CS251: Homework #6

Due on December 3, 2019 at 2:00pm

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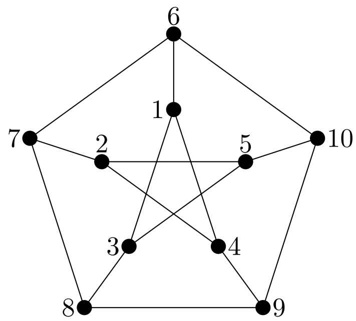
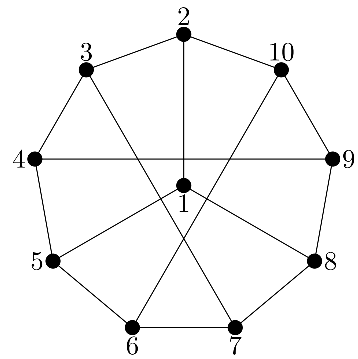
Problem 1

The table provides a function that defines an isomorphism between the graphs.

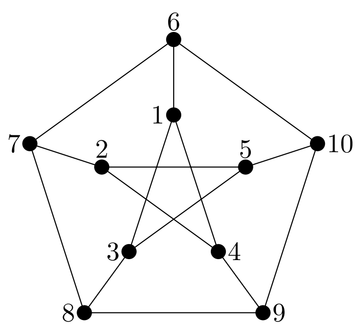




Left	**Right**
1	6
2	3
3	5
4	7
5	4
6	10
7	2
8	1
9	8
10	9

Problem 2

The table provides a 3 coloring of the graph using the colors a, b, and c.
The chromatic number cannot be 2 because C_5 is a subgraph.



Vertex	**Color**
1	a
2	c
3	c
4	b
5	b
6	b
7	a
8	b
9	c
10	a

Problem 3

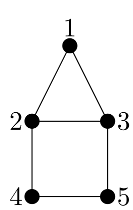
Given the prolog program

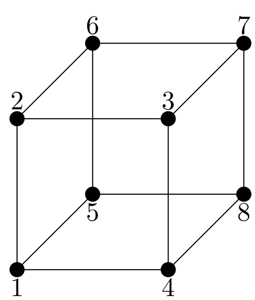
```
p :- a, b, c.
a :- b, d.
a :- b, e.
b :- x.
b :- y.
c.
y.
d :- x.
e.
```

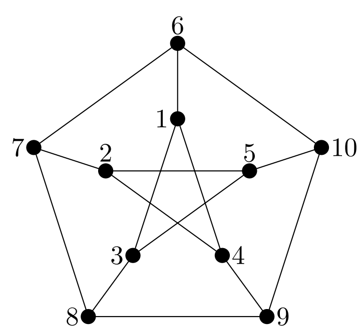
Use SLD resolution to evaluate the query

```
?- p.
GS = p
GS = a, b, c
GS = ?<(b, d), (b, e)>, b, c
GS = b, d, ?<b, e>, b, c
GS = ?<x, y>, d, ?<b, e>, b, c
GS = x, ?<y>, d, ?<b, e>, b, c
GS = fail, ?<y>, d, ?<b, e>, b, c
GS = ?<y>, d, ?<b, e>, b, c
GS = y, d, ?<b, e>, b, c
GS = d, ?<b, e>, b, c
GS = x, ?<b, e>, b, c
GS = fail, ?<b, e>, b, c
GS = b, ?<e>, b, c
GS = y, ?<e>, b, c
GS = ?<e>, b, c
GS = e, b, c
GS = b, c
GS = ?<x, y>, c
GS = x, ?<y>, c
GS = fail, ?<y>, c
GS = ?<y>, c
GS = y, c
GS = c
```

Problem 4



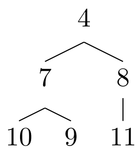
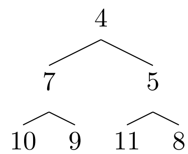
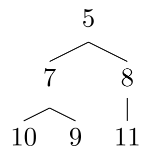
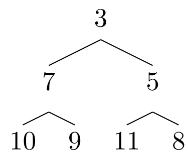
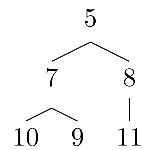




Problem 4 continued on next page...

