CS251: Homework #1

Due on October 8, 2019 at 2:00pm

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Problem 1

Define variables, and write the following sentences as logical statements.

Part One
Sentence: If it's not cloudy, then it's not raining.
Variables:

- A — It is cloudy.
- B — It is raining.

Statement: $\neg A \to \neg B$

Part Two
Sentence: If we won the big game, then either we scored more points, or the other team didn't show up.
Variables:

- A — We won the big game.
- B — We scored more points.
- C — The other team showed up.

Statement: $A \to (B \vee \neg C)$

Part Three
Sentence: This is a sentence.
Variables:

- A — This is a sentence.

Statement: A

Part Four
Sentence: If you don't study for tests, then you won't pass the class.
Variables:

- A — You study for tests.
- B — You pass the class.

Statement: $\neg A \to \neg B$

Part Five
Sentence: A graph is planer if it contains neither a minor of $K_{3,3}$ nor K_5.
Variables:

- A — A graph contains a minor of $K_{3,3}$.
- B — A graph contains a minor of K_5.
- C — A graph is planer.

Statement: $(\neg A \wedge \neg B) \to C$

Problem 2

Draw truth tables for the following formulas.

$a \oplus b$

a	b	$a \oplus b$
1	1	0
1	0	1
0	1	1
0	0	0

$\neg(\neg a)$

a	$\neg(\neg a)$
1	1
0	0

$\neg b \to \neg a$

a	b	$\neg a \to \neg b$
1	1	1
1	0	1
0	1	0
0	0	1

$\neg a \wedge \neg b$

a	b	$\neg a \wedge \neg b$
1	1	0
1	0	0
0	1	0
0	0	1

$a \leftrightarrow (b \leftrightarrow c)$

a	b	c	$a \leftrightarrow (b \leftrightarrow c)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

$(a \vee c) \wedge (b \vee c)$

a	b	c	$(a \vee c) \wedge (b \vee c)$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

Problem 3

Reduce the following to the shortest form.
Determine if it's satisfiable, a tautology, or neither.

Part One
$(a \wedge \neg b) \vee \neg(\neg a \vee b)$: satisfiable

$(a \wedge \neg b) \vee \neg(\neg a \vee b)$	
$(a \wedge \neg b) \vee (a \wedge \neg b)$	$DM_\vee$
$a \wedge \neg b$	$Item_\wedge$
$\neg\neg(a \wedge \neg b)$	$\neg\neg$
$\neg(\neg a \vee b)$	$DM_\wedge$
$\neg(a \to b)$	Imp

Part Two
$a \wedge b \equiv \neg(\neg a \wedge \neg b)$: satisfiable

$a \wedge b \equiv \neg(\neg a \wedge \neg b)$	
$((a \wedge b) \to \neg(\neg a \wedge \neg b)) \wedge (\neg(\neg a \wedge \neg b) \to (a \wedge b))$	$Def \equiv$
$((a \wedge b) \to (a \vee b)) \wedge (\neg(\neg a \wedge \neg b) \to (a \wedge b))$	$DM_\wedge$
$(\neg(a \wedge b) \vee (a \vee b)) \wedge (\neg(\neg a \wedge \neg b) \to (a \wedge b))$	Imp
$((\neg a \vee \neg b) \vee (a \vee b)) \wedge (\neg(\neg a \wedge \neg b) \to (a \wedge b))$	$DM_\wedge$
$(\neg a \vee a \vee \neg b \vee b) \wedge (\neg(\neg a \wedge \neg b) \to (a \wedge b))$	$Com_\vee$
$(T \vee T) \wedge (\neg(\neg a \wedge \neg b) \to (a \wedge b))$	$LEM * 2$
$\neg(\neg a \wedge \neg b) \to (a \wedge b)$	$Id_\wedge$
$\neg\neg(\neg a \wedge \neg b) \vee (a \wedge b)$	Imp
$(\neg a \wedge \neg b) \vee (a \wedge b)$	$\neg\neg$
$((\neg a \wedge \neg b) \vee a) \wedge ((\neg a \wedge \neg b) \vee b)$	$Dis_\vee$
$(a \vee (\neg a \wedge \neg b)) \wedge (b \vee (\neg a \wedge \neg b))$	$Com_\vee * 2$
$((a \vee \neg a) \wedge (a \vee \neg b)) \wedge (b \vee (\neg a \wedge \neg b))$	$Dis_\vee$
$(T \wedge (a \vee \neg b)) \wedge (b \vee (\neg a \wedge \neg b))$	LEM
$(a \vee \neg b) \wedge (b \vee (\neg a \wedge \neg b))$	$Id_\wedge$
$(\neg b \vee a) \wedge (b \vee (\neg a \wedge \neg b))$	$Com_\vee$
$(b \to a) \wedge (b \vee (\neg a \wedge \neg b))$	Imp
$(b \to a) \wedge ((b \vee \neg a) \wedge (b \vee \neg b))$	$Dis_\vee$
$(b \to a) \wedge (b \vee \neg a) \wedge T$	LEM
$(b \to a) \wedge (b \vee \neg a)$	$Id_\wedge$
$(b \to a) \wedge (\neg a \vee b)$	$Com_\vee$
$(b \to a) \wedge (a \to b)$	Imp
$a \equiv b$	$Def \equiv$

