English Auction with Option to Buy: Study of The Risk in the Auction of WPI Students

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Outline

1. English Auction with Option to Buy?
2. Related Experiments
3. Theoretical Modeling
4. Experiment Design

English Auction with Option to Buy?

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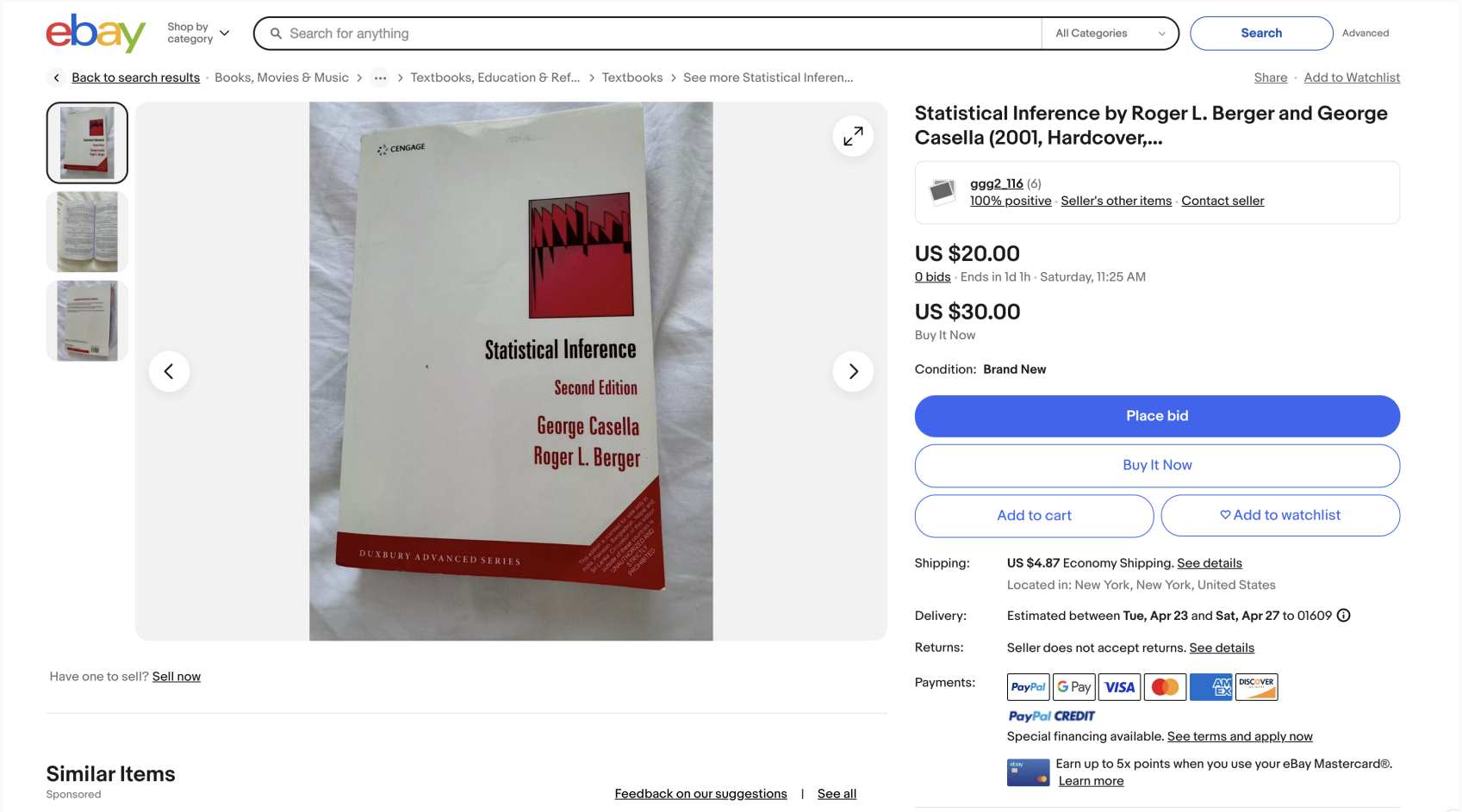


Figure 1: Example of English auction with the option to buy

English Auction with Option to Buy?

