# Computer Vision I

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# 1 Filter Algebra

#### 1.1

Without loss of generality choose an image  $I \in [0, 255]$  with size  $3 \times 3$ .

The maximum value for  $I_{22}$  is achieved by weighting all positive values in the filter with 255 and all negative values with 0. This results in image (1). With this image the maximum output value is  $2 \cdot 255 + 2 \cdot 255 + 1 \cdot 255 = 1275$ .

$$I_{max} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{pmatrix} \tag{1}$$

The minimum value for  $I_{22}$  is achieved by weighting all positive values in the filter with 0 and all negativ values with 255. This results in image (2). With this image the maximum output value is  $-2 \cdot 255 + (-2) \cdot 255 + (-1) \cdot 255 = -1275$ .

$$I_{max} = \begin{pmatrix} 255 & 255 & 0\\ 255 & 0 & 0\\ 0 & 0 & 0 \end{pmatrix} \tag{2}$$

$$\begin{array}{rcl} I & = & (1\ 2) \\ H & = & (1\ -1) \\ \alpha = 255 \end{array}$$

With these values calculate the convolution (the values are continued with 0):

```
\begin{array}{rcl} (\alpha \cdot I) * H & = & (255\ 255) * (1\ -1) = (255\ 0\ 0) \\ \alpha \cdot (I * H) & = & 255 \cdot (1\ 1\ 0) = (255\ 255\ 0) \\ & \Rightarrow & (\alpha \cdot I) * H\alpha \cdot (I * H) \end{array}
```

As shown by the contradiction above the linearity does not hold for clamped values.

#### 1.3

Matlab Code:

```
I = imread("lena.tif");
_2 sigma = 3;
G = fspecial("gaussian", 2*ceil(2*sigma)+1, sigma);
_{4} H = \begin{bmatrix} -1 & -2 & 0; & -2 & 0 & 2; & 0 & 2 & 1 \end{bmatrix};
   \begin{array}{lll} T1 = imfilter\,(I\,,\,G,\,\,\,'replicate\,\,'\,,\,\,\,'conv\,\,')\,;\\ R1 = imfilter\,(T1,\,H,\,\,\,'replicate\,\,'\,,\,\,\,'conv\,\,')\,; \end{array}
   figure();
  subplot (1,3,1);
imshow(I);
   title ("Original image");
subplot(1,3,2);
imshow(T1);
   title ("Blurred Image");
   subplot (1,3,3);
   imshow(R1);
   title ("Diagonal edges of the blurred image");
19
   print("lenaEdge1.eps", "-depsc");
```





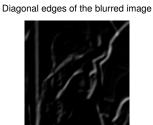


Abbildung 1: Output of the Matlab script

The filter highlights strong edges in the diagonal axis (north-west to south-east) by calculating an approximitation of the derivative in this direction.

```
T2 = imfilter(I, H, 'replicate', 'conv');
  R2 = imfilter (T2, G, 'replicate', 'conv');
23
24
  figure();
25
  subplot(2,3,1);
  imshow(I);
  title (" Original image");
  subplot(2,3,2);
29
  imshow(T1);
  title ("Blurred Image");
  subplot(2,3,3);
32
  imshow(R1);
  title ("Diagonal edges of the blurred image");
  subplot(2,3,4);
  imshow(abs(R2-R1));
  title ("Difference of both images");
37
  subplot (2,3,5);
  imshow(T2);
  title ("Diagonal edges of the original image");
  subplot (2, 3, 6);
```

```
42 imshow(R2);
43 title("Blurred image of the diagonal edges");
44
45 print("lenaEdge2.eps", "-depsc");
```

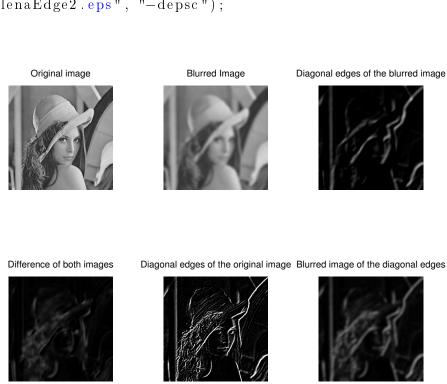


Abbildung 2: Output of the second part of the Matlab script

The second image looks slighly more detailed. When using clamped images the convolution product is no longer commutative.

