Predictive Model Comparison:

If we score model $y \sim [y|\underline{\theta}]$ with

$$D(y, \underline{\theta}) = -2 \log [y|\underline{\theta}]$$

("Deviance" smaller is better)

- $\underline{\theta}$ is unknown
- This score is optimistic if we fit and score model w/ y.

Out-of-sample validation

for new data $\tilde{y}$, score model with $D(\tilde{y}, \underline{\theta})$, but we don't know $\underline{\theta}$.

Two options:

1.) $\hat{D}(\tilde{y}) = D(\tilde{y}, \hat{\underline{\theta}})$, $\hat{\underline{\theta}} = \int \underline{\theta} [\underline{\theta}|y] d\underline{\theta}$

2.) $\bar{D}(\tilde{y}) = \int D(\tilde{y}, \underline{\theta}) [\underline{\theta}|y] d\underline{\theta}$

accounts for uncertainty associated w/ $\underline{\theta}$ based on info in y.

Cross-Validation:

$\bar{D}_{cv} = \sum_{l=1}^{L} \bar{D}(y_l)$, where $y = (y_1, \ldots, y_L)'$ and

$$\bar{D}(y_l) = \int D(y_l|\underline{\theta}) [\underline{\theta}|y_{-l}] d\underline{\theta}$$

y_{-l}: all data excluding y_l

In-sample validation: Need to account for optimism if we use $\hat{D}(y)$ as score.

Estimate for optimism: $2p_D = 2(\bar{D}(y) - \hat{D}(y))$

Thus, a bias-corrected score is

$$DIC = \hat{D}(y) + 2p_D$$

$$= \bar{D} + \underbrace{p_D}_{\text{"penalty" for model complexity}}$$

Question: why not use $p_D = p$?

DIC is valid when:

1.) regular models, non-hierarchical
2.) $p_D << n$
3.) $\underline{\hat{\theta}} = E(\underline{\theta} \mid y)$ is good for central tendency

using MC integration:

$$\hat{\underline{\theta}} = \frac{\sum_{k=1}^{K} \underline{\theta}^{(k)}}{K}, \quad \bar{D}(y) = \frac{\sum_{k=1}^{K} D(y, \underline{\theta})}{K}$$