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Biform Games

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Both noncooperative and cooperative game theory have been applied to business strategy. We propose a hybrid noncooperative-cooperative game model, which we call a biform game. This is designed to formalize the notion of business strategy as making moves to try to shape the competitive environment in a favorable way. (The noncooperative component of a biform game models the strategic moves. The cooperative component models the resulting competitive environment.) We give biform models of various well-known business strategies. We prove general results on when a business strategy, modelled as a biform game, will be efficient.

Key words: noncooperative game; efficiency; business strategy; cooperative game
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1. Introduction

There have been a number of applications of game theory to the field of business strategy in recent years. Reflecting the two branches of game theory itself—the so-called noncooperative and cooperative branches—these applications have taken two forms. Noncooperative applications, which are the more numerous, use the more familiar language of game matrices and trees. A leading recent example of this approach is Ghemawat (1997). Cooperative applications use the less-familiar characteristic-function language, and to date have been much fewer in number. For this approach, see Brandenburger and Stuart (1996), Lippman and Rumelt (2003), MacDonald and Ryall (2002, 2004), Oster (1999), Spulber (2004), and Stuart (2001).

The two approaches address different questions. The noncooperative model is useful for analyzing strategic moves in business—say the decision whether to enter a market, where to position a product, what brand to build, how much capacity to install, or how much money to devote to R&D. The cooperative model is useful for addressing the basic question of how much power the different players—firms, suppliers, customers, etc.—have in a given setting.

Both models have a role to play in understanding business strategy. In a cooperative game, no player is given any price-setting power a priori, and all players are active negotiators. Buyers compete for sellers and sellers compete for buyers, without any specific "protocol."[1] This direct analysis of the power of the different players in the marketplace has been basic to the business-strategy field, going back at least to the Five Forces framework (Porter 1980). One can see the Five Forces—and other related frameworks that we mention later—as a tool for assessing how much value is created in a certain environment, and how that "pie" is divided up among the players. (It is notable that these frameworks embody a "free-form" view of competition. One reason is empirical. For example, in business-to-business relationships—such as supplier to firm, and firm to distributor—prices and other terms are often negotiated rather than posted by one or another party.)

Cooperative theory, then, offers a model of the competitive environment.[2] However, this is only the starting point for business strategy. The next step is to find ways to shape the environment in a favorable way. A good strategic move is one that brings about favorable economics to a player—one that enables the player to capture more value. This is where the noncooperative theory is important because it gives us a formal language in which to write down such strategic moves (and countermoves).

In this paper, we put the two models together to create a hybrid noncooperative-cooperative model

[1] This term is from Kreps (1990, pp. 92–95).

[2] We remind the reader that although standard, the terms "noncooperative" and "cooperative" game theory are perhaps unfortunate. In particular, cooperative theory can indeed be used to analyze the implications of competition among the players.

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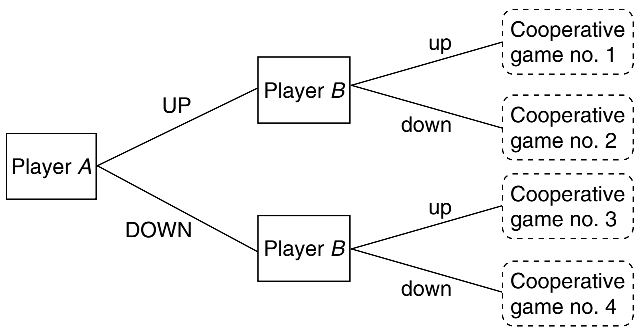
Figure 1 **A Biform Game**



that we call the *biform*[3] *game model*. A biform game is a two-stage game. The first stage is noncooperative and is designed to describe the strategic moves of the players. (These might be entry, position, brand, capacity, R&D, etc., as above. More broadly, we can think of the players' first-stage moves as building their various "strategic capabilities."[4]) However, the consequences of these moves are not payoffs (at least not directly). Instead, each profile of strategic choices at the first stage leads to a second-stage, cooperative game. This gives the competitive environment created by the choices that the players made in the first stage. Analysis of the second stage then tells us how much value each player will capture (i.e., gives us the payoffs of the players). In this way, the biform model is precisely a formalization of the idea that business strategies shape the competitive environment—and thereby the fortunes of the players. See Figure 1 for a schematic of a biform game.

We note that at the colloquial level, phrases such as "changing the game" or "choosing the game" are often used when conveying the idea of business strategy. The biform model fits with this language: In the first stage, the players are each trying to choose the best game for themselves, where by "game" is meant the subsequent (second-stage) game of value. Equally, they are trying to change the game, if we define one of the second-stage games as the status quo.

2. Examples

We now give two examples of biform games. (More examples can be found in §5.)

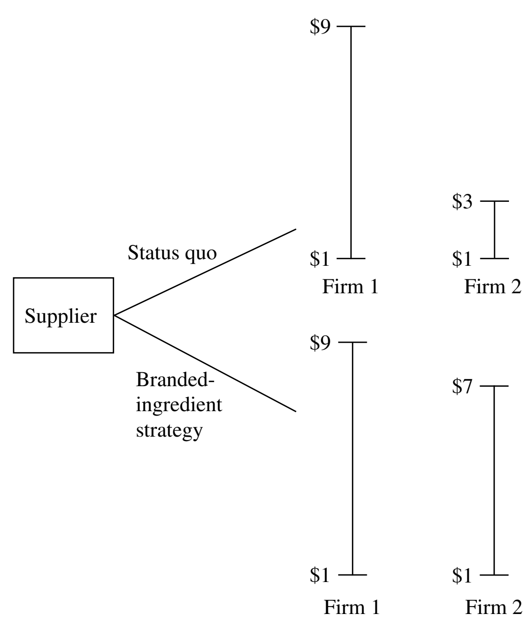
Example 2.1 (A Branded Ingredient Game). We start with a biform analysis of the idea of a "branded-ingredient" strategy.[5] In the game, there are two firms, each able to produce a single unit of a certain product. There is one supplier that can supply the necessary input to at most one firm, at a cost of \$1. There are numerous buyers, each interested in buying a single unit of the product from one of the two firms. Every buyer has a willingness-to-pay of \$9 for Firm 1's product, and a willingness-to-pay of \$3 for Firm 2's product.

The supplier has the option of incurring an upfront cost of \$1 to increase the buyers' willingness-to-pay for Firm 2's product to \$7. This is the branded-ingredient strategy, under which the supplier puts its logo on its customers' products. (To make our point most clearly, we assume that this does not affect the buyers' willingness-to-pay for Firm 1, the stronger firm.)