Problem 3 continued on next page. . .

Part Three

$a \wedge b \equiv \neg(\neg a \vee \neg b)$: tautology

$a \wedge b \equiv \neg(\neg a \vee \neg b)$	
$((a \wedge b) \rightarrow \neg(\neg a \vee \neg b)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$Def \equiv$
$(\neg(a \wedge b) \vee \neg(\neg a \vee \neg b)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	Imp
$((\neg a \vee \neg b) \vee \neg(\neg a \vee \neg b)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$DM_\wedge$
$((\neg a \vee \neg b) \vee (a \wedge b)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$DM_\vee$
$(((\neg a \vee \neg b) \vee a) \wedge ((\neg a \vee \neg b) \vee b)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$Dis_\vee$
$((\neg a \vee a \vee \neg b) \wedge (\neg a \vee \neg b \vee b)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$Com_\vee$
$((T \vee \neg b) \wedge (\neg a \vee T)) \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$LEM * 2$
$T \wedge T \wedge (\neg(\neg a \vee \neg b) \rightarrow (a \wedge b))$	$Anul_\vee * 2$
$\neg(\neg a \vee \neg b) \rightarrow (a \wedge b)$	$Id_\wedge$
$(a \wedge b) \rightarrow (a \wedge b)$	$DM_\vee$
T	$Def \rightarrow$

Part Four

$a \wedge (b \vee c) \rightarrow a \wedge (b \wedge c)$: satisfiable

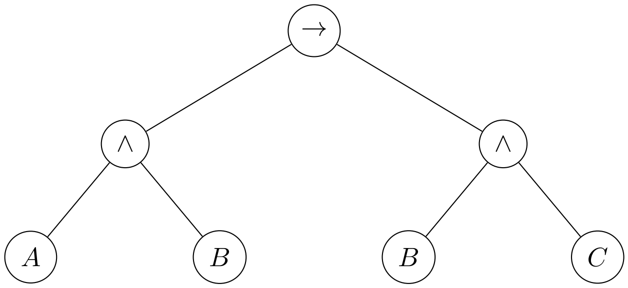
On the left hand side of the implication we have $a \wedge E$, where E is some expression. Because false always implies true, and when a is false $a \wedge E = F$ ($Anul_\wedge$), the expression $a \wedge (b \vee c) \rightarrow a \wedge (b \wedge c)$ will always be true when a is false. Using this knowledge we can reduce the expression to $\neg a \vee (T \wedge (b \vee c) \rightarrow T \wedge (b \wedge c))$.

