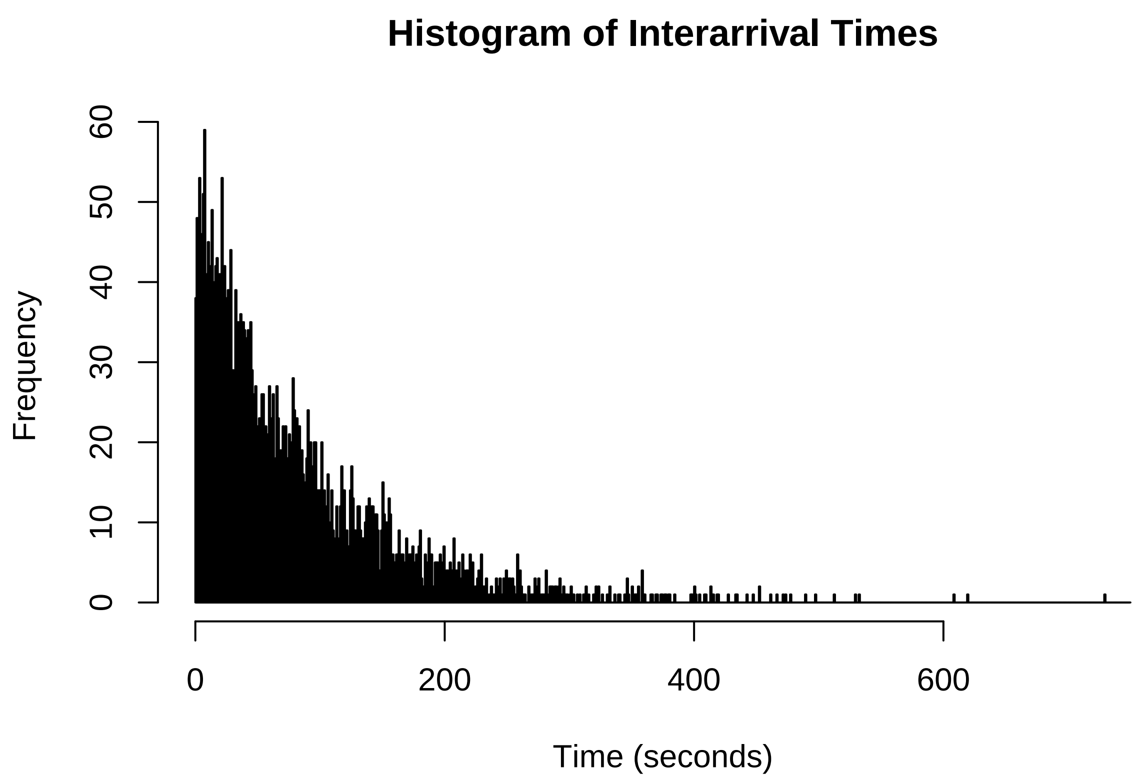
Problem 43

Gamma Ray Interarrival Times

Initial Histogram

The data consists of 3935 observations of interarrival times for gamma rays, with units in seconds.

```
hist(data, breaks = seq(from = 0, to = 750, by = 1), main = 'Histogram of Interarrival Times', xlab = '
```



The interarrival times look roughly exponential, or at least gamma, since the exponential distribution is a special case of the gamma distribution. For the moment, we will assume that interarrival times are distributed via a gamma distribution.

Parameter Estimates

Method of Moments

The method of moments estimators for α and γ are

$$\hat{\alpha}_{MoM} = \frac{\bar{X}^2}{\bar{\sigma}^2}$$

$$\hat{\lambda}_{MoM} = \frac{\bar{X}}{\bar{\sigma}^2}$$

```
#Computes the MoM estimates of alpha and lambda for a gamma distribution, respectively
gammaMoM <- function(data) {
  output <- vector(mode = 'numeric', length = 2)
  output[1] <- (mean(data)^2) / var(data)
  output[2] <- mean(data) / var(data)
  return(output)
}
foo <- gammaMoM(data)
aMoM <- foo[1]
lMoM <- foo[2]
rm(foo)
```

Thus the method of moments estimates are $\hat{\alpha}_{MoM} = 1.0120949$ and $\hat{\lambda}_{MoM} = 0.0126614$.

Maximum Likelihood

The maximum likelihood estimator for α satisfies the following equation

$$nln(\hat{\alpha}) - nln(\bar{X}) + \sum_{i=1}^{n} ln(X_i) - n\psi(\hat{\alpha}) = 0$$

Where $\psi(x)$ is the digamma function, $\frac{\Gamma'(x)}{\Gamma(x)}$. Then an estimator for λ is

$$\hat{\lambda}_{MLE} = \frac{\hat{a}}{\bar{X}}$$

We will use the method of moments estimator as an initial guess to solve for $\hat{\alpha}_M LE$.

```
#Computes the MLE estimates of alpha and lambda for a gamma distribution, respectively
gammaMLE <- function(data) {
  aMoM <- gammaMoM(data)[1]
  #Initialize variables
  n <- length(data)
  lnXBar <- log(mean(data))
  sumLnX <- sum(log(data))
  aMLEFun <- function(a) {
    n*log(a) - n*lnXBar + sumLnX - n*digamma(a)
  }


  #x <- seq(from = aMoM - .5, to = aMoM + .5, by = 0.01)
  #plot(x, aMLEFun(x))
  aMLE <- uniroot(aMLEFun, interval = c(aMoM - .5, aMoM + .5))$root
  lMLE <- aMLE / mean(data)
  return(c(aMLE, lMLE))
}
```

```
foo <- gammaMLE(data)
aMLE <- foo[1]
lMLE <- foo[2]
rm(foo)
```

Thus the maximum likelihood estimates are $\hat{\alpha}_{MLE} = 1.026308$ and $\hat{\lambda}_{MLE} = 0.0128392$.