Homework 2

MA 590 Special Topics: Causal Inference

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Problem 1

A

In a completely randomized experiment in which $n_1 = n_0 = n/2$, say $S_t^2 = S_c^2$ (the sets of potential outcomes have the same variance in the experimental sample). What is the smallest possible value for the true sampling variance of the difference in means estimator ($\mathbb{V}_{fp}(\hat{\tau}^{diff})$)? Justify your answer (with math)

(*hint:* $S_{tc}^2 = S_t^2 + S_c^2 - 2\rho S_t S_c$, *where* $\rho = corr\{Y(1), Y(0)\}$, *the sample correlation of potential outcomes,* $S_t = \sqrt{S_t^2}$, *and* $S_c = \sqrt{S_c^2}$, *and recall that* $-1 \leq \rho \leq 1$)

Solution

something

B

With $n = 6$ and binary Y , write out a table of potential outcomes for which $\mathbb{V}_{fp}(\hat{\tau}^{diff})$ attains its lowest value. What is the ATE?

Solution

something

Problem 2

In a Bernoulli randomized trial, each subject is randomized between $Z = 1$ and $Z = 0$ independently. Hence, n0 and n1 are random variables, and since $Pr(n_0 = 0) > 0$ and $Pr(n_1 = 0) > 0$, the difference-in-means estimator is not always defined. (The total sample size n remains fixed, not random.) If $\mathbb{E}[Z_i] = Pr(Z_i = 1) = p$ for every subject i, show that the following is an unbiased estimator for $\bar{\tau}$:

$$\hat{\tau}_{IPW} = \frac{1}{n}\sum_{i=1}^{n}\left\{\frac{Z_i Y_i}{p} - \frac{(1 - Z_i)Y_i}{1 - p}\right\}$$

(IPW stands for “inverse probability weighted” and will become important when we talk about propensity scores.)

Solution

something

Problem 3

Had the Lady Tasing Tea experiment been a Bernoulli randomized trial, so that each cup would be randomized to tea-first or milk-first independently, what would be the p-value (using Fisher's method) had the lady gotten all 8 cups right?

Solution

something