Linear Algebra **(Lecture 1: Linear System)**

Problem Set #1

Instructor: Aukkawut Ammartayakun

Problem 1: Solving a system of linear equations (and its substitution) (2 points)

Solve for x, y, z from the system of non-linear equation. Parameterize your answer if needed.

$$\begin{aligned} 3\cos(x) + 4\sin(y) - 5\sin(z) &= 0 \\ \cos(x) - 2\sin(y) &= -1 \end{aligned}$$

Problem 2: Kirchhoff's rule (2 points)

In physics class, we have learned about "Kirchhoff's rule" on the closed circuit that:

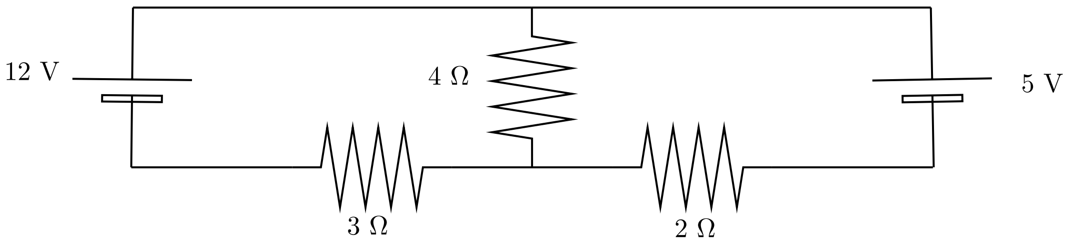
- The algebraic sum of currents in a network of conductors meeting at a point is zero
- The directed sum of the potential differences (voltages) around any closed loop is zero.

We also learned about Ohm's law that

$$V = IR$$

where V is a voltage, I is a current, and R is a resistance.

(a) Calculate the current at each resistance in this circuit



(Solution: $I_{3\Omega} = 2A, I_{4\Omega} = 1.5A, I_{2\Omega} = 0.5A$)

Linear Algebra **(Lecture 2: Vector Space)**

Problem Set #2

Instructor: Aukkawut Ammartayakun

Problem 1: We need more space (2 points)

Decide if each is a subspace of the vector space of real-valued functions of one real variable.
(a) The even functions $F_{odd} = \{f : \mathbb{R} \to \mathbb{R} | f(-x) = f(x), \forall x \in \mathbb{R}\}$ for example, $g(x) = x^2$ is an even function.
(a) The odd functions $F_{even} = \{f : \mathbb{R} \to \mathbb{R} | f(-x) = -f(x), \forall x \in \mathbb{R}\}$ for example, $h(x) = x^3$ is a odd function.

Problem 2: Symmetric? (2 points)

A square matrix is *symmetric* if for all indices i and j, entry i, j equals entry j, i.
(a) Is a set of symmetric matrices with the normal operation set $(+, \cdot)$ over the field $\mathbb{F} = \mathbb{R}$ a vector space? If it is a vector space, find the basis of a vector space of a symmetric $n \times n$ matrices. If not, indicate the reason why.

Linear Algebra **(Lecture 3: Mapping)**

Problem Set #3

Instructor: Aukkawut Ammartayakun

Problem 1: We are lost in the Calculus woods (4 points)

Given a polynomial space $\mathcal{P}_n = \{[a_0 \ a_1 \ ... \ a_n] \in \mathbb{R}^n | P(x) = [a_0 \ a_1 \ ... \ a_n] \cdot [x^0 \ x^1 \ ... \ x^n]\}$ with the basis $B = \langle 1, x, x^2, ..., x^n \rangle$. Create the mapping $d/dx : \mathcal{P}_n \to \mathcal{P}_n$ which is basically a derivative of a input polynomial using the same basis as an input. Then answer the following questions.

(a) What is a representation matrix of mapping d/dx?
(b) What is the rank of d/dx?
(c) What is the "real" range space dimension of the mapping d/dx?
(d) Is this mapping has an inverse? Why/Why not?