Einführung in die Neuroinformatik

Tim Luchterhand, Paul Nykiel (Gruppe P)

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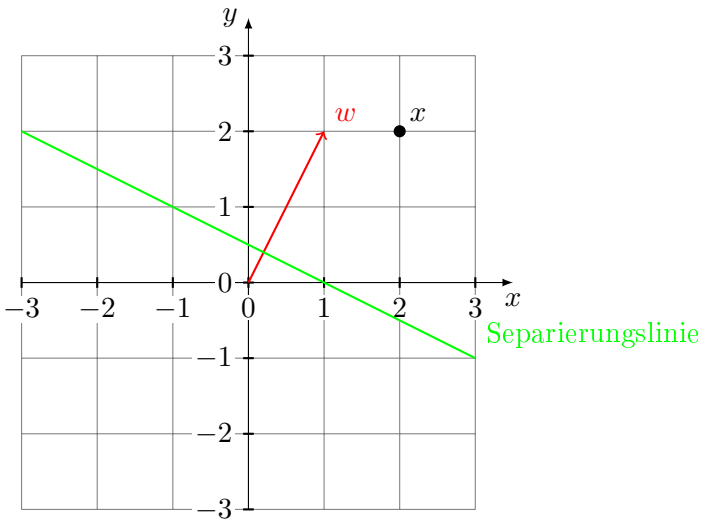
1 Lernschritt im Perzeptron-Lernalgorithmus

(a)

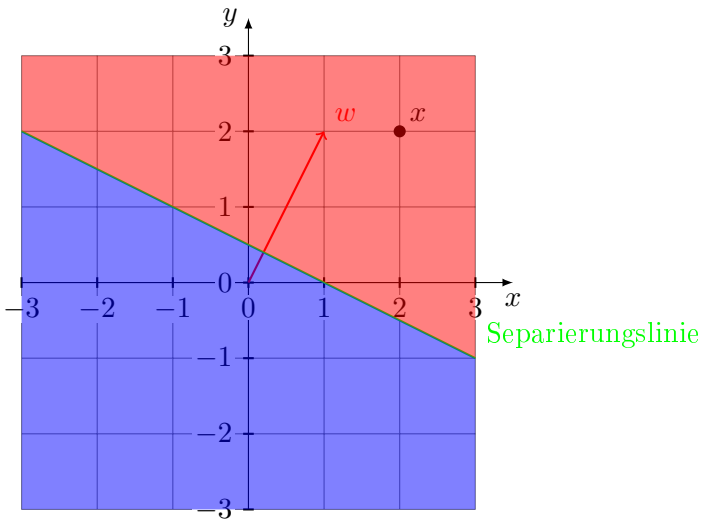
$$\begin{aligned} w_1 \cdot x_1 + w_2 \cdot x_2 + w_0 &= 0 \\ \Leftrightarrow x_2 &= -\frac{w_1 \cdot x_1 + w_0}{w_2} \\ x_2 &= -\frac{1}{2}x_1 + \frac{1}{2} \end{aligned}$$



(b)

$$w^* = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$
$$x^* = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

(c) w_1 in rot, w_{-1} in blau



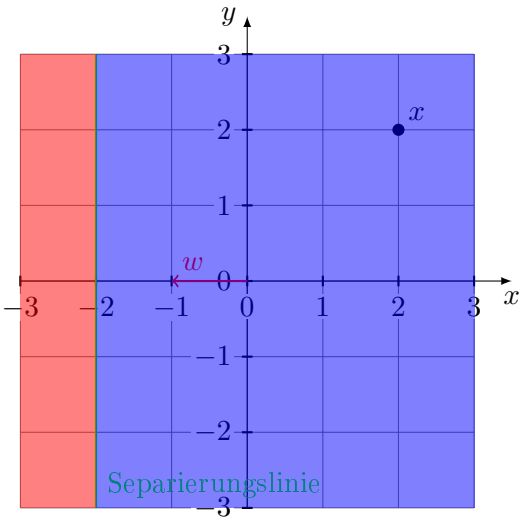
(d) Überprüfen ob bereits korrekt klassifiziert:

$$(w^*)^T \cdot x^* = 2 + 4 - 1 = 5 \geq 0$$
$$\Rightarrow \quad x^* \in \omega_1$$

