Introduction to Bayes

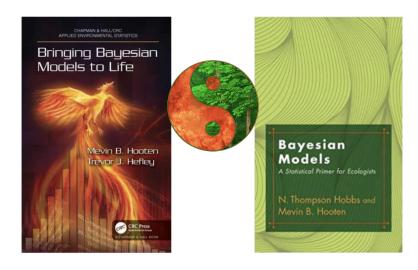
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8 Steps to Become a Better Bayesian

- 1 Build model
- 2 Write posterior
- 3 Derive full-conditionals
- 4 Code MCMC algorithm
- 5 Make predictions
- 6 Check model
- 7 Validate model
- 8 Obtain inference

Books



1.) Build Model

$$y_i \sim [y_i | \boldsymbol{\theta}], \quad i = 1, \dots, n$$

 $\boldsymbol{\theta} \sim [\boldsymbol{\theta}]$

- Gelman, A., et al. (2013). Bayesian Data Analysis, Third Edition. Chapman & Hall/CRC.
- · Hobbs, N.T. and M.B. Hooten (2015). Bayesian Models: A Statistical Primer for Ecologists. Princeton University Press.

2.) Write Posterior

$$egin{aligned} [m{ heta}|\mathbf{y}] &= rac{[\mathbf{y}|m{ heta}][m{ heta}]}{[\mathbf{y}]} \ &\propto [\mathbf{y}|m{ heta}][m{ heta}] \end{aligned}$$

 Hobbs, N.T. and M.B. Hooten (2015). Bayesian Models: A Statistical Primer for Ecologists. Princeton University Press.

3.) Derive Full-Conditionals

$$[\theta_j|\cdot] = [\theta_j|\boldsymbol{\theta}_{-j}, \mathbf{y}]$$

$$\propto [\mathbf{y}|\boldsymbol{\theta}][\theta_j]$$

Hooten, M.B. and T.J. Hefley (2019). Bringing Bayesian Models to Life. Chapman & Hall/CRC.

4.) Code MCMC Algorithm

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\begin{array}{l} \text{for } k=1,\ldots,K \ \\ \text{for } j=1,\ldots,J \ \{ \\ \theta_j^{(k)} \sim [\theta_j|\cdot] \\ \} \end{array}
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Intro

5.) Make Predictions

$$[\mathbf{y}_u|\mathbf{y}] = \int [\mathbf{y}_u, \boldsymbol{\theta}|\mathbf{y}] d\boldsymbol{\theta}$$
$$= \int [\mathbf{y}_u|\boldsymbol{\theta}, \mathbf{y}] [\boldsymbol{\theta}|\mathbf{y}] d\boldsymbol{\theta}$$

- Gelman, A., et al. (2013). Bayesian Data Analysis, Third Edition. Chapman & Hall/CRC.
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6.) Check Model

$$\int 1_{\{g(\mathbf{y}_u) > g(\mathbf{y})\}} [\mathbf{y}_u | \mathbf{y}] d\mathbf{y}_u = \mathsf{P}(g(\mathbf{y}_u) > g(\mathbf{y}) | \mathbf{y})$$

- Conn, P.B., D.S. Johnson, P.J. Williams, S. Melin, and M.B. Hooten. (2018). A guide to Bayesian model checking for ecologists. Ecological Monographs, 88: 526-542.
- Gelman, A., et al. (2013). Bayesian Data Analysis, Third Edition. Chapman & Hall/CRC.

7.) Validate Model

$$\int g(\mathbf{y}_u, \mathbf{y})[\mathbf{y}_u|\mathbf{y}]d\mathbf{y}_u$$

- Hooten, M.B. and N.T. Hobbs. (2015). A guide to Bayesian model selection for ecologists. Ecological Monographs, 85: 3-28.
- Gelman, A., et al. (2013). Bayesian Data Analysis, Third Edition. Chapman & Hall/CRC.

8.) Obtain Inference

$$\int g(\boldsymbol{\theta})[\boldsymbol{\theta}|\mathbf{y}]d\boldsymbol{\theta} \approx \frac{\sum_{k=1}^{K} g(\boldsymbol{\theta}^{(k)})}{K}$$

- Gelman, A., et al. (2013). Bayesian Data Analysis, Third Edition. Chapman & Hall/CRC.
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References

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