The game is depicted in Figure 2. Here, there is a simple game tree, in which only one player (the supplier) gets to move. The endpoints of the tree are the cooperative games induced by the supplier's choices. The two vertical bars at the upper endpoint summarize the cooperative game that results if the supplier chooses the status-quo strategy. The left bar shows the value created if the supplier supplies Firm 1 (and therefore not Firm 2), and vice versa for the right bar. The interpretation of the bars at the lower endpoint is similar. (We do not show the upfront cost here, but it is included in the numbers below.)

How will this game be played? To see, we start by analyzing each of the cooperative games, and then work back to find the optimal strategy for the supplier. Formally, we will analyze cooperative games using the core, a solution concept that embodies

Figure 2 **A Branded-Ingredient Game**



[3] Having, or partaking of, two distinct forms (Oxford English Dictionary, 2nd edition).

[4] The authors are grateful to a referee for suggesting this wording.

[5] This example is taken from teaching materials written by Adam Brandenburger and Ken Corts. A well-known instance of a branded-ingredient strategy is the Intel Inside campaign. See Botticelli et al. (1997) for an account; also see footnote 8.

competition among the players (see §4). The cores in the current example are as follows.[6]

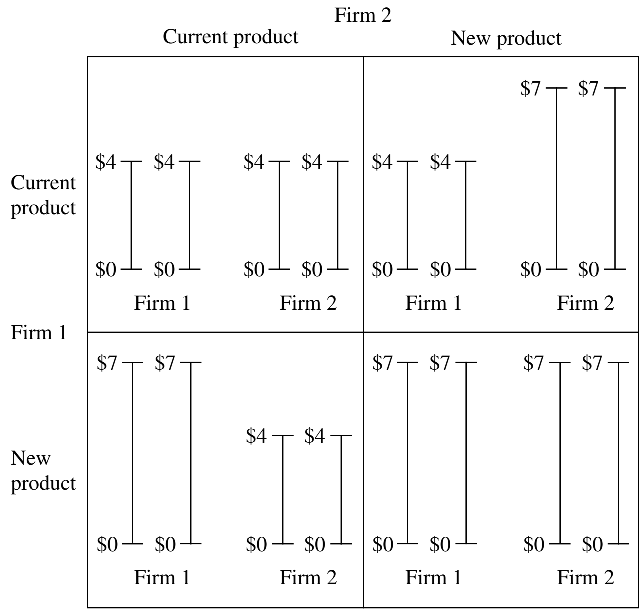
In the status-quo game, a total of $\$9 - \$1 = \$8$ of value will be created. Firm 2 and the buyers will not capture any value. The supplier will receive between \$2 and \$8, and Firm 1 will receive between \$0 and \$6 (where the sum of what the supplier and Firm 1 get must be \$8). This is, of course, the intuitive answer: Competition among buyers ensures that the firms can get their full willingness-to-pay. Firm 2 can then bid up to \$3 for the input from the supplier. Firm 1 has an advantage over Firm 2 (because it commands the higher willingness-to-pay), and so will be the one to secure the input. However, because of the presence of Firm 2, it will have to pay at least \$3 for it. Thus, the supplier gets a minimum of $\$3-\$1=\$2$ of value, and the remaining $\$9 - \$3 = \$6$ is subject to negotiation between the supplier and Firm 1, and this could be split in any way.

The analysis of the branded-ingredient game is very similar: \$8 of value will be created gross of the \$1 upfront cost, or \$7 net. Again, Firm 2 and the buyers will not capture any value. This time, the supplier is guaranteed \$5, and the remaining \$2 of value will be split somehow between the supplier and Firm 1.

We see that paying \$1 to play the branded-ingredient strategy may well be worthwhile for the supplier. For example, if the supplier is cautious about how the "residual pies" will get divided between it and Firm 1, then it will prefer the guaranteed \$5 in the bottom game to the guaranteed \$2 in the top game.

The "aha" of the strategy is that it would not be worthwhile for Firm 2 to pay \$1 to increase willingness-to-pay for its product from \$3 to \$7. It would still be at a competitive disadvantage. However, it is worthwhile for a supplier (at least if cautious) to pay the \$1 to increase this willingness-to-pay and thereby level the playing field. It gains by creating more equal competition among the firms.[7] This may be at least one effect of a branded-ingredient strategy in practice.[8]

Figure 3 **An Innovation Game**



The next example involves strategic moves by more than one player.

Example 2.2 (An Innovation Game). Consider the following game of innovation between two firms. Each firm has a capacity of two units, and (for simplicity) zero unit cost. There are three buyers, each interested in one unit of product. A buyer has a willingness-to-pay of \$4 for the current-generation product, and \$7 for the new-generation product. Each firm has to decide whether to spend \$5 to bring out the new product. The biform game is depicted in Figure 3. (Each vertical bar represents one unit. Again, upfront costs are not shown.)

The cores of the four cooperative games are as follows: In the top-left game, each firm gets \$0 and each buyer gets \$4.[9] (This is intuitive because supply exceeds demand.) In the bottom-left game, Firm 1 gets \$6 gross (and \$1 net of its upfront cost), Firm 2 gets \$0, and each buyer gets \$4. The same answer—with Firms 1 and 2 reversed—holds in the upper-right game. These two cases are a bit more subtle. The core effectively says that what the two firms offer in common is competed away by them to the buyers, but what Firm 1 (respectively, Firm 2) uniquely offers is competed away to it by the buyers. Finally, in the bottom-right game, each firm gets \$0 gross (and thus $-\$5$ net of its upfront cost) and each buyer gets \$7. (Supply again exceeds demand.)

[6] Core calculations for this and subsequent examples can be found in Appendix A, which is provided in the e-companion. An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

[7] In Porter terminology, we could say that the supplier's strategy has reduced buyer power. (Its buyers are the firms, of course.)

[8] Some facts on Intel (taken from Botticelli et al. 1997): In 1990, Intel created its Intel Inside campaign, which reimbursed PC makers for some portion of ad spending in return for their using the Intel Inside logo on PCs and in their ads. By 1993, Intel had spent \$500 million cumulatively on the campaign. In 1994, IBM and Compaq both opted out of Intel Inside. IBM said: "There is one brand, and it's IBM as far as IBM is concerned. We want to focus on what makes IBM computers different, not what makes them the same" (Johnson 1994, p. 52). Compaq rejoined the campaign in 1996; IBM in 1997. (If we take IBM to be like Firm 1 in our example, then we can interpret IBM's leaving the campaign as an attempt to undermine its effectiveness.)

[9] The structure of this cooperative game comes from Postlewaite and Rosenthal (1974).

