

M-H for parameters w/ constraints:

Example:  $y_i \sim \text{Bern}(\theta)$ ,  $i=1, \dots, n$   
 $\theta \sim \text{Beta}(\alpha, \beta)$ ,  $0 < \theta < 1$

$$[\theta | y] \propto [y | \theta] [\theta] \\ = \text{Beta}(\sum_i y_i + \alpha, \sum_i (1 - y_i) + \beta)$$

Suppose we want/need MCMC w/ M-H updates:

Proposals:

1.) prior:  $\theta^* \sim [\theta]_x = [\theta]$

2.) RW:  $\theta^* \sim [\theta | \theta^{(k-1)}]_x$  and reject  
if  $\theta^* < 0$  or  $\theta^* > 1$ .

If  $0 < \theta^* < 1$ , use

$$mh = \frac{[y | \theta^{(*)}] [\theta^{(*)}]}{[y | \theta^{(k-1)}] [\theta^{(k-1)}]}$$

where  $[\theta | \theta^{(k-1)}]_x = N(\theta^{(k-1)}, \sigma_{\text{tune}}^2)$

3.) TNRW:  $\theta^* \sim [\theta | \theta^{(k-1)}] = \text{TN}(\theta^{(k-1)}, \sigma_{\text{tune}}^2)_p^A$

$$mh = \frac{[y | \theta^{(*)}] [\theta^{(*)}] [\theta^{(k-1)} | \theta^{(*)}]}{[y | \theta^{(k-1)}] [\theta^{(k-1)}] [\theta^{(*)} | \theta^{(k-1)}]}$$

Other models:

1.)  $y_i \sim N(\mu, \sigma^2)$   
 $\mu \sim N(\mu_0, \sigma_0^2)$   
 $\sigma^2 \sim \text{Gamma}(a, b)$

2.)  $y_i \sim \text{Beta}(\alpha, \beta)$   
 $\alpha \sim \text{Gamma}(a_\alpha, b_\alpha)$   
 $\beta \sim \text{Gamma}(a_\beta, b_\beta)$