Electrical Power and Energy Systems 60 (2014) 45–52

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A method for evaluating the accuracy of power system state estimation results based on correntropy

Ye Guo, Wenchuan Wu *, Boming Zhang, Hongbin Sun

Department of Electrical Engineering, State Key Laboratory of Power Systems, Tsinghua University, China

ARTICLE INFO

Article history:
Received 27 March 2013
Received in revised form 29 January 2014
Accepted 25 February 2014
Available online 21 March 2014

Keywords:
Power systems
State estimation
Residuals
Correntropy
Uncertainty

ABSTRACT

Power system state estimation (SE) is a crucial basic function in energy management systems (EMSs) which offers the basic load flow models. It is a critical problem to check whether the SE results are credible enough to be used for online decision making or close-loop control. However, there are few published works focus on this topic till now. The accuracy of SE results can be affected by various factors, many of them are very difficult to quantify. Hence a feasible method is to estimate the accuracy of SE results based on residuals. Due to the huge number of measurements in real power systems, it is necessary to establish a reasonable scalar index that represents the residuals of all measurements and directly indicates the accuracy of the SE results. This paper provides a systematic research on the SE accuracy evaluation problem and proposes a new SE accuracy evaluation index based on correntropy. Some conventional SE accuracy evaluation indices are also introduced and compared theoretically and also through extensive numerical tests.

1. introduction

There are mainly three essential problems evolved in power system state estimation: measurement accuracy quantification, SE algorithms and result accuracy evaluation. Extensive researches focus on SE algorithms have been published, especially on the suppression of gross errors in topology [1,2], analog measurements [3–6] and network parameters [7–9]. Some effective studies on the measurement accuracy quantification have also been reported [10,11]. However, there are few systematic researches focus on the SE results accuracy evaluation problem. Since the performances of most functions in EMS depend on the SE results, it is a critical problem to check whether the SE results are credible enough to be used for online decision making or close-loop control.

The accuracy of the SE results can be affected by various factors including errors in PT/CT, AC sampling errors, errors from information channels, device malfunctioning and parameter errors, SE algorithm, etc. Many of them are very difficult to quantify. Hence it is very difficult to give a deductive combination for all these factors to evaluate the accuracy of SE results, and a residual-based accuracy estimation method should be employed instead. Generally speaking, the smaller the measurement residuals are, the more accurate the SE result is. Since there are huge number of measurements in a real power system, the various measurement residuals must be represented by a scalar index to quantify the SE accuracy intuitively.

There are several conventional SE accuracy evaluation indices. The most common method is using the first or second order norms of residual vector to build evaluation indices. Such indices sum up the absolute values or square values of the residual vector [12,13]. These indices lack clear physical meanings, and maybe infected by gross errors. In metrology, measurement uncertainty is a textbook method to evaluate the accuracy of SE [14,15]. The calculation of measurement uncertainty is a deductive method, which is very difficult for implementation in large-scale practical power systems. Refs. [16,17] suggest using a threshold value of 3% or 5% of the measurement value as a confidence bound for each measurement. In China, the State Grid Corporation of China (SGCC) proposes an official standard for evaluating the accuracy of the SE, which named as acceptance rate (*AR*). *AR* counts the rate for the measurement whose residual is less than a artificial set threshold value. In theoretically, *AR* is loosely connected with measurement uncertainty, but its crucial threshold values are totally determined by manual experience.

This paper provides a systemic research on the problem of SE result accuracy evaluation, but the measurement accuracy quantification and SE algorithms are beyond this paper. An important contribution of this paper is that it proposes a new SE accuracy evaluation method, the scalar index of correntropy (*COE*), and

* Corresponding author.
E-mail address: wuwench@tsinghua.edu.cn (W. Wu).

http://dx.doi.org/10.1016/j.ijepes.2014.02.025
0142-0615/

makes a comprehensive comparison of the proposed *COE* with several common scalar indices, including the mean absolute value of weighted residuals (*MAR*), mean squares of the weighted residuals (*MSR*), and acceptance rate (*AR*).

The remainder of this paper is organized as follows. Section 2 provides an introduction to the traditional indices. The correntropy index is introduced in Section 3. Section 4 introduces the comparison method used in this paper. Extensive numerical tests used to verify the performances of the various indices, and their results are described in Section 5.

2. Common scalar indices

2.1. Mean absolute value of weighted residuals (MAR)

The mean absolute value of weighted residuals (*MAR*) can be expressed as:

$$MAR = \frac{1}{m}\sum_{i=i}^{m}|r_i/\delta_i| \tag{1}$$

where m denotes the number of measurements, r_i and δ_i are the measurement residual and standard deviation for the ith measurement. A smaller *MAR* index indicates more accurate SE results.

2.2. Mean squares of weighted residuals (MSR)

The mean squares of weighted residuals (*MSR*) can be defined as:

$$MSR = \frac{1}{m}\sum_{i=i}^{m}(r_i/\delta_i)^2 \tag{2}$$

A smaller *MSR* index indicates more accurate SE results.