Figure 4 **Innovation Game Payoffs**

		Firm 2: Current product	Firm 2: New product
Firm 1	Current product	0, 0	0, 1
	New product	1, 0	−5, −5

We see that analysis of the second-stage, cooperative games yields an induced noncooperative game, which is the Battle of the Sexes (Figure 4). Here then, unlike Example 2.1, finding the best strategy reduces to a game problem, not a decision problem. If Firm 1 thinks that Firm 2 will not innovate, then its best strategy is to innovate (and vice versa). Also, both firms may innovate, and lose overall, if each thinks the other will not. Buyers will win in this case.

Examples 2.1 and 2.2 highlight two issues that arise in biform analysis. In Example 2.2, each player gets a unique payoff under the core, whereas in Example 2.1, the core gives some players a range of payoffs. That is, competition is determinate in Example 2.2 and (partially) indeterminate in Example 2.1. In §4, we give the general definition of a biform game and a method of analysis that covers both the determinate and indeterminate cases.

The two examples also differ in terms of efficiency. In Example 2.2, the efficient outcome is when one firm innovates and the other does not. The (net) value created is then $\$7 + \$7 + \$4 - \$5 = \$13$, as opposed to \$12 when neither innovates and \$11 when both innovate. The efficient outcome can therefore arise in this game, at least if we look at the two (pure-strategy) Nash equilibria. By contrast, in Example 2.1 we saw that the supplier might optimally choose the branded-ingredient strategy. This is inefficient—it costs \$1 and does not increase the total (gross) value. In §5, we give a general biform analysis of the efficiency or inefficiency of strategies. We prove that three conditions on a biform game—*adding up*, *no externalities*, and *no coordination*—are sufficient for the outcome to be efficient. (In getting to this result, we also give some additional specific examples of biform games.) This analysis gives a general way of doing a kind of "audit" of business strategies—if modelled as biform games—to investigate their efficiency properties.

In §6, we conclude with a discussion of some conceptual aspects of the paper.

3. Related Models

The biform model is related to the two-stage models that are common in the game-theoretic industrial organization (IO) literature. Typically, these models are purely noncooperative. Firms first make strategic choices that define a resulting noncooperative subgame, in which they then set prices and thereby compete for customers. Shapiro (1989) uses this setup to give a taxonomy of a wide range of IO models. A similar formulation is also central to Sutton's (1991) theory of market structure.

The difference between these models and the biform model is in the treatment of the second stage. We have cooperative rather than noncooperative second-stage games for two related reasons, both mentioned in the introduction. First, the cooperative model directly addresses the question of the power of the different players (implied by the first-stage strategic choices). This is the central question of a number of frameworks in the business-strategy literature. (We have already mentioned Porter 1980, and can add the imitation-substitution-holdup-slack framework of Ghemawat 1991, and the framework in Spulber 2004, Chapter 7, among others.) Second, the cooperative model embodies an "institution-free" concept of competition.[10] This is clear in the examples above. In Example 2.1, we did not say whether the firms set prices that buyers must take or leave, or whether the buyers name prices, nor did we specify how the supplier and the firms interact. The prices we mentioned were the consequences of free-form competition among the players. The same was true of Example 2.2—both the firms and the buyers were free to negotiate.

Of course, these two features of the cooperative approach—the direct analysis of power and the free-form modelling—are related. The free-form modelling ensures that the power of a player comes solely from the structure of the second-stage game, and not from any procedural assumptions. A good (first-stage) strategy is one that creates a favorable (second-stage) structure for that player, in accordance with our conception of business strategy.

Two-stage models are used in the economics of organizations; see, in particular, Grossman and Hart (1986) and Hart and Moore (1990). Both of these papers have a noncooperative first stage and a cooperative second stage, just as we have.[11] However, then the approaches diverge, reflecting the different contexts being studied. In particular, Grossman and Hart (respectively, Hart and Moore) use the Shapley value (respectively, the Nash bargaining solution) to

[10] Aumann (1985, p. 53) writes that "the core expresses the idea of unbridled competition."

[11] To be quite precise, Grossman and Hart (1986) start with a noncooperative second-stage game, from which they then define an associated cooperative game.

say how players who jointly create some value might agree to divide it. The difference is more than a technical one. For example, the core and Shapley value can give very different predictions.[12] The Shapley value (and Nash bargaining solution) always gives a determinate answer, whereas, by definition, the core does so only when competition is determinate.

Next, we briefly mention other work that applies cooperative game theory to business strategy. In an earlier paper (Brandenburger and Stuart 1996), we proposed using cooperative theory—the core in particular—to give foundations for business-strategy frameworks. Stuart (2001) summarizes some applications different from the ones we present in this paper. MacDonald and Ryall (2002, 2004) generate results based on the minimum and maximum values a player can get in the core, and relate these results to business strategy. Lippman and Rumelt (2003) examine differences between cooperative game theory and general- or partial-equilibrium price theory, and discuss the benefits of the first as a basis for business-strategy research. (They consider several cooperative solution concepts apart from the core—the Shapley value, the Nash bargaining solution, and the nucleolus.) Adner and Zemsky (2006) employ some cooperative concepts in analyzing factors affecting the sustainability of competitive advantage. Two recent textbooks on strategy (Oster 1999 and Spulber 2004) use ideas from cooperative theory.[13] Finally, we mention work by Makowski and Ostroy (1994, 1995), which, while formally in the general-equilibrium setting, was a very important influence on this and our earlier paper (Brandenburger and Stuart 1996). (We say more about this connection in §5.)

4. General Formulation

We now give the general definition of a biform game. Some notation: Given a set X, let $\mathcal{P}(X)$ denote the power set of X, i.e., the set of all subsets of X. Also, write $N = \{1, \ldots, n\}$.

Definition 4.1. An *n-player biform game* is a collection

$$(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n),$$

where

(a) for each $i = 1, \ldots, n$, S^i is a finite set;

(b) V is a map from $S^1 \times \cdots \times S^n$ to the set of maps from $\mathcal{P}(N)$ to the reals, with $V(s^1, \ldots, s^n)(\varnothing) = 0$ for every $s^1, \ldots, s^n \in S^1 \times \cdots \times S^n$; and

(c) for each $i = 1, \ldots, n$, $0 \le \alpha^i \le 1$.

The set N is the set of *players*. Each player i chooses a strategy s^i from *strategy set* S^i. The resulting profile of strategies $s^1, \ldots, s^n \in S^1 \times \cdots \times S^n$ defines a *transferable utility* (TU) *cooperative game* with *characteristic function* $V(s^1, \ldots, s^n)$: $\mathcal{P}(N) \to \mathbb{R}$. That is, for each $A \subseteq N$, $V(s^1, \ldots, s^n)(A)$ is the value created by the subset A of players, given that the players chose the strategies $s^1, \ldots, s^n$. (As usual, we require $V(s^1, \ldots, s^n)(\varnothing) = 0$.) Finally, the number α^i is player i's *confidence index*. Roughly speaking, it indicates how well player i anticipates doing in the resulting cooperative games. The precise way the indices α^i are used, and a justification of the indices, are given below.

