# Problem 63

The proportion of items that are defective in a production run is $\theta$. 100 items are randomly sampled from the items, of which 3 are found to be defective. Let $X$ denote the number of defective items in the sample. Thus $X|\theta \sim Bin(100, \theta)$.

## Uniform(0, 1) Prior

In this part, the prior distribution of $\theta$ is assumed to be Beta(1, 1), which is equivalent to a Uniform(0, 1) distribution. Thus the joint distribution is

$$f_{\Theta|X}(\theta|x) \propto f_{X|\Theta}(x|\theta) f_{\Theta}(\theta) = \theta^x (1-\theta)^{100-x}, \theta \in [0, 1]$$

Using the knowledge that $X = 3$,

$$f_{\Theta|X}(\theta|x = 3) \propto \theta^3 (1-\theta)^{97}, \theta \in [0, 1]$$

which we recognize as a Beta(4, 98) variable.

## Beta(1/2, 5) Prior

Letting the prior of $\theta$ be Beta(1/2, 5),

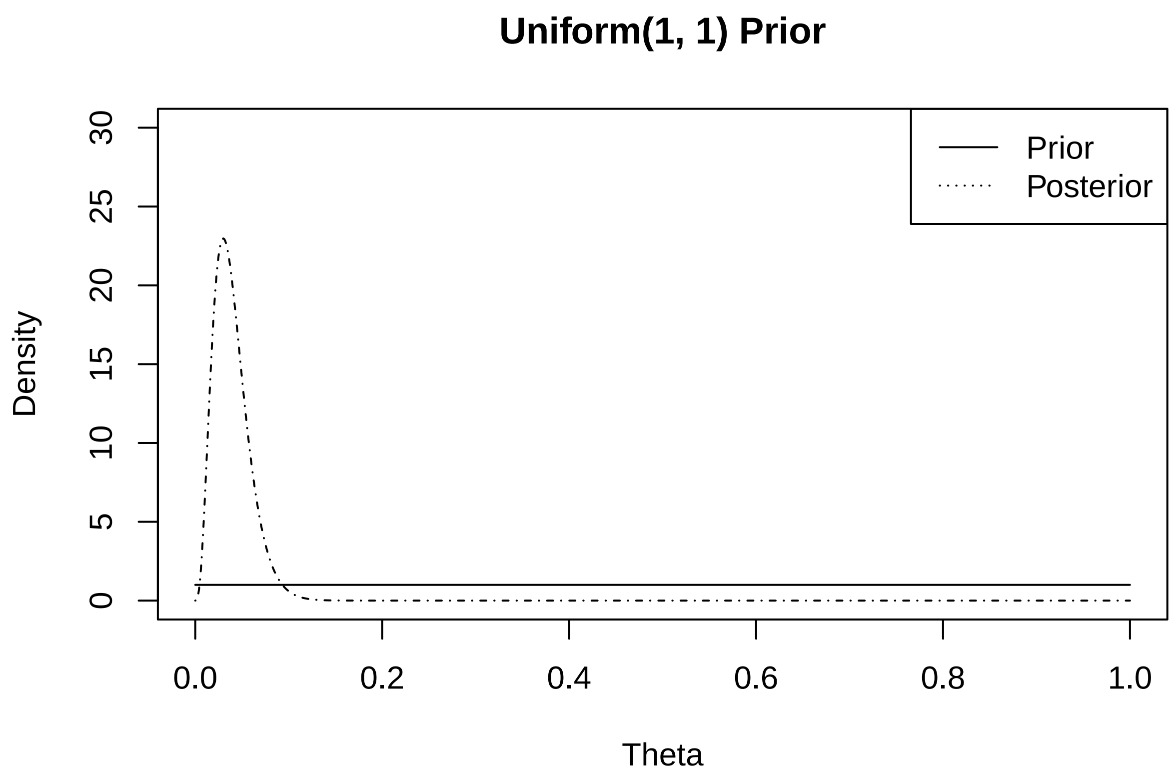
$$f_{\Theta|X}(\theta|x) \propto \theta^{x-\frac{1}{2}} (1-\theta)^{104-x}, \theta \in [0, 1]$$