2.3. Measurement uncertainty and acceptance rate (AR)

Measurement uncertainty is a textbook method to measure the accuracy of the SE results in metrology. The International Standards Organization (ISO) has defined two types of measurement uncertainties: standard and extended. Standard measurement uncertainty U_{ISO} is defined as [15]:

$$U_{ISO} = \pm K[U_A^2 + U_B^2]^{1/2} \tag{3}$$

where U_A is type A uncertainty, which is measured by statistical methods, and U_B is type B uncertainty, which measured more subjectively using non-statistical methods. K is a multiplier used to obtain the confidence of interest. The calculation of measurement uncertainty is deductive which depends on the measurement standard deviations and artificial justice. In real power systems, however, the measurement standard deviations comprise various factors and it is very difficult to estimate their standard deviations. Although some control centers have their manually specified values for measurement standard deviation, δ_i, they are usually inaccurate. The quantification of type B uncertainty for large-scale real power systems is even more difficult. Hence, the application of standard measurement uncertainty becomes impractical.

Instead of standard measurement uncertainty, extended measurement uncertainty can be used to evaluate the accuracy of the SE results. Extended measurement uncertainty is defined as:

$$P(|r_i| < kU_{ISO}) = \gamma \tag{4}$$

where k is the coverage factor and γ is the confidence level, which can be 99%. Assume that the measurement errors are Gaussian distributed, the probability of the measurement errors falling in the interval $[-U_{ISO}, +U_{ISO}]$ is 68.3%, and the probability of the measurement errors falling in the interval $[-3U_{ISO}, +3U_{ISO}]$ is 99.7%. The corresponding coverage factor k is about 3.0 for $\gamma = 99\%$. For a reasonable SE result, most measurement residuals must fall into the interval defined by the extended measurement uncertainty with an appropriate confidence level or coverage factor. Therefore, the acceptance rate (*AR*) index can be used to evaluate the accuracy of the SE results:

$$AR = \frac{1}{m}\sum_{i=1}^{m}\alpha_i \times 100\% \tag{5}$$

where

$$\alpha_i = \begin{cases} 1 & \text{if } |r_i| < \varepsilon_i \\ 0 & \text{else} \end{cases} \tag{6}$$

$\varepsilon_i = kU_{ISOi}$ stands for the extended measurement uncertainty and defines the confidence interval. The *AR* index in (6) has been adopted by the SGCC as an official standard for evaluating the SE accuracy. However, since the standard measurement uncertainty U_{ISO} is difficult to estimate in practical power systems, the value of the extended measurement uncertainty ε_i is also difficult to quantify, and is always specified according to manual experience. Although the acceptance rate is derived from measurement uncertainty theory, it is very subjective since the threshold value ε_i is set manually.

3. Correntropy based scalar index

3.1. Brief introduction

Correntropy is a generalized similarity measure between two random variables in signal processing [18–20]. Assume that x_1 and x_2 are two random variables with probability density functions f_1 and f_2, respectively. Their Renyi's quadric correntropy can be defined as:

$$\begin{aligned} &H(x_1, x_2) = -\log I \\ &I = \int f_1(x)f_2(x)\mathrm{d}x \end{aligned} \tag{7}$$

where I is the cross information potential and $H(x_1, x_2)$ is Renyi's quadric correntropy of x_1 and x_2. The probability density function can be estimated from the kernel function based on the samples, and the cross information potential can be estimated as:

$$\hat{I} = \frac{1}{m}\sum_{i=1}^{m}\kappa(x_{1i}, x_{2i}) \tag{8}$$

where m is the number of samples, x_{1i} and x_{2i} are the ith sample values for x_1 and x_2, respectively, and κ is the kernel function, which should be symmetric non-negative definite. According to the Moore–Aronszajn theorem [21,22], for any symmetric non-negative definite function κ, there exists a corresponding reproducing kernel Hilbert space F defined by mapping φ: $X \to F$ with κ as a kernel function. This yields:

$$\langle\varphi(x_1), \varphi(x_2)\rangle = \kappa(x_1, x_2) \tag{9}$$

where $\langle\cdot\rangle$ stands for the inner product, which measures the information potential in Hilbert space. Hence, one can conclude that correntropy in fact maps the random variables from their original space to another Hilbert space in which their cross information potential is calculated. An interesting characteristic is that all of the reproducing kernel Hilbert spaces with definite dimensions are omorphism, so we can choose any symmetric non-negative definite kernel function to construct the reproducing kernel Hilbert space F. The entire analysis can be carried out in space F without knowing its exact meaning.

The most commonly used kernel function is the Gaussian kernel:

$$\kappa(x_1, x_2) = \frac{1}{\sqrt{2\pi}\sigma}\sum_{i=1}^{m}\exp\left(-\frac{(x_{1i}-x_{2i})^2}{2\sigma^2}\right) \tag{10}$$

where σ is the kernel size. According to Silverman's rule [23], when the measurement errors are Gaussian distributed, the optimal choice for the kernel size σ_{opt} is:

$$\sigma_{opt} = \left(\frac{4}{2n+1}\right)^{1/(n+4)} \times m^{-1/(n+4)} \times \delta \tag{11}$$

where n is the dimension of state vector. As discussed, the measurement error distribution is not strictly Gaussian distributed; consequently, (11) is not a strict optimal choice for parameter σ. However, some researches have shown that correntropy is insensitive to the value of σ as long as its value is within the proper range [19]. In addition, Eq. (11) can be viewed as a reference choice for parameter σ.

