

CS251: Homework #2

Due on October 15, 2019 at 2:00pm

Steven Libby Section A

Austen Nelson

Problem 1

last week we showed that nand \odot is a universal operator. That is, we can write all operators in terms of nand. Show that \rightarrow is a universal operator by writing \neg , \wedge , and \vee with only \rightarrow .

- $A \rightarrow \perp = \neg A$
- $(A \rightarrow \perp) \rightarrow B = A \wedge B$
- $(A \rightarrow B) \rightarrow B = A \vee B$

Problem 2

Convert the following to CNF

Part One

$$\begin{array}{l} (c \wedge a) \vee (b \wedge c) \\ (c \vee b) \wedge (c \vee c) \wedge (a \vee b) \wedge (a \vee c) \end{array} \quad FOIL_{\vee}$$

Part Two

$$\begin{array}{l} (a \wedge \neg a) \vee (b \wedge \neg b) \\ \perp \vee \perp \\ \perp \end{array} \quad \begin{array}{l} CTR * 2 \\ Item_{\vee} \end{array}$$

Part Three

$$\begin{array}{l} a \rightarrow (b \equiv c) \\ a \rightarrow ((b \rightarrow c) \wedge (c \rightarrow b)) \\ \neg a \vee ((\neg b \vee c) \wedge (\neg c \vee b)) \\ (\neg a \vee \neg b \vee c) \wedge (\neg a \vee \neg c \vee b) \end{array} \quad \begin{array}{l} Def_{\equiv} \\ Imp * 3 \\ Dis_{\vee} \end{array}$$

Part Four

$$\begin{array}{l} (a \rightarrow b) \wedge (b \rightarrow c) \\ (\neg a \vee b) \wedge (\neg b \vee c) \end{array} \quad Imp * 2$$

Part Five

$$\begin{array}{l} \neg(a \vee b) \\ \neg a \wedge \neg b \end{array} \quad DM_{\vee}$$

Part Six

$$\begin{aligned}
& (a \equiv b) \equiv c \\
& (((a \rightarrow b) \wedge (b \rightarrow a)) \rightarrow c) \wedge (c \rightarrow ((a \rightarrow b) \wedge (b \rightarrow a))) & Def_{\equiv} \\
& (\neg((\neg a \vee b) \wedge (\neg b \vee a)) \vee c) \wedge (\neg c \vee ((\neg a \vee b) \wedge (\neg b \vee a))) & Imp * 6 \\
& ((a \wedge \neg b) \vee (b \wedge \neg a) \vee c) \wedge (\neg c \vee ((\neg a \vee b) \wedge (\neg b \vee a))) & DM_{\wedge}, DM_{\vee} * 2 \\
& (((a \vee b) \wedge (a \vee \neg a) \wedge (\neg b \vee b) \wedge (\neg b \vee \neg a)) \vee c) \wedge (\neg c \vee ((\neg a \vee b) \wedge (\neg b \vee a))) & FOIL_{\vee} \\
& ((c \vee a \vee b) \wedge (c \vee a \vee \neg a) \wedge (c \vee \neg b \vee b) \wedge (c \vee \neg b \vee \neg a)) \wedge (\neg c \vee \neg a \vee b) \wedge (\neg c \vee \neg b \vee a) & Dis_{\vee} * 5 \\
& (c \vee a \vee b) \wedge (c \vee \neg b \vee \neg a) \wedge (\neg c \vee \neg a \vee b) \wedge (\neg c \vee \neg b \vee a) & LEM * 2, Anul_{\vee} * 2
\end{aligned}$$

Problem 3

Prove the following:

Part One

$a \vee b \vdash b \vee a$:

$$\frac{A \vee B \quad \frac{[A] \quad \frac{B \vee A}{\rightarrow I} \vee I2}{A \rightarrow B \vee A} \rightarrow I \quad \frac{[B] \quad \frac{B \vee A}{\rightarrow I} \vee I1}{B \rightarrow B \vee A} \rightarrow I}{B \vee A} \vee E$$

