Homework 5
Bayesian Statistical Methods

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1 Using JAGS, develop the R code to fit the Bayesian hierarchical model:

$$y_{i,j} \sim N(x_{i,j}^{'}\beta_i, \sigma^2), \text{for i=1,...,n and j=1,...,J} , \tag{1}$$

$$\boldsymbol{\beta_i} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) , \tag{2}$$

$$\boldsymbol{\mu} \sim N(\boldsymbol{\mu_{\mu}}, \boldsymbol{\Sigma_{\mu}}) , \tag{3}$$

$$(\sigma_0)^{-1} \sim Gamma(a_0, b_0), \tag{4}$$

$$(\sigma_1)^{-1} \sim Gamma(a_1, b_1), \tag{5}$$

$$(\sigma)^{-1} \sim Gamma(a, b) \tag{6}$$

Where $x_{i,j} = (1, x_{i,j})^{'}$ and β_i= $\begin{pmatrix} \beta_{0,i} \\ \beta_{1,i} \end{pmatrix}$, μ= $\begin{pmatrix} \mu_0 \\ \mu_1 \end{pmatrix}$, Σ= $\begin{pmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{pmatrix}$

Based on the provided model statement above, we write JAGS model as below:

```
ox.jags <-"
  model{
    for(i in 1:n){
      y[i,] ~ dmnorm(X[,c(1,i+1)]%*%beta,tau*I.matrix)
    }
    invtau ~ dgamma(a,b)
    tau = 1/invtau
    beta ~ dmnorm(mu.beta, taubeta.matrix)
    mu.beta ~ dmnorm(mu_u, tau_u)

    invtau0 ~ dgamma(a0,b0)
    taubeta.matrix[1,1] <- 1/invtau0

    invtau1 ~ dgamma(a1,b1)
    taubeta.matrix[2,2] <- 1/invtau1
    taubeta.matrix[1,2] <- 0
    taubeta.matrix[2,1] <- 0
}
"
```

2 Use JAGS to fit the Bayesian hierarchical model above to the oxboys.csv data set using height as the response variable and age as the predictor variable. Provide 'ridgeline' plots, made using R, to display the marginal posterior distributions of β_i and μ (see Figure 1 and Figure 2 below for examples).