$\neg a \vee (T \wedge (b \vee c) \rightarrow T \wedge (b \wedge c))$	
$\neg a \vee (b \vee c \rightarrow b \wedge c)$	$Id_\wedge * 2$
$\neg a \vee (\neg(b \vee c) \vee (b \wedge c))$	Imp
$\neg a \vee ((\neg b \wedge \neg c) \vee (b \wedge c))$	$DM_\vee$
$\neg a \vee (((\neg b \wedge \neg c) \vee b) \wedge ((\neg b \wedge \neg c) \vee c))$	$Dis_\vee$
$\neg a \vee ((b \vee (\neg b \wedge \neg c)) \wedge (c \vee (\neg b \wedge \neg c)))$	$Com_\vee * 2$
$\neg a \vee (((b \vee \neg b) \wedge (b \vee \neg c)) \wedge ((c \vee \neg b) \wedge (c \vee \neg c)))$	$Dis_\vee * 2$
$\neg a \vee ((T \wedge (b \vee \neg c)) \wedge ((c \vee \neg b) \wedge T))$	$LEM * 2$
$\neg a \vee ((b \vee \neg c) \wedge (c \vee \neg b))$	$Id_\wedge * 2$
$\neg a \vee ((\neg c \vee b) \wedge (\neg b \vee c))$	$Com_\vee * 2$
$\neg a \vee ((c \rightarrow b) \wedge (b \rightarrow c))$	$Imp * 2$
$\neg a \vee (b \equiv c)$	$Def \equiv$

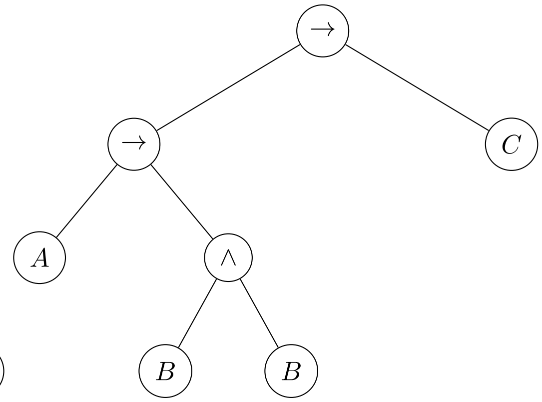
Problem 4

Draw the following abstract syntax trees:

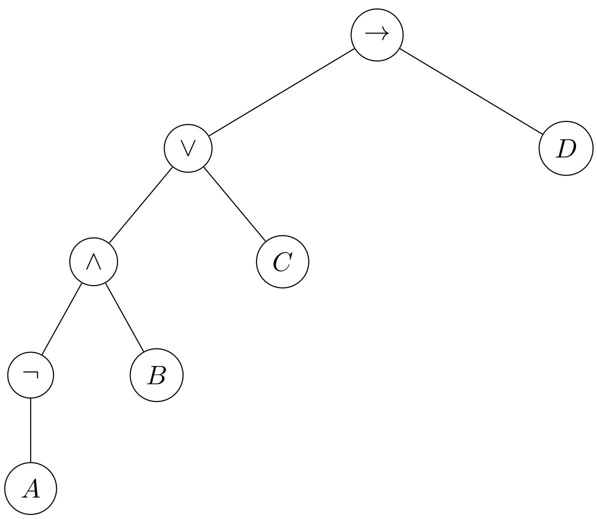
$A \land B \to B \land C$



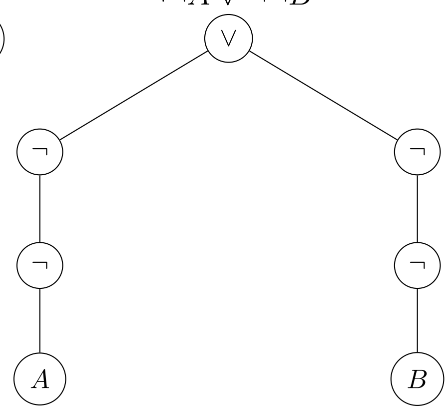
$A \to B \land B \to C$



$\neg A \land B \lor C \to D$



$\neg\neg A \lor \neg\neg B$



$(A \to B) \lor (B \to A)$