Bernoulli model with different prior:

$y_i \sim \text{Bern}(p), \; i = 1, \dots, n$

$\text{logit}(p) \sim N(\mu, \sigma^2)$

Note:

$x = \text{logit}(p) = \log\left(\frac{p}{1-p}\right)$

$p = \text{logit}^{-1}(x) = \frac{e^x}{1+e^x} = \frac{1}{1+e^{-x}}$

Posterior:

$$[\text{logit}(p) \mid y] \propto [y \mid \text{logit}(p)][\text{logit}(p)]$$

Use M-H w/ proposal: $\text{logit}(p)^{(*)} \sim N\left(\text{logit}(p)^{(k-1)}, \sigma^2_{tune}\right)$

which is called a <u>random walk proposal</u>.

we can tune σ^2_{tune} to improve MCMC mixing.

$$mh = \frac{[y \mid \text{logit}(p)^{(*)}][\text{logit}(p)^{(*)}]\cancel{[\text{logit}(p)^{(k-1)} \mid \text{logit}(p)^{*}]}_{*}}{[y \mid \text{logit}(p)^{(k-1)}][\text{logit}(p)^{(k-1)}]\cancel{[\text{logit}(p)^{(*)} \mid \text{logit}(p)^{(k-1)}]}_{*}}$$

Note: $[\text{logit}(p)^{(*)} \mid \text{logit}(p)^{(k-1)}] \propto \exp\left\{-\frac{1}{2\sigma^2_{tune}}\left(\text{logit}(p)^{*} - \text{logit}(p)^{(k-1)}\right)^2\right\}$

$[\text{logit}(p)^{(k-1)} \mid \text{logit}(p)^{(*)}] \propto \exp\left\{-\frac{1}{2\sigma^2_{tune}}\left(\text{logit}(p)^{(k-1)} - \text{logit}(p)^{(*)}\right)^2\right\}$

symmetric

* <u>Important Note</u>: prior and proposal have to be in same transformation!