Graph	**Edge List**	**Adjacency List**	**Adjacency Matrix**
left	$\begin{bmatrix} 1 \leftrightarrow 2 \\ 1 \leftrightarrow 3 \\ 2 \leftrightarrow 3 \\ 2 \leftrightarrow 4 \\ 3 \leftrightarrow 5 \\ 4 \leftrightarrow 5 \end{bmatrix}$	$\begin{bmatrix} 1 & \rightarrow & 2,3 \\ 2 & \rightarrow & 1,3,4 \\ 3 & \rightarrow & 1,2,5 \\ 4 & \rightarrow & 2,5 \\ 5 & \rightarrow & 3,4 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$
middle	$\begin{bmatrix} 1 \leftrightarrow 2 \\ 1 \leftrightarrow 4 \\ 1 \leftrightarrow 5 \\ 2 \leftrightarrow 3 \\ 2 \leftrightarrow 6 \\ 3 \leftrightarrow 4 \\ 3 \leftrightarrow 7 \\ 4 \leftrightarrow 8 \\ 5 \leftrightarrow 6 \\ 5 \leftrightarrow 8 \\ 6 \leftrightarrow 7 \\ 7 \leftrightarrow 8 \end{bmatrix}$	$\begin{bmatrix} 1 & \rightarrow & 2,4,5 \\ 2 & \rightarrow & 1,3,6 \\ 3 & \rightarrow & 2,4,7 \\ 4 & \rightarrow & 1,3,8 \\ 5 & \rightarrow & 1,6,8 \\ 6 & \rightarrow & 2,5,7 \\ 7 & \rightarrow & 3,6,8 \\ 8 & \rightarrow & 4,5,7 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$
right	$\begin{bmatrix} 1 \leftrightarrow 3 \\ 1 \leftrightarrow 4 \\ 1 \leftrightarrow 6 \\ 2 \leftrightarrow 4 \\ 2 \leftrightarrow 5 \\ 2 \leftrightarrow 7 \\ 3 \leftrightarrow 5 \\ 3 \leftrightarrow 8 \\ 4 \leftrightarrow 9 \\ 5 \leftrightarrow 10 \\ 6 \leftrightarrow 7 \\ 6 \leftrightarrow 10 \\ 7 \leftrightarrow 8 \\ 8 \leftrightarrow 9 \\ 9 \leftrightarrow 10 \end{bmatrix}$	$\begin{bmatrix} 1 & \rightarrow & 3,4,6 \\ 2 & \rightarrow & 4,5,7 \\ 3 & \rightarrow & 1,5,8 \\ 4 & \rightarrow & 1,2,9 \\ 5 & \rightarrow & 2,3,10 \\ 6 & \rightarrow & 1,7,10 \\ 7 & \rightarrow & 2,6,8 \\ 8 & \rightarrow & 3,7,9 \\ 9 & \rightarrow & 4,8,10 \\ 10 & \rightarrow & 5,6,9 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$

Problem 5

	Step 0	**Step 1**	**Step 2**	**Step 3**	**Step 4**
Operation	Start	Push(5)	Pop	Push(3)	Pop
Heap					
Array	$\begin{bmatrix}4\\7\\8\\10\\9\\11\end{bmatrix}$	$\begin{bmatrix}4\\7\\5\\10\\9\\11\\8\end{bmatrix}$	$\begin{bmatrix}5\\7\\8\\10\\9\\11\end{bmatrix}$	$\begin{bmatrix}3\\7\\5\\10\\9\\11\\8\end{bmatrix}$	$\begin{bmatrix}5\\7\\8\\10\\9\\11\end{bmatrix}$

Problem 6

Prove that if a graph contains no odd cycles, then it is bipartite.

