Dependence Discovery via Multiscale Generalized Correlation

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Joint Work with Joshua T. Vogelstein & Mauro Maggioni & Carey E. Priebe

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Overview

1. Motivations

2. Results
   - Multiscale Generalized Correlation
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Section 1

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Modern data sets may be **high-dimensional, nonlinear, noisy, of small sample size, from disparate spaces**, etc.

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MGC combines distance correlation and nearest-neighbor, and is able to satisfy all of the above.

Section 2

Results

Generalized Correlation Coefficient

Start with n pairs of observations $(\boldsymbol{x}_i, \boldsymbol{y}_i)$ for $i = 1, \ldots, n$, where $\boldsymbol{x}$'s and $\boldsymbol{y}$'s both might be vectors of arbitrary dimensions, shapes, networks, etc.

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Then $A = \{a_{ij}\}$ and $B = \{b_{ij}\}$ are the $n \times n$ interpoint comparison matrices for $X = \{\boldsymbol{x}_i\}$ and $Y = \{\boldsymbol{y}_i\}$, respectively. Assuming $\{a_{ij}\}$ and $\{b_{ij}\}$ have zero mean, a generalized correlation coefficient can then be written:

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$$c = \frac{1}{z}\sum_{i,j=1}^{n} a_{ij} b_{ij}, \tag{1}$$

where z is proportional to standard deviations of A and B, that is $z = n^2 \sigma_a \sigma_b$.

Examples

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- The modified distance correlation [*Szekely and Rizzo (2013)*] [4] by slightly tweaking a_{ij}/b_{ij} of dcorr.

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$$\widetilde{a}_{ij}^{k} = \begin{cases} a_{ij}, & \text{if } R(a_{ij}) \leq k, \\ 0, & \text{otherwise;} \end{cases} \qquad \widetilde{b}_{ij}^{l} = \begin{cases} b_{ij}, & \text{if } R(b_{ji}) \leq l, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

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Then let $a_{ij}^{k} = \widetilde{a}_{ij}^{k} - \bar{a}^{k}$, where $\bar{a}^{k}$ is the mean, and b_{ij}^{k} similarly.

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There are a maximum of n^2 different local correlations, and they are always symmetric, i.e. $c^{kl}(X, Y) = c^{lk}(Y, X)$, no matter A and B are symmetric or not.

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Each local correlation requires $O(n^2)$ to compute. However, once A and B are sorted, **all local correlations can be simultaneously calculated in $O(n^2)$ as well!!!**

The Testing Framework

The formal testing scenario is as follows: assume that $\boldsymbol{x}_i, i = 1, \ldots, n$ are identically independently distributed (i.i.d.) as $\boldsymbol{x} \sim f_x$; similarly each $\boldsymbol{y}_i$ are i.i.d. as $\boldsymbol{y} \sim f_y$.

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But we all want a test with high power in finite-sample rather than asymptotically!

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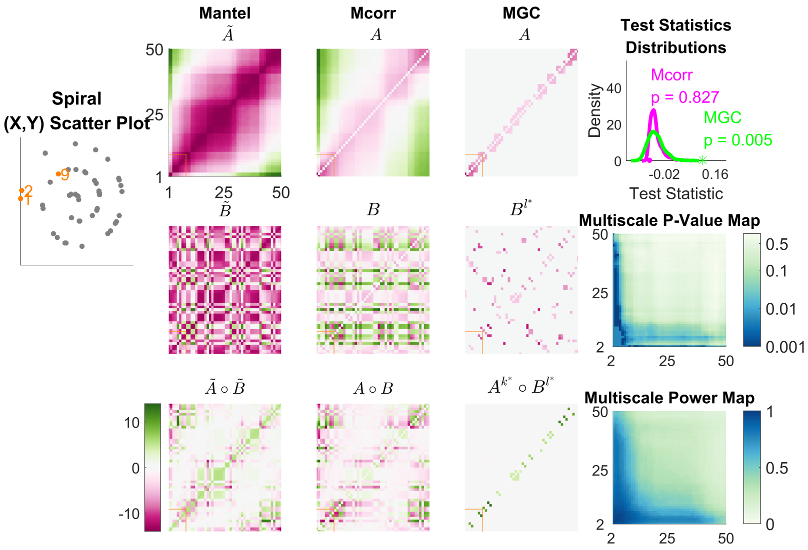
Theorem 3

There exists f_{xy} *and* n *such that* MGC *is better than its global counterpart in testing power.*

Illustration of Utilizing locality



Advantage of Utilizing Locality

	Mantel	Mcorr	MGC
$\delta_x(1,2)$	-2.42	-5.21	-5.07
$\delta_y(1,2)$	-1.58	-0.91	-0.12
$\delta_x \times \delta_y$	3.82	4.74	0.61
$\delta_x(2,9)$	0.70	0.61	0.14
$\delta_y(2,9)$	-0.91	-0.28	0.12
$\delta_x \times \delta_y$	-0.63	-0.17	0.02
$\sum \delta_x \times \delta_y$	-162.14	-93.04	116.41
$\sum \delta_x \times \delta_y / \sum \delta_x^2 \sum \delta_y^2$	-0.02	-0.02	0.16

Simulation Set-Up

In total 20 different joint distributions f_{xy} are considered, including linear and nearly linear (1-5), polynomial (6-12), trigonometric (13-17), uncorrelated but nonlinearly dependent (18-19), and an independent relationship (20).

