(t-t_0)-\lambda_0 t_0} - \frac{\mu}{\mu-\lambda_1} e^{-\lambda_1(t-t_0)-\lambda_0 t_0} \\ +\frac{\lambda_0}{\mu-\lambda_0} e^{-\mu t}, & t \geqslant t_0 \end{cases} \tag{8}$$

where $G(t)$ is the probability distribution function of the repair duration Y_i.

The renewal period distribution curves for a two-step aging failure rate and a constant aging failure rate are shown in Fig. 7. The blue squares curves represent the model with a two-step aging failure rate, and the green circles curves indicate the results for the model with a constant aging failure rate. The sudden change in the blue squares at time t_0 is due to the effect of component aging. If the number of steps of the staircase function increases, this curve will become smoother.

3.5. Derivation of the availability function for the component renewal process outage model

We define a random process $X(t)$, which denotes the state of a component at time t. $X(t) = 1$ means the component is working at time t, while $X(t) = 1$ means the component is being repaired at time t. Let $A(t)$ denote the transient component availability function for the component renewal process outage model, based on the precondition that the initial state of the component is working. Hence, $A(t) = P\{X(t) = 0|X(0) = 0\}$. According to renewal process theory, the following equation can be obtained:

$$A(t) = 1 - F(t) + A(t) * q(t) \tag{9}$$

Applying the Laplace transform to Eq. (9) over the frequency domain, we obtain

$$A(s) = \frac{\frac{1}{\lambda_0+s} - \frac{1}{\lambda_0+s} e^{-(\lambda_0+s)t_0} + \frac{1}{\lambda_1+s} e^{-(\lambda_1+s)t_0+(\lambda_1-\lambda_0)t_0}}{1 - \frac{\mu\lambda_0}{(\lambda_0+s)(\mu+s)} - \frac{\mu s(\lambda_1-\lambda_0)}{(\lambda_0+s)(\mu+s)(\lambda_1+s)} e^{-(s+\lambda_0)t_0}} \tag{10}$$

From Eq. (10), the steady-state value of the availability function can be derived as follows:

$$\lim_{t\to\infty} A(t) = \frac{\mu\lambda_1 - \mu(\lambda_1 - \lambda_0)e^{-\lambda_0 t_0}}{\lambda_1(\mu + \lambda_0) - \mu(\lambda_1 - \lambda_0)e^{-\lambda_0 t_0}} \tag{11}$$

From Eq. (11), we find that:

(1) If $\lambda_0 = \lambda_1$, the steady-state value reduces to $A(t) = \frac{\mu}{\mu+\lambda_0}$.
(2) If $t_0 \to \infty$, the steady-state value reduces to $A(t) = \frac{\mu}{\mu+\lambda_0}$.

Therefore, if there is no aging effect, the steady-state availability of the renewal process outage model is equal to the value for the traditional two-state Markov outage model.

4. Generalized component outage model based on a renewal process

4.1. Component outage model with an N-step staircase function for approximating the aging failure rate

A more accurate component outage model can be obtained by extending the two-step staircase function to an N-step staircase function. An N-step staircase function for approximating the aging failure rate is shown in Fig. 8.

The component life probability distribution function in this model can be written as

$$F(t) = \left\{ 1 - e^{-\lambda_k t + \sum_{i=1}^{k} (\lambda_i - \lambda_{i-1}) t_{i-1}}, \quad t_{k-1} \leqslant t < t_k \right\},$$
$$k = 0, 1, \ldots, n-1 \tag{12}$$

