CS251 homework 3

name:__________________________

Due: 10/29/19

1. Convert the following sentences into FOPC, remember to define sensible predicates and domains.

   (a) $\mathbb{R}$ is the set of real numbers

   For any two real numbers, there is a real number between them.

   (b) $a|b$ is read as "a divides b". (meaning $\frac{b}{a}$ is an integer)
   $\mathbb{N}$ is the set of natural numbers $\{1, 2, 3 \ldots\}$

   A natural number is prime if, and only if, it has no divisors other than 1 and itself.

   (c) A compiler is a function between two languages $C : L_1 \rightarrow L_2$
   A Language is Turing Complete if it can encode Turing Machines $L = TM$

   There is a compiler between any two Turing Complete languages.

(d) the set of closed balls (in n dimensional space) is B^n
a fixed point is a point that doesn't change after applying a function.

Brouwer fixed-point theorem:
Any continuous function on a closed ball has at least one fixed point.

(e) A Graph is a set of vertices, V, and a set of edges between vertices $E \subset V \times V$.
A Graph $G = V \times E$ is n-colorable if we can pick n colors, and assign every vertex a color, such that no two vertices connected by edges have the same color.

every planer graph is 4-colorable.

2. Restate the following theorems in English.
Try to find the most natural way to write them.
Example:
$\forall x \in \mathbb{N}.\exists y \mathbb{N}.x < y$
could be translated as "for all x in $\mathbb{N}$, there is a y in $\mathbb{N}$ such that, x is less than y."
However, it sounds much more natural as "There is no largest natural number."
   (a) $(\forall n, a, b \in \mathbb{N}.n|a \wedge n|b \rightarrow \exists x, y \in \mathbb{N}.n|ax + by$

   (b) $(\forall n \in \mathbb{N}, n > 2 \rightarrow (\nexists a, b, c \in \mathbb{N}.a^n + b^n = c^n)$

(c) $\nexists p, q \in \mathbb{N} \left(\frac{p}{q}\right)^2 \neq 2$

(d) $\exists x \in \mathbb{R}. x^2 + 1 = 0$

3. Determine if the following propositions are well formed. If not, give a reason why. Assume P is a predicate with arity 3, Q is a predicate with arity 2, f is a function with arity 2, and g is a function with arity 1. Also assume our only domain is $\mathbb{N}$. (so $=, <, \leq$ are possible predicates and $+, *$ are possible functions)

   (a) $\forall x. P(f(x, x), g(x), x) = x$

   (b) $\forall x. (\exists y. Q(x, y) \wedge P(x, y, x)) \to f(x, y) = g(x)$

   (c) $\nexists y. y = f(y, y)$

(d) $\forall x.\forall y.\forall z.P(x, y, z) \rightarrow Q(x, y, z)$.

(e) $\forall z.1 + 1 = 2$

4. Reduce the following propositions to their shortest form.

(a) $\nexists x \exists y.(\neg x \vee \neg y)$

(b) $\forall q.((\forall x.x) \wedge (\forall y.y) \rightarrow (\forall z.q))$

(c) $(\forall x.x) \to (\exists y.\neg y)$

5. Convert the following into PNF

(a) $(\forall a \exists b.a \to b) \land (\forall b \exists a.a \to b)$

(b) $\forall a.P(a) \to (\exists a.R(a, a))$

(c) $\forall x.(\forall y.x \to y) \land (\forall y.y \to x)$

(d) $\forall x.(\forall y.x \to y) \to (\forall z.z)$

(e) $\forall x.\neg(\forall y.y) \to \neg(\forall z.x \land z)$

6. Get the predicate logic package from `https://github.com/slibby05/pred` Implement the `sub` method for all of the classes in AST.