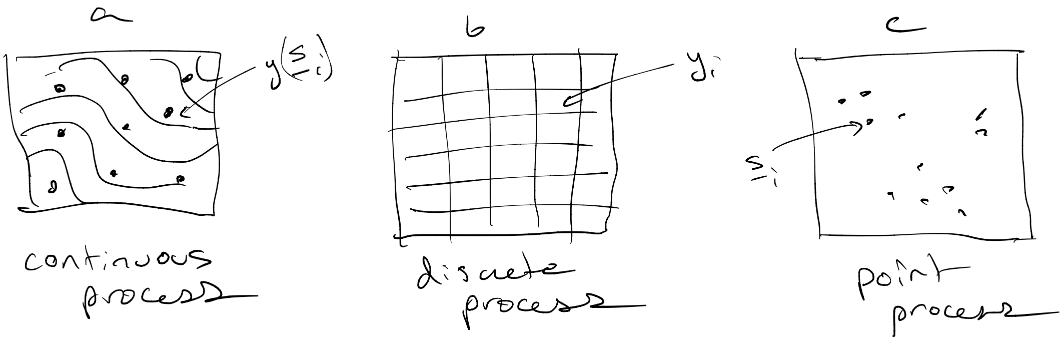
Spatial Dependence:

3 types of spatial processes:



a.) Continuous spatial process:

- underlying spatial field exists everywhere in domain of interest (e.g., temperature)
- Our observations $y(\underline{s}_i)$ are recorded at locations $\underline{s}_i$ for $i=1,\ldots,n$
- For univariate cts. $y(\underline{s}_i)$, ~~we~~ can use a geostatistical model:

$$\underline{y} = (y(\underline{s}_1), \ldots, y(\underline{s}_n))' \sim N(X\underline{\beta}, \Sigma)$$

$$\underline{\beta} \sim N(\underline{\mu}_\beta, \Sigma_\beta) \qquad \Sigma = \sigma^2 R(\phi)$$

$$\sigma^2 \sim IG(q, r)$$

$$\phi \sim \text{Gamma}(a, b)$$

<u>covariance matrix</u>: $\Sigma = \sigma^2 R(\phi)$

for correlation matrix R parameterized.

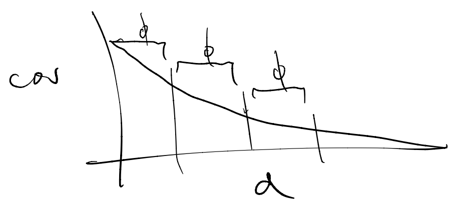
<u>For example</u>:

$R(\phi) = \exp\left(-\frac{D}{\phi}\right)$ for distance

matrix $d_{ij} = \sqrt{(\underline{s}_i - \underline{s}_j)'(\underline{s}_i - \underline{s}_j)}$

$$D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & & \vdots \\ \vdots & & \ddots & \\ d_{n1} & \cdots & \cdots & d_{nn} \end{bmatrix}.$$

covariance function w/ distance:



3ϕ is <u>effective range</u> of spatial dependence.
(95% of cov)

<u>Reasonable priors for ϕ</u>:

$\log(\phi) \sim N(\mu_\phi, \sigma^2_\phi)$

$\phi \sim \text{Gamma}(a, b)$

$\phi \sim \text{Unif}(0, u)$

can set $u = \frac{\max(D)}{2}$

b/c we won't be able to learn about large ϕ.

Posteriors:

$$[\underline{\beta}, \sigma^2, \phi \mid \underline{y}] \propto \overset{\text{no product!}}{[\underline{y} \mid \underline{\beta}, \sigma^2, \phi]}[\underline{\beta}][\sigma^2][\phi]$$

full-conditional distn's:

$$[\underline{\beta} \mid \cdot] \propto [\underline{y} \mid \underline{\beta}, \sigma^2, \phi][\underline{\beta}]$$

$$\propto \exp\left\{-\frac{1}{2}(\underline{y} - X\underline{\beta})' \Sigma^{-1} (\underline{y} - X\underline{\beta})\right\} \exp\left\{-\frac{1}{2}(\underline{\beta} - \underline{\mu}_\beta)' \Sigma_\beta^{-1} (\underline{\beta} - \underline{\mu}_\beta)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left(-2\underbrace{(\underline{y}'\Sigma^{-1}X + \underline{\mu}_\beta' \Sigma_\beta^{-1})}_{\underline{b}'}\underline{\beta} + \underline{\beta}'\underbrace{(X'\Sigma^{-1}X + \Sigma_\beta^{-1})}_{A^{-1}}\underline{\beta}\right)\right\}$$

$$= N(A^{-1}\underline{b}, A^{-1})$$

$$[\sigma^2 \mid \cdot] \propto [\underline{y} \mid \underline{\beta}, \sigma^2, \phi][\sigma^2]$$

$$\propto |\Sigma|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\underline{y} - X\underline{\beta})' \Sigma^{-1} (\underline{y} - X\underline{\beta})\right\} (\sigma^2)^{-(q+1)} \exp\left\{-\frac{1}{r\sigma^2}\right\}$$

$$\propto (\sigma^2)^{-\underbrace{\left(\frac{n}{2} + q + 1\right)}_{\tilde{q}}} \exp\left\{-\frac{1}{\sigma^2}\underbrace{\left(\frac{(\underline{y} - X\underline{\beta})' R^{-1} (\underline{y} - X\underline{\beta})}{2} + \frac{1}{r}\right)}_{\tilde{r}^{-1}}\right\}$$

$$= IG(\tilde{q}, \tilde{r})$$

$$[\phi \mid \cdot] \propto [\underline{y} \mid \underline{\beta}, \sigma^2, \phi][\phi] \qquad \text{use M-H}$$