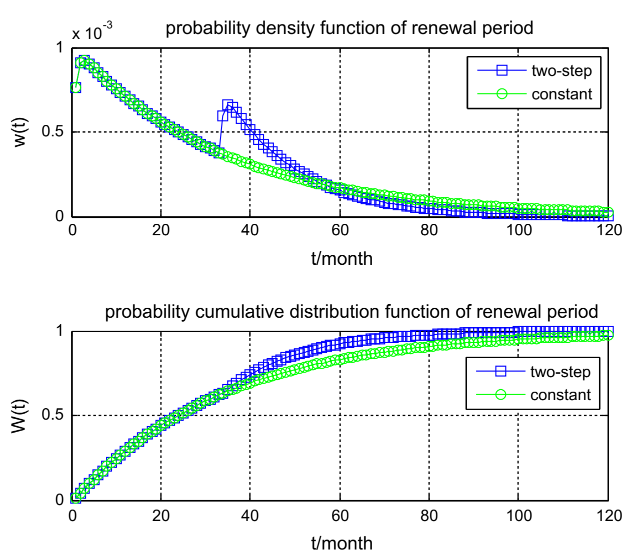

Fig. 7. Component renewal period distribution curves for a two-step aging failure rate and a constant aging failure rate.

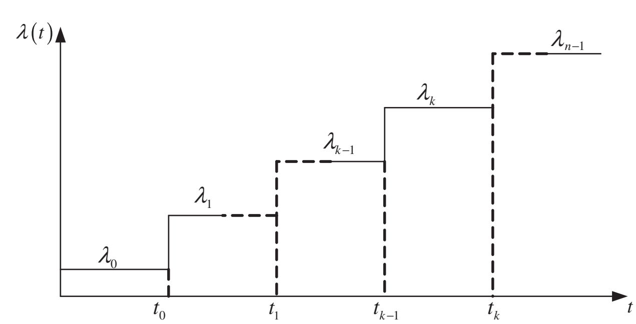

Fig. 8. *N*-step staircase function for approximating the aging failure rate.

The component renewal period probability distribution function is

$$Q(t)=\left\{\begin{aligned}&1+\sum_{i=1}^{k}\left[\frac{\lambda_i}{\mu-\lambda_i}-\frac{\lambda_{i-1}}{\mu-\lambda_{i-1}}\right]e^{-\mu(t-t_{i-1})-\sum_{j=0}^{i-1}\lambda_j(t_j-t_{j-1})}\\&-\frac{\mu}{\mu-\lambda_k}e^{-\lambda_k(t-t_{k-1})-\sum_{j=0}^{k-1}\lambda_j(t_j-t_{j-1})}+\frac{\lambda_0}{\mu-\lambda_0}e^{-\mu t},t_{k-1}\leqslant t<t_k\end{aligned}\right\},$$
$$k=0,1,\ldots,n-1 \tag{13}$$

The steady-state value of the availability function is

$$\lim_{t\to\infty}A(t)=\frac{\frac{1}{\lambda_0}-\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_{i-1}}-\frac{1}{\lambda_i}\right)e^{-\sum_{j=0}^{i-1}\lambda_j(t_j-t_{j-1})}}{\frac{\lambda_0+\mu}{\lambda_0\mu}-\sum_{i=1}^{n-1}\left[\frac{1}{\lambda_{i-1}}-\frac{1}{\lambda_i}\right]e^{-\sum_{j=0}^{i-1}\lambda_j(t_j-t_{j-1})}} \tag{14}$$

A component renewal process outage model based on an N-step staircase function is suitable for any aging mode of a component, and as the number of intervals N increases, the model becomes more accurate.

4.2. Component outage model based on a delayed renewal process

In practice, maintenance after outages causes the subsequent life periods of a component to differ from the initial life period. For example, the component failure rate decreases to a low level after maintenance, as shown in Fig. 9. Here, the shaded interval is the repair time. The staircase curve representing the aging failure rate during the first life period differs from those of subsequent life periods. Therefore, a delayed renewal process is used to formulate this type of component outage model.

In a delayed renewal process, the first period and subsequent periods are independent, and the probability distribution function for the first period is different from that of subsequent periods, while the probability distribution functions for the subsequent periods are identical. Let $H(t)$ represent the probability distribution function for the first life period of a component, and let $w(t)$ be the probability density function for the first renewal period of the component. $F(t)$ is the probability distribution function for the subsequent life periods of the component, and $q(t)$ is the probability density function for the subsequent renewal periods of the component. The variables used for the first life and renewal period are denoted by $\tilde{t}_0,\ldots,\tilde{t}_{\tilde{n}-2}$ and $\tilde{\lambda}_0,\ldots,\tilde{\lambda}_{\tilde{n}-1}$, while the variables used for the subsequent life and renewal periods are denoted by $t_0,\ldots,t_{n-2}$ and $\lambda_0,\ldots,\lambda_{n-1}$. Then, the following formulas can be obtained in a manner analogous to that of Section 3.