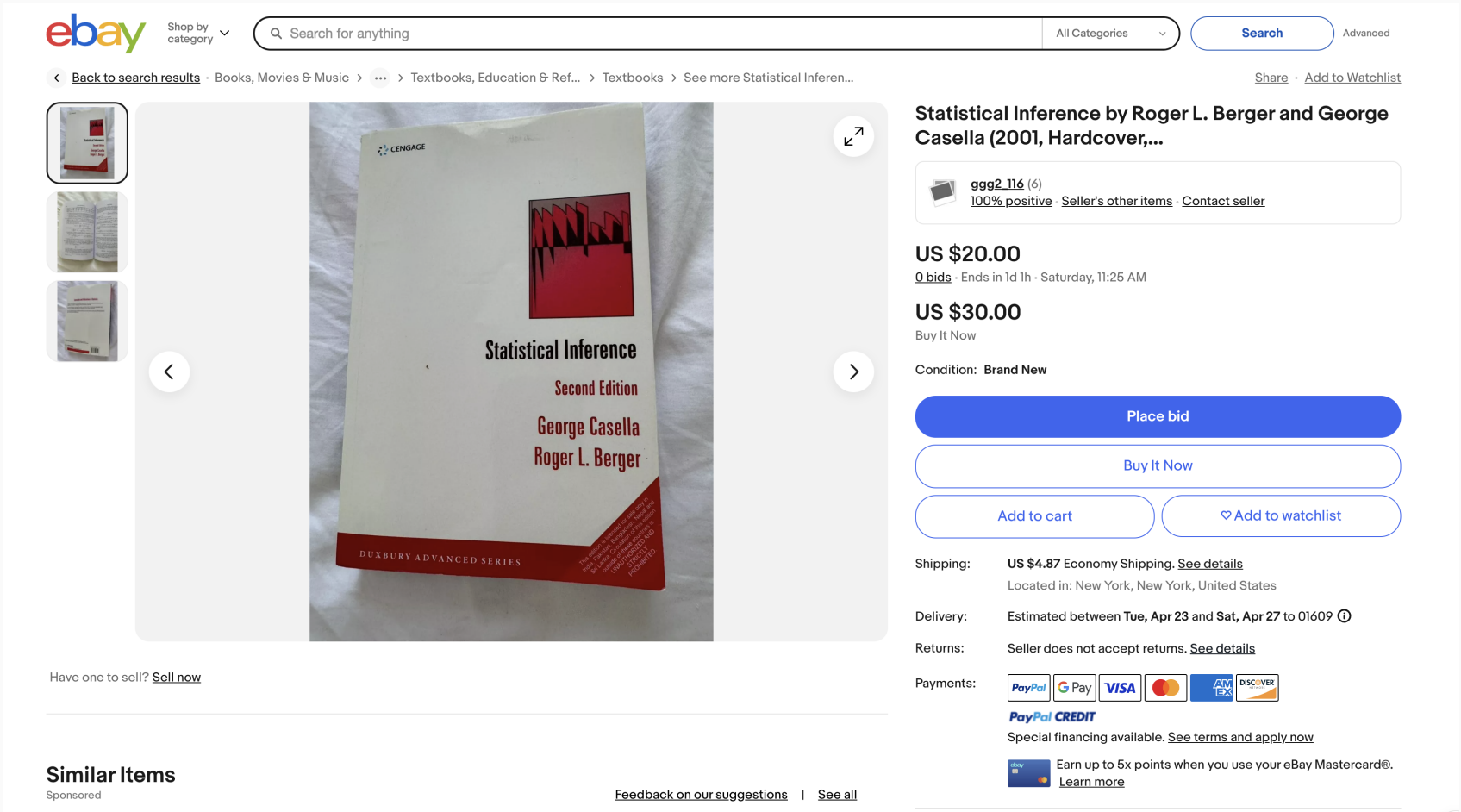


Figure 1: Example of English auction with the option to buy

Question: Should we buy it now or participate in the auction?

Related Experiments

Related Experiments

- Coppinger et al. (1980): 48 participants in a series of auctions. Comparing the difference in the treatment effect of English and Dutch auctions. This work uses the induced value. This work concludes that English and Dutch auctions are not equivalent.
- Cox et al. (1988): From 48 experiments, 80% of the variance of the bidding price of the individual by the value change can be explained using a linear function.

