Estimating the Paramater of the Poisson Distribution with MLE and Bayes

Maximum Likelihood Estimation

For a Poisson distribution, the MLE estimator matches the method of moments estimator with $\hat{\lambda} = \bar{X}$, while the standard error of $\hat{\lambda}$ is $s_{\hat{\lambda}} = \sqrt{\frac{\hat{\lambda}}{n}}$.

```
mleLambdaBar <- round(mean(data), 1)
mleSLambdaBar <- round(sqrt(mleLambdaBar / length(data)), 2)
```

Thus the MLE estimates of $\bar{\lambda}$ and $s_{\bar{\lambda}}$ are 24.9 and 1.04, respectively, as in the book.

Bayesian Estimation, Uniform Prior

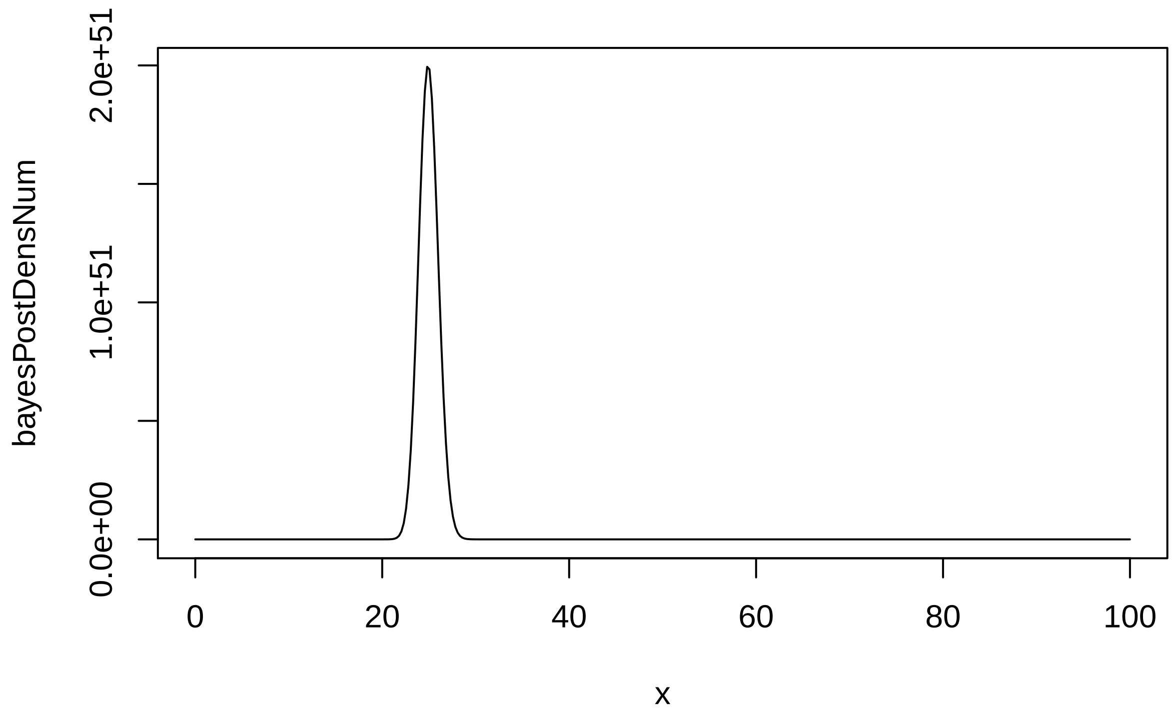
Here, we assume that the prior distribution of λ follows a uniform distribution on $[0, 100]$. Thus, the posterior density of λ takes the form

$$f_{\Lambda|X}(\lambda|x) \propto \lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}, 0 \leq \lambda \leq 100$$

First we compute the numerator of the posterior density, up to a proportionality constant. To prevent the production of infinities, we will shrink the posterior density of lambda by a factor of 10^{-500}.

```
bayesPostDensNum <- function(lambda) {
  xsum <- sum(data)
  n <- length(data)
  return(
    exp(
      -500 * log(10) +
        (xsum * log(lambda) - (n * lambda))
    )
  )
}
```

```
plot.function(bayesPostDensNum, from = 0, to = 100, n = 400)
```


bayesPostDensNum
0.0e+00
1.0e+51
2.0e+51
0
20
40
60
80
100
x