3.2. Power system state estimation accuracy evaluation based on correntropy

In power systems, the cross-information potential and correntropy between the measurement value vector $\boldsymbol{z}$ and the estimated value vector $\boldsymbol{h}(\boldsymbol{x})$ can be estimated by:

$$\begin{aligned}\hat{I} &= \tfrac{1}{\sqrt{2\pi}\sigma m}\sum_{i=1}^{m}\exp\left(-\tfrac{(z_i-h_i(\mathbf{x}))^2}{2\sigma^2}\right)\\ \hat{H}(\boldsymbol{z},\boldsymbol{h}(\boldsymbol{x})) &= -\log\left[\tfrac{1}{\sqrt{2\pi}\sigma m}\sum_{i=1}^{m}\exp\left(-\tfrac{(\boldsymbol{z}_i-\boldsymbol{h}_i(\boldsymbol{x}))^2}{2\sigma^2}\right)\right]\end{aligned} \tag{12}$$

Neglecting the constant terms in (12) and converting its value to percentage, a new SE accuracy evaluation index based on correntropy (*COE* for short) can be defined as:

$$COE = \frac{1}{m}\sum_{i=1}^{m}\exp\left(-\frac{(\mathbf{z}_i-\mathbf{h}_i(\mathbf{x}))^2}{2\sigma^2}\right) \times 100\% \tag{13}$$

In practical power systems, the dimension of the state vector n has a large value, and the kernel size σ_{opt} in (11) can be approximated as

$$\sigma_{opt} \approx \delta \tag{14}$$

3.3. Characteristics of the index based on correntropy

Except for solid theoretical foundation, the index of *COE* in (13) has several novel characteristics which guarantee its outstanding performance.

(1) Continuous and has clear physical meanings

It can be observed from (13) that the *COE* index is continuous. In the index of *COE* in (13), each measurement will give its own *COE* index value, which in fact means its marking to the given SE result according to the residual. The *COE* index in (13) is in fact a summary of the markings of all the measurements. Such markings range from 0% to 100%, which clearly demonstrate how reliable the SE result is.

(2) Nonparametric estimation method

By expanding *COE* into a Taylor series:

$$COE = \frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{\infty}\frac{(-1)^k}{(2\sigma^2)^k}(\mathbf{z}_i-\mathbf{h}_i(\mathbf{x}))^{2k} \times 100\% \tag{15}$$

From (15), one can observe that all of the even-order moment will affect the kernel function in (13). Consequently, *COE* is a nonparametric estimation method.

(3) Suppression the influence of gross errors

It should be noticed that all the measurements which belong to the input of SE should be considered in SE accuracy evaluation, including the susceptive measurements identified by the SE program but excluding the artificial pre-filtered measurements (a detailed explanation is presented in Section 4.3). Accurate SE results should clearly distinguish normal measurements and gross errors. Consequently, a residual vector with major very small values and a few large residuals indicates accurate SE results; and another residual vector with a lot of medium values indicates inaccurate SE results. The former vector is called a distinguished residual vector, and the latter is called as an infected residual vector for short. The *COE* index value with single residual is illustrated in Fig. 1. From 1 one can observe that the *COE* index curve is very flat in the large residual area, which makes a distinguished residual vector to have a better *COE* index than an infected residual vector.

3.4. Comparison with MAR, MSR and AR

It has been shown that the index of *COE* has several novel characteristics: It is a continuous index with clear meaning; it is a nonparametric method and can suppress the influence of gross errors. In this section, we will compare *COE* with other indices including *MAR*, *MSR* and *AR*, to demonstrate its effectiveness.

MAR and *MSR* are continuous indices, but they do not have clear physical meanings. In other words, given a value of *MAR* or *MSR* index (0.1 or 0.2), one can hardly determine how accurate the SE result is. On contrary, the value of *COE* (80%, 90%, etc.) directly indicates the accuracy of SE result.

The value of *AR* is also in percentage and has a clear physical meaning, but this index is not continuous. Furthermore, *COE* has an important advantage compared with *AR*: *AR* uses a threshold value ε to distinguish acceptable and unacceptable measurements. However, different acceptable measurements have individual residuals, and so do unacceptable measurements, but these different residuals are not reflected in the *AR* index. In comparison, *COE* scores continuously each measurement from 0% to 100% according to their residuals.

MSR is a parametric method, and it is very sensitive to gross errors. The *MSR* index of an infected residual vector maybe better than a distinguished residual vector. On the other hand, *MAR*, *AR* and *COE* are less sensitive to gross errors.

Furthermore, *AR* is very sensitive to the accuracy of the measurement standard deviations due to its crisp division of accept-

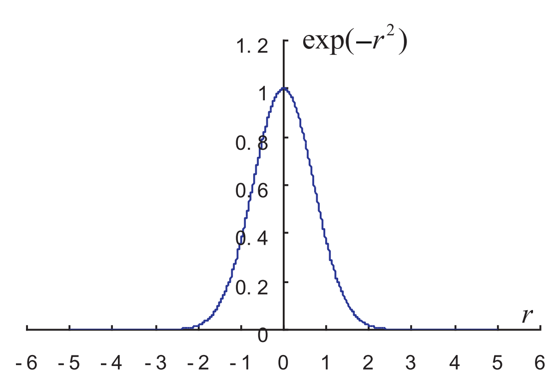

Fig. 1. *COE* index value with a single residual.

