CS251: Homework #2

Due on October 15, 2019 at 2:00pm

Steven Libby Section A

Austen Nelson

Problem 1

last week we showed that nand $\odot$ is a universal operator. That is, we can write all operators in terms of nand. Show that $\rightarrow$ is a universal operator by writing $\neg, \wedge$, and $\vee$ with only $\rightarrow$.

- $A \rightarrow \bot = \neg A$
- $(A \rightarrow \bot) \rightarrow B = A \wedge B$
- $(A \rightarrow B) \rightarrow B = A \vee B$

Problem 2

Convert the following to CNF

Part One

$$\begin{aligned} & (c \wedge a) \vee (b \wedge c) & \\ (c \vee b) \wedge (c \vee c) \wedge (a \vee b) \wedge (a \vee c) & & FOIL_{\vee} \end{aligned}$$

Part Two

$$\begin{aligned} (a \wedge \neg a) \vee (b \wedge \neg b) & & \\ \bot \vee \bot & & CTR * 2 \\ \bot & & Item_{\vee} \end{aligned}$$

Part Three

$$\begin{aligned} a \rightarrow (b \equiv c) & & \\ a \rightarrow ((b \rightarrow c) \wedge (c \rightarrow b)) & & Def_{\equiv} \\ \neg a \vee ((\neg b \vee c) \wedge (\neg c \vee b)) & & Imp * 3 \\ (\neg a \vee \neg b \vee c) \wedge (\neg a \vee \neg c \vee b) & & Dis_{\vee} \end{aligned}$$

Part Four

$$\begin{aligned} (a \rightarrow b) \wedge (b \rightarrow c) & & \\ (\neg a \vee b) \wedge (\neg b \vee c) & & Imp * 2 \end{aligned}$$

Part Five

$$\begin{aligned} \neg(a \vee b) & & \\ \neg a \wedge \neg b & & DM_{\vee} \end{aligned}$$

Problem 2 continued on next page...

Part Six

$$
\begin{aligned}
&(a \equiv b) \equiv c && \\
&(((a \to b) \wedge (b \to a)) \to c) \wedge (c \to ((a \to b) \wedge (b \to a))) && Def_{\equiv} \\
&(\neg((\neg a \vee b) \wedge (\neg b \vee a)) \vee c) \wedge (\neg c \vee ((\neg a \vee b) \wedge (\neg b \vee a))) && Imp * 6 \\
&((a \wedge \neg b) \vee (b \wedge \neg a) \vee c) \wedge (\neg c \vee ((\neg a \vee b) \wedge (\neg b \vee a))) && DM_{\wedge},\ DM_{\vee} * 2 \\
&(((a \vee b) \wedge (a \vee \neg a) \wedge (\neg b \vee b) \wedge (\neg b \vee \neg a)) \vee c) \wedge (\neg c \vee ((\neg a \vee b) \wedge (\neg b \vee a))) && FOIL_{\vee} \\
&((c \vee a \vee b) \wedge (c \vee a \vee \neg a) \wedge (c \vee \neg b \vee b) \wedge (c \vee \neg b \vee \neg a)) \wedge (\neg c \vee \neg a \vee b) \wedge (\neg c \vee \neg b \vee a) && Dis_{\vee} * 5 \\
&(c \vee a \vee b) \wedge (c \vee \neg b \vee \neg a) \wedge (\neg c \vee \neg a \vee b) \wedge (\neg c \vee \neg b \vee a) && LEM * 2,\ Anul_{\vee} * 2
\end{aligned}
$$

Problem 3

Prove the following:

Part One
$a \vee b \vdash b \vee a$:

$$\dfrac{A \vee B \qquad \dfrac{[A]}{A \to B \vee A} \to I \qquad \dfrac{[B]}{B \to B \vee A} \to I}{B \vee A} \vee E$$

Part Two
$(a \vee b), \neg b \vdash a$:

$$\dfrac{A \vee B \qquad \dfrac{[A]}{A \to A} \to I \qquad \dfrac{[B] \qquad \dfrac{\neg B \qquad [B]}{\dfrac{\bot}{A} \bot E}}{B \to A} \to I}{A} \vee E$$

Part Three
$\neg a \vee \neg b \vdash \neg(a \wedge b)$:

$$\dfrac{\neg A \vee \neg B \qquad \dfrac{[\neg A]}{\neg A \to \neg(A \wedge B)} \to I \qquad \dfrac{[\neg B]}{\neg B \to \neg(A \wedge B)} \to I}{\neg(A \wedge B)} \vee E$$

$$\dfrac{[\neg A]}{\neg A \to \neg(A \wedge B)} \to I = \dfrac{\dfrac{[A \wedge B]}{\dfrac{A \wedge B \to \bot}{\neg(A \wedge B)} \neg I} \to I}{\neg A \to \neg(A \wedge B)} \to I$$

$$\dfrac{[A \wedge B]}{A \wedge B \to \bot} \to I = \dfrac{\dfrac{[A \wedge B]}{A} \wedge E2 \qquad \dfrac{[\neg A]}{\bot} \neg E}{A \wedge B \to \bot} \to I$$

$$\dfrac{[\neg B]}{\neg B \to \neg(A \wedge B)} \to I = \dfrac{\dfrac{[A \wedge B]}{\dfrac{A \wedge B \to \bot}{\neg(A \wedge B)} \neg I} \to I}{\neg B \to \neg(A \wedge B)} \to I$$

$$\dfrac{[A \wedge B]}{A \wedge B \to \bot} \to I = \dfrac{\dfrac{[A \wedge B]}{B} \wedge E1 \qquad \dfrac{[\neg B]}{\bot} \neg E}{A \wedge B \to \bot} \to I$$

Part Four
DL1: $\neg(\neg a \vee \neg b) \vdash a$
Part Five
$\neg(a \wedge b) \vdash \neg a \vee \neg b$:

Problem 3 continued on next page...

(Hint: you can use the previous problem, and a theorem from class.)

$\frac{\neg(\neg a \vee \neg b)}{a}\, DL1 \quad \frac{\neg(\neg a \vee \neg b)}{b}\, DL2 \quad \frac{\neg\neg a}{a}\, \neg\neg E$