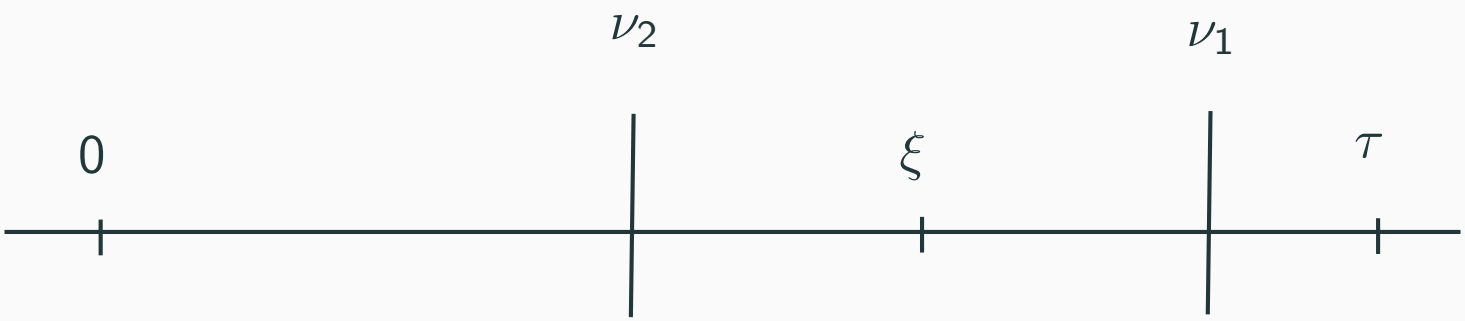
Theoretical Modeling

eBay Auction: Naive Model

- Suppose we have N bidders: $P_1, P_2, \ldots, P_N$. They can choose to
  - Buy the item now
  - Play the auction
- Suppose ith player has the value of a particular item ν_i and suppose the seller is proposing the selling price ξ.
- For simplicity, let $\nu_1, \ldots \nu_N \overset{\text{iid}}{\sim} \text{Uniform}(0, \tau)$ where $\tau > \xi$ and ξ is fixed at $\tau - \kappa$ for some $0 < \kappa < \tau$.
  - This ensures that immediate needs or happiness is captured within the model without explicitly defining it.
- Assume the risk neutrality.
- Only P_i can choose to buy the item. (Why?)

Question: What is the optimal action and optimal bid (if we choose to)?

eBay Auction: Naive Model



eBay Auction: Naive Model

Question: What is the optimal action and optimal bid (if we choose to)?

- We don't know the ending price. Thus, we want to bid until we reach the target valuation.
  - We either lose or win with no profit.
- The probability of P_i winning the auction is

$$p = \left(\frac{\nu_i}{\tau}\right)^{N-1}$$

eBay Auction: Naive Model

- If $\nu_i < \xi$, P_i always auction.
- If $\nu_i \geq \xi$, P_i can choose to buy or to auction.
  - Expected cost of bidding is

$$\begin{aligned}\mathbb{E}[B|B < \nu_i] &= \int_0^{\nu_i} \frac{b}{\tau}\, db \\ &= \frac{\nu_i^2}{2\tau}\end{aligned}$$

  - That means if $\nu_i \geq \sqrt{2\tau\xi}$ then you should buy. If not, you should auction.

eBay Auction: Naive Model

Expected payoff is

$$\mathbb{E}[\pi_i] = \underbrace{\left(1 - \sqrt{\frac{2\xi}{\tau}}\right)(\nu_i - \xi)}_{\text{Payoff if buy}} + \underbrace{\sqrt{\frac{2\xi}{\tau}}(\nu_i - b_i)}_{\text{Payoff if auction}}$$

Experiment Design

Experiment Design: Number of Experiments

- There are 3 variables that can be tuned
  - N: Increase/Decrease the number of participants
  - κ: Increase/Decrease the threshold for the valuation (equivalent to moving ξ)
  - τ: Increase/Decrease the value
- Suppose we are looking at
  - $N = 2, 5, 10, 20$
  - $\kappa = 0, 5, 10, 20$
  - $\tau = 10, 20, 50, 100$

  We have to perform at least $4^3 = 64$ experiments with different parameters (320 if we perform five repeated experiments)
- Target group: WPI students
- (Worst case) estimated budget: \$150K

Experiment Design: Auction

- Participants use the website to perform the auction and start with τ as money for every auction round.
- Participants are auctioning the gift card with value ξ with the option to buy it for ξ.
    - If the item is brought, that experiment ends.
- Random valuation is assigned for each participant; specifically, it is how much they can redeem after the experiment ended (induced value)
- Compensate the participant with 40% of what they earned from selling back the item and the number of hours they participated.

Experiment Design: Observations

- Observing the following values:
  - The probability of choosing to buy it now.
  - How much they bid
- Comparing this value with the theoretical results.
  - Should bid higher than the theoretical prediction.
  - Should pursue on the auction more than they should.

Reference

References i

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