Covariate standardization:

For covariates $\underline{x}_j$, $j = 1, \dots, p-1$

let $\underline{z}_j = \dfrac{\underline{x}_j - \bar{x}_j}{s_j}$ where

$$\bar{x}_j = \frac{\sum_{i=1}^{n} x_{ji}}{n} \quad \text{and} \quad s_j^2 = \frac{\sum_{i=1}^{n} (x_{ji} - \bar{x}_j)^2}{n-1}$$

and $\underline{z}_0 = \underline{1}$,

Then for regression:

$y \sim N(Z\underline{\alpha}, \sigma^2 I)$, we have

$$X\underline{\beta} = Z\underline{\alpha} \Rightarrow (X'X)^{-1}X'X\underline{\beta} = (X'X)^{-1}X'Z\underline{\alpha}$$
$$\Rightarrow \underline{\beta} = (X'X)^{-1}X'Z\underline{\alpha}$$

Thus, for every $\underline{\alpha}^{(k)}$ MCMC realization, we can get $\underline{\beta}^{(k)}$.

Specifying vague prior for $\underline{\alpha}$

is $\underline{\alpha} \sim N(\underline{0}, \sigma_\alpha^2 I)$ which implies prior for $\underline{\beta}$:

$\underline{\beta} \sim N(\underline{0}, \Sigma_\beta)$ with prior covariance

$$\Sigma_\beta = \sigma_\alpha^2 (X'X)^{-1} X'ZZ'X(X'X)^{-1}$$

Standardize Response:

Suppose $y_{std} = \frac{y - \bar{y}}{s_y}$, Then if

we fit $y_{std} \sim N(X\underline{\gamma}, \sigma^2_{std} I)$,

$$\Rightarrow \frac{y - \bar{y}}{s_y} = X\underline{\gamma} + \underline{\varepsilon}, \quad \underline{\varepsilon} \sim N(\underline{0}, \sigma^2_{std} I)$$

$$\Rightarrow y = \bar{y} + s_y X \underline{\gamma} + s_y \underline{\varepsilon}$$

$$\Rightarrow y \sim N\Big(\underbrace{(\bar{y} + s_y \gamma_0)}_{\beta_0} + X_{-0} \underbrace{s_y \underline{\gamma}_{-0}}_{\underline{\beta}_{-0}}, \underbrace{s_y^2 \sigma^2_{std}}_{\sigma^2} I\Big)$$

Thus, convert $\underline{\gamma}^{(k)}$ to $\underline{\beta}^{(k)}$ by

$$\beta_0^{(k)} = \bar{y} + s_y \gamma_0^{(k)}$$
$$\underline{\beta}_{-0}^{(k)} = s_y \underline{\gamma}_{-0}^{(k)}$$
$$\sigma^{2(k)} = s_y^2 \sigma^{2(k)}_{std}$$