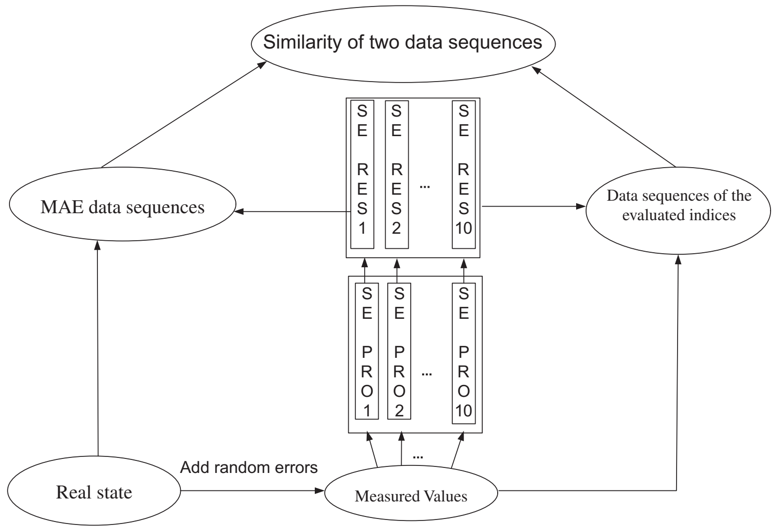

Fig. 2. Comparison procedure using the same system.

able and unacceptable measurements. *COE* is much more insensitive to the measurement standard deviations than *AR*, since they affect only the kernel size. Since measurement standard deviations are difficult to quantify accurately in real power systems, *COE* also is more suitable for evaluating the SE accurately from this perspective.

A summary of the comparison of *MAR*, *MSR*, *AR* and *COE* is given in Table 1. We use *italic* characters to denote the advantageous characteristics. Table 1 shows that in theoretical, *COE* is a better evaluation index than existing *MAR*, *MSR* and *AR* indices.

4. Method for evaluating the performance of different indices

4.1. Procedure

The real states of a real power system are unknown, and it is difficult to judge which index provides the most reasonable evaluation result. Therefore, test systems for which the real states can be known should be used to compare the performance of these indices. The mean absolute estimated error (*MAE*) of the SE results of test systems can be calculated using the differences between the real and estimated values of state variables:

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|\dot{V}_i^{se} - \dot{V}_i^{real}| \tag{16}$$

where N denotes the bus number, and $\dot{V}_i^{se}$ and $\dot{V}_i^{real}$ are the estimated and real values for the complex voltage on the ith bus, respectively

The comparison procedure used here is illustrated in Fig. 2. The real states for a given test system are known. A group of measurements can be generated by adding random measurement errors (possibly containing some manually set gross errors). Ten SE programs based on the weighted least-square principle with different manual parameters and bad data identification functions were developed, and expressed as SE PRO1, SE PRO2,...,SE PRO10. Using these ten SE programs, ten groups of SE results can be calculated based on the same group of measurements, expressed as SE RES1, SE RES2,...,SE RES10. The data sequences of the accuracy evaluation indices for *MAR*, *MSR*, *AR*, and *COE* can be calculated using these ten SE results. The data sequence of *MAE* can also be obtained using Eq. (16). The performances of these accuracy evaluation indices can be evaluated using the similarity between their data sequences and the data sequence of *MAE*.

4.2. Sequences similarity

The similarity of the index and *MAE* sequences is measured using the length of the longest common sequence (LCS) [24,25]. The LCS is widely used in many disciplines to measure the similarity of two sequences. In this paper, the LCS index is transformed into a percentage using:

$$\text{LCS index} = l_{LCS}/10 \times 100\% \tag{17}$$

where l_{LCS} is the length of the LCS; the lengths of the index data sequence and data sequence for *MAE* are both 10 in the comparison procedure.

Obviously, the LCS index reflects the similarity between two data sequences. The *MAE* reflects the true accuracy of the SE results. Therefore, a higher value of the LCS index means that the corresponding evaluation index is more reasonable.

4.3. Measurement set to be considered

An important problem in SE accuracy evaluation is which kinds of measurements should be considered. For SE, the erroneous measurements can be identified in two steps including artificial pre-filtering and automatically compressing by bad data identification function as shown in Fig. 3. In the artificial pre-filtering step, some

Table 1
Comparison of *MAR*, *MSR*, *AR* and *COE*.

	MAR	*MSR*	*AR*	*COE*
Continuity	Continuous	Continuous	Not continuous	Continuous
If has clear physical meanings	No	No	Yes	Yes
If sensitive to gross errors	No	Yes	No	No
If sensitive to measurement standard errors	No	No	Yes	No

erroneous measurements are removed according to experiences. The artificial filtered measurements are removed before SE calculation, so they should be excluded in the SE accuracy evaluation. On the other hand, the susceptive measurements identified by the SE program, should be considered in the evaluation, because these measurements in fact have taken part in the SE calculation with bad data identification function. In fact, these identified susceptive measurements may differ from the real gross errors, different SE and bad data identification algorithms have different performances to compress bad data.