$$H(t)=\left\{1-e^{-\tilde{\lambda}_k t+\sum_{i=1}^{k}(\tilde{\lambda}_i-\tilde{\lambda}_{i-1})\tilde{t}_{i-1}},\quad \tilde{t}_{k-1}\leqslant t<\tilde{t}_k\right\},$$
$$k=0,1,\ldots,\tilde{n}-1 \tag{15}$$

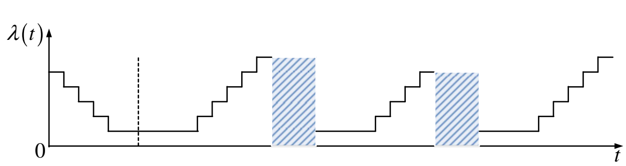

Fig. 9. Aging failure rate in a delayed renewal process.

$$w(t)=\left\{\begin{aligned}&\sum_{i=1}^{k}\left[\frac{\mu\tilde{\lambda}_{i-1}}{\mu-\tilde{\lambda}_{i-1}}-\frac{\mu\tilde{\lambda}_i}{\mu-\tilde{\lambda}_i}\right]e^{-\mu(t-\tilde{t}_{i-1})-\sum_{j=0}^{i-1}\tilde{\lambda}_j(\tilde{t}_j-\tilde{t}_{j-1})}\\&+\frac{\mu\tilde{\lambda}_k}{\mu-\tilde{\lambda}_k}e^{-\tilde{\lambda}_k(t-\tilde{t}_{k-1})-\sum_{j=0}^{k-1}\tilde{\lambda}_j(\tilde{t}_j-\tilde{t}_{j-1})}-\frac{\mu\tilde{\lambda}_0}{\mu-\tilde{\lambda}_0}e^{-\mu t},\quad \tilde{t}_{k-1}\leqslant t<\tilde{t}_k\end{aligned}\right\},$$
$$k=0,1,\ldots,\tilde{n}-1 \tag{16}$$

$$F(t)=\left\{1-e^{-\lambda_k t+\sum_{i=1}^{k}(\lambda_i-\lambda_{i-1})t_{i-1}},t_{k-1}\leqslant t<t_k\right\},\quad k=0,1,\ldots,n-1 \tag{17}$$

$$q(t)=\left\{\begin{aligned}&\sum_{i=1}^{k}\left[\frac{\mu\lambda_{i-1}}{\mu-\lambda_{i-1}}-\frac{\mu\lambda_i}{\mu-\lambda_i}\right]e^{-\mu(t-t_{i-1})-\sum_{j=0}^{i-1}\lambda_j(t_j-t_{j-1})}\\&+\frac{\mu\lambda_k}{\mu-\lambda_k}e^{-\lambda_k(t-t_{k-1})-\sum_{j=0}^{k-1}\lambda_j(t_j-t_{j-1})}-\frac{\mu\lambda_0}{\mu-\lambda_0}e^{-\mu t},\quad t_{k-1}\leqslant t<t_k\end{aligned}\right\},$$
$$k=0,1,\ldots,n-1 \tag{18}$$

Let $H(s)$, $w(s)$, $F(s)$, and $q(s)$ denote the Laplace transforms of the quantities given in Eqs. (15)–(18). Let $A_{delay}(t)$ denote the transient component availability function for the delayed renewal process outage model. The component availability can be obtained from the delayed renewal process as follows:

$$\begin{aligned}A_{delayed}(t)&=P\{X(t)=0|X(0)=0\}\\&=P\{X(t)=0,\quad X_1>t|X(0)=0\}+P\{X(t)=0,\\&\quad X_1\leqslant t<X_1+Y_1|X(0)=0\}\\&\quad+P\{X(t)=0,\quad X_1+Y_1\leqslant t|X(0)=0\}\\&=P\{X_1>t|X(0)=0\}+0+P\{X(t)=0,\quad Z_1\leqslant t|X(0)=0\}\\&=1-H(t)+\int_0^t P\{X(t)=0|Z_1=u,\quad X(0)=0\}w(u)du\\&=1-H(t)+\int_0^t A(t-u)w(u)du\\&=1-H(t)+A(t)*w(t)\end{aligned} \tag{19}$$