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Our MGC implementation is based on modified distance correlation with single centering.

Visualizations of Simulation Settings

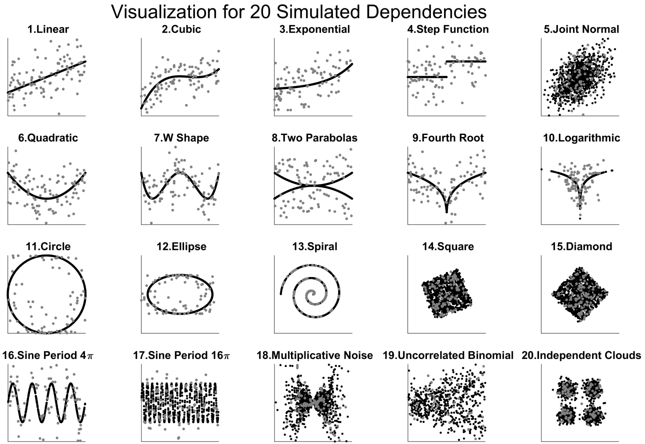

Figure: Visualization of the 20 dependencies for one-dimensional simulations.

Simulation Powers

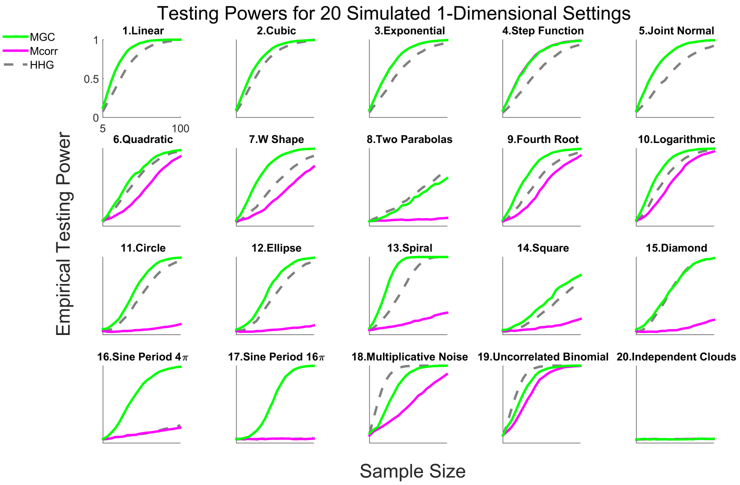

Figure: Powers of different methods for 20 different one-dimensional dependence structures.

Simulation Power Map

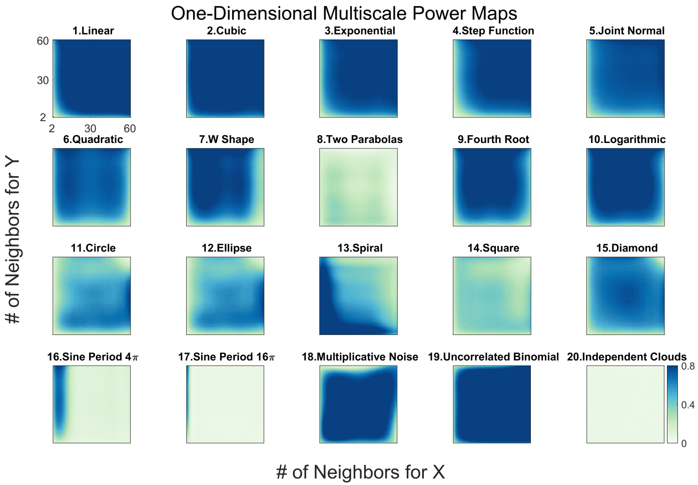

Figure: Multiscale Power Maps indicating the influence of neighborhood size on MGC testing power.

Brain vs Personality

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For the five-factor personality data, we directly use the Euclidean distance.

Permutation Test P-value map

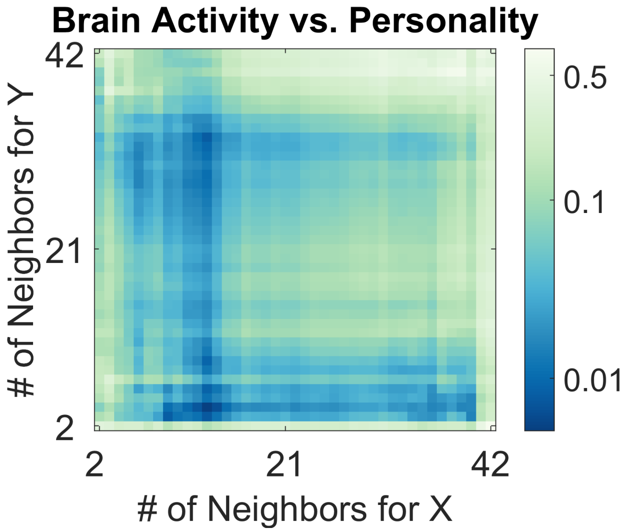

Figure: P-value map (log scale) for brain fMRI scan vs five-factor personality.

Section 3

Conclusion

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- 3) MGC is easy and efficient to implement for any generalized correlation.
- 4) MGC not only outputs a p-value in testing, but also provides useful information on the local scale where the dependency is the strongest, and implies the geometry of the underlying dependency.

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And many other details...

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