An extreme example is that one can identify all the measurements as "susceptive" except for a group of key measurements. In such case, all the "non-susceptive" measurements (the rested key measurements) will have zero residuals. If the susceptive measurements are not included in accuracy evaluation, these zero residuals will indicate very accurate SE results erroneously.

5. Numerical tests

5.1. Tests of different values of parameter ε for AR

In this section, numerical tests are used to find a reasonable choice for parameter ε in preparation for comparing the performance of different indices, since this parameter affects the performance of *AR* significantly. Generally, every measurement might have an individual value for ε. For convenience, however, the standard deviations of all of the power measurements are set equal and the standard deviations for voltage amplitude measurements are set to 0.1 times the power measurements in the subsequent tests. Consequently, the same ε is used for all of the power measurements, and the ε for voltage amplitude measurements is set to 0.1 times the value for the power measurements.

The IEEE 118-bus system was used for the tests. The gross error rate was set to 3%, and no parameter or topology errors exist in the tests. Let the standard deviation δ of the power measurements change from 0.001 to 0.1; then, the LCS index curves with variable $\varepsilon = k\delta$ and constant ε are plotted in Figs. 4 and 5, respectively. To realize a comprehensive comparison with statistical meaning, the procedure in Fig. 2 was repeated 100 times, and each point in Figs. 4 and 5 is the average LCS index for these 100 tests. From Figs. 4 and 5, we can conclude that the value of ε affects the performance of *AR* significantly. On average, the LCS index for $\varepsilon = k\delta$ is greater when ε takes a fixed value, the LCS index is relatively high when $\varepsilon = 4\delta$. Therefore, $\varepsilon = 4\delta$ is a reasonable choice for setting the value of ε.

5.2. Illustration on 9-bus system

To clearly demonstrate the advantages of the proposed *COE* index, the 9-bus system in Fig. 6 is studied in this section as an example. Linear SE model is used here, in which only use active power measurements and consider bus voltage angles as state variables. The standard deviations of measurements are set as 1.50 MW. Single measurement set and two groups of SE results are constructed in this test. The measurements are shown by black square in Fig. 6. For each measurement, its real value, measurement value, error are shown in Table 2. The power injection of

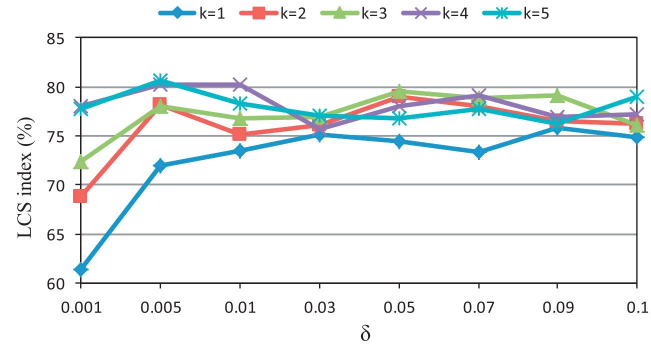

Fig. 4. LCS index for *AR* when $\varepsilon = k\delta$.

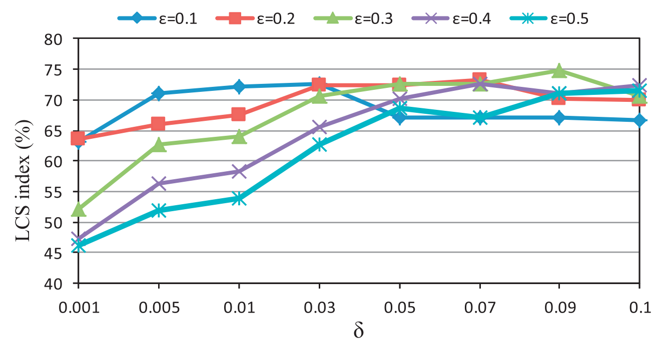

Fig. 5. LCS index for *AR* when ε takes a fixed value.

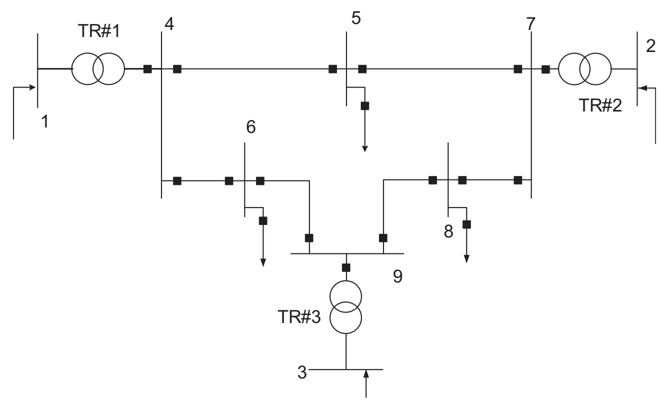

Fig. 6. Configuration and measurement distribution of 9-bus system.