We note that the strategy sets $S^1, \ldots, S^n$ can, of course, come from a general extensive-form game, so that the definition of a biform game is certainly not restricted to a simultaneous-move first-stage game.[14]

Write $S = S^1 \times \cdots \times S^n$, with typical element s.

Definition 4.2. A biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ will be called *inessential* (respectively, *superadditive*) if the cooperative game $V(s)$ is inessential (respectively, superadditive) for each $s \in S$.[15]

The biform model is a strict generalization of both the strategic-form noncooperative and TU cooperative game models. Formally, write an n-player strategic-form noncooperative game as a collection $(S^1, \ldots, S^n; \pi^1, \ldots, \pi^n)$, where the sets S^i are as above and, for each i, player i's payoff function π^i maps S to the reals. The following two remarks are then immediate:

Remark 4.1. Fix confidence indices $\alpha^1, \ldots, \alpha^n$. There is a natural bijection between the subclass of n-player biform games $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ that are inessential and superadditive, and the class of n-player strategic-form noncooperative games $(S^1, \ldots, S^n; \pi^1, \ldots, \pi^n)$.

Remark 4.2. Fix confidence indices $\alpha^1, \ldots, \alpha^n$. There is a natural bijection between the subclass of n-player biform games $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ in which the sets S^i are singletons, and the class of n-player TU cooperative games.

We now turn to the analysis of a biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$, and adopt the following procedure:

(4.1) For every profile $s \in S$ of strategic choices and resulting cooperative game $V(s)$,

(4.1.1) compute the core of $V(s)$,[16] and, for each player $i = 1, \ldots, n$,

[12] A case in point is monopoly, where the seller chooses capacity in the first stage, and the second stage is a cooperative game between seller and buyers. See Stuart (2006).

[13] For an application to the field of operations management, see Anupindi et al. (2001), who employ a hybrid noncooperative-cooperative model similar to a biform game.

[14] Also, incomplete information about the characteristic functions V could be incorporated (along standard lines) by adding first-stage moves by nature that (along with the players' choices $s^1, \ldots, s^n$) determine $V(s^1, \ldots, s^n)$.

[15] See, e.g., Owen (1995, pp. 145–147) for definitions of inessential and superadditive games.

[16] We assume that for each $s \in S$, the core of $V(s)$ is nonempty. See §6.

(4.1.2) calculate the projection of the core onto the ith coordinate axis, and

(4.1.3) calculate the α^i: $(1-\alpha^i)$ weighted average of the upper and lower endpoints of the projection.[17]

(4.2) For every profile $s \in S$ of strategic choices, and each player $i = 1, \ldots, n$,

(4.2.1) assign to i a payoff equal to i's weighted average as in (4.1.3) above, and

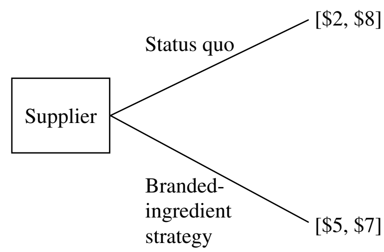
(4.2.2) analyze the resulting strategic-form noncooperative game.

Given a profile of strategic choices $s \in S$, the first step is to restrict attention to core allocations in the resulting cooperative game $V(s)$ (Step 4.1.1). Second, we calculate the implied range of payoffs to each player i (Step 4.1.2). Third, each player i uses confidence index α^i to evaluate the given cooperative game $V(s)$ as a weighted average of the largest and smallest amounts of value that i can receive in the core (Step 4.1.3). Use of the confidence indices reduces a biform game to a strategic-form noncooperative game (Step 4.2.1). This game may now be analyzed in standard fashion—say, by computing Nash equilibria, iteratively eliminating dominated strategies, or some other method (Step 4.2.2).

Conceptually, this method of analysis starts by using the core to calculate the effect of competition among the players at the second stage of the game—i.e., given the strategic choices made in the first stage. This determines how much value each player can capture. The core might be a single point (as in Example 2.2). If so, competition fully determines the division of value. However, there may also be a range of values in the core, so that competition alone is not fully determinate, and (at least some) players face a "residual" bargaining problem. For instance, in Example 2.1, competition narrowed down the outcomes, but left a range of value to be negotiated between the supplier and Firm 1. In the status-quo game, the range for the supplier was [\$2, \$8]; in the branded-ingredient game it was [\$5, \$7]. The supplier's decision problem was then a choice between ranges of payoffs rather than single payoffs. (See Figure 5.) In general, under some basic axioms, the players' preferences over such intervals can be represented by confidence indices. An optimistic player i will have a confidence index α^i close to one, indicating that player i anticipates capturing most of the value to be divided in the residual bargaining. A pessimistic player i will have an α^i close to zero, indicating that player i anticipates getting little of this residual value. Thus, a player's confidence index represents a view of the game—optimistic if the confidence index is large, and pessimistic if the confidence index is small.

[17] Note that the projection is a closed bounded interval of the real line.

Figure 5 **Branded-Ingredient Game–Payoff Ranges**



(In Example 2.1, the supplier would choose the branded-ingredient strategy if $\alpha 7 + (1-\alpha)5 > \alpha 8 + (1-\alpha)2$, or $\alpha < 3/4$, so the supplier would choose this strategy unless it was very optimistic.)

Use of the confidence indices gives us an induced noncooperative game among the players. Analysis of this game indicates which strategies the players will choose. Let us repeat that we do not insist on a particular solution concept here. We can use Nash equilibrium, or some other concept.

A final comment on the formulation: Appendix B in the e-companion gives axioms on a player's preferences over intervals of outcomes, from which the confidence index is derived. The axioms are *order*, *dominance*, *continuity*, and *positive affinity*. The first three axioms are standard. The fourth is what accounts for the specific weighted-average representation, but we show that this axiom is implied by our cooperative game context, so the structure of a biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ is a consistent whole. In §6, we comment further on how the confidence indices represent the players' subjective views of the game.

5. Efficiency and Business Strategy

We now give more examples of biform games, beyond those in §2. The examples, like the earlier ones, show how to analyze business-strategy ideas using a biform game. They also lead to a general analysis of the efficiency and inefficiency of business strategies.

We start with some notation and a definition. Write $N\backslash\{i\}$ for the set $\{1, \ldots, i-1, i+1, \ldots, n\}$ of all players except i.

