CS251: Homework #4

Due on November 5, 2019 at 2:00pm

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Problem 1

Part One

$$\dfrac{x = y \qquad \dfrac{}{y = y}\ REFL}{y = x} = E2$$

Part Two

$$\dfrac{x = y \qquad y = z}{x = z} = E1$$

Part Three

$\forall y.y = 0 \lor (\exists z.y = z + 1)$,
$\forall x.x + 0 = x$,
$\forall x.x = 0 + x$,
$\forall xy.x + (y + 1) = (y + 1) + x$,
$\vdash \forall nm.n + m = m + n$

1.	$\forall y.y = 0 \lor (\exists z.y = z + 1)$	*Premise*
2.	$\forall x.x + 0 = x$	*Premise*
3.	$\forall x.x = 0 + x$	*Premise*
4.	$\forall xy.x + (y + 1) = (y + 1) + x$	*Premise*
5.	$[c], [d]$	*Assumptions*
6.	$[d = e + 1]$	*Assumption*
7.	$c + (e + 1) = (e + 1) + c$	$\forall E, 4$
8.	$c + (e + 1) = d + c$	$= E2, 6, 7$
9.	$c + d = d + c$	$= E1, 6, 8$
10.	$(d = e + 1) \to (c + d = d + c)$	$\to I, 6 - 10$
11.	$[d = 0]$	*assumption*
12.	$c + 0 = c$	$\forall E, 2$
13.	$c = 0 + c$	$\forall E, 3$
14.	$c = d + c$	$= E2, 11, 13$
15.	$c + 0 = d + c$	$= E1, 12, 14$
16.	$c + d = d + c$	$= E1, 11, 16$
17.	$(d = 0) \to (c + d = d + c)$	$\to I, 11 - 16$
18.	$d = 0 \lor = e + 1$	$\forall E, 1$
19.	$c + d = d + c$	$\lor E, 18, 17, 10$
20.	$\forall n, m.n + m = m + n$	$\forall I, 5 - 19$

Problem 2

$$\forall(U \in \mathbb{C}^{n\times n})(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).U(\langle a|b\rangle) = \langle a|b\rangle,$$
$$\forall(U \in \mathbb{C}^{n\times n})(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).U(ab) = U(a)U(b),$$
$$\forall a \in \mathbb{R}.\ (a \cdot a = a) \to (a = 0 \vee a = 1),$$
$$\forall(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).\langle a|b\rangle = ((\langle a| \otimes \langle 0|)(|b\rangle \otimes |0\rangle)),$$
$$\forall(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).((\langle a| \otimes \langle a|)(|b\rangle \otimes |b\rangle)) = \langle a|b\rangle \cdot \langle a|b\rangle,$$
$$\vdash$$
$$((U(|a\rangle \otimes |0\rangle) = |a\rangle \otimes |a\rangle) \wedge (U(|b\rangle \otimes |0\rangle) = |b\rangle \otimes |b\rangle)) \to \langle a|b\rangle = 0 \vee \langle a|b\rangle = 1$$

1.	$\forall(U \in \mathbb{C}^{n\times n})(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).U(\langle a\|b\rangle) = \langle a\|b\rangle$	*Premise*
2.	$\forall(U \in \mathbb{C}^{n\times n})(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).U(ab) = U(a)U(b)$	*Premise*
3.	$\forall a \in \mathbb{R}.\ (a \cdot a = a) \to (a = 0 \vee a = 1)$	*Premise*
4.	$\forall(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).\langle a\|b\rangle = ((\langle a\| \otimes \langle 0\|)(\|b\rangle \otimes \|0\rangle))$	*Premise*
5.	$\forall(a \in \mathbb{C}^n)(b \in \mathbb{C}^n).((\langle a\| \otimes \langle a\|)(\|b\rangle \otimes \|b\rangle)) = \langle a\|b\rangle \cdot \langle a\|b\rangle$	*Premise*
6.	$[((U(\|a\rangle \otimes \|0\rangle) = \|a\rangle \otimes \|a\rangle) \wedge (U(\|b\rangle \otimes \|0\rangle) = \|b\rangle \otimes \|b\rangle))]$	*Assumption*
7.	$U(\|a\rangle \otimes \|0\rangle) = \|a\rangle \otimes \|a\rangle$	$\wedge E1, 6$
8.	$U(\|a\rangle \otimes \|0\rangle) = U(\|a\rangle) \otimes U(\|0\rangle)$	$\forall E, 2$
9.	$U(\|a\rangle) \otimes U(\|0\rangle) = \|a\rangle \otimes \|a\rangle$	$= E1, 7, 8$
10.	$U(\|a\rangle) = \|a\rangle$	$\forall E, 1$
11.	$U(\|0\rangle) = \|0\rangle$	$\forall E, 1$
12.	$\|a\rangle \otimes \|0\rangle = \|a\rangle \otimes \|a\rangle$	$= E1 * 2, 9, 10, 11$
13.	$U(\|b\rangle \otimes \|0\rangle) = \|b\rangle \otimes \|b\rangle$	$\wedge E2, 6$
14.	$U(\|b\rangle \otimes \|0\rangle) = U(\|b\rangle) \otimes U(\|0\rangle)$	$\forall E, 2$
15.	$U(\|b\rangle) \otimes U(\|0\rangle) = \|b\rangle \otimes \|b\rangle$	$= E1, 13, 14$
16.	$U(\|b\rangle) = \|b\rangle$	$\forall E, 1$
17.	$U(\|0\rangle) = \|0\rangle$	$\forall E, 1$
18.	$\|b\rangle \otimes \|0\rangle = \|b\rangle \otimes \|b\rangle$	$= E1 * 2, 15, 16, 17$
19.	$\langle a\|b\rangle = ((\langle a\| \otimes \langle 0\|)(\|b\rangle \otimes \|0\rangle))$	$\forall E, 4$
20.	$\langle a\|b\rangle = ((\langle a\| \otimes \langle 0\|)(\|b\rangle \otimes \|b\rangle))$	$= E1, 18, 19$
21.	$((\langle a\| \otimes \langle a\|)(\|b\rangle \otimes \|b\rangle)) = \langle a\|b\rangle \cdot \langle a\|b\rangle$	$\forall E, 5$
22.	$((\langle a\| \otimes \langle 0\|)(\|b\rangle \otimes \|b\rangle)) = \langle a\|b\rangle \cdot \langle a\|b\rangle$	$= E1, 12, 21$
23.	$\langle a\|b\rangle \cdot \langle a\|b\rangle = \langle a\|b\rangle$	$= E1, 20, 22$
24.	$(\langle a\|b\rangle \cdot \langle a\|b\rangle = \langle a\|b\rangle) \to (\langle a\|b\rangle = 0 \vee \langle a\|b\rangle = 1)$	$\forall E, 3$
25.	$\langle a\|b\rangle = 0 \vee \langle a\|b\rangle = 1$	$\to E, 23, 24$
26.	$((U(\|a\rangle \otimes \|0\rangle) = \|a\rangle \otimes \|a\rangle) \wedge (U(\|b\rangle \otimes \|0\rangle) = \|b\rangle \otimes \|b\rangle)) \to \langle a\|b\rangle = 0 \vee \langle a\|b\rangle = 1$	$\to I, 6 - 25$