清華大學

Production Planning and Control

Lecture 6 Inventory Control Subject to Known Demand (1)

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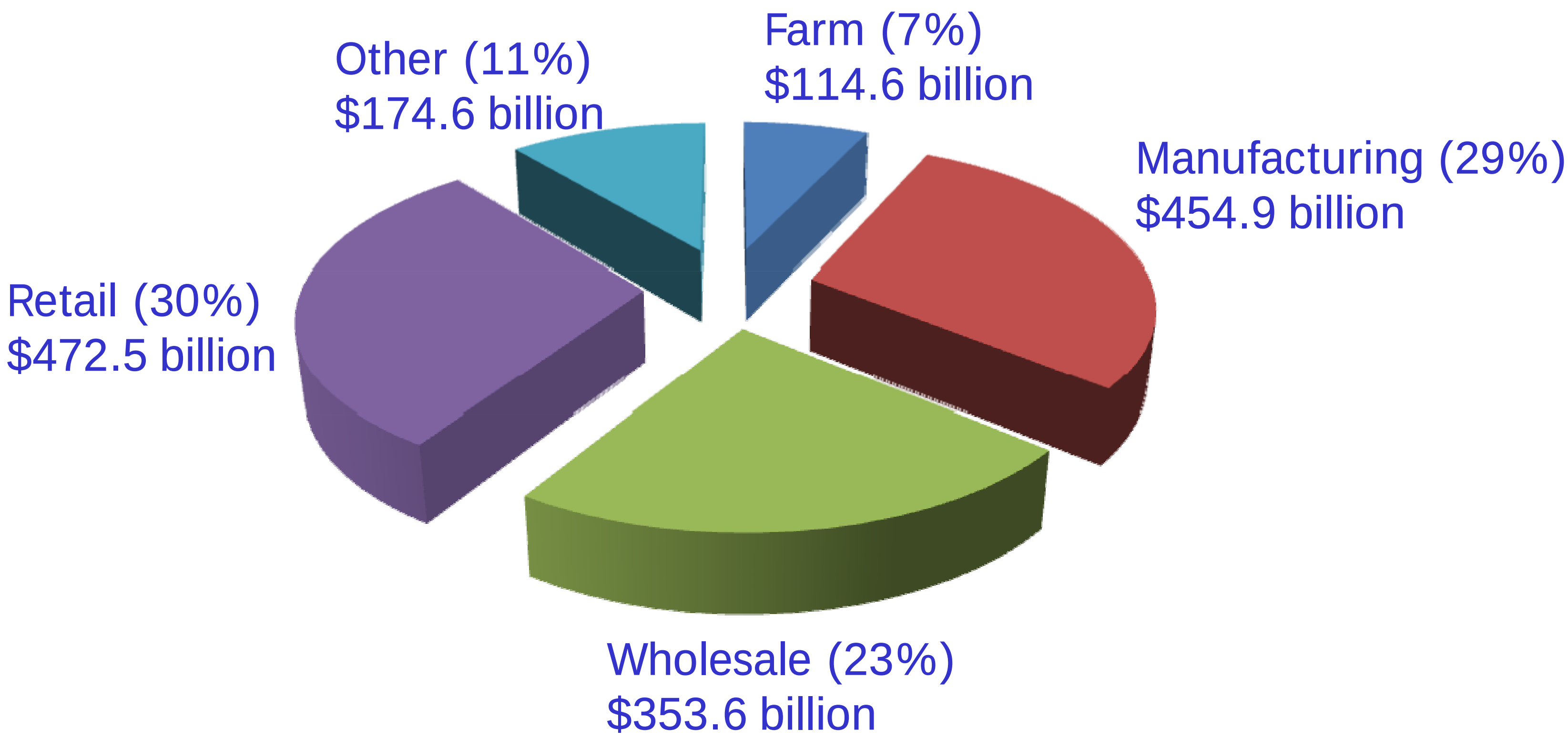
Breakdown of the Total Investment in Inventories in the U.S. Economy (2007)