Eq. (19) can be solved via the Laplace transform. The component availability is then given by

$$A_{delayed}(t)=L^{-1}\left[\frac{1}{s}-H(s)+\frac{\frac{1}{s}-F(s)}{1-q(s)}w(s)\right] \tag{20}$$

The steady-state availability of the component is

$$\lim_{t\to\infty}A(t)=\frac{\frac{1}{\lambda_0}-\sum_{i=1}^{n-1}\left(\frac{1}{\lambda_{i-1}}-\frac{1}{\lambda_i}\right)e^{-\sum_{j=0}^{i-1}\lambda_j(t_j-t_{j-1})}}{\frac{\lambda_0+\mu}{\lambda_0\mu}-\sum_{i=1}^{n-1}\left[\frac{1}{\lambda_{i-1}}-\frac{1}{\lambda_i}\right]e^{-\sum_{j=0}^{i-1}\lambda_j(t_j-t_{j-1})}} \tag{21}$$

Thus, the steady-state availability from the delayed renewal process is the same as the value obtained from the ordinary renewal process. This means that the difference in the first period of the component has no effect on steady-state availability of component.

Table 1
Weibull parameters for the aging failure rate of a transformer.

Component	Failure causes	Rating	Weibull parameters		MTBF/yr
			Shape	Scale	
Load tap changer	Damage	115/22KV, 25MVA	2.93	14.83	13.24
Load tap changer	Leakage	115/22KV, 25MVA	1.99	17.61	15.61
Bushing	Leakage	115/22KV, 25MVA	2.29	24.94	22.1
Tank	Leakage	115/22KV, 25MVA	2.98	22.03	19.66
Tank	Leakage	230/115KV, 200MVA	2.78	22.65	20.17
Bushing	Leakage	230/115KV, 200MVA	3.99	18.82	17.06

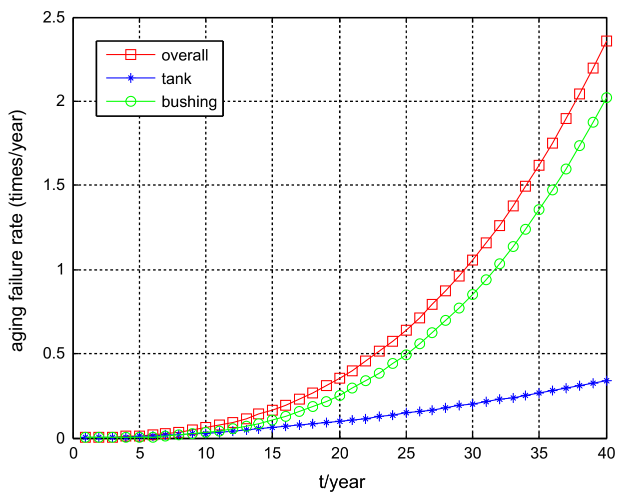

Fig. 10. Transformer aging failure rate curves.

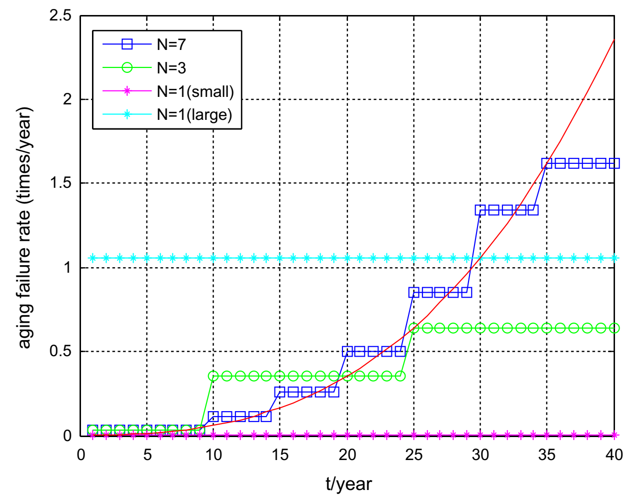

Fig. 11. Step curves for approximating the transformer aging failure rate.