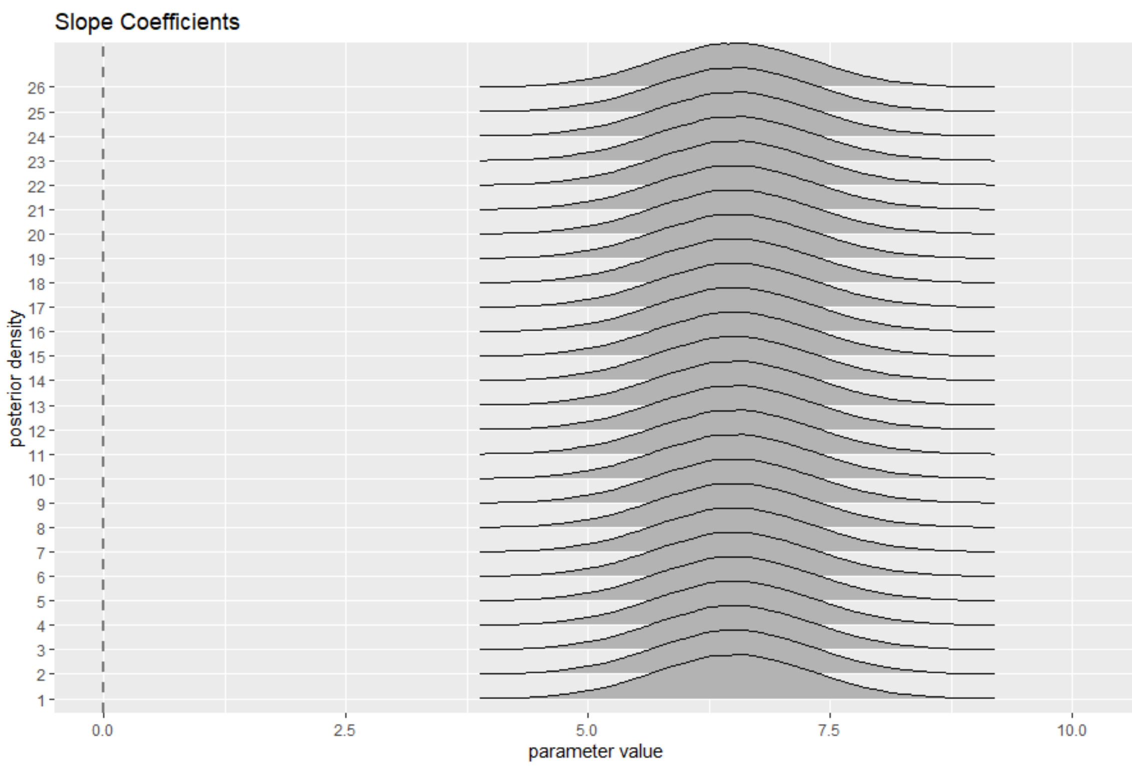

Figure 1: β Ridgeline Plot

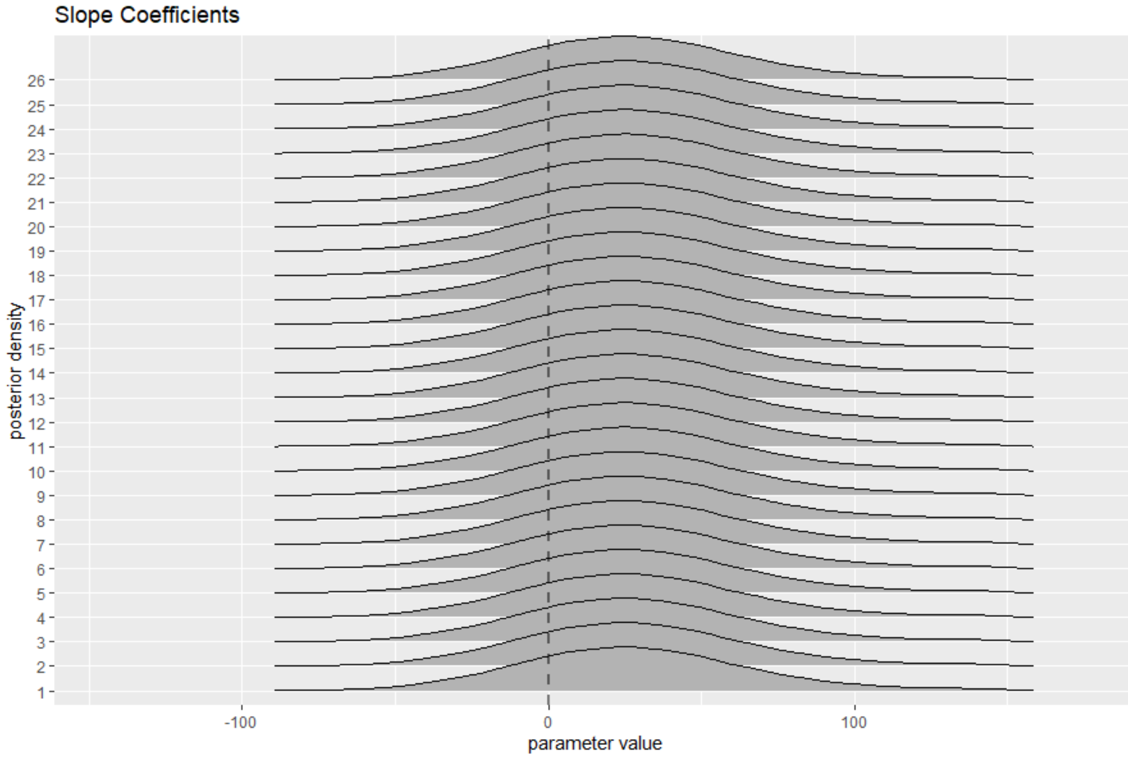

Figure 2: μ Ridgeline Plot

3 Use JAGS to calculate the ‘potential scale reduction factors’ for all model parameters(including the ‘process’ variables). You will need to obtain multiple MCMC chains to do this. Report the resulting ˆR statistics in a table.

3 chains are used to calculate the potential scale reduction factors for all model parameters. Table 1 shows that these values are close to 1, indicating a good convergence for all parameters.

