Section 8.4, Example C - Method of Moments and MLE Estimation of the Parameters of the Gamma Distribution

Method of Moments Estimation

First we load the data.

```
getwd()
```

```
## [1] "C:/Users/arthu/Desktop/Stat 135/Chapter 8"
```

```
length(data)
```

```
## [1] 227
```

The data has been loaded correctly.

We will model the data with a gamma distribution with parameters α and λ. From the text, the method of moments estimates of the parameters are

$$\hat{\alpha}_{MoM} = \frac{\bar{X}^2}{\hat{\sigma}^2}$$

and

$$\hat{\lambda}_{MoM} = \frac{\bar{X}}{\hat{\sigma}^2}$$

Now we calculate $\hat{\alpha}_{MoM}$ and $\hat{\lambda}_{MoM}$.

```
xbar = mean(data)
sigma2hat = var(data)
lhatMoM = xbar / sigma2hat
ahatMoM = lhatMoM * xbar
```

The data has $\bar{X} = 0.2243921$ and $\hat{\sigma}^2 = 0.1338252$. This produces estimates of $\hat{\alpha}_{MoM} = 0.3762506$ and $\hat{\lambda}_{MoM} = 1.6767555$.

Now we will calculate the standard errors for these estimates. We will generate 1000 samples of 227 draws from a gamma distribution with $\alpha = 0.3762506$ and $\lambda = 1.6767555$, then calculate 1000 instances of $\hat{\alpha}_{MoM}$ and $\hat{\lambda}_{MoM}$, then take the standard deviations of those estimates and use them as estimates for the standard deviations of $\hat{\alpha}_{MoM}$ and $\hat{\lambda}_{MoM}$.

```r
# Initialize the vectors
ahat_est <- vector(mode = 'numeric', length = 1000)
lhat_est <- vector(mode = 'numeric', length = 1000)

gen_ahat <- function(data) {
  xbar <- mean(data)
  sigma2hat <- var(data)
  ahat <- xbar^2 / sigma2hat
  return(ahat)
}

gen_lhat <- function(data) {
  xbar <- mean(data)
  sigma2hat <- var(data)
  lhat <- xbar / sigma2hat
  return(lhat)
}

#Generate 1000 draws from a gamma distribution and calculate the resulting a hat and b hat
set.seed(1000)
for(i in 1:1000){
  data <- rgamma(227, shape = ahatMoM, rate = lhatMoM)
  ahat_est[i] <- gen_ahat(data)
  lhat_est[i] <- gen_lhat(data)
}

ahatMoM_ste <- sd(ahat_est)
lhatMoM_ste <- sd(lhat_est)

rm('ahat_est', 'lhat_est', 'i')
```

Thus the estimates of the standard errors for $\hat{\alpha}_{MoM}$ and $\hat{\lambda}$ are $s_{\hat{\alpha}_{MoM}} = 0.0664075$ and $s_{\hat{\lambda}_{MoM}} = 0.3648682$. The corresponding estimates in the book are 0.06 and 0.34, respectively.

Maximum Likelihood Estimation

According to the book, the maximum likelihood estimator of α, $\hat{\alpha}_{MLE}$