## Newton (a)

We have the following functions:

Note that I have replaced with to avoid any confusion. Differentiating the second function with respect to gives:

By using the main equations, we can simplify the RHS by replacing them with their equivalents:

Leaving the term with at the RHS, we get:

We can right LHS as a single derivation after multiplying both sides with :

Now, we divide it back to to get rid of at the RHS. This gives:

From now on, we have to use other equalities given to us to simplify further. The following are given:

Then, we can get . Letting , we can write as follows:

Using chain rule, we can calculate that is in the equation above.

Putting this and into the equation above, we get:

Further simplification on the RHS leads to:

The last thing is to scale the radius . Let where . Note that this value depends on out choice of eventually, which works for this case. A generic solution could be which will be required in the upcoming parts. Noting

, we get:

Performing the simplifications and moving the terms at the RHS to the LHS, we get the Lane-Emden equation:

For , the following command solves the differential equation:

Note that I have expanded the equation before using Mathematica. FullSimplify command is to simplify the output result, otherwise it gives it in the form of imaginary numbers . The result is:

Put it into the main equation to check.

Therefore, the solution is correct.

Using the following equation that is provided to us, we can find the total mass of the star.

Since . The boundaries are also must be modified. and similarly, .

Using the fundamental theorem of Calculus, we get:

This is the same result in the project manual. Let us denote with , by letting . Also noting , the equation for we found becomes:

At last, writing as , which are equal, we get the version in the manual:

Denoting the already defined as:

Then, as calculated above,

Putting this into the equation above, we get:

The final observation is the following two:

Therefore,

More precisely,

So,

(The term before M is the constant of proportionality.)

## Newton (b)