## Negative Binomial Regression:

$$y_i \sim NB(\mu_i, N) = \frac{\Gamma(y_i+N)}{\Gamma(N)\, y_i!}\left(\frac{N}{N+\mu_i}\right)^N \left(1-\frac{N}{N+\mu_i}\right)^{y_i}$$

$$\log(\mu_i) = \underline{x}_i' \underline{\beta}$$

$$\underline{\beta} \sim N(\underline{\mu}_\beta, \Sigma_\beta)$$

$$\log(N) \sim N(\mu_N, \sigma_N^2)$$

## Posterior:

$$[\underline{\beta}, \log(N) \mid \underline{y}] \propto [\underline{y} \mid \underline{\beta}, \log N][\underline{\beta}][\log N]$$

## Full-conditional Distns:

$$[\underline{\beta} \mid \cdot] \propto \left(\prod_{i=1}^{n} [y_i \mid \underline{\beta}, \log N]\right)[\underline{\beta}]$$

$$[\log N \mid \cdot] \propto \left(\prod_{i=1}^{n} [y_i \mid \underline{\beta}, \log N]\right)[\log N]$$

## MCMC Algorithm:

1.) Set $\underline{\beta}^{(0)}$, $\log N^{(0)}$, $k=0$
2.) $k=k+1$
3.) Use M-H to update $\underline{\beta}$ w/ $\underline{\beta}^{(*)} \sim N(\underline{\beta}^{(k-1)}, \sigma^2_{\beta\text{tune}} I)$
4.) Use M-H to update $\log N$ w/ $\log N^{(*)} \sim N(\log N^{(k-1)}, \sigma^2_{N\text{tune}})$
5.) Goto 2, until $k=K$.

## Binary Regression:

For $y_i \in \{0, 1\}$, $i = 1, \ldots, n$

model
$$\begin{cases} y_i \sim \text{Bern}(\theta_i) \\ g(\theta_i) = \underline{x}_i' \underline{\beta} \\ \underline{\beta} \sim N(\underline{\mu}_\beta, \Sigma_\beta) \end{cases}$$

link function

### Options for link functions:

1.) $g(\theta) = \text{logit}(\theta) = \log\left(\frac{\theta}{1-\theta}\right)$ "logit"

2.) $g(\theta) = \Phi^{-1}(\theta)$ "probit"

inverse std. normal CDF. (quantile fcn)

### Posterior Distribution:

$$[\underline{\beta} \mid \underline{y}] \propto \left(\prod_{i=1}^{n} [y_i \mid g^{-1}(\underline{x}_i' \underline{\beta})]\right)[\underline{\beta}]$$

### MCMC Algorithm:

1.) Set $\underline{\beta}^{(0)}$ at init. values, $k=0$

2.) $k = k+1$

3.) Sample $\underline{\beta}^{(*)} \sim N(\underline{\beta}^{(k-1)}, \sigma^2_{tune} I)$

4.) Let $\underline{\beta}^{(k)} = \underline{\beta}^{(*)}$ w.p. $\min\left(\frac{[\underline{y} \mid \underline{\beta}^{*}][\underline{\beta}^{*}]}{[\underline{y} \mid \underline{\beta}^{(k-1)}][\underline{\beta}^{(k-1)}]}, 1\right)$

5.) Goto 2 until $k = K$.

# Logistic Regression Inference:

Recall <u>Odds</u>: $\dfrac{\theta}{1-\theta}$

Thus w/ logit link:

$$\text{logit}(\theta_i) = \underset{\sim}{x}_i'\underset{\sim}{\beta}$$
$$= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots$$

$\ast$ A 1-unit change in $x_1$ increases the odds by a multiplicative factor of $e^{\beta_1}$.

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If covariates are standardized, then

$$\text{logit}(\theta_i) = \beta_0 + \beta_1 \overline{x}_i \quad , \text{ for } \quad \overline{x} = \frac{x - \bar{x}}{s}$$
$$= \beta_0 + \beta_1\left(\frac{x_i - \bar{x}}{s}\right)$$
$$= \left(\beta_0 - \frac{\beta_1 \bar{x}}{s}\right) + \frac{\beta_1}{s} x_i$$

Thus, a 1-unit change in $x$ increases odds by a factor of $e^{\frac{\beta_1}{s}}$