Consider a graph G containing only even cycles with edges E and vertices V. Choose an arbitrary vertex $v \in V$. Create a partition of the vertices by first, constructing the shortest path between each vertex and v. The vertices with even length paths go into E and the rest go into O. Vertex v goes into E. Assume that this partition is not a bipartition, that $o_1, o_2 \in O$ and $(o_1, o_2) \in E$. Take the shortest paths from o_1 to v and o_2 to v, and call these Q and L. Q and L must merge at some vertex, v_1 (could be same as v). The paths from o_1 and o_2 to v_1 will be called Q_1 and L_1. Because Q and L are both odd length, and the same portion was removed from each to create L_1 and Q_1, these new paths must be the same parity. This results in the cycle: o_1, v_1, o_2, o_1, which is of odd length, hence contradicting the assumption. The partition must be a bipartition. The same can be done for two vertices in E.

Problem 7

Prove that if a graph has a Eulerian circuit, then all of the vertices have even degree.

Consider a graph G with a Eulerian circuit. Traverse the circuit and count how many times each vertex is visited. Every time a vertex is visited, one edge is used to enter the vertex and one edge is used to leave the vertex. As a result, every vertex except the starting vertex has a degree of $2n$, with n being the number of visits. The starting vertex has degree of twice the number of visits minus two (not counting start and finish as visits). $2x - 2$ is an even number.

Problem 8

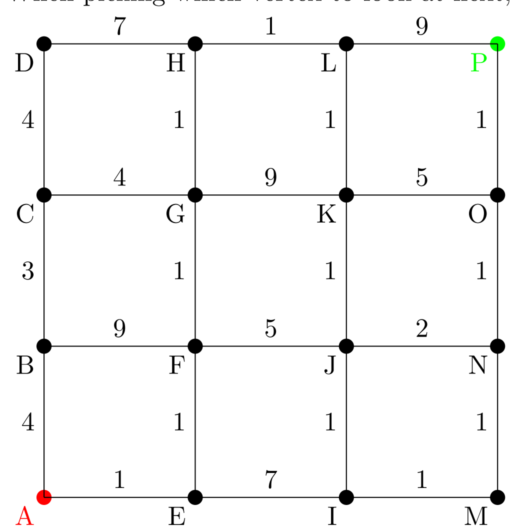
Prove Euler's formula for connected planar graphs, $V - E + F = 2$.

Problem 8 continued on next page. . .

Base case: Graph has only a single vertex. $1 - 0 + 1 = 2$
Inductive hypothesis: $V - E + F = 2$ works on all graphs with n edges.
Inductive case 1: The graph is a tree. Remove one of the terminal vertices (with only one edge) and its edge. In the process, you have removed one vertex and one edge. The new equation is $(v - 1) - (n - 1) + f = 2$ which is the same as $V - E + F = 2$.
Inductive case 2: The graph has a cycle. Remove an edge on a cycle. Because this edge was part of a cycle, its removal results in the combining of two faces resulting in the loss of a single face. The new equation is $v - (n - 1) + (f - 1) = 2$ which is the same as $V - E + F = 2$.

Problem 9

Run depth first search, breadth first search, and Dijkstra's on the following graph.
When picking which vertex to look at next, pick the next vertex alphabetically.



Breadth first results in the neighborhoods:
$A \rightarrow B, E \rightarrow C, F, I \rightarrow D, G, J, M \rightarrow H, K, N \rightarrow L, O \rightarrow P$
Depth first (alphabetically) visits the vertices:
$A \rightarrow B \rightarrow C \rightarrow D \rightarrow H \rightarrow G \rightarrow F \rightarrow E \rightarrow I \rightarrow J \rightarrow K \rightarrow L \rightarrow P$
Dijkstra's results in the path:
$A \rightarrow E \rightarrow F \rightarrow J \rightarrow N \rightarrow O \rightarrow P$
With a total length of 11.