24.10.08 Mithril MSP and Benchmark

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1 Mithril MSP

There are two versions of MSP algorithms in the Mithril paper. We omitted the check of MSP-PoP(a multisignature based on Boneh Lynn Shacham(BLS) signatureswith proofs of possession), since this part will not be included in the relations.

The first one is $\mathsf{MSP.AKey}(\mathbf{mvk})$, and $\mathsf{MSP.ASig}(\boldsymbol{\sigma})$:

- $\mathsf{MSP.AKey}(\mathbf{mvk})$: Takes a vector $\mathbf{mvk}$ of (previously checked) verification keys and returns an intermediate aggregate public key $ivk = \prod mvk_i$
- $\mathsf{MSP.ASig}(\boldsymbol{\sigma})$: Takes as input a vector $\boldsymbol{\sigma}$ and returns $\mu \leftarrow \prod_1^d \sigma_i$,

The second one is is $\mathsf{MSP.BKey}(\mathbf{mvk}, \boldsymbol{e_\sigma})$, and $\mathsf{MSP.BSig}(\boldsymbol{\sigma})$:

- $\mathsf{MSP.BKey}(\mathbf{mvk}, \boldsymbol{e_\sigma})$: Takes a vector $\mathbf{mvk}$ of (previously checked) verification keys and weighting seed $\boldsymbol{e_\sigma}$, and returns an intermediate aggregate public key $ivk = \prod mvk_i^{e_i}$, where $e_i \leftarrow \mathrm{H}(i, e_\sigma)$.
- $\mathsf{MSP.BSig}(\boldsymbol{\sigma})$: Takes as input a vector of signatures $\boldsymbol{\sigma}$ and returns $(\mu, \boldsymbol{e_\sigma})$ where $\mu \leftarrow \prod \sigma_i^{e_i}$, where $e_i \leftarrow \mathrm{H}_\lambda(i, e_\sigma)$ and $e_\sigma \leftarrow \mathrm{H}_p(\sigma)$.

The reason why Mithril use $\mathsf{MSP.BKey}(\mathbf{mvk}, \boldsymbol{e_\sigma})$ and $\mathsf{MSP.BSig}(\boldsymbol{\sigma})$ is that:

which enforce more stringent checking than that of standard multisignatures by utilizing the short random exponent batching of Bellare et al. [5]. The difference from standard multisignature aggregation (via MSP.AKey *and* MSP.ASig*), is that the randomized check will fail with overwhelming probability if any of the individual signatures is invalid, whereas standard aggregation allows for spurious individual signatures as long as they sum up to the correct aggregate. Furthermore,* MSP.BKey *uses a weighting seed* $e_{\boldsymbol{\sigma}}$ *as input; in practice this is produced by the signature set to be verified and cannot be run ahead of time. In our use case, this can be overcome by having* MSP.BKey *be evaluated inside a proof system.*

2 Relations for Mithril(latest version)

Here we post the latest version of Mithril relations(which we plan to prove in SNARK). It is somewhat different from the initial version:

- $ivk = \mathsf{MSP.BKey}(\mathbf{mvk}, \boldsymbol{e_\sigma})$ and $ivk_{\mathsf{body}} = \mathsf{MSP.AKey}(\mathbf{mvk})$
- $(\mu, e_{\boldsymbol{\sigma}}) = \mathsf{MSP.BSig}(\boldsymbol{\sigma})$
- $\forall i : \mathsf{index}_i \leq m$ and $\forall i \neq j : \mathsf{index}_i \neq \mathsf{index}_j$.
- For $i \in \{1 \dots k\}$: $(mvk_i, \mathsf{stake}_i)$ lies in Merkle tree AVK, N following path $\boldsymbol{p}_i$.
- For $i \in \{1 \dots k\}$: $\mathsf{MSP.Eval}(\mathsf{topic}, \mathsf{index}_i, \sigma_i) = ev_i$.
- For $i \in \{1 \dots k\}$: $ev_i \leq \phi(\mathsf{stake}_i)$.

Concretely, statements are $x = (\mathsf{AVK}, ivk, ivk_{\mathsf{body}}, \mu, e_{\boldsymbol{\sigma}}, \mathsf{mesg})$ and witnesses are of the form $w = (mvk_i, \mathsf{stake}_i, p_i, ev_i, \sigma_i, \mathsf{index}_i)$ for $i \in \{1 \dots k\}$.