Total investment = $1,570.2 billion

Fundamental Problems of Inventory Management

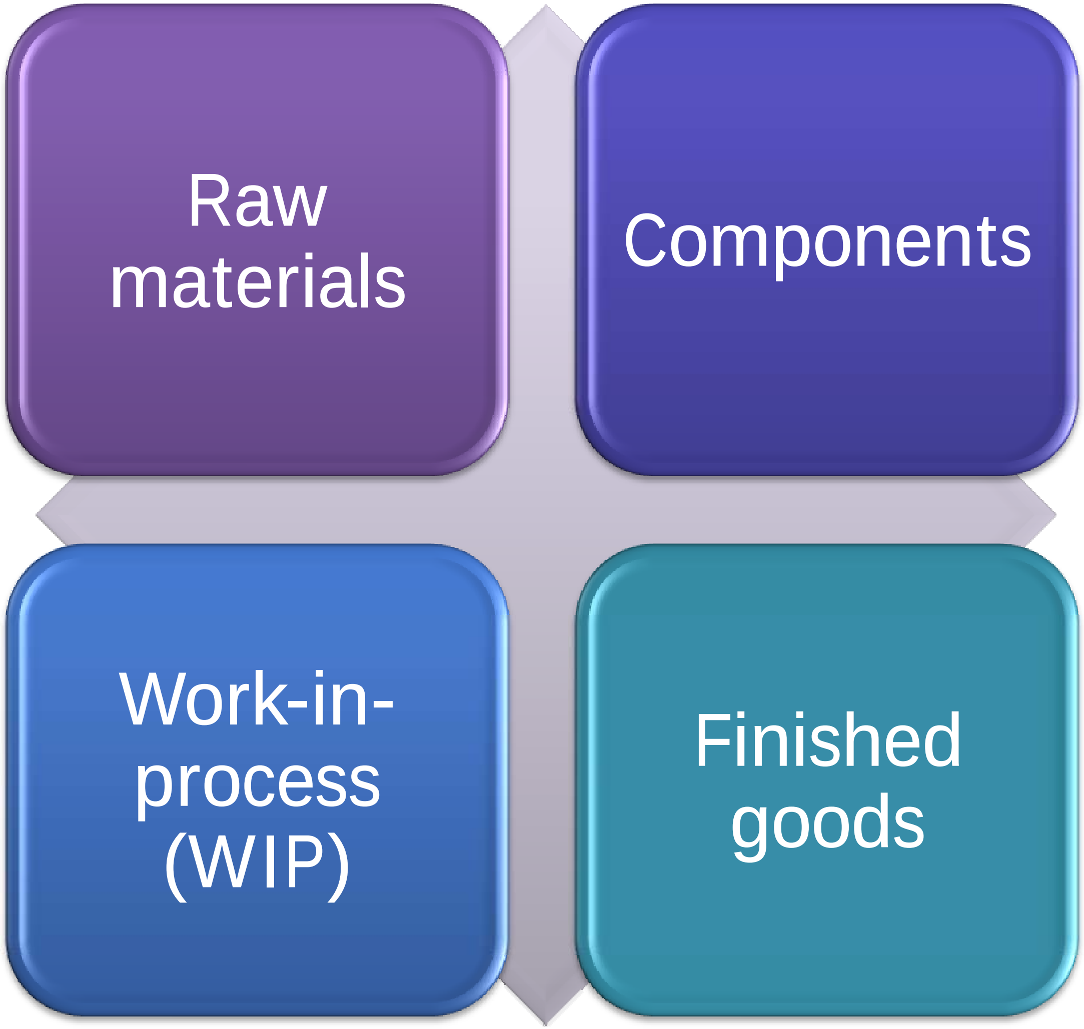
When should an order be placed?

How much should be ordered?

Model assumptions

- Known demand
- Uncertain demand

Types of Inventories



Motivation for Holding Inventories

Economies of scale

- Amortize fixed setup costs

Uncertainties

- Demand
- Lead time
- Resource supply
  - Material, labor, capital

Speculation

- Value increase

Transportation

- In-transit or pipeline inventories

Smoothing

- Inventory for anticipated peak demand

Logistics

- Constraints in the purchasing, production, or distribution

Control cost

Characteristics of Inventory Systems (1/2)

Demand	• Constant vs. variable • Known vs. random
Lead time	• Order from the outside • Produced internally
Review time	• Continuous review • Periodic review

Characteristics of Inventory Systems (2/2)