Definition 5.1. Fix a biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ and a strategy profile $s \in S$. Then, the number

$$V(s)(N) - V(s)(N\backslash\{i\})$$

is called player i's *added value* in the cooperative game $V(s)$.

In words, player i's added value is the difference, in the given cooperative game, between the overall value created by all the players and the overall value created by all the players except i. It is, in this sense, what player i adds to the value of the game. This is the standard cooperative-game concept of marginal contribution. Here we use the added value terminology from our previous paper (Brandenburger and Stuart 1996).[18]

The following well-known observation is immediate, and will be used below.[19]

Lemma 5.1. *Fix a biform game* $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$, *a strategy profile* $s \in S$, *and suppose that the core of* $V(s)$ *is nonempty. Then, if*

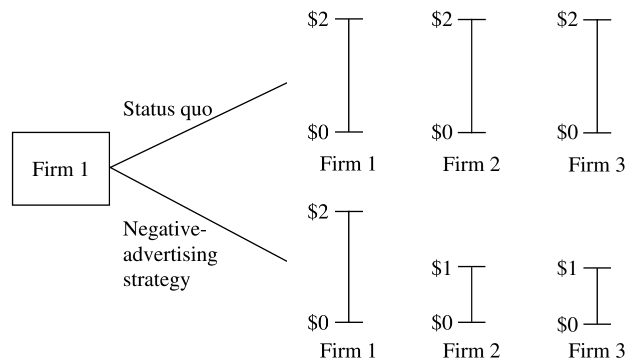
$$\sum_{i=1}^{n}[V(s)(N) - V(s)(N\backslash\{i\})] = V(s)(N),$$

the core of $V(s)$ *consists of a single point, in which each player* i *receives* $V(s)(N) - V(s)(N\backslash\{i\})$.

Proof. Given an allocation $x \in \mathbb{R}^n$ and $T \subseteq N$, write $x(T) = \sum_{j\in T} x^j$. Fix some i, and note that the core conditions $x(N) = V(s)(N)$ and $x(N\backslash\{i\}) \geq V(s)(N\backslash\{i\})$ imply that $x^i \leq V(s)(N) - V(s)(N\backslash\{i\})$. Summing over i yields $x(N) \leq \sum_{i\in N}[V(s)(N) - V(s)(N\backslash\{i\})]$, with strict inequality if $x^i < V(s)(N) - V(s)(N\backslash\{i\})$ for some i. However, the right side is equal to $V(s)(N)$ by assumption, so strict inequality implies $x(N) < V(s)(N)$, a contradiction. □

Example 5.1 (A Negative-Advertising Game). Here we look at a "generic" strategy (Porter 1980), with a difference. Consider the biform game depicted in Figure 6.[20] There are three firms, each with one unit to sell at $0 cost. There are two buyers, each interested in one unit of product from some firm. Firm 1 alone has a strategic choice, which is whether or not to engage in negative advertising. If it does not, then each buyer has a willingness-to-pay of $2 for each firm's product. If it does, then willingness-to-pay for its own product is unchanged, but that for Firm 2's and Firm 3's products falls to $1. (The negative advertising has hurt the image of Firms 2 and 3 in the eyes of the buyers.)

Figure 6 **A Negative-Advertising Game**



Along the status-quo branch, the overall value is $4. Each firm's added value is $0; each buyer's added value is $2. By Lemma 5.1, each firm will get $0 in the core, and each buyer will get $2. (This is the intuitive answer because the firms are identical, and supply exceeds demand.) Along the negative-advertising branch, the overall value is $3. Firm 1's added value is $1, Firm 2's and Firm 3's added values are $0, and each buyer's added value is $1. Again using Lemma 5.1, Firms 2 and 3 will get $0 in the core, and Firm 1 and each of the buyers will get $1. (Similar to Example 2.2, the core says that what the three firms offer in common is competed away to the buyers, while what Firm 1 uniquely offers is competed away to it by the two buyers.)

Firm 1 will optimally choose the negative-advertising example, thereby capturing $1 versus $0. This is the opposite of a differentiation strategy, where a firm creates added value for itself by raising willingness-to-pay for its product (perhaps at some cost to it). Here, Firm 1 creates added value by lowering willingness-to-pay for its rivals' products. Note that the strategy is inefficient: It shrinks the overall value created from $4 to $3.

Example 5.2 (A Coordination Game). There are three players, each with two strategies, labelled No and Yes. Player 1 chooses the row, Player 2 chooses the column, and Player 3 chooses the matrix. Figure 7 depicts the cooperative game associated with each strategy profile, where the value of all one-player subsets is taken to be zero.[21] This example can be thought of as a model of switching from an existing technology standard (the strategy No) to a new standard (the strategy Yes). The new technology costs $1 more per player, and is worth $2 more per player, provided at least two players adopt it.[22]

Using Lemma 5.1, it is easy to check that in each of the four cooperative games, the core will give the players exactly their added values. We get the induced noncooperative game in Figure 8, which is a kind of coordination game. There are two (pure-strategy) Nash equilibria: (No, No, No) and (Yes, Yes, Yes). The first is inefficient (the total value is $6), while the second is efficient (the total value is $9).

We now show that Examples 5.1 and 5.2, together with Example 2.1, actually give a complete picture,

[18] We do so for the same reason as there: One frequently sees the term "added value" in business-strategy writing, and the usage fits with the formal definition above.

[19] See, e.g., Moulin (1995, Remark 2.3).

[20] Different formalisms aside, the example is essentially the same as Example 1 in Makowski and Ostroy (1994), which they kindly say was prompted by discussions with one of us.

[21] Properly, we should write $V(\text{No}, \text{No}, \text{No})(N) = 6$, $V(\text{No}, \text{No}, \text{No})(\{1, 2\}) = 4$, etc. However, the cell identifies the strategy profile, so we use the simpler notation in the matrix.

[22] For more detail on this interpretation of the characteristic functions, see Appendix A in the e-companion.

Figure 7 **A Coordination Game**

No

	No	Yes
No	$v(N)=6$ $v(1,2)=4$ $v(2,3)=4$ $v(3,1)=4$	$v(N)=5$ $v(1,2)=3$ $v(2,3)=3$ $v(3,1)=4$
Yes	$v(N)=5$ $v(1,2)=3$ $v(2,3)=4$ $v(3,1)=3$	$v(N)=6$ $v(1,2)=6$ $v(2,3)=3$ $v(3,1)=3$

Yes

	No	Yes
No	$v(N)=5$ $v(1,2)=4$ $v(2,3)=3$ $v(3,1)=3$	$v(N)=6$ $v(1,2)=3$ $v(2,3)=6$ $v(3,1)=3$
Yes	$v(N)=6$ $v(1,2)=3$ $v(2,3)=3$ $v(3,1)=6$	$v(N)=9$ $v(1,2)=6$ $v(2,3)=6$ $v(3,1)=6$

within the biform model, of how strategies can cause inefficiencies. (Example 2.2 will also fit in.) We start with some definitions. As usual, we write S^{-i} for $S^1 \times \cdots \times S^{i-1} \times S^{i+1} \times \cdots \times S^n$.