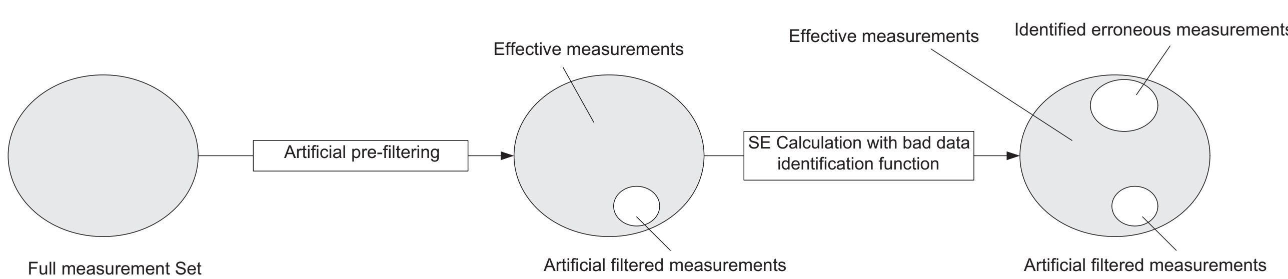

Fig. 3. Measurements flow in the whole SE procedure.

Table 2
Measurement estimation values, residuals and indices of two groups of SE results for 9-bus system.

Meas.	Real value	Meas. value	SE results A				SE results B			
			SE value	Residual	If acceptable	*COE*	SE value	Residual	If acceptable	*COE*
P_{1-4}	66.96	65.48	65.67	−0.19	Yes	0.9920	66.22	−0.74	Yes	0.8854
P_{4-5}	38.01	39.22	43.31	−4.09	Yes	0.0242	39.38	−0.16	Yes	0.9943
P_{4-6}	28.95	27.15	22.24	4.91	Yes	0.0047	26.85	0.30	Yes	0.9801
P_{2-7}	163.04	161.22	160.39	0.83	Yes	0.8580	161.80	−0.58	Yes	0.9279
P_{7-5}	86.99	87.21	83.46	3.75	Yes	0.0439	87.04	0.17	Yes	0.9935
P_{7-8}	76.05	75.12	76.58	−1.46	Yes	0.6227	74.75	0.37	Yes	0.9700
P_{3-9}	85.00	84.32	79.45	4.87	Yes	0.0051	84.11	0.21	Yes	0.9902
P_{9-6}	61.05	58.33	52.41	5.92	Yes	0.0004	59.94	−1.61	Yes	0.5621
P_{9-8}	23.95	24.77	26.01	−1.24	Yes	0.7105	24.16	0.61	Yes	0.9206
P_5	−125.0	−126.2	−126.7	0.50	Yes	0.9459	−126.42	0.22	Yes	0.9893
P_6	−90.0	−70.0	−74.65	4.65	Yes	0.0081	−86.79	16.79	No	0.0000
P_8	−100.0	−97.6	−102.59	4.99	Yes	0.0039	−98.91	1.31	Yes	0.6829

Table 3
Measurement estimation values, residuals and indices of two groups of SE results for 9-bus system.

	Mean error of state variables dθ	*MAR*	*MSR*	*AR* (%)	*COE* (%)
SE result A	0.0047	1.3022	6.1048	100	35.17
SE result B	7.44e−4	0.9383	10.6608	91.67	82.47

bus 6 is set as a gross errors, and is highlighted by bold characters. For the two groups of SE results, their estimated values, residuals, if determined as acceptable by *AR*, and the value of *COE* are also listed in Table 2. The overall indices values for these two groups of SE results are shown in Table 3:

The second column in Table 3 gives the mean error of the bus voltage angles, defined by

$$d\theta = \frac{1}{n}\sum_{i=1}^{n}\left|\theta_i^{SE} - \theta_i^{Real}\right| \tag{18}$$

where n is the bus number, the superscripts *SE* and *Real* denote estimated value and real value, respectively. It can be observed that the error of SE result A is much larger than that of SE result B, indicating B is much more accurate than A. In fact, according to Table 2, it can be concluded that A failed to suppress the gross error and got an infected residual vector (as the 5th column in Table 2). On the other hand, B has successfully suppressed the gross error and got a distinguished residual vector (9th column).

However, on the issue of evaluating indices, *MSR* and *AR* give erroneous determination that A is more accurate than B. For *MSR*, it is sensitive to gross errors and the infected residual vector has a better index. For *AR*, its disadvantage is the equal treatment for all the acceptable measurements. The residual for SE result A has some large value, but none of them violates the threshold value (4δ = 6.0 MW). On contrary, *COE* gives continuous markings for each measurement which clearly shows its accuracy. As a result, the *COE* index for B is much higher than A, which correctly suggests that B is much more accurate.

MAR also suggests that B is more accurate than A, but these values (1.3022 and 0.9383) do not have clear physical meanings and

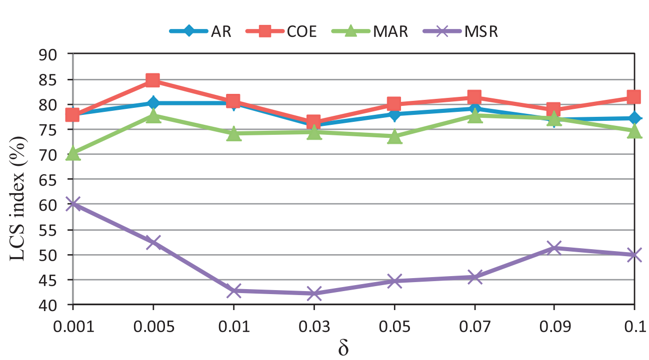

Fig. 7. Comparison of the different evaluation indices on a 118-bus system.