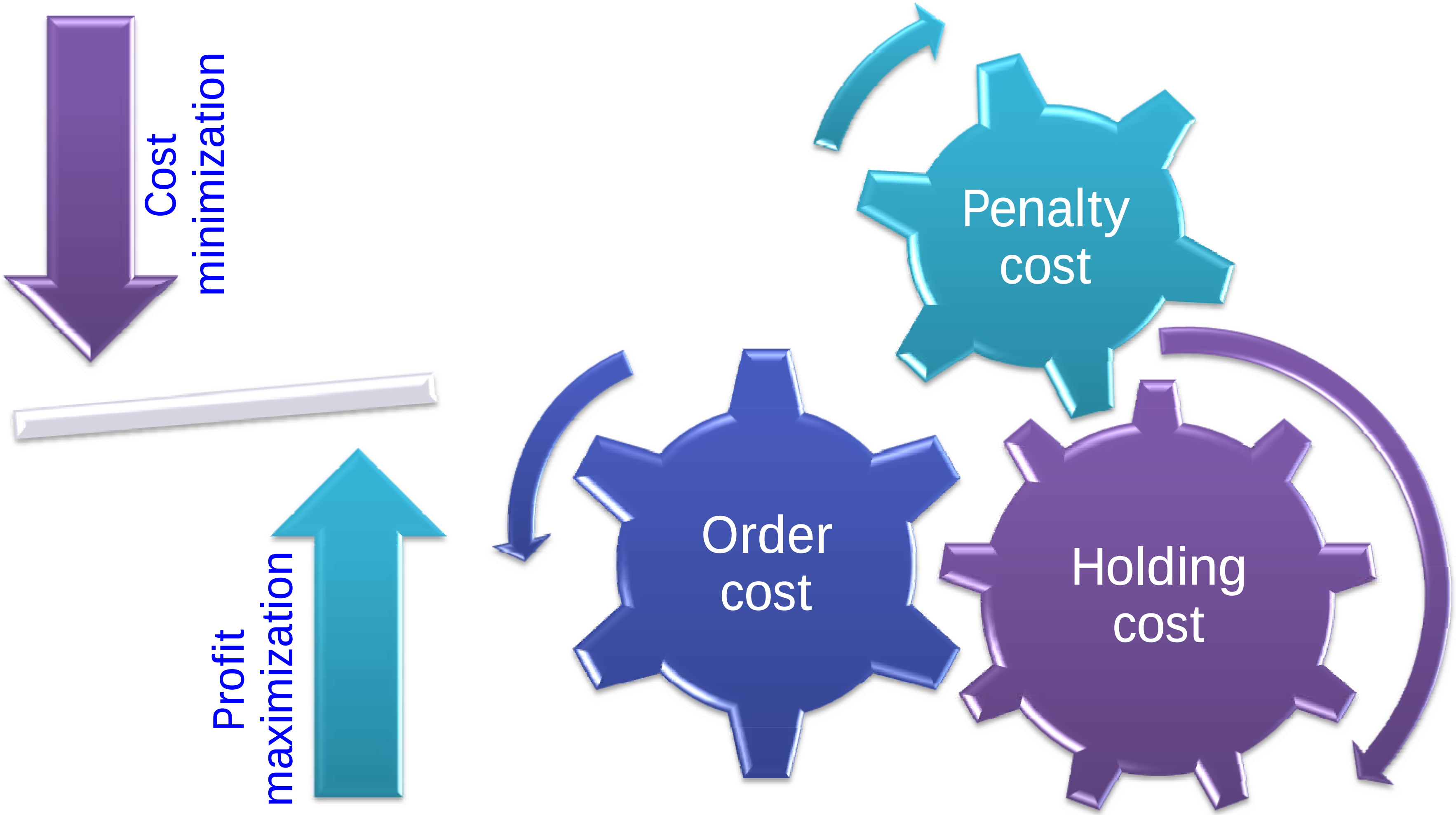
Excess demand

- Backorder
  - Satisfied at a future time
- Lost
  - Satisfied from outside the system
- Partial back-ordering
  - Part of the demand is lost
- Customer impatience
  - Customer cancels order

Changing inventory

- Inventory undergoes changes over time that may affect its utility

Optimization Criterion and Relevant Costs



Holding Cost

Cost of storage

Taxes and insurance

Holding Cost

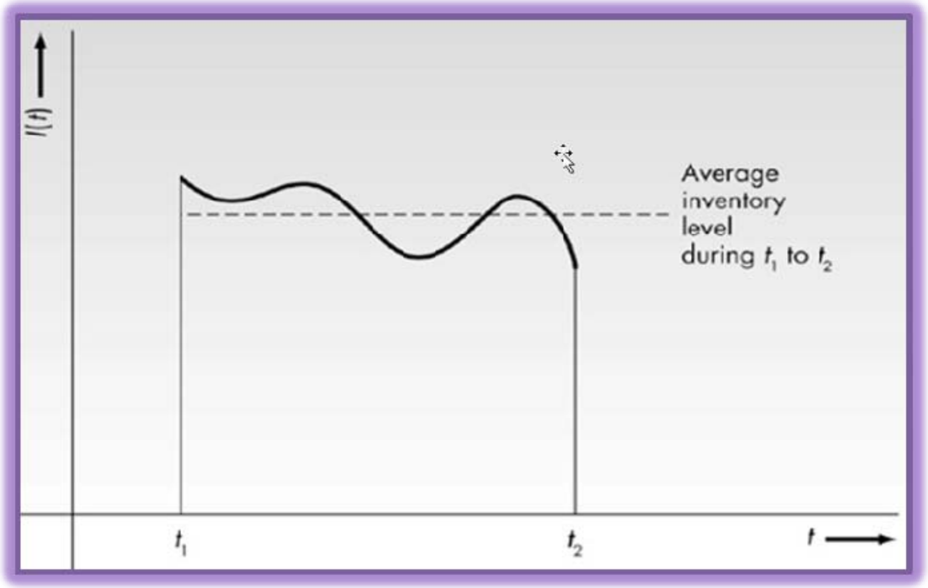
Breakage, spoilage, deterioration, obsolescence

Opportunity cost

- Cost of capital

$$h = Ic$$

- h, the holding cost in terms of dollars per unit per year
- c, the dollar value of one unit of inventory
- I, the annual interest rate