Lernschritt durchführen:

$$\tilde{w}^* = w^* - \eta \cdot x^* = \begin{pmatrix} -1 \\ 0 \\ -2 \end{pmatrix}$$
$$\Rightarrow \tilde{w} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$
$$\tilde{w}_0 = -2$$

(e) w_1 in rot, w_{-1} in blau



(f) Überprüfen ob bereits korrekt klassifiziert:

$$\begin{aligned}(\tilde{w}^*)^T \cdot x^* &= -2+0-2=-4 \\ &\Rightarrow \quad x^* \in \omega_{-1}\end{aligned}$$

Kein Lernschritt ist notwendig $\Rightarrow w$ wird nicht verändert

2 Perzeptron-Lernalgorithmus

Matlab script:

```
%% Initialization
data = [-3  1  -1;
        -3  3   1;
        -2  1  -1;
        -2  4   1;
        -1  3   1;
        -1  4   1;
         2  2  -1;
         2  4   1;
         3  2  -1;
         4  1  -1;];


% Create all vectors
inputs = data(:,1:2);
inputsExtended = [inputs ones(size(inputs,1),1)];
```

```
17 classes = data(:,3);
18 w = [0,0,0];
19
20 % Extended weight vectors; each iteration adds one more row
21 % Since we don't know the exact number of rows in advance, we
     preallocate the matrix with a maximum size and crop the
     result in the end
22 maxVectors = 100;
23 vectorDimension = 3;
24 wExtendedMat = zeros(maxVectors, vectorDimension);
25 L = size(data, 1);
26 numberOfOptimizations = 1;
27 wExtendedMat(numberOfOptimizations,:) = w;
28
29 changesInLastIteration = 1;
30 while changesInLastIteration > 0
31     changesInLastIteration = 0;
32     for c = 1:size(inputs,1)
33         currentInput = inputsExtended(c,:);
34         desiredClass = classes(c,:);
35         calculatedOutput = w * transpose(currentInput);
36
37         % Wrong class
38         if calculatedOutput <= 0 && desiredClass > 0
39             w += currentInput;
40         elseif calculatedOutput >= 0 && desiredClass < 0
41             w -= currentInput;
42         end
43
44         % Plot the updated weights
45         if calculatedOutput * desiredClass <= 0
46             changesInLastIteration += 1;
47             numberOfOptimizations += 1;
48             wExtendedMat(numberOfOptimizations,:) = w;
49
50             x = y = 0;
51             if w(2) == 0
52                 y = -6:0.1:6;
53                 x = -w(3)/w(1) * ones(size(y));
54             else
55                 x = -6:0.1:6;
56                 y = -(w(1)*x + w(3))/w(2);
57             end
58             clf();
```

```
59              p = plot(x,y);
60              set(p, "linewidth", 1.5, "color", "green");
61              hold on;
62              scatter(data(:,1), data(:,2), [], data(:,3),'filled'
                    );
63              xlabel("x_1");
64              ylabel("x_2");
65              title("Punkte in der Ebene und Separierungslinie");
66              axis([-6 6 -6 6]);
67              grid on;
68
69              if numberOfOptimizations <= 2
70                  print("initial", "-depsc", "-color");
71                  print(gcf, "initial.epsc");
72              else
73                  print("separated", "-depsc", "-color");
74                  print(gcf, "separated.epsc");
75
76              end
77
78              while waitforbuttonpress ~= 1
79              end
80          end
81
82      end
83  end
84
85  % Remove unused weight vector entries
86  wExtendedMat = wExtendedMat(1:numberOfOptimizations, :);
87  L = 0;
```