SAGS:

Binomial model:

$$y_i \sim \text{Binom}(N_i, p), \quad i = 1, \ldots, n$$

$$p \sim \text{Beta}(\alpha, \beta)$$

Use $m > 1$ chains to assess convergence.

Potential scale reduction factor (Gelman and Rubin, 1992)

$$\hat{r} = \sqrt{\frac{\hat{\text{var}}(p|y)}{w}} \quad \text{for}$$

w/in chain variance → $w = \frac{\sum_{j=1}^{m} s_j^2}{m}$, $s_j^2 = \frac{\sum_{k=1}^{K} (p_j^{(k)} - \bar{p}_j)^2}{K-1}$, $\bar{p}_j = \frac{\sum_{k=1}^{K} p_j^{(k)}}{K}$

and

est. of marginal post. variance → $\hat{\text{var}}(p|y) = \frac{K-1}{K} w + \frac{1}{K} b$

b/w chain variance → $b = K \frac{\sum_{j=1}^{m} (\bar{p}_j - \bar{p})^2}{m-1}$, $\bar{p} = \frac{\sum_{j=1}^{m} \bar{p}_j}{m}$

Note: $\hat{r} \to 1$ as $K \to \infty$, Thus $\hat{r}$ close to 1 implies convergence. ✗