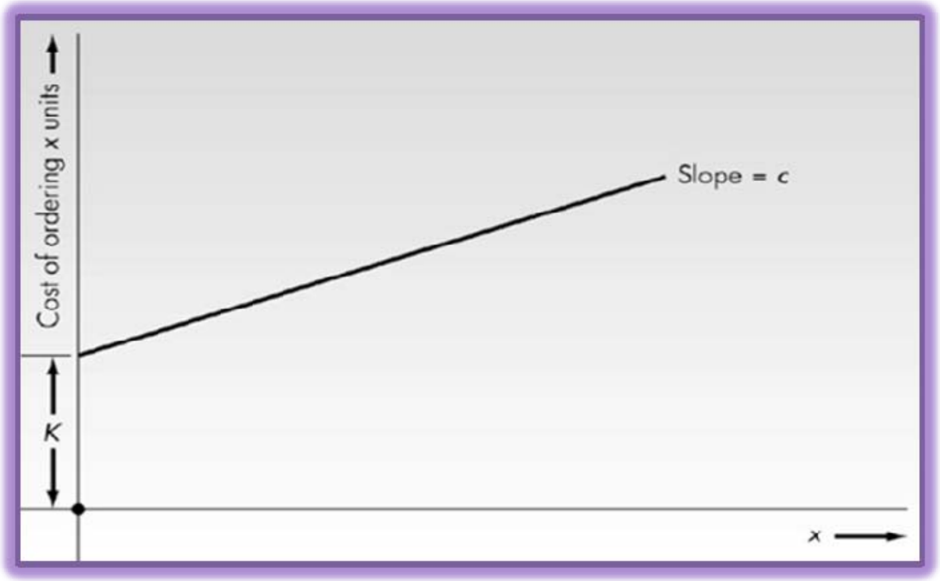


Order Cost

Fixed cost — K Setup cost

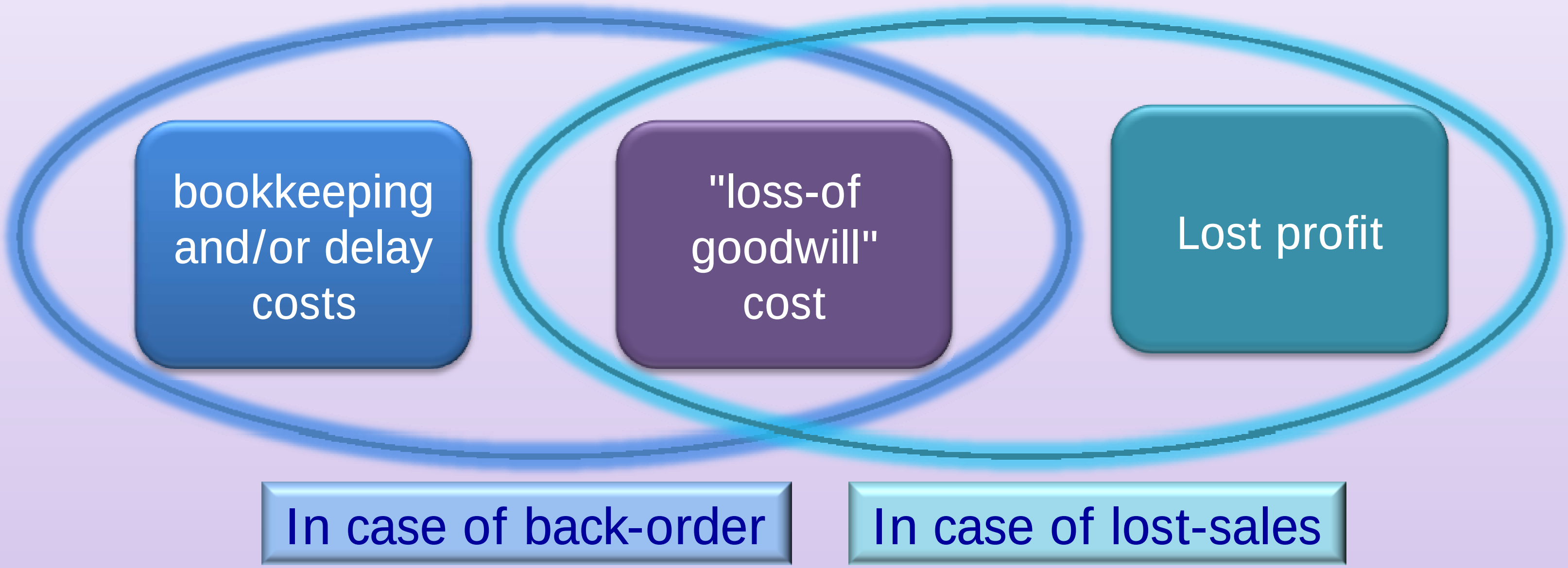
Variable cost — c Proportional order cost

$$C(x) = \begin{cases} 0 & \text{if } x = 0 \\ K + cx & \text{if } x > 0 \end{cases}$$