Part Two

$(a \vee b), \neg b \vdash a$:

$$\frac{A \vee B \quad \frac{[A] \quad [A]}{A \rightarrow A} \rightarrow I \quad \frac{[B] \quad \frac{\frac{\perp}{A} \perp E}{\neg B} \neg E}{B \rightarrow A} \rightarrow I}{A} \vee E$$

Part Three

$\neg a \vee \neg b \vdash \neg(a \wedge b)$:

$$\frac{\neg A \vee \neg B \quad \frac{[A \wedge B] \quad \frac{[A] \quad \frac{[A \wedge B]}{A} \wedge E2}{\perp} \perp}{A \wedge B \rightarrow \perp} \rightarrow I \quad \frac{[A \wedge B] \quad \frac{[B] \quad \frac{[A \wedge B]}{B} \wedge E1}{\perp} \perp}{A \wedge B \rightarrow \perp} \rightarrow I}{\neg(A \wedge B)} \neg I}{\neg A \rightarrow \neg(A \wedge B)} \rightarrow I \quad \frac{[A \wedge B] \quad \frac{[B] \quad \frac{[A \wedge B]}{B} \wedge E1}{\perp} \perp}{A \wedge B \rightarrow \perp} \rightarrow I}{\neg B \rightarrow \neg(A \wedge B)} \rightarrow I}{\neg(A \wedge B)} \vee E$$

Part Four

DL1: $\neg(\neg a \vee \neg b) \vdash a$

$$\begin{array}{c}
\frac{\frac{\frac{A \vee \neg A}{LEM} \quad \frac{\frac{[A] \quad [A]}{A \rightarrow A} \rightarrow I \quad \frac{[\neg A]}{\neg A \rightarrow A} \vee E}{A} \rightarrow I}{A \vee \neg A} \vee E \quad \frac{\frac{[\neg A \vee \neg B]}{\neg A \vee \neg B} \vee I2 \quad \frac{\frac{[\neg A \vee \neg B]}{\neg A \vee \neg B \rightarrow A} \rightarrow E \quad \frac{\frac{[\neg A \vee \neg B]}{\neg(\neg A \vee \neg B)} \neg E \quad \frac{\perp}{A} \perp E}{\neg A \vee \neg B \rightarrow A} \rightarrow E}{A} \rightarrow I
\end{array}$$

Part Five

$\neg(a \wedge b) \vdash \neg a \vee \neg b$:

$$\begin{array}{c}
\frac{\frac{\frac{[A] \quad [B]}{A \wedge B} \wedge I \quad \frac{\neg(A \wedge B)}{\neg E} \quad \frac{\perp}{\neg A \vee \neg B} \perp E}{\frac{[B]}{B \rightarrow \neg A \vee \neg B} \rightarrow I} \rightarrow I \quad \frac{\frac{[\neg B]}{\neg A \vee \neg B} \vee I1 \quad \frac{[\neg B]}{\neg B \rightarrow \neg A \vee \neg B} \rightarrow I}{\neg A \vee \neg B} \vee E \\
\frac{[A] \quad \frac{B \vee \neg B}{LEM} \quad \frac{B \rightarrow \neg A \vee \neg B}{\neg A \vee \neg B} \rightarrow I}{A \rightarrow \neg A \vee \neg B} \rightarrow I \\
\vdots \\
\vdots \\
\vdots \\
\frac{\frac{A \vee \neg A}{LEM} \quad \frac{A \rightarrow \neg A \vee \neg B}{\neg A \vee \neg B} \rightarrow I \quad \frac{\frac{[\neg A]}{\neg A \vee \neg B} \vee I2 \quad \frac{[\neg A]}{\neg A \rightarrow \neg A \vee \neg B} \rightarrow I}{\neg A \vee \neg B} \vee E
\end{array}$$