Definition 5.2. A biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ satisfies *adding up* (AU) if for each $s \in S$,

$$\sum_{i=1}^{n}[V(s)(N) - V(s)(N\backslash\{i\})] = V(s)(N).$$

The game satisfies *no externalities* (NE) if for each $i = 1, \ldots, n$, $r^i, s^i \in S^i$, and $s^{-i} \in S^{-i}$,

$$V(r^i, s^{-i})(N\backslash\{i\}) = V(s^i, s^{-i})(N\backslash\{i\}).$$

The game satisfies *no coordination* (NC) if for each $i = 1, \ldots, n$, $r^i, s^i \in S^i$, and $r^{-i}, s^{-i} \in S^{-i}$,

$$V(r^i, r^{-i})(N) > V(s^i, r^{-i})(N) \quad \text{if and only if } V(r^i, s^{-i})(N) > V(s^i, s^{-i})(N).$$

The AU condition says that in each second-stage cooperative game, the sum of the players' added values is equal to the overall value created in that game. (This is just the condition of Lemma 5.1 applied to each second-stage game.) The NE condition says that each player's strategic choice does not affect the value that the remaining players can create (without that player). The NC condition says that when one player switches strategy, the sign of the effect on the overall value created is independent of the other players' strategic choices.

We need two additional (obvious) definitions: A profile of strategies $s \in S$ in a biform game will be called a (*pure-strategy*) *Nash equilibrium* if it is a (pure-strategy) Nash equilibrium of the induced noncooperative game. (This is the noncooperative game induced as in §4. That is, given a profile $r \in S$, we assign each player i a payoff equal to the α^i: $(1-\alpha^i)$ weighted average of the upper and lower endpoints of the projection onto the ith coordinate axis of the core of the cooperative game $V(r)$.) A strategy profile $s \in S$ will be called *efficient* if it solves $\max_{r \in S} V(r)(N)$.[23]

We can now state and prove two results on efficiency in biform games.

Lemma 5.2. *Consider a biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ satisfying AU and NE, and that for each $r \in S$, the game $V(r)$ has a nonempty core. Then, a strategy profile $s \in S$ is a Nash equilibrium if and only if*

$$V(s)(N) \geq V(r^i, s^{-i})(N) \tag{1}$$

for every $r^i \in S^i$.

Proof. By Lemma 5.1, AU implies that for each $r \in S$, the core of $V(r)$ consists of a single point, in which each player i receives $V(r)(N) - V(r)(N\backslash\{i\})$. Therefore, a profile s is a Nash equilibrium iff for each i,

$$V(s)(N) - V(s)(N\backslash\{i\}) \geq V(r^i, s^{-i})(N) - V(r^i, s^{-i})(N\backslash\{i\}) \tag{2}$$

for every $r^i \in S^i$. However, NE implies that

$$V(s)(N\backslash\{i\}) = V(r^i, s^{-i})(N\backslash\{i\})$$

for every $r^i \in S^i$. Thus, inequality (2) holds if and only if inequality (1) holds. □

Proposition 5.1. *Consider a biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ satisfying AU, NE, and NC, and that for each $r \in S$, the game $V(r)$ has a nonempty core. Then, if a strategy profile $s \in S$ is a Nash equilibrium, it is efficient.*

Proof. Write

$$\begin{aligned} V(s)(N) - V(r)(N) &= V(s^1, r^2, \ldots, r^n)(N) - V(r)(N) \\ &\quad + V(s^1, s^2, r^3, \ldots, r^n)(N) - V(s^1, r^2, r^3, \ldots, r^n)(N) \\ &\quad + \cdots + V(s)(N) - V(s^1, \ldots, s^{n-1}, r^n)(N). \end{aligned}$$

NC and inequality (1) in Lemma 5.2 together imply that each pair of terms on the right-hand side of this equation is nonnegative, from which $V(s)(N) \geq V(r)(N)$. □

Figure 8 **Coordination Game Payoffs**

No

	No	Yes
No	2, 2, 2	2, 1, 2
Yes	1, 2, 2	3, 3, 0

Yes

	No	Yes
No	2, 2, 1	0, 3, 3
Yes	3, 0, 3	3, 3, 3

[23] An efficient profile always exists because we are assuming that the strategy sets S^i are finite.

Proposition 5.2. *Consider a biform game* $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ *satisfying AU and NE, and that for each* $r \in S$, *the game* $V(r)$ *has a nonempty core. Then, if a strategy profile* $s \in S$ *is efficient, it is a Nash equilibrium.*

Proof. We are given that $V(s)(N) \geq V(r)(N)$ for every $r \in S$, so certainly inequality (1) in Lemma 5.2 holds. Thus, the profile s is a Nash equilibrium. □

Propositions 5.1 and 5.2 provide, respectively, conditions for any Nash-equilibrium profile of strategies to be efficient and for any efficient profile of strategies to be a Nash equilibrium. They can thus be viewed as game-theoretic analogs to the first and second welfare theorems of general-equilibrium theory. Both Propositions 5.1 and 5.2 are closely related to results in Makowski and Ostroy (1994, 1995), as we will discuss below. First, we give some intuition for the results, and also tie them back to our earlier examples of business strategies.

For some intuition, note that Lemma 5.2 shows that AU and NE ensure a kind of "social alignment." Under AU and NE, a player's payoff increases if and only if the overall value created (the "pie") increases. In the presence of such social alignment, NC then ensures that there is a unique equilibrium.[24] (If either AU or NE does not hold, then there can be multiple equilibria even under NC.)

Next, back to our earlier examples: Table 1 summarizes which of the conditions—AU, NE, and NC—are satisfied in Examples 2.1, 5.1, and 5.2.

Start with Example 2.1 (the branded-ingredient game). In the status-quo second-stage game, the supplier has an added value of \$8, and Firm 1 has an added value of \$6 (Firm 2 and the buyers have zero added value). The overall value is \$8, so AU fails. (We could equally have looked at the branded-ingredient second-stage game.) NE holds: Only the supplier has a strategic choice, and the value created without the supplier is constant—at \$0—regardless of which strategy the supplier follows. Finally, it is easy to see that NC is automatically satisfied in any game where only one player has a strategic choice.[25] We conclude that the possible inefficiency in this game (which we already noted in §2) comes from the failure of AU. In plain terms, there is a bargaining problem between the supplier and Firm 1 over the \$6 of value that must be divided between them. To do better in this bargaining, the supplier may well adopt a strategy (the branded-ingredient strategy) that decreases the pie.