Penalty Cost

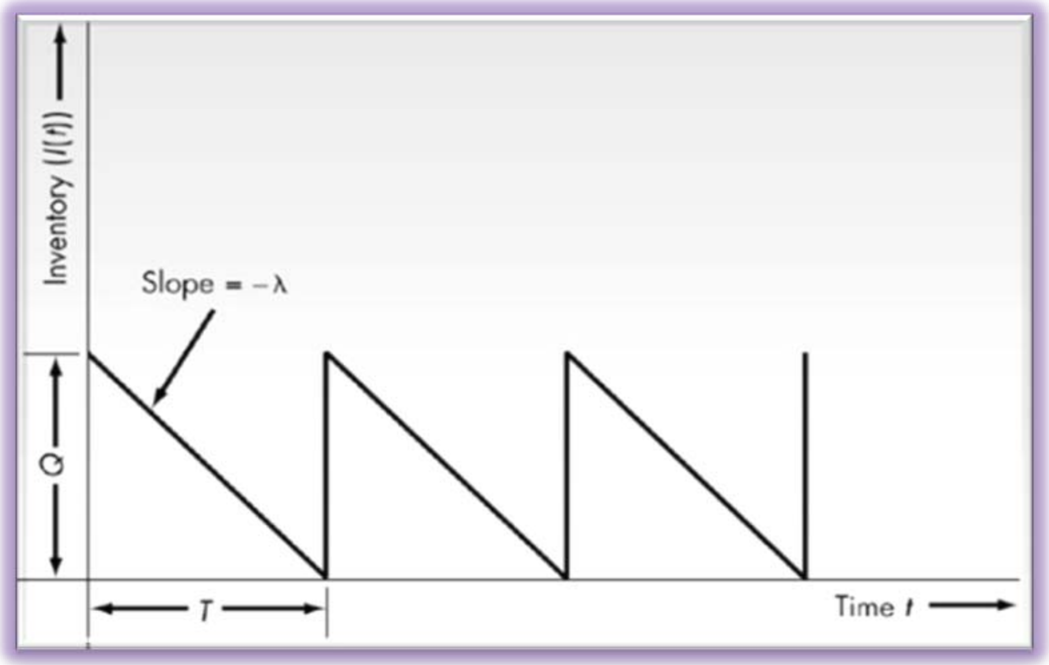
Shortage cost / Stock-out cost



- Penalty cost: p
  - Charged on a per-unit basis
  - Charged on a per-unit-per-unit-time basis

The Economic Order Quantity Model (1/4)

- Assumptions
  - Fixed demand rate
    - λ units per unit time
  - No shortage
  - Zero lead time
  - Costs
    - Setup cost: K
    - Order cost: c
    - Holding cost: h
  - Order size: Q



Objective: choose Q to minimize the average cost per unit time

The Economic Order Quantity Model (2/4)

- Order cost in each cycle: $C(Q) = K + cQ$
- Cycle time: $T = Q / \lambda$
- Cycle number per unit time: $1 / T$
- Average inventory level: $Q / 2$

- Average annual cost

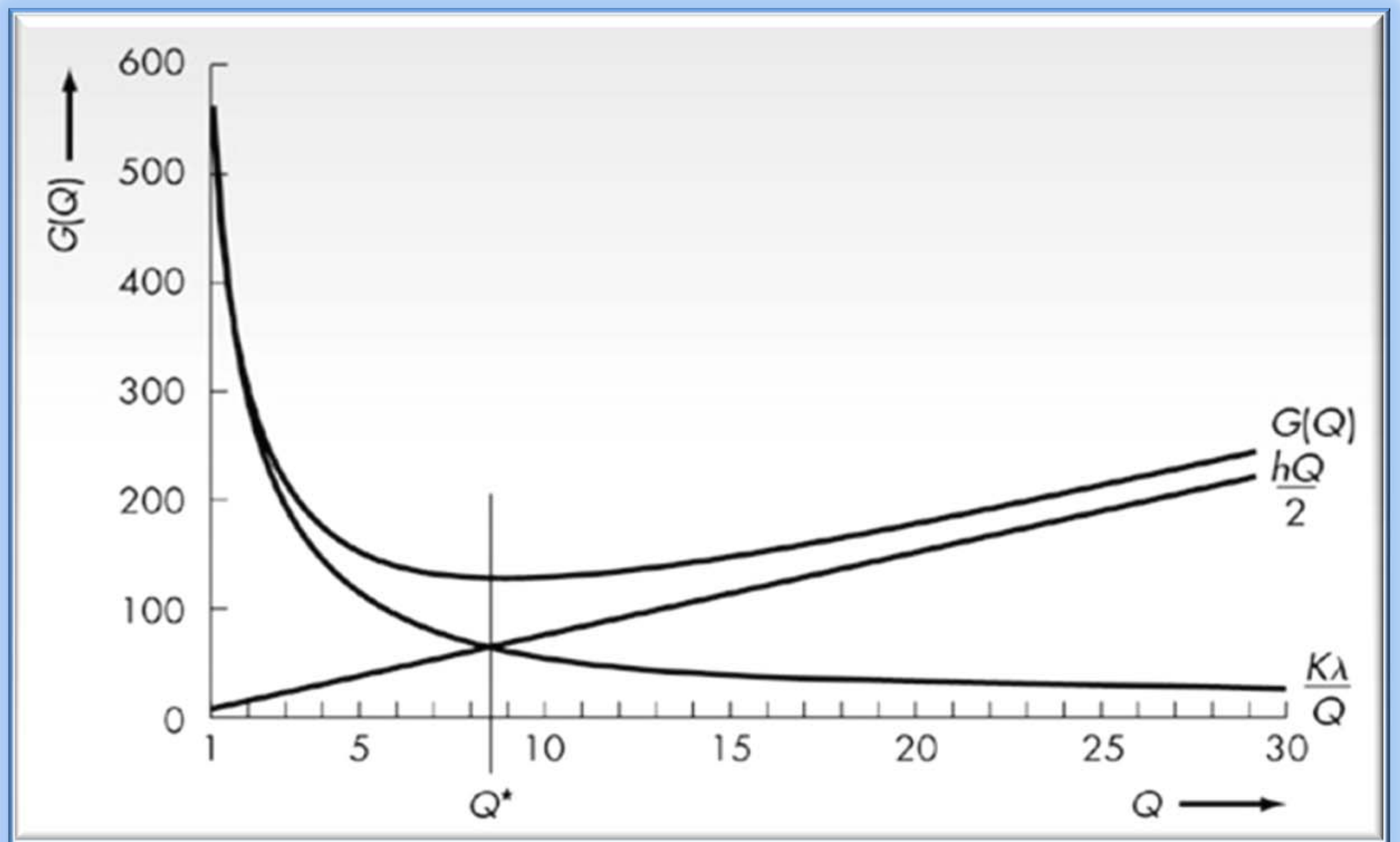
$$G(Q) = \frac{K + cQ}{T} + \frac{hQ}{2} = \frac{K + cQ}{Q/\lambda} + \frac{hQ}{2} = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2}$$