5. Practical application to an actual transformer

A Weibull distribution function is always used to approximate the bathtub curve for an aging failure rate. The failure rate of a Weibull distribution function is

$$\lambda(t) = \frac{\beta}{\alpha^{\beta}} t^{\beta-1} \tag{22}$$

where α is the scale parameter and β is the shape parameter. The reliability index of an outage model based on a Weibull distribution failure rate is usually obtained from a Monte Carlo simulation.

The parameters of the Weibull distribution failure rate for a transformer are listed in Table 1 [25]. We choose a 230/115 kV, 200 MVA transformer as an example. The aging failure rate curve for this kind of transformer is shown in Fig. 10. The blue[1] asterisks curves represent the failure rate of the tank. The green circles curves represent the failure rate of the bushing, and the red squares curves represent the overall failure rate of the transformer.

After the transformer has been in operation for 10 years, the failure rate begins to increase significantly. To approximate the transformer aging process, four renewal process outage models are constructed as follows:

1. $N = 7$; the time intervals are divided as follows: $\begin{cases} 0 \leqslant t < 10, \\ 10 \leqslant t < 15, \\ 15 \leqslant t < 20, \\ 20 \leqslant t < 25, \\ 25 \leqslant t < 30, \\ 30 \leqslant t < 35, \\ 35 \leqslant t < +\infty \end{cases}$.

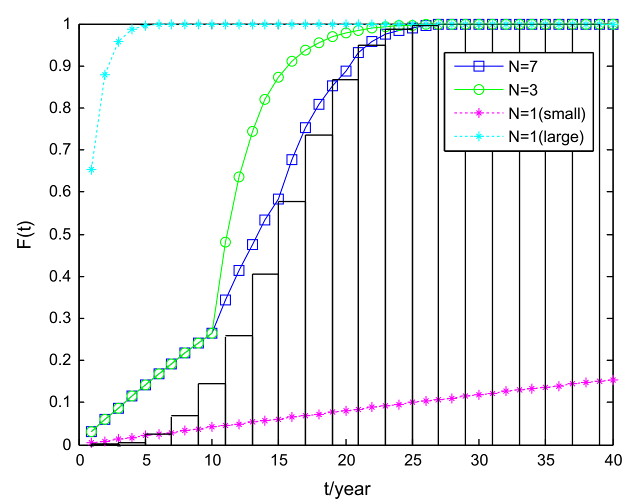

Fig. 12. Probability distribution functions for transformer life.

2. $N = 3$; the time intervals are divided as follows: $\begin{cases} 0 \leqslant t < 10 \\ 10 \leqslant t < 25 \\ 25 \leqslant t < +\infty \end{cases}$.
3. $N = 1$; the constant aging failure rate is the value for the 3rd year.
4. $N = 1$; the constant aging failure rate is the value for the 30th year.

The aging failure rates of the four models are shown in Fig. 11. The blue squares curves represent the $N = 7$ model. The green circles curves represent the $N = 3$ model. The pink asterisks curves represent the $N = 1$ model with the small aging failure rate, and

[1] For interpretation of color in Figs. 6, 7, 9–12, the reader is referred to the web version of this article.

Table 2
Steady-state transformer availability values.

Model	Steady-state availability	Deviation
1 ($N = 7$)	0.9957	–
2 ($N = 3$)	0.9948	−0.09%
3 ($N = 1$)	0.9998	0.41%
4 ($N = 1$)	0.9445	−5.14%
Monte Carlo	0.9957	0

the light blue asterisks curves represent the $N = 1$ model with the large aging failure rate. The red curve is the transformer aging failure rate curve from Fig. 10.

Suppose that the repair rate of the transformer is $\mu = 18$/year. The transformer life probability distribution curve is shown in Fig. 12. The black histogram is the result of a Monte Carlo simulation that sampled 50,000 times.