# 2 Discrete Fourier Transform

```
1 %% Part a  
2 N = 4;  
3 ti = \begin{bmatrix} 0 & 1/30 & 2/30 & 3/30 \end{bmatrix};  
4 si = \begin{bmatrix} 2 & 3 & 0 & 1 \end{bmatrix};  
5  
6 S = zeros(size(ti));  
7 f = zeros(size(ti));
```

```
\begin{array}{lll} {}_{9} & \text{for } u\!=\!0\!:\!(N\!-\!1) \\ {}_{10} & \text{for } t\!=\!0\!:\!(N\!-\!1) \\ {}_{11} & S(u\!+\!1) \;+\!=\; si\,(\,t\!+\!1) \;*\; exp(-2\!*pi\!*i\!*u/N\!*t\,)\,; \\ {}_{12} & \text{end}\,; \\ {}_{13} & f(u\!+\!1) \;=\; 2\!*pi\!*u/N\,; \\ {}_{14} & \text{end}\,; \\ {}_{15} & S\;./\!=\; sqrt\,(N)\,; \end{array}
```

#### 2.2

The uth coefficient (starting at zero) corresponds to a frequency of

$$f(u) = 2 \cdot \pi \cdot \frac{u}{N} = \frac{\pi \cdot u}{2}$$

```
17 % Part b

18 x = 0:0.01:1;

19 y = \sin(2*pi*x*1/N*3);

20 figure();

21 plot(x,y);

22 title("A sine-wave with the frequency of S(3)");

23 xlabel("t");

24 ylabel("y(t)");

25 print("frequency.eps", "-depsc");
```

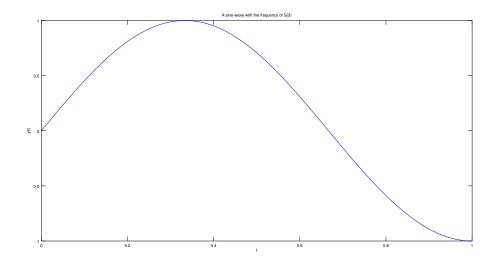


Abbildung 3: Output of the second part of the Matlab script

```
27 % Part c
   sr = zeros(size(ti));
   for u=0:N-1
         for t = 0: (N-1)
              \label{eq:sr}  \text{sr} \, (\, u + 1) \ + = \ S \, (\, t + 1) \ * \ \exp \, (\, 2 * p \, i * i * u / N * \, t \,) \; ; 
31
         end;
^{32}
зз end;
sr \cdot /= sqrt(N);
   2.4
36 % Part d
37 figure();
  subplot(3,1,1);
39 plot (ti, si);
_{40} _{x \, la \, b \, e \, l \, (" \, t \, ")};
41 ylabel("s i(t)");
subplot (3,1,2);
43 plot (f,S);
44 xlabel("f");
ylabel("S(f)");
subplot(3,1,3);
47 plot(ti,sr);
48 xlabel("t");
49 ylabel("s_r(t)");
print("transforms.eps", "-depsc");
```

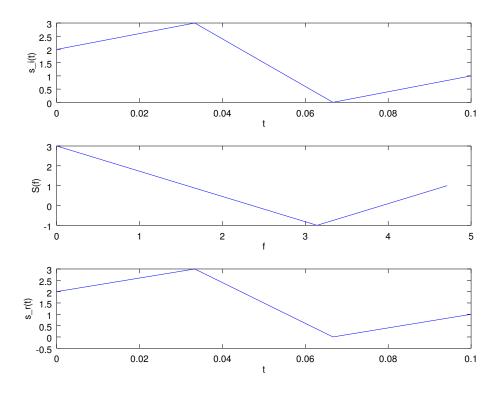


Abbildung 4: Output of the last part of the Matlab script

# 3 Fourier Transform for Image Quality Assessment

```
sigma = size(I1,1);
  G = fspecial ('gaussian', sigma, sigma/2);
  G = padarray(G, (size(F1)-size(G))/2, 0, 'replicate');
  G /= max(max(G));
  H = ones(size(G)) - G;
17
_{18} H1 = F1 .* H;
_{19} H2 = F2 .* H;
  3.3
 % Part c
  H1 .*= H1;
  H2 .*= H2;
  E1 = sum(sum(H1));
  E2 = sum(sum(H2));
  3.4
28 % Part d
  figure();
  subplot(3,3,1);
  imshow(I1);
  title ("flower01.png");
  subplot(3,3,2);
  imshow(F1);
   title ("Fourier transform of flower01.png");
  subplot(3,3,3);
  imshow (H1);
   title ("Filtered fourier transform of flower01.png");
38
39
  subplot (3,3,4);
  imshow (I2);
   title ("flower02.png");
43
  subplot (3,3,5);
  imshow(F2);
  title ("Fourier transform of flower02.png");
  subplot (3, 3, 6);
  imshow(H2);
48
   title ("Filtered fourier transform of flower02.png");
49
  subplot(3,3,7);
```

```
title("H");
56
    subplot(3,3,9);
57
    bar ([E1 E2]);
58
    title ("Energy of both images");
   print("Quality.eps","-depsc");
                     flower01.png
                                         Fourier transform of flower01.png Filtered fourier transform of flower01.png
                     flower02.png
                                         Fourier transform of flower02.png Filtered fourier transform of flower02.png
                         G
                                                     Н
                                                                          Energy of both images
                                                                  300000
                                                                  250000
                                                                  200000
                                                                  150000
                                                                  100000
```

imshow(G);
title("G");
subplot(3,3,8);

imshow(H);

Abbildung 5: Output of the Matlab script

50000

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