Annual setup cost

Annual purchase cost

Annual holding cost

The Economic Order Quantity Model (3/4)



The Economic Order Quantity Model (4/4)

- Find Q to minimize $G(Q)$

$$G'(Q) = -K\lambda / Q^2 + h/2$$

$$G''(Q) = 2K\lambda / Q^3 > 0 \quad \text{for } Q > 0$$

  - Let $G'(Q) = 0$

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

The economic order quantity (EOQ)

Example of EOQ Application

- A campus bookstore also sells pencils
  - Sold at a fairly steady rate
    - 60 pencils per week
  - Each pencil
    - Cost 2 cents
    - Sell for 15 cents
  - initiate an order
    - \$12
  - Holding cost
    - 25 percent / year

- Optimal order quantity?
- Time between orders?
- Yearly holding and setup costs?

- Solution
  - The annual demand rate
    - λ = (60)(52) = 3,120
  - The holding cost
    - $\boldsymbol{h}$ = (0.25)(0.02) = 0.005
  - The setup cost
    - $\boldsymbol{K}$ = 12
  - EOQ

$$Q^* = \sqrt{\frac{2\times 12\times 3120}{0.005}} = 3870$$

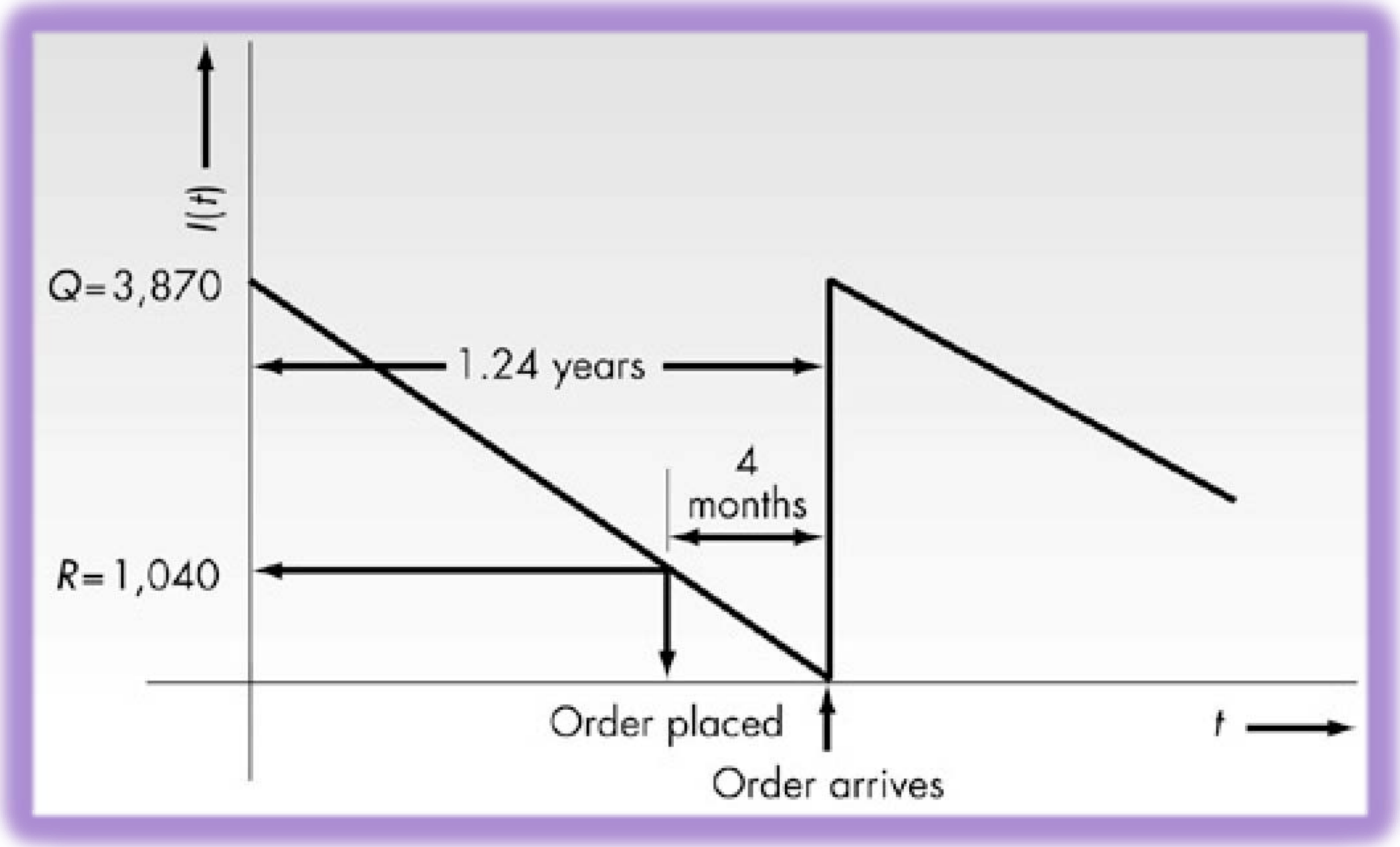
$$T = Q/\lambda$$

$$= 3870/3120 = 1.24 \quad \text{(years)}$$

$$h(Q/2) = K\lambda/Q$$

$$= 0.005(3870/2) = \$9.675$$

Inclusion of Order Lead Time

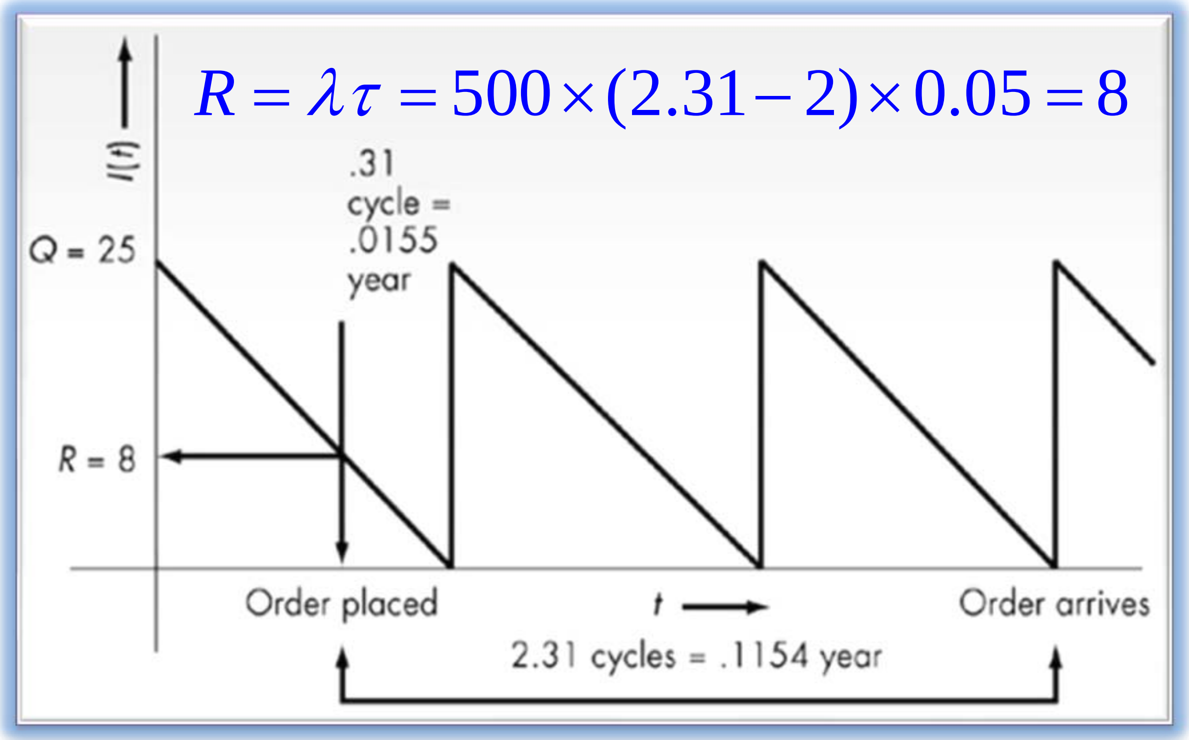


$$R = \lambda\tau = 3870 \times 0.3333 = 1040$$

If Lead Time Exceeds A Cycle……

- Example
  - EOQ：25 units
  - Demand rate：500 units per year
  - Lead time: 6 weeks
  - Cycle time: $T = 25/500 = 0.05$ **year** (2.6 weeks)

$$\tau / T = 6/2.6 = 2.31$$



Sensitivity

Sensitivity of the annual cost to errors in *Q*

- Optimal average annual holding and setup cost

$$G^*(Q) = K\lambda / Q^* + hQ^* / 2$$

$$= \frac{K\lambda}{\sqrt{2K\lambda / h}} + \frac{h}{2}\sqrt{\frac{2K\lambda}{h}} = 2\sqrt{\frac{K\lambda h}{2}} = \sqrt{2K\lambda h}$$