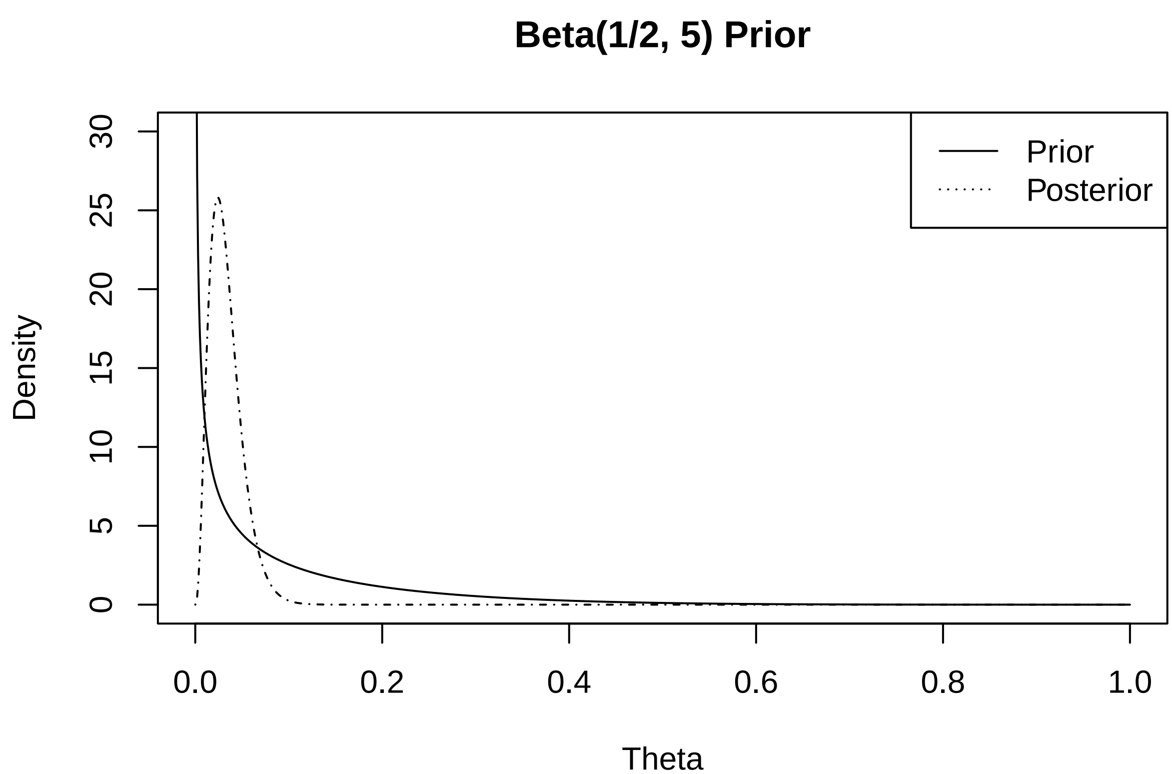
Evaluating at $x = 3$,

$$f_{\Theta|X}(\theta|x = 3) \propto \theta^{\frac{5}{2}} (1-\theta)^{101}, \theta \in [0, 1]$$

Which we recognize as a Beta(7/2, 102) variable.

**Graphs and Analysis**



The posterior mean of theta with a Uniform(1, 1) prior is 0.0392157, while the posterior mean of theta with a Beta(1/2, 5) prior is 0.0343137. As can be seen the histograms, the differences in posterior mean can be attributed to differences in the prior distributions. The Beta(1/2, 5) prior has more mass near zero, so that makes the posterior distribution's peak closer to zero.