Equivalences

$$
\begin{array}{rcll}
a \wedge \top & = & a & Id_{\wedge} \\
a \vee \bot & = & a & Id_{\vee} \\
a \wedge b & = & b \wedge a & Com_{\wedge} \\
a \vee b & = & b \vee a & Com_{\vee} \\
a \wedge (b \vee c) & = & (a \wedge b) \vee (a \wedge c) & Dis_{\wedge} \\
a \vee (b \wedge c) & = & (a \vee b) \wedge (a \vee c) & Dis_{\vee} \\
a \wedge a & = & a & Item_{\wedge} \\
a \vee a & = & a & Item_{\vee} \\
a \vee \top & = & \top & Anul_{\vee} \\
a \wedge \bot & = & \bot & Anul_{\wedge} \\
a & = & \neg\neg a & \neg\neg \\
\neg(a \wedge b) & = & \neg a \vee \neg b & DM_{\wedge} \\
\neg(a \vee b) & = & \neg a \wedge \neg b & DM_{\vee} \\
a \to b & = & \neg a \vee b & Imp \\
a \vee \neg a & = & \top & LEM \\
a \wedge \neg a & = & \bot & CTR \\
(\forall x.a) \wedge b & = & \forall x.(a \wedge b) & \forall\wedge \\
(\exists x.a) \wedge b & = & \exists x.(a \wedge b) & \exists\wedge \\
(\forall x.a) \vee b & = & \forall x.(a \vee b) & \forall\vee \\
(\exists x.a) \vee b & = & \exists x.(a \vee b) & \exists\vee \\
a \to (\forall x.b) & = & \forall x.(a \to b) & Cov_{\forall} \\
a \to (\exists x.b) & = & \exists x.(a \to b) & Cov_{\exists} \\
(\forall x.a) \to b & = & \exists x.(a \to b) & Cont_{\forall} \\
(\exists x.a) \to b & = & \forall x.(a \to b) & Cont_{\exists} \\
\forall xy.a & = & \forall yx.a & Com_{\forall} \\
\exists xy.a & = & \exists yx.a & Com_{\exists} \\
\neg(\forall x.a) & = & (\exists x.\neg a) & DM_{\forall} \\
\neg(\exists x.a) & = & (\forall x.\neg a) & DM_{\exists} \\
\forall x.a & = & a \text{ if } x \notin a & Item_{\forall} \\
\exists x.a & = & a \text{ if } x \notin a & Item_{\exists} \\
\forall x.a & = & \forall z.a[x \mapsto z] & Rep_{\forall} \\
\exists x.a & = & \exists z.a[x \mapsto z] & Rep_{\exists}
\end{array}
$$

Inference Rules

$$\frac{A \wedge B}{A}\wedge E1 \qquad \frac{A \wedge B}{B}\wedge E2 \qquad \frac{A \quad B}{A \wedge B}\wedge I$$

$$\frac{A}{A \vee B}\vee I1 \qquad \frac{B}{A \vee B}\vee I2 \qquad \frac{A \vee B \quad A \to C \quad B \to C}{C}\vee E$$

$$\frac{\begin{matrix}[A] & \begin{matrix}[A]\\ \vdots \\ B\end{matrix}\end{matrix}}{A \to B}\to I \qquad \frac{A \quad A \to B}{B}\to E$$

$$\frac{A \to \bot}{\neg A}\neg I \qquad \frac{A \quad \neg A}{\bot}\neg E$$

$$\frac{}{\top}\top I \qquad \frac{\bot}{A}\bot E \qquad \frac{}{A \vee \neg A}LEM$$

$$\frac{\forall x.A(x)}{A(c)}\forall E \qquad \frac{A(c)}{\exists x.A(x)}\exists I$$

$$\frac{\begin{matrix}[c] & \begin{matrix}[c]\\ \vdots \\ A(c)\end{matrix}\end{matrix}}{\forall x.A(x)}\forall I$$

If $c \notin B$

$$\frac{\exists x.A(x) \quad A(c) \to B}{B}\exists E$$

$$\frac{\forall x.P(x)}{\forall z.P(z)}\forall[x \mapsto z] \qquad \frac{\exists x.P(x)}{\exists z.P(z)}\exists[x \mapsto z]$$

$$\frac{}{x = x}Refl \qquad \frac{a = b \quad P(a)}{P(b)} = E1 \qquad \frac{a = b \quad P(b)}{P(a)} = E2$$