Problem 8

The MLE estimator of p for a geometric distribution is

$$\hat{p} = \frac{1}{\bar{X}} = \frac{n}{\sum_{i=1}^{n} X_i}$$

To find the asymptotic variance, $I(p)$ equals (assuming the usual smoothness conditions)

$$I(p) = -E\left[\frac{\partial^2}{\partial p^2} \ln p + (x-1)\ln(1-p)\right] = -E\left[-\frac{1}{p^2} - \frac{X-1}{(1-p)^2}\right] = \frac{1}{p^2} + \frac{E(X)-1}{(1-p)^2} = \frac{1}{p^2} + \frac{1-p}{p(1-p)^2} = \frac{1}{p^2(1-p)}$$

Thus the asymptotic variance of $\hat{p}$ is

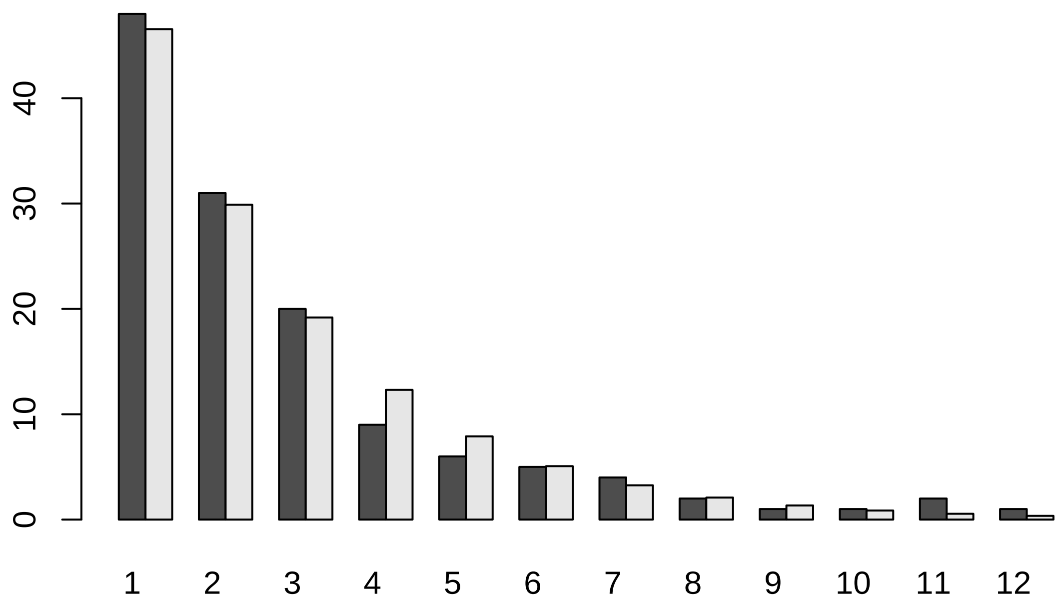
$$Var(\hat{p}) \to \frac{p^2(1-p)}{n}$$

And an estimator for the variance of $\hat{p}$ is

$$s_{\hat{p}}^2 = \frac{\hat{p}^2(1-\hat{p})}{n}$$

Estimating using the data, we estimate p to be 0.3581267, with a standard error of 0.0251646 and a 95% confidence interval of (0.3088041, 0.4074493).

Visually examining the histogram, we find



The left bars are the observed results, while the right bars are the predicted results. The fit is quite good. With a prior distribution of P being $f_P(p) \sim Unif(0,1)$, the posterior distribution of P is

$$f_{P|\boldsymbol{X}}(p|\boldsymbol{x}) = \frac{p^n(1-p)^{\sum X_i - n}}{\int_0^1 p^n(1-p)^{\sum X_i - n} dp} = \frac{p^{130}(1-p)^{363-130}}{\int_0^1 p^{130}(1-p)^{363-130} dp} = \frac{p^{130}(1-p)^{233}}{\int_0^1 p^{130}(1-p)^{233} dp}, p \in [0,1]$$

The mean of the posterior distribution of p is 0.3589382, while the standard deviation is 0.0248294.