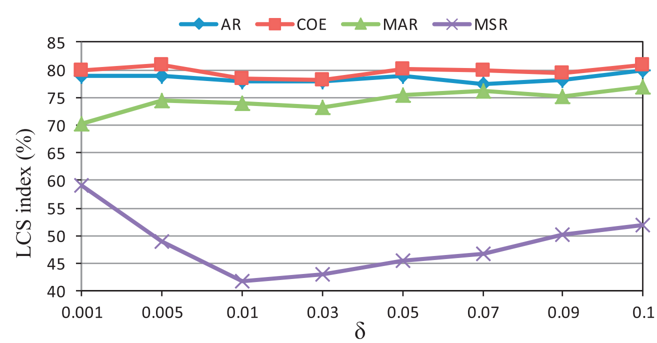

Fig. 8. Comparison of the different evaluation indices on a 145-bus system.

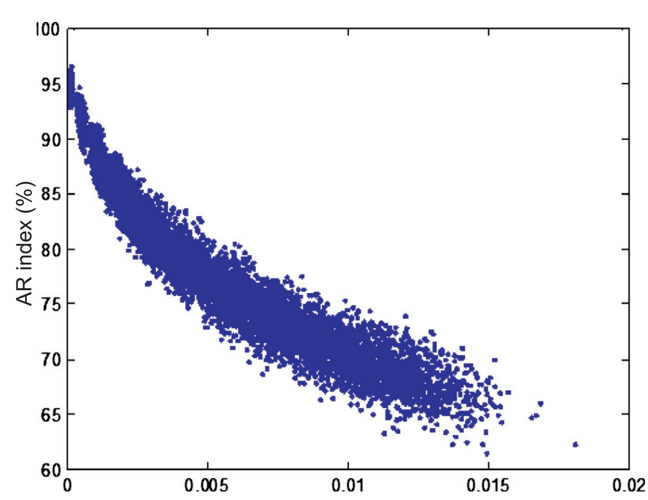

Fig. 9. Scatter plot for the *AR* index in the 118-bus system test.

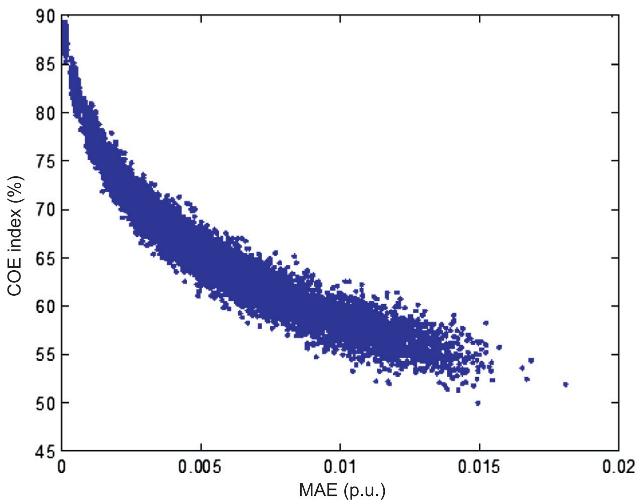

Fig. 10. Scatter plot for the *COE* index in the 118-bus system test.

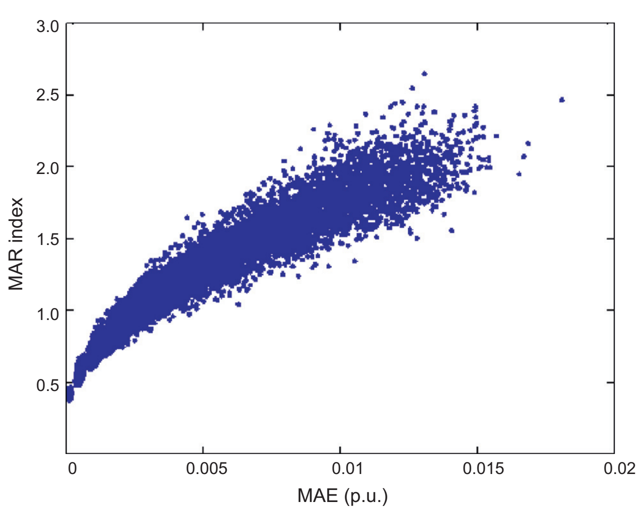

Fig. 11. Scatter plot for the *MAR* index in the 118-bus system test.

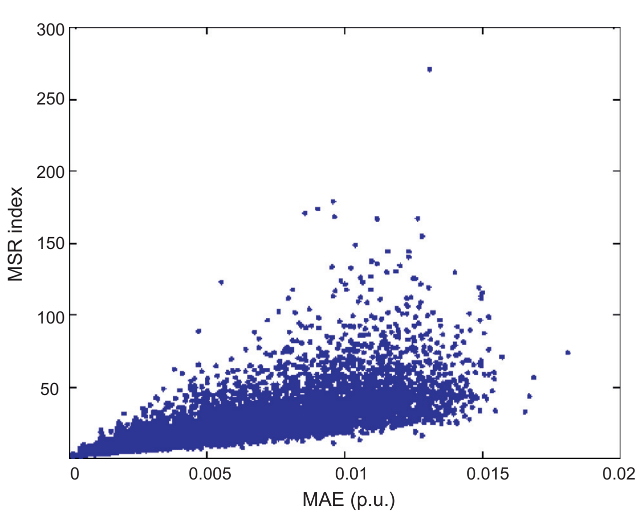

Fig. 12. Scatter plot for the *MSR* index in the 118-bus system test.