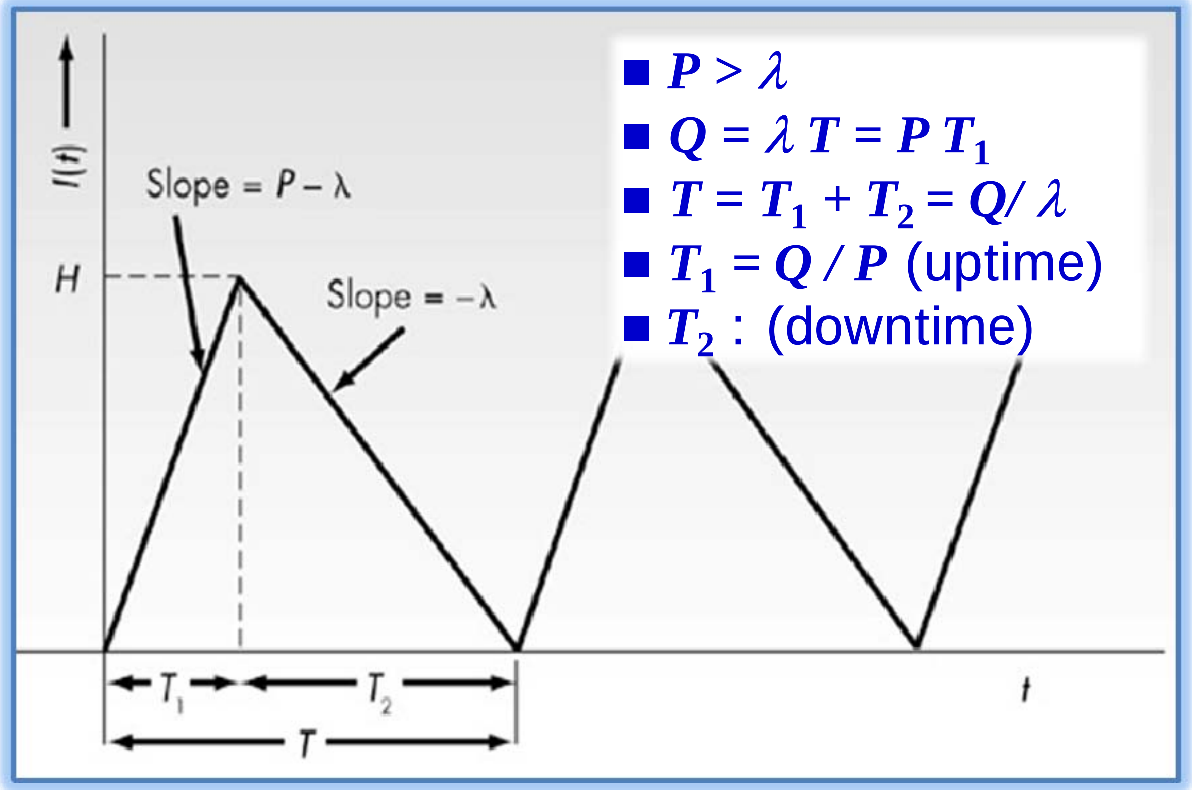
- For any *Q*

$$\frac{G(Q)}{G^*} = \frac{K\lambda / Q + hQ / 2}{\sqrt{2K\lambda h}} = \frac{1}{2Q}\sqrt{\frac{2K\lambda}{h}} + \frac{Q}{2}\sqrt{\frac{h}{2K\lambda}}$$

$$= \frac{Q^*}{2Q} + \frac{Q}{2Q^*} = \frac{1}{2}\left[\frac{Q^*}{Q} + \frac{Q}{Q^*}\right]$$

Example: $Q^* / Q = 3.87$ → $G(Q) / G^* = 2.06$

Extension to a Finite Production Rate



- $P > \lambda$
- $Q = \lambda\, T = P\, T_1$
- $T = T_1 + T_2 = Q / \lambda$
- $T_1 = Q / P$ (uptime)
- T_2 : (downtime)

- *H*: The maximum level of on-hand inventory
  - $H / T_1 = P - \lambda \rightarrow H = (P - l)T_1 = Q(1 - l / P)$
- Average inventory level : $H / 2$

- Average annual cost function

$$G(Q) = \frac{K}{T} + \frac{hH}{2}$$

$$= \frac{K\lambda}{Q} + \frac{hQ}{2}(1 - \frac{\lambda}{P})$$

- Define

$$h' = h\,(1 - \lambda / P)$$

- Get revised EOQ

$$Q^* = \sqrt{\frac{2K\lambda}{h'}}$$

Example: EOQ with Finite Production Rate (1/2)

- A local company produces a programmable EPROM
  - Relatively flat demand of 2,500 units per year
  - Produced at a rate of 10,000 units per year
  - It costs $50 to initiate a production run
  - Each unit costs the company $2 to manufacture
  - 30 percent annual interest rate

- Optimal size of a production run?
  - Length of each production run
  - The average annual cost of holding and setup
  - Maximum level of the on-hand inventory

Example: EOQ with Finite Production Rate (1/2)

Solution

- h = (0.3)(2) = 0.6 per unit per year
  - The modified holding cost

    $h' = h\,(1 - l\ / P)$ = (0.6)(1 -2,500/10,000) = 0.45
- Q^* = 745 (Note: the simple EOQ is 645)
- The time between production runs

  $T = Q / \lambda$ = 745 / 2,500 = .298 year
  - The uptime: $T_1 = Q / P$ = 745/10,000 = 0.0745 year
  - The downtime: $T_2 = T - T_1$ = 0.2235 year
- The average annual cost of holding and setup

$$G(Q^*) = \frac{K\lambda}{Q^*} + \frac{h'Q^*}{2} = \frac{(50)(2500)}{745} + \frac{(0.45)(745)}{2} = 335.41$$

- The maximum level of on-hand inventory

  $H = Q^*\,(1 - \lambda\ / P)$ = 559 units

Summary

- Inventory control
  - Basic problems, types of inventories
  - Motivation for holding inventories
  - Characteristics of inventory systems
- Costs of inventory
  - Holding cost, order cost, penalty cost
- The EOQ Model
  - Assumptions
  - Basic model
  - consideration of order lead time
  - Sensitivity
  - Extension to a finite production rate

Assignment 06

Problem 9	Problem 12
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