# *n*-Sample Test Classifier on Binary Outcomes of Stratified Randomized Experiments

MA 590 Special Topics: Causal Inference

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# Two-Sample Case

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Let say we have two group of data D:  $D_t$  and  $D_c$ . We want to test whether there is a difference between the two groups.

# Classifier Two-Sample Test (Lopez-Paz and Oquab 2017)

- Combined two dataset into one dataset D.
- Split the dataset into training and testing set.
- Fit the classifier (like logistic regression) to the training set and predict the testing set.
- Calculate the empirical loss  $l_e$  of the classifier. If  $|l_e 0.5| < \epsilon$ , then  $\bar{\tau} = 0$ .

```
set.seed(590)
# generate random multivariate gaussian data
n <- 1000
d <- 2
X <- matrix(rnorm(n*d), n, d)
y <- c(rep(0, n/2), rep(1, n/2))
c2st(X, y, echo = TRUE)
[1] Empirical loss: 0.57
[1] p-value: 0.378949753441706</pre>
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It is the *impossible* classification problem. Thus, the result should be close to near-chance level.

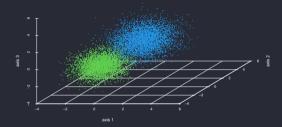
axis 1

Generalization of n-Sample Case

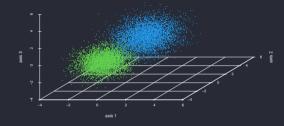
# Testing the Algorithm

```
set.seed(590)
# generate two dataset: two gaussians
n <- 5000
d <- 3
X0 <- matrix(rnorm(n*d, -1,0.8), n, d)
X1 <- matrix(rnorm(n*d, 1,1), n, d)
y <- c(rep(0, n), rep(1, n))
c2st(rbind(X0, X1), y, echo = TRUE)</pre>
```

- [1] Empirical loss: 0.0285
- [1] p-value: 0



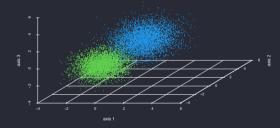
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[1] p-value: 0

Test statistic diverges from near-chance level as there is an exist of "linear" decision boundary. The dimensionality problem in homogenity test is solved.

**Question**: Does changing from GLM to other models increases (widen the range of) the testing power,  $\beta$ ?



# Comparison to Maximum Mean Discrepancy

## Use C2ST on Causal Inference

• But, how can we use C2ST on causal inference?

# Example:

## Generalization of *n*-Sample Case

#### Possible Solution

- Combine both treatment and control group within each stratum into one dataset  $D_i^s$ .
- For each group, fit the classifier (like logistic regression) to the training set and predict the testing set.
- ullet Calculate the empirical loss  $l_e$  of the classifier. If  $|l_e-0.5|<\epsilon$ , then  $ar{ au}_{
  m within}=0$
- ullet Find the way to infers  $ar{ au}_{\mathsf{between}}$

### References

#### References

Lopez-Paz, David, and Maxime Oquab. 2017. "Revisiting Classifier Two-Sample Tests." In *International Conference on Learning Representations*. https://openreview.net/forum?id=SJkXfE5xx.