Optimizing Information Propagation in Social Network

OIE 559 Advanced Prescriptive Analytics

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Why?

- A lot of information
- Let say that we know the truth that we are living in the simulation; who should spread the news so that most people are *aware*?

Why not just look into the cascade or influence maximization problem?

- Traditionally, we look into the spread by looking at the behavior of vertices.
- Suppose we have a friend who has a different political view. They don't need to share the news from their side!
    - They can also share the news that opposes them and criticize that too!
    - "We are living in the simulation" I agree.
    - Who is dumb enough to trust this news "Scientist confirmed! We are not living in the simulation".
- In this case, looking at the side the vertices are on is not enough.

If that is the case, why don't we say it is "independent"?

- Although the flow of information now seems to be independent, the decision of the spreading is depending on the side they are in!
  - That is, there will be more tendency to spread the information they are sided on.

Assumptions

- Each person in the social network has their own probability of spreading the information and adopting the information on their own.
  - That is, even if you don't join the force to break the simulation (or even if you join the enemy team), you can still spread the information about fighting against it.
  - However, the probability of spreading information will depend on which side they are in given the information.
- Unlike game theory problem, enemy will not just choose the best candidate for this problem. Instead, they will choose the node randomly.

Problem Statement

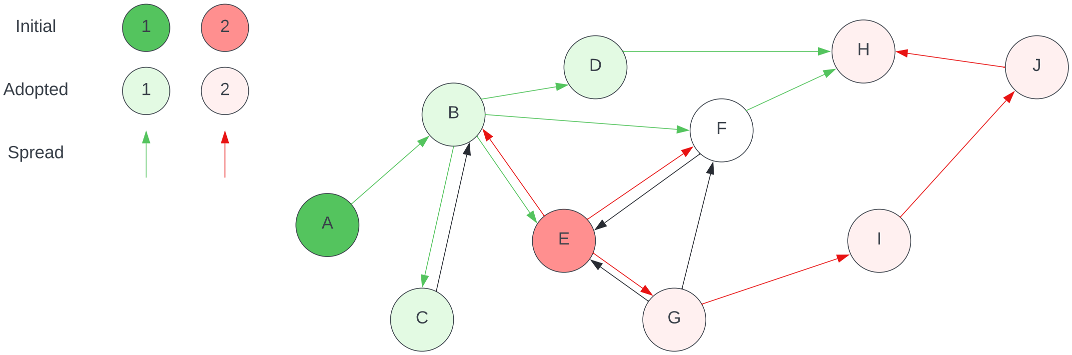

Figure: Problems

Exploration

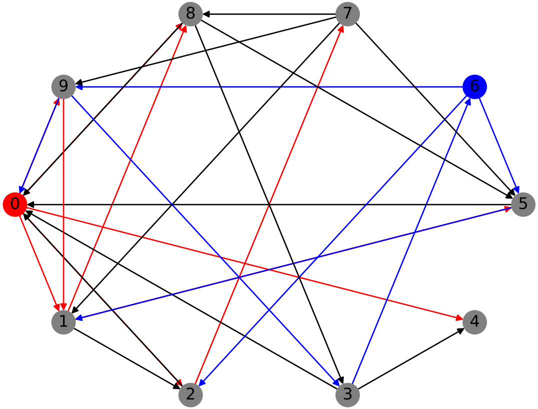

Figure: Example run with $ER(10, 0.3)$ and $p_P = 0.5$. Node 0 is picked as our initial propagator. In this case $\mathbb{E}_G\left[|E_A|\,|v_A = 0, v_B = 6\right] = 8$ and $\mathbb{E}_G\left[|E_B|\,|v_B = 6, v_A = 0\right] = 7$

Exploration

- This game is not fair!
  - That means, for some graph, uniformly random strategy will result in a lose.
  - In 500 random trials (of 100 graphs, i.e., 50000 runs) of size 10 with $p_P = 0.5$ will favors on one player (88%) compared to the tie game (12%).