In Example 5.1 (the negative-advertising game), it is immediate from our earlier analysis that AU is satisfied. NC is satisfied for the same reason as in Example 2.1 (only Firm 1 has a strategic choice). However, NE fails: The value created by Firm 2, Firm 3, and the two buyers is \$4 when Firm 1 chooses the status-quo strategy, but changes to \$2 when Firm 1 chooses the negative-advertising strategy. Again, the outcome is inefficient.

Table 1 Conditions Satisfied in Examples

	Adding up	No externalities	No coordination
Branded-ingredient game	×	✓	✓
Negative-advertising game	✓	×	✓
Coordination game	✓	✓	×

Finally, in Example 5.2 (the coordination game), AU is satisfied. So is NE, as the reader can check from Figure 7. However, NC fails: For example, when Player 1 changes strategy from No to Yes, the overall value falls from \$6 to \$5 when Players 2 and 3 are both playing No, but rises from \$6 to \$9 when Players 2 and 3 are both playing Yes. There is an inefficient Nash equilibrium (No, No, No). Note that the efficient profile (Yes, Yes, Yes) is also a Nash equilibrium, as Proposition 5.2 says it must be.

We see how the general results enable us to identify the source of the inefficiency of the business strategy in each of our examples. We also know that if a biform model of a business strategy satisfies our three conditions, then we will get efficiency. Of course, our examples are meant only to be suggestive of how one might model many other business strategies as biform games. However, we now have a general way of doing a kind of "audit" of any business strategy—if modelled as a biform game—to discover its efficiency properties.[26]

Here is one more example, just to "redress the balance," because we have given only one example of efficiency so far (Example 2.2). This time, AU, NE, and NC will all hold.

Example 5.3 (A Repositioning Game). Consider the biform game depicted in Figure 9. There are three firms, each with one unit to sell. There are two identical buyers, each interested in one unit of product from some firm. Under the status quo, the firms have the costs, and the buyers have the willingness-to-pay numbers, on the upper branch. Firm 2 has the possibility of repositioning, as shown by its vertical bar on the lower branch. Specifically, it can spend \$1 to raise "quality" (willingness-to-pay) and lower cost as shown.

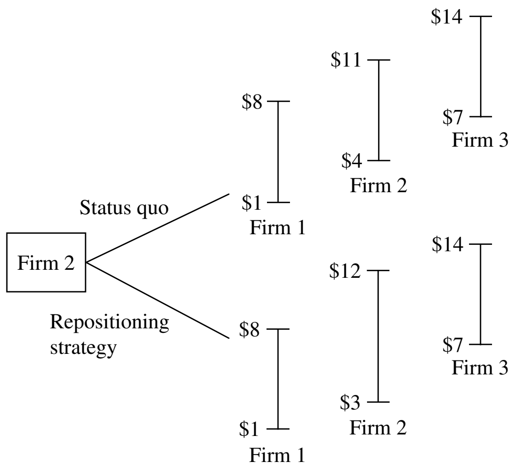
Along the status-quo branch, the overall value is \$14. Each firm's added value is \$0; each buyer's added value is \$7. Along the repositioning branch, the overall value is \$15 (\$16 minus the \$1 repositioning

[24] The authors are grateful to a referee for providing this intuition.

[25] This is what we want, of course. The possibility of inefficiency due to coordination issues should arise only when at least two players have (nontrivial) choices.

[26] In Brandenburger and Stuart (2006), we use the same efficiency analysis to frame some of the ideas in the corporate-strategy literature.

Figure 9 **A Repositioning Game**



cost). Firm 2's added value is \$1; Firm 1's and Firm 3's added values are \$0; each buyer's added value is \$7. AU is satisfied, as clearly are NE and NC. By Proposition 5.1, we know that Firm 2 must optimally make the efficient choice of the lower branch, as indeed it will, to net \$1.

This is a simple biform model of a (re)positioning strategy. Note that on the lower branch, Firm 2 still does not have either the lowest cost or the highest quality among the three firms. However, it does command the largest gap between quality and cost. This is the source of its added value.

Some further comments on Propositions 5.1 and 5.2: First, note that Table 1 establishes the independence of the AU, NE, and NC conditions. It is possible for any two to hold, but not the third.[27] Next, we emphasize that the efficiency conditions in Proposition 5.1 are sufficient, not necessary. For instance, in Example 2.2, the (pure) Nash equilibria were efficient (as we noted), but it is easily checked that AU and NC fail (NE holds).[28] This said, the conditions do seem "tight." To maintain sufficiency, none of them can be left out in Proposition 5.1, as our examples showed, and we do not see how they could be weakened in general.

Finally, we note the considerable debt we owe to Makowski and Ostroy (1994, 1995). Makowski and Ostroy (henceforth M-O) present welfare theorems to which our Propositions 5.1 and 5.2 are closely related. The M-O model is a reformulation of general-equilibrium theory, in which the agents make choices ("occupational" choices in their terminology) that define the subsequent economy in which they then produce and trade. It therefore has a two-stage structure, just as our purely game-theoretic model does. There are some differences. We define added value (marginal contribution) in the standard game-theoretic way, while M-O's definition effectively assumes that NE holds. Our NC condition is weaker than the corresponding condition in M-O (they call it "no complementarities"), ruling out coordination issues.[29]

6. Discussion of the Model

The goal of this paper is to combine the virtues of both branches of game theory—the noncooperative and the cooperative—into a hybrid model that could be used to analyze business strategies. We conclude in this section with comments on some conceptual aspects of the model.

6.1. Emptiness of the Core

We have used the core to analyze the various cooperative games that result from the players' strategic choices. We did this because we wanted to capture the idea of free-form competition among the players. What if the core is empty?