Note that we implemented the $\mathsf{MSP.vers}(msghash, ivk, \mu)$ in the circuit, so the actually relations will be different from the paper. But we can adjust it for free.

3 MSP.B Implementation Details

Here we give the details of MSP.BKey function.

For $\mathsf{MSP.BKey}(\mathbf{mvk}, \boldsymbol{e_\sigma})$: Takes a vector $\mathbf{mvk}$ of (previously checked) verification keys and weighting seed $\boldsymbol{e_\sigma}$, and returns an intermediate aggregate public key.

The code is is as follows:

```
// ivk = \sum_{i=0}^{n-1} e_i * mvk_i
let ivk_assigned = self.bls_signature_chip.pairing_chip.load_private_g1(ctx, ivk);
let mvks = pubkeys.iter().map(|pt| self.bls_signature_chip.pairing_chip.load_private_g1(ctx, *pt)).collect::<Vec<_>>();
let products = mvks.iter().zip(e_is.iter()).map(|(mvk, &e_i)| {
    let e_vec = vec![e_i];
    g1_chip.scalar_mult::<G1Affine>(ctx, mvk.clone(), e_vec,254,4)
}).collect::<Vec<_>>();

//println!("ivk_assigned x:{:?}", ivk_assigned.clone().x());
//println!("ivk_assigned y:{:?}", ivk_assigned.clone().y());

let ivk_comp = g1_chip.sum::<G1Affine>(ctx, products);
```

For $\mathsf{MSP.BSig}(\boldsymbol{\sigma})$: Takes as input a vector of signatures $\boldsymbol{\sigma}$ and returns $(\mu, e_{\boldsymbol{\sigma}})$ where $\mu \leftarrow \prod \sigma_i^{e_i}$, where $e_i \leftarrow \mathrm{H}_\lambda(i, e_\sigma)$ and $e_\sigma \leftarrow \mathrm{H}_p(\sigma)$.

The code is is as follows:

```
// isig = \sum_{i=0}^{n-1} e_i * sig_i
let isig_assigned = self.bls_signature_chip.pairing_chip.load_private_g2(ctx, isig);
let sigs = signatures.iter().map(|pt| self.bls_signature_chip.pairing_chip.load_private_g2(ctx, *pt)).collect::<Vec<_>>();
let products = sigs.iter().zip(e_is.iter()).map(|(sig, &e_i)| {
    let e_vec = vec![e_i];
    g2_chip.scalar_mult::<G2Affine>(ctx, sig.clone(), e_vec,254,4)
}).collect::<Vec<_>>();
let isig_comp = g2_chip.sum::<G2Affine>(ctx, products);
```

We use the the method named "scalar_mult" in halo2-lib. The encapsulated code is as follows:

```
/// See [`scalar_multiply`] for more details.
pub fn scalar_mult<C>(
    &self,
    ctx: &mut Context<F>,
    P: EcPoint<F, FC::FieldPoint>,
    scalar: Vec<AssignedValue<F>>,
    max_bits: usize,
    window_bits: usize,
) -> EcPoint<F, FC::FieldPoint>
where
    C: CurveAffineExt<Base = FC::FieldType>,
{
    scalar_multiply::<F, FC, C>(self.field_chip, ctx, P, scalar, max_bits, window_bits)
}
```

We use the parameters of this function as follows:

- max_bits: 254. Since we work on the curve BN254, the poseidon function is also on BN254, so the output of hash function is a field element of BN254. We set this to 254 bits, which means the max vlaue of scalar is less than 2^{254}.
- window_bits: 4. We set the window size of multi-scalar multiplication to 4. Usually this value is set to 2 or 4. Different settings may result in different proving efficiencies, which will be tested later.

4 MSP Benchmark

4.1 MSP.A Benchmark

It is basically a BLS multi-signature, we have benched before, here is the result:

Note: The configuration with Degree , Advice , Lookup , Fixed , Lookup Bits , Limb Bits , Num Limbs is 17, 25, 3, 1, 16, 88, and 3, respectively.

Num Aggregation	Proof Time	Proof Size	Verify Time
2	17.4515s	11520	72.7713ms
200	22.3638s	15008	89.0890ms
2000	67.0237s	45952	286.869ms
4000	116.439s	79680	327.100ms
6000	165.035s	113760	551.478ms
8000	210.119s	147840	664.927ms
10000	256.993s	181920	841.507ms

Table 1: Benchmark results for varying aggregation sizes (Version 1)

4.2 MSP.B Benchmark

4.2.1 Server Benchmark

We benchmarked the MSP.BKey and MSP.BSig, as well as pairing verification(MSP.Ver).