We can draw following conclusions from Fig. 12:

(1) Compared to the black histogram, the curve from the $N = 7$ model was the most accurate among the four models. The precision of the renewal process model could be infinitely refined by increasing number of intervals N.
(2) The $N = 7$ model concurred with $N = 3$ model over the period 1–10 years, while the difference between them increased after 10 years. Since the $N = 7$ model had more intervals than the $N = 3$ model, it approximated the transformer life more accurately.
(3) The curve for the $N = 7$ model was above the curve for the $N = 1$ model with the small aging failure rate, which means that a small constant aging failure rate model could lead to overly optimistic results and lack of maintenance over a long period.
(4) The curve for the $N = 7$ model was below the curve for the $N = 1$ model with the large aging failure rate, which means that a large constant aging failure rate model could lead to unduly pessimistic results and excessive maintenance.

The steady-state availabilities of the four models are listed in Table 2. The steady-state availability of the $N = 7$ model was the same as that of the Monte Carlo simulation. Using the steady-state availability of the $N = 7$ model as a reference value, the deviation of the $N = 1$ model with the small aging failure rate was 0.41%, and the deviation of the $N = 1$ model with the large aging failure rate was 5.14%. It would seem that the deviation of the constant aging failure rate model was not very large for a single component. However, the deviation of the overall system state probability is magnified in accordance with the number of components.

For example, suppose that the $N = 1$ model with the small aging failure rate is used. The availability of a single component is $\widetilde{A} = 0.9998$. The actual availability of a single component is $A = 0.9957$. If there are 30 components in the system, then the probability that all components are working is $\widetilde{P} = \widetilde{A}^{30}$ and $P = A^{30}$. The deviation of the overall system state probability is 13.12%. If the $N = 1$ model with the large aging failure rate is used, the deviation of the overall system state probability is 79%. Therefore, an accurate component outage model is very important for maintenance scheduling in a power system.

Here the transformer is taken as an example to show the efficiency of the proposed model. The model can also be used in other components as long as the aging characteristics are known. In the power system reliability evaluation, this model can be used to calculate the probability of each state of power system based on the steady-state availability of each component and the reliability indices can be calculated further.

6. Conclusion

A staircase function was used to approximate the aging failure rate in power systems, and a component renewal process outage model based on a time-varying failure rate was proposed. This model was able to reflect the effects of component aging and repair activities on the aging failure rate. Compared to traditional constant-rate models, the new model offered the following advantages.

(1) The model yielded an analytical solution, and no Monte Carlo simulation was required, greatly simplifying the calculations.
(2) The model was able to reflect the effects of component aging and repair activities on failure rates, and was more accurate than traditional constant-rate models.
(3) The model had high form agility. As the number of intervals N increased, the model more closely approximated practical situations. The traditional Markov model is a special case of the proposed model with $N = 1$.
(4) The new model was suitable for any components, as long as their aging rate curves were known.

In recent years, the condition monitoring systems have been used widely in power systems which provide the basic data for the operating state evaluation of the component, and the accurate failure rates needed in the proposed outage model in this paper can be estimated according to on-line monitoring data with precise time stamps. An algorithm has been presented to estimate the failure rate from dissolved gas analysis (DGA) data in our recent work [24] and the failure rate curve of the transformer can be obtained, which make the renewal-process-based outage model applicable. The future work will focus on the more accurate failure rate estimation method and the reliability analysis method based on the proposed model.

Acknowledgements

This work was supported in part by the National Science Foundation of China (51177080), National High Technology Research Program (2011AA05A118) and New century excellent talents in university (NCET-11-0281).