Table 4
Average LCS index declines.

AR (%)	*COE* (%)	*MAR* (%)
1.55371	0.907143	0.87500

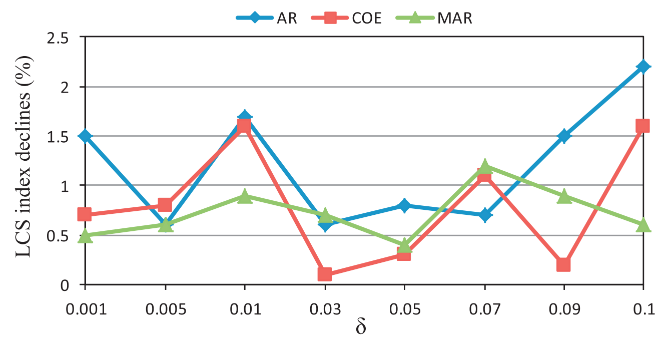

Fig. 13. The LCS index declines when δ is inaccurate.

seemed to be similar. While *COE* clearly demonstrates that *B* is quite accurate and *A* is very inaccurate.

In summary, this example clearly shows the conventional indices have their disadvantages which may lead to a erroneous accuracy evaluation. On contrary, the proposed *COE* does not have such disadvantages, and that is why it can be more reasonable than conventional indices.

5.3. Performance comparison among different evaluation indices

General comparisons of the four indices *AR*, *COE*, *MAR*, and *MSR* were carried out on 118- and 145-bus systems [26]. For the *AR* index, the threshold value ε was set as 4δ, based on the previous test results, and the kernel size σ for *COE* was set to equal to δ according to (13). Using the procedures in Fig. 2 and 100 repetitions for each point, the LCS curves for these four indices are plotted in Figs. 7 and 8 for the 118- and 145-bus systems, respectively. Figs. 7 and 8 show that *COE* had the largest LCS index, which indicates that it is the most reasonable index for evaluating the SE accuracy. This is consistent with the theoretical analysis. *AR* and *MAR* are suboptimal options, while *MSR* has the smallest LCS index, making it a poor SE accuracy evaluation standard.

Scatter plots are shown in Figs. 9–12 to illustrate the relationship between *MAE* and the four evaluation indices on the IEEE 118-bus system. The points are concentrated most densely in the scatter plot for *COE*, which indicates that *COE* is the most reasonable of the four indices. The shape of the scatter plot for *AR* is similar to that for *COE*, but the points are more dispersed. The scatter plot for *MAR* is similar to that for *AR* flipped vertically. The samples in *MSR* were the most widely dispersed, with some outliers, which means *MSR* cannot indicate the accuracy of the SE results in a robust manner.

5.4. Sensitivity analysis

According to (1), (2), (5), and (13), the performance of all four indices depends on the measurement standard deviations δ, which are very difficult to quantify accurately in practical power systems. Therefore, it is necessary to check the sensitivity of these indices to δ. If the index is very sensitive to δ, its performance will deteriorate significantly in a practical application when the standard deviations are inaccurate.

To verify how much the LCS index is affected by the inaccuracy of δ, a normally distributed random error was added to δ in this test:

$$\delta' = \left(1 + N(0, 0.5^2)\right) \times \delta \tag{19}$$

where $N(0, 0.5^2)$ denotes a normally distributed number with 0 expectation and 0.5 standard deviation. Since in practice, the accurate value of δ is unknown, when calculating the index values and their index sequences, only an inaccurate δ' can be used. Under such conditions, the LCS index will fall due to the inaccuracy of δ. Repeating the comparison procedure in Fig. 2 based on the 118-bus system, the LCS index declined compared with the results in Fig. 7 for the *AR*, *COE*, and *MAR* indices, which are plotted in Fig. 13 for comparison. Table 4 lists the average declines in the LCS index for these three indices.

From Fig. 13 and Table 4, when δ had an inaccuracy of about 50%, the LCS index for *AR* decreased by about 1.55% on average; this was the most sensitive parameter to the accuracy of δ. The performances of *COE* and *MAR* were much more robust. Considering the LCS index and its sensitivity, *COE* is the most practical SE accuracy evaluation standard.

6. Conclusions

It is important to evaluate the accuracy of the SE results for practical power systems. This is a technical challenge because the real states of power systems are unknown. In this paper, we proposed a practical SE accuracy evaluation index based on correntropy (*COE*). Theoretical analysis has shown that the proposed index is more reasonable than traditional indices, such as the *AR*, *MAR*, and *MSR*. Extensive numerical tests were used to compare the performances of these evaluation indices, and the results proved that correntropy most reasonably indicates the true accuracy of the SE. *AR* is another evaluation index, but its performance depends heavily on the accuracy of the measurement standard deviations, which are usually inaccurate for power systems. *COE* is much less sensitive to the accuracy of the standard deviation measurements than *AR*, making it more suitable for practical applications.

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