Analysis of the Game

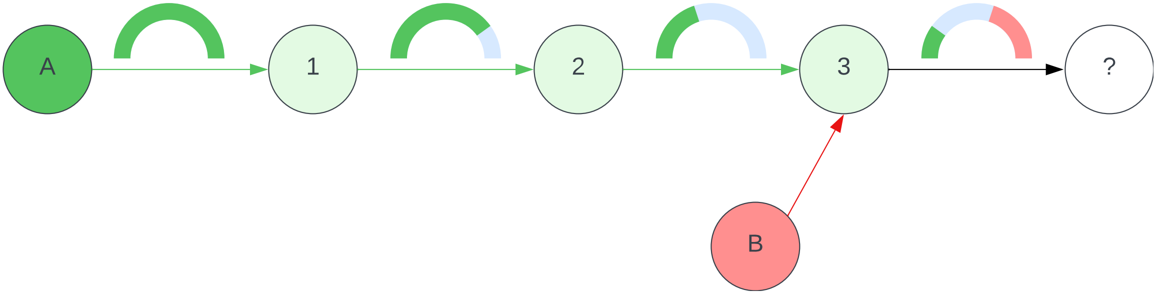

Figure: Toy example

- The information will diffused. i.e., the longer the chain is, the less chance of propagating the information I_A

Naive Approach

- We want to maximize the propagation probability of the furthermost node that is connected to the source.

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- We want to maximize the propagation probability of the furthermost node that is connected to the source.
- **Decision variable**: an initial node $v^{\text{init}} \in V_A$

Naive Formulation

This method is the simplified version of the stochastic programming proposed in (Wu *et al.*, 2018)

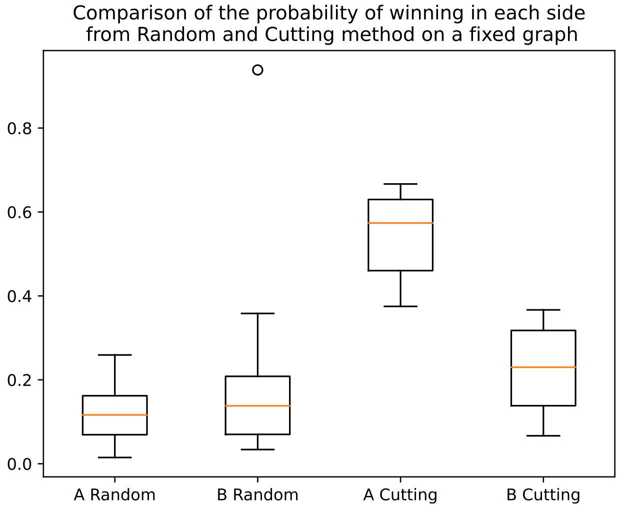
- Consider the graph G with initial node s. We initialize the flow $f = 1$ to the network from x. Our goal is to maximize the flow. This is the max-flow problem.

$$
\begin{aligned}
\max \quad & v \\
\text{s.t.} \quad & \sum_{j:(s,j)\in E} x_{sj} \leq v \\
& \sum_{i:(i,t)\in E} x_{it} \leq v \\
& \sum_{(i,j)\in E} x_{ij} = \sum_{(j,i)\in E} x_{ji} \\
& x_{ij} > 0
\end{aligned}
$$

(Naive) Competitive Max flow Formulation

- Given the graph G, we run the sets of simulations to see the probability distribution for each edge $e_{ij} \in E$. Then, we run the same max flow algorithm with edges with a high probability that will adopt the enemy's information removed.
  - This can be done with binary variables to prune flow.
  - The bounding for a probability distribution is arbitrary.
    - In this case, we consider the bounding as $\mathbb{P}[e_{ij} = I_A | G] > \mathbb{P}[e_{ij} = I_B | G]$

Simulations



Future Works

- Prove the optimality (or suboptimality).
  - This heuristic model is sub-optimal. i.e., Not guaranteed to provide an optimal solution
- Optimize the model computational complexity

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