CS251: Homework #1

Due on October 8, 2019 at 2:00pm

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Problem 1

last week we showed that nand $\odot$ is a universal operator. That is, we can write all operators in terms of nand. Show that $\to$ is a universal operator by writing $\neg, \wedge$, and $\vee$ with only $\to$.

- $A \to \bot = \neg A$
- $(A \to \bot) \to B = A \wedge B$
- $(A \to B) \to B = A \vee B$

Problem 2

Convert the following to CNF

Part One

$(c \wedge a) \vee (b \wedge c)$

$(c \vee b) \wedge (a \vee b) \wedge (a \vee c)$

Part Two

$(a \wedge \neg a) \vee (b \wedge \neg b)$

$\bot$

Part Three

$a \to (b \equiv c)$

Part Four

$(a \to b) \wedge (b \to c)$

Part Five

$\neg(a \vee b)$

Part Six

$(a \equiv b) \equiv c$

Problem 3

Prove the following:

Part One

$a \vee b \vdash b \vee a$:

$$\dfrac{A \vee B \qquad \dfrac{[A] \quad \dfrac{[A]}{B \vee A}\vee I2}{A \to B \vee A}\to I \qquad \dfrac{[B] \quad \dfrac{[B]}{B \vee A}\vee I1}{B \to B \vee A}\to I}{B \vee A}\vee E$$

Part Two

$(a \vee b), \neg b \vdash a$:

Problem 3 continued on next page...

$$\dfrac{A \lor B \qquad \dfrac{[A] \qquad [A]}{A \to A}\to I \qquad \dfrac{[B] \qquad \dfrac{\dfrac{[B] \qquad \neg B}{\perp}\neg E}{A}\perp E}{B \to A}\to I}{A}\lor E$$

Part Three

$\neg a \lor \neg b \vdash \neg(a \land b)$:

$$\dfrac{\neg A \lor \neg B \qquad \dfrac{[\neg A] \qquad \dfrac{\dfrac{[A \land B] \qquad \dfrac{[\neg A] \qquad \dfrac{[A \land B]}{A}\land E2}{\perp}\neg E}{A \land B \to \perp}\to I}{\neg(A \land B)}\neg I}{\neg A \to \neg(A \land B)}\to I \qquad \dfrac{[\neg B] \qquad \dfrac{\dfrac{[A \land B] \qquad \dfrac{[\neg B] \qquad \dfrac{[A \land B]}{B}\land E1}{\perp}\neg E}{A \land B \to \perp}\to I}{\neg(A \land B)}\neg I}{\neg B \to \neg(A \land B)}\to I}{\neg(A \land B)}\lor E$$

Part Four

DL1: $\neg(\neg a \lor \neg b) \vdash a$

Part Five

$\neg(a \land b) \vdash \neg a \lor \neg b$:

(Hint: you can use the previous problem, and a theorem from class.

$$\dfrac{\neg(\neg a \lor \neg b)}{a}DL1 \qquad \dfrac{\neg(\neg a \lor \neg b)}{b}DL2 \qquad \dfrac{\neg\neg a}{a}\neg\neg E$$