First, note that there are various important classes of TU cooperative games known to have nonempty cores. Examples of such classes that are of economic interest include the "market games" and "assignment games" of Shapley and Shubik (1969, 1971). Stuart (1997) proves nonemptiness of the core for a class of three-way assignment games (think of suppliers, firms, and buyers) that satisfy a local additivity condition.[30]

Next, we observe that a biform model can restore nonemptiness to at least one scenario that is known to produce an empty core when modelled purely cooperatively. The point is essentially made by Example 2.2. Consider the following cooperative game: There are two firms, each with capacity of two units, and zero unit (marginal) cost. There are three buyers, each interested in one unit of product, and each with a willingness-to-pay of \$4. If a firm is "active," then it incurs a fixed cost of $\$\varepsilon$, for some small $\varepsilon > 0$. (Formally, the value of any subset containing that firm and one or more buyers is reduced by $\$\varepsilon$.) This game has an empty core. (Telser 1994 contains other examples of empty cores.) However, there is a natural biform model of the scenario with nonempty cores: Make paying the $\$\varepsilon$ cost a first-stage strategic choice for each firm, and write down the resulting

[27] It is easy to prove that if in a two-player game AU and NE are satisfied, then so is NC. This is why we needed three players to get a failure of NC alone (Example 5.2).

[28] The same example shows that it is also true that the conditions of Proposition 5.2 are not necessary. Both efficient profiles are Nash equilibria, despite the failure of AU.

[29] Brandenburger and Stuart (2006) contains more details on the relationship to M-O.

[30] Example 2.1 is a three-way assignment game, and Examples 5.1 and 5.3 are two-way assignment games.

Figure 10 Battle of the Sexes

		Firm 2: Don't pay	Firm 2: Pay $\$\varepsilon$
Firm 1	Don't pay	0, 0	0, 8−ε
	Pay $\$\varepsilon$	8−ε, 0	−ε, −ε

second-stage cooperative games. (If a firm does not pay $\$\varepsilon$ in the first stage, then it does not increase the value of any subset at the second stage.) Similarly to Example 2.2, the cores of the second-stage games will be singletons (in particular, nonempty!), and we will get an induced noncooperative game that is the battle of the sexes (Figure 10).

Arguably, finding an empty core in a model is a "positive" finding, telling us something important about the instability of the situation being studied. This seems true of the example of emptiness above, although we also showed a way to avoid emptiness. Interestingly, in our biform resolution, the instability of the empty core is replaced by a kind of strategic instability: Each firm will want to be the one to pay $\$\varepsilon$, hoping the other will not, and thereby netting $\$(8-\varepsilon)$. However, both might end up losing overall.

In sum, there are important classes of TU games that have nonempty cores. The biform model may enable us to circumvent the emptiness of the core in at least some other cases. (We just gave one example of this.) Finally, emptiness of the core may be a valuable insight in any case. This last possibility does raise the interesting question of how players at the first stage of a biform game might evaluate a second-stage game with an empty core. We do not have an answer at present, and believe that this question merits further study.

6.2. Efficiency

A criticism that is sometimes made of cooperative game theory is that it presumes efficiency. All potential value is created. This criticism is moot if the biform model is used. The biform model does incorporate what might be called conditional efficiency: Given a profile $s \in S$ of strategic choices, use of the core says that all of the value $V(s)(N)$ that can then be created will, in fact, be created. However, overall efficiency would require that the strategy profile s that the players actually choose must maximize $V(s)(N)$, i.e., that the profile be efficient as defined in §5. We have seen several times in the paper that this need not be so. The biform model permits inefficiency.

6.3. Externalities

Another longstanding issue in cooperative game theory is how to deal with situations where the value created by a subset A of players—call it $v(A)$—may depend on what players outside A do. Prima facie, $v(A)$ does not depend on what the players outside A do, for the simple reason that a cooperative model has no concept of action or strategy in it. The approach in von Neumann and Morgenstern (1944) was to start with a noncooperative game, which does have actions, of course, and to define an induced cooperative game from it. They then had to make a specific assumption about how the players outside A would behave. (They assumed minimax behavior.[31]) The biform model avoids this difficulty by simply positing that each strategy profile in the first-stage noncooperative game leads to a different, and independently specified, second-stage cooperative game. The quantity $v(A)$ is then a function of the first-stage strategic choices of all the players—including, in particular, the choices of players outside A. However, this dependence is without prejudice, so to speak. There is no built-in assumption of minimax or any other kind of behavior.[32] Of course, the biform model also allows for the special case in which $v(A)$ does not depend on what the players outside A do. This is precisely the situation of NE, as in Definition 5.2.[33]

6.4. Interpretation of the Confidence Index

The biform model makes no prediction about how the value created at the second stage will be divided, beyond saying it must be in accordance with the core. It does say that players form views, as represented by their confidence indices, as to how much value they will get.

Sometimes the confidence indices will play a small role. In particular, under AU, the core consists of a unique point (Lemma 5.1), in which case the division of value is independent of the indices. In Example 2.1, the supplier's confidence index played a big role—determining whether it played the status-quo or branded-ingredient strategy.

[31] Define $v(A)$ to be the maximin payoff to A in the two-person zero-sum game in which the players are A and the complement of A, and the payoff to A from a pair of strategies is the sum of the payoffs to the players in A from those strategies. In effect, von Neumann and Morgenstern (1994) imagine that the players in A "assume the worst" about the players outside A.

[32] Zhao (1992) and Ray and Vohra (1997) have interesting alternative proposals for how to make what the players in subset A can achieve depend in a "neutral" way on what the complementary players do. Unlike us, they derive cooperative games from noncooperative games. Also, their models are one stage, not two stage.

[33] That definition was restricted to subsets $A = N\backslash\{i\}$. However, it generalizes in the obvious way to other subsets—simply require that $V(s)(A)$ be independent of the s^j for $j \notin A$.

We can think of the confidence indices as describing the players' views on how good they think they are at dealing with negotiation "intangibles" like persuasion, bluffing, holding out, etc. These are subjective views, not necessarily correct in any sense. The actual outcome of the game for player i might be quite different from an α^i: $(1-\alpha^i)$ weighted average of the upper and lower endpoints of the core projection. Thus, in Example 2.1, the supplier might forego the branded-ingredient strategy, anticipating doing very well in the bargaining with Firm 1, and then end up with less than \$5, the minimum it would get under the branded-ingredient strategy. Appendix B in the e-companion formalizes this foundation for the confidence indices: Proposition B1 gives a characterization of the confidence index in terms of a player's (subjective) preferences over intervals of outcomes.

We emphasize that because the confidence indices are subjective, they may or may not be mutually consistent, in the following sense. Fix a second-stage game. Assign to each player i an α^i: $(1-\alpha^i)$ weighted average of the upper and lower endpoints of the corresponding core projection. Of course, the resulting tuple of points may or may not itself lie in the core. If it does for each second-stage game, we can say that the players' confidence indices are "mutually consistent." If not, they are "mutually inconsistent," and in at least one second-stage game, one or more players would definitely end up with an outcome different from that anticipated. Appendix B shows how mutual consistency can be achieved, if desired, but we stress that there is no logical or conceptual difficulty with the inconsistent case, which is also natural given the subjectivity of the players' views of the game.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

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