The following benchmark is on the server:

OS	Ubuntu 20.04.6 LTS (x86_64)
Kernel	5.4.0-169-generic
CPU	Intel Xeon Silver 4214 (48 cores) @ 3.200GHz
GPU	4 x NVIDIA GeForce RTX 2080 Ti
Memory	128546 MiB

Note: The configuration with Degree , Advice , Lookup Bits , Limb Bits , Num Limbs is 19, 6, 18, 90, and 3, respectively.

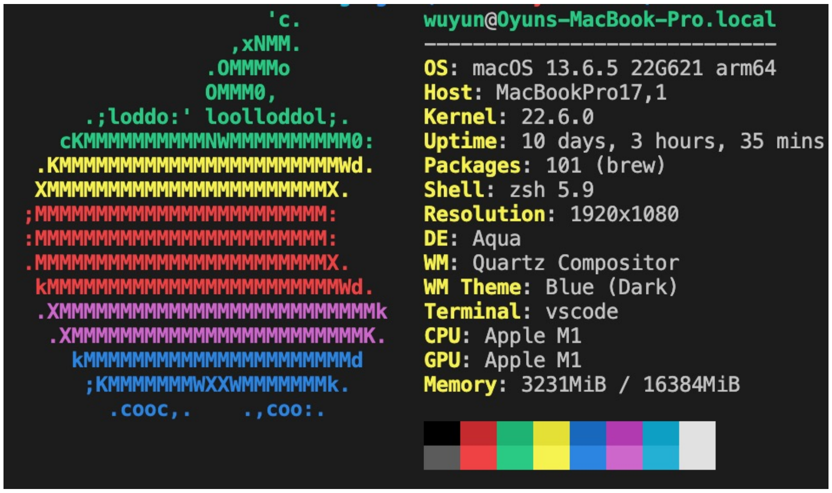
num_aggregation	proving_time	verification_time
4	70.5080s	14.4204ms
8	100.3936s	17.5523ms
16	145.7191s	18.6467ms
32	251.0654s	21.7318ms
64	471.5117s	30.2121ms

Our program can only handle signing with up to 64 public keys. When attempting to sign with 128 public keys, the server's 126 GB of RAM and 8 GB swap partition become full, leading to the program being killed. Therefore, the program cannot continue.

4.2.2 PC Benchmark

We benchmarked the MSP.BKey and MSP.BSig, as well as pairing verification(MSP.Ver).

The PC's configuration is shown in the following figure.



The following benchmark is on the PC:

num_advice	degree	lookup_bits	limb_bits	num_agg	proving	proof_size	verification
25	17	16	88	4	73.2230s	36192	14.5060ms
25	17	16	88	8	114.6814s	49856	20.6187ms
6	19	18	90	4	75.5567s	8864	5.3844ms
6	19	18	90	8	115.3011s	12576	8.0946ms
2	21	20	88	4	118.3032s	2848	4.2537ms
2	21	20	88	8	159.2993s	3776	5.5756ms

Table 2: MSP Benchmark on PC(original)

We also benchmark the MSP in the different group, this situation is consistent with the Mithril paper:

num_advice	degree	lookup_bits	limb_bits	num_agg	proving	proof_size	verification
25	17	16	88	4	73.8717s	36192	13.4883ms
25	17	16	88	8	124.6366s	49856	22.3612ms
6	19	18	90	4	75.7733s	8864	5.7507ms
6	19	18	90	8	118.0805s	12576	8.3911ms
2	21	20	88	4	117.6121s	2848	14.2941ms
2	21	20	88	8	163.3018s	3776	6.1740ms

Table 3: MSP Benchmark on PC(swap group,where pk is on G2)

Similarly, when we attempt to run 32 signature aggregations on the PC, the 16 GB of RAM and approximately 40 GB of swap partition (which varies) become full, ultimately resulting in the program being killed.

5 Next Step

We plan to do more tests and benchmarks on MSP.

- Adjust the configs(degree, advice) of MSP circuit.
- Optimize the scalar_multi function in halo2-lib.
- Manege the RAM on the server to run the large scale data.