Table 1: Potential scale reduction factors

Parameters	Point est.	Upper C.I.
β_0	1.000060	1.000225
β_1	1.000032	1.000118
μ_0	1.000041	1.000109
μ_1	1.000046	1.000140
σ_0^2	1.083661	1.0837405
σ_1^2	1.062568	1.062616

APPENDIX A: R Script

```
###   Homework 5

###   Libraries
library(rjags)
library(vioplot)
library(readr)
library(mvnfast)
library(forcats)
library(ggridges)
library(ggplot2)
library(coda)

###
###   Read the Data
###

setwd("~/UT Austin/Spring 2023/Bayesian Stats/hw5/hw5") #Set WD
oxboys <- read_csv("oxboys.csv")
#View(oxboys)

n <- max(oxboys$Subject)
J <- sum(oxboys$Subject==1)

#plot(oxboys$age,oxboys$height,type="p",ylab="height",xlab="age")

#Setup Data
y <- matrix(1,n,J)
for (i in 1:n){
  y[i,] <- oxboys[which(oxboys$Subject==i),]$height
}

X <- matrix(1,n,J)

for (i in 1:n){
  X[i,] <- oxboys[which(oxboys$Subject == i),]$age
}
X <- cbind(1,t(X))

###
###  Setup Models
###
```

```
ox.jags <-"
  model{
    for(i in 1:n){
      y[i,] ~ dmnorm(X[,c(1,i+1)]%*%beta,tau*I.matrix)
    }
    tau ~ dgamma(a,b)
    beta ~ dmnorm(mu.beta, taubeta.matrix)
    mu.beta ~ dmnorm(mu_u, tau_u)

    tau0 ~ dgamma(a0,b0)
    taubeta.matrix[1,1] <- tau0

    tau1 ~ dgamma(a1,b1)
    taubeta.matrix[2,2] <- tau1
    taubeta.matrix[1,2] <- 0
    taubeta.matrix[2,1] <- 0
}
"
mod<-textConnection(ox.jags)



n.mcmc=100000
n.burn=round(.2*n.mcmc)


#Set Priors
mu_u=rep(0,2)
I.matrix = diag(J)
taubeta.matrix = matrix(0,2,2)
tau_u=1/1000*diag(2) #Precision. 1 / sigma^2
a  = 1
b  = 3
a0 = 1
b0 = 3
a1 = 1
b1 = 3

m.out<-jags.model(mod,data=list('y'=y, 'X'=X, 'n'=n,
    'I.matrix'=I.matrix,'mu_u'=mu_u,'tau_u'=tau_u,'a'=a,'b'=b,'a0'=a0,'b0'=b0,'a1'=a1,'b1'=b

update(m.out,n.burn) # perform burn-in
m.samples=jags.samples(m.out,c('beta','mu.beta','taubeta.matrix[1,1]','taubeta.matrix[2,2]'
    # fit model post-burn-in
```

```
#Trace Plot for Betas and Sigma
beta.post.mat=cbind(m.samples$beta[1,,1],m.samples$beta[2,,1],m.samples$mu.beta[1,,1])
sig.post.mat=cbind(m.samples$`taubeta.matrix[1,1]`[1,,1],m.samples$`taubeta.matrix[2,2]`[1,
   #Need to do 1/taubetamatrix

matplot(beta.post.mat,type="l",lty=1,ylab=bquote(beta))
matplot(sig.post.mat,type="l",lty=1,ylab=bquote(sigma^2))

#Ridgeline Plots



dist.labels=as.factor(rep(as.character(1:n),each=n.mcmc))
slopes.mat=as.matrix(cbind(m.samples[[1]][1,,1], m.samples[[1]][2,,1]))
slopes.df=data.frame(x=c(slopes.mat),y=dist.labels)
ggplot(slopes.df,
   aes(x=x,y=fct_reorder(dist.labels,rep(1:n,each=n.mcmc))))+geom_density_ridges(rel_min_he
   = 0.005)+labs(y="posterior density",x="parameter value",title="Slope
   Coefficients")+geom_vline(xintercept=0,size=1,linetype="dashed",col=rgb(0,0,0,.5))
```