Remarks on Certain Graph Forms

Neal Foster, 2012

The directed acyclic graph (DAG) is a graph which conforms to the following properties:

1. *Directed* – All edges point from a starting vertex to an end vertex. It is not possible to traverse an edge from the end to the start.

2. *Acyclic* – No closed loops or cycles can be created from edges.

Such a graph presents several novel considerations on the question of minimum path calculations. In particular, it greatly simplifies path calculations, since the graph can be represented as a topological tree with a single origin point and all possible paths branching out from that point. All shortest paths can be derived from that origin point, or as subpaths of paths from the origin point to the destination point. Compared to the algorithms used to derive shortest path length for cyclic or ordinary graphs (see Djikstra's Algorithm), any possible shortest path can therefore be calculated with a time complexity of $O(n.\ Vertices + n.\ Edges)$.

Of course, DAGs also present several interesting philosophical properties. In order to properly examine these it is useful to examine an application of DAGs in a practical context, namely that of decision trees (DT). A DT consists of the results of several nested choices beginning from a single decision point, with each choice outcome leading to further future choices. Decision points form vertices and possible choices directed edges. But why are such decision trees acyclic? Consider Heraclitus' famous invocation, *panta rhea*. Further consider that, even if you reach seemingly the same decision day after day (e.g. what to eat for breakfast on Tuesday vs. Monday), you (the body making the decision) have changed, and your predilections and disgusts commensurately so, even if they change in imperceptible increments. Therefore, we may safely say that at least in our lives DTs are DAGs.

I wish to introduce a final constraint on DTs in our world, namely that they are Markov decision trees (MDTs). A MDT presents not choices but a set of probabilistic outcomes – you are most likely to choose fried eggs with your breakfast, but sometimes you may choose to eat scrambled eggs. But what determines the probability? Certainly, your preferences have something to do with it, and your knowledge of the situation plays some indeterminate part (you are unlikely to step into the road knowing a car is approaching), yet humans nevvertheless make decisions that are surprising, even to themselves. To borrow some parlance from a neighbouring field, we may say that there exists a reward function *R* which evaluates each branch of the decision tree and all possible following choices (which we may shorten to a "policy"), and from there assigns each policy *p* an estimated reward r_p. The normalised probability of any given policy being chosen is therefore

$$\hat{\underline{v}} := \frac{\underline{v}}{||\underline{v}||_\infty}$$

Where $\underline{v} = \{r_1, r_2, r_3 \dots r_n\}$ and *n* is the total number of policies available at that decision point.

Of course, such a function and the subsequent outcomes of each decision point are fundamentally unpredictable despite our best efforts. Some choices in our lives are more constrained than they appear, some less so. Our limited minds cannot possibly foresee with any accuracy the certain outcomes of any choice, thus the rewards assigned are mere flawed estimates. And sometimes, the right to choose is taken away from us, rendering our calucations moot. We are each of us peering into the future, attempting to estimate some unknown reward, and subjecting ourselves to the vicissitudes of fate at every moment.