References

[1] Billinton R, Allan RN. Reliability evaluation of engineering systems: concepts and techniques. New York and London: Plenum Press; 1983.
[2] Li W. Risk assessment of power systems-models, methods, and applications. USA and Canada: IEEE Press and Wiley; 2005.
[3] Billinton R, Allan RN. Reliability evaluation of power systems. 2nd ed. New York: Plenum Press; 1996.
[4] Bowles JB. Commentary-caution: constant failure rate models may be hazardous to your design. IEEE Trans Reliab 2002;51(3):375–7.
[5] He Jian, Sun Yuanzhang, Wang Peng, Cheng Lin. A hybrid conditions-dependent outage model of a transformer in reliability evaluation. IEEE Trans Power Del 2009;24(4):2025–33.
[6] Wan H, Mccalley JD, Vittal V. Increasing thermal rating by risk analysis. IEEE Trans Power Syst 1999;14(3):815–28.
[7] Pathak J, Jiang Yong, Honavar V, Mccalley J. Condition data aggregation with application to failure rate calculation of power transformers. In: Presented at the Hawaii international conference on system sciences, Hawaii, USA; 2006.
[8] Kim Hagkwen, Singh Chanan. Reliability modeling and simulation in power systems with aging characteristics. IEEE Trans Power Syst 2010;25(1):21–8.
[9] Ning Liaoyi, Wu Wenchuan, Zhang Boming. Generator random outage model for risk-based monthly maintenance scheduling. In: Presented at the international conference on probabilistic methods applied to power systems, Singapore; 2010.
[10] Zheng Xitong, Fard Nasser. A maintenance policy for repairable systems based on opportunistic failure-rate tolerance. IEEE Trans Reliab 1991;40(2):237–44.
[11] Sefidgaran Mobin, Mirzaie Mohammad, Ebrahimzadeh Ataollah. Reliability model of the power transformer with ONAF cooling. Int J Electr Power Energy Syst 2012;35:97–104.

[12] Lisnianski Anatoly, Elmakias David, Laredo David, Haim Hanoch Ben. A multi-state Markov model for a short-term reliability analysis of a power generating unit. Int J Reliab Eng Syst Safety 2012;98:1–6.
[13] Abeygunawardane Saranga K, Jirutitijaroen Panida. New state diagrams for probabilistic maintenance models. IEEE Trans Power Syst 2011;26(4):2207–13.
[14] Arya LD, Choube SC, Arya Rajesh, Tiwary Aditya. Evaluation of reliability indices accounting omission of random repair time for distribution systems using Monte Carlo simulation. Int J Electr Power Energy Syst 2012;42:533–41.
[15] Chu CM, Moon JF, Lee HT, Kim JC. Extraction of time-varying failure rates on power distribution system equipment considering failure modes and regional effects. Int J Electr Power Energy Syst 2010;32:721–7.
[16] Bollen Math HJ. Effects of adverse weather and aging on power system reliability. IEEE Trans Ind Appl 2001;37(2):452–7.
[17] Abu-Elanien AEB, Salama MMA, Bartnikas R. A techno-economic method for replacing transformers. IEEE Trans Power Del 2011;26(2):817–29.
[18] Retterath B, Venkata SS, Chowdhury AA. Impact of time-varying failure rates on distribution reliability. In: Presented at the international conference on probabilistic methods applied to power systems, Ames, IA, USA; 2004.
[19] Chen Yi, Wang Zhonglai, Qiu Jing, Zheng Bin, Huang Hong-Zhong. Adaptive bathtub hazard rate curve modelling via transformed radial basis functions. In: Presented at the international conference on quality, reliability, risk, maintenance, and safety engineering, Xi'an, China; 2011.
[20] Wondmagegnehu Eshetu T, Navarro Jorge, Hernandez Pedro J. Bathtub shaped failure rates from mixtures: a practical point of view. IEEE Trans Reliab 2005;54(2):270–5.
[21] Li Wenyuan. Incorporating aging failures in power system reliability evaluation. IEEE Trans Power Syst 2002;17(3):918–23.
[22] Hassett Thomas F, Dietrich Duane L, Szidarovszky Ferenc. Time-varying failure rates in the availability and reliability analysis of repairable systems. IEEE Trans Reliab 1995;44(1):155–60.
[23] Sheldon MR. Introduction to probability models. Harcourt Academic Press; 2000.
[24] Ning Liaoyi, Wenchuan Wu, Zhang Boming, Zhang Pei. A time-varying transformer outage model for on-line operational risk assessment. Int J Electr Power Energy Syst 2011;33:600–7.
[25] Chaidee E, Tippachon W. Failure statistics and condition evaluation for power transformer maintenance. In: Presented at the power and energy engineering conference, Asia-Pacific (APPEEC), Wuhan, China; 2011.