Dependence Discovery via Multiscale Generalized Correlation

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Section 1

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Modern data sets may be high-dimensional, nonlinear, noisy, of small sample size; and different features may come from disparate spaces.

A particular application is to investigate the association between brain activities and various phenotypes, such as brain vs disease, or brain vs personality. Despite recent inventions of many consistent test statistics, existing methods fall short for our real data.

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MGC is theoretically consistent and efficient to compute, achieves excellent testing powers in a comprehensive set of simulations, sheds light into the underlying dependency structures, and works well for our real data.

Section 2

Results

Generalized Correlation Coefficient

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$$c = \tfrac{1}{z}\textstyle\sum_{i,j=1}^{n} a_{ij} b_{ij}, \tag{1}$$

where z is proportional to standard deviations of A and B, that is $z = n^2 \sigma_a \sigma_b$.

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- An unbiased version of dcorr can be achieved by slightly tweaking a_{ij}/b_{ij} of dcorr, which yields the modified distance correlation [*Szekely and Rizzo (2013)*] [4].

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$$\widetilde{a}_{ij}^{k} = \begin{cases} a_{ij}, & \text{if } R(a_{ij}) \leq k, \\ 0, & \text{otherwise;} \end{cases} \qquad \widetilde{b}_{ij}^{l} = \begin{cases} b_{ij}, & \text{if } R(b_{ji}) \leq l, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

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Then let $a_{ij}^{k} = \widetilde{a}_{ij}^{k} - \bar{a}^{k}$, where $\bar{a}^{k}$ is the local mean, and b_{ij}^{k} similarly.

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There are a maximum of $(n^2 - n)/2$ different local correlations, one for each possible combination of k and l. And our definition always yields symmetric local correlations, i.e. $c^{kl}(X, Y) = c^{lk}(Y, X)$, no matter A and B are symmetric or not.

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This allows the optimal scale to be efficiently determined for $\textsc{Mgc}$, unlike many other applications by nearest-neighbor (say knn classification, manifold learning, etc.).

The Testing Framework

The formal testing scenario is as follows: assume that $\boldsymbol{x}_i, i = 1, \ldots, n$ are identically independently distributed (i.i.d.) as $\boldsymbol{x} \sim f_x$; similarly each $\boldsymbol{y}_i$ are i.i.d. as $\boldsymbol{y} \sim f_y$.

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The null and the alternative hypothesis for testing independence are

$$\begin{aligned} H_0 &: f_{xy} = f_x f_y, \\ H_A &: f_{xy} \neq f_x f_y, \end{aligned}$$

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The power of a test is defined as the probability that it correctly rejects the null when the null is indeed false, and has power equal to the type 1 error level when the null is true. And a test is universally consistent if its power converges to 1 as $n \to \infty$ whenever $f_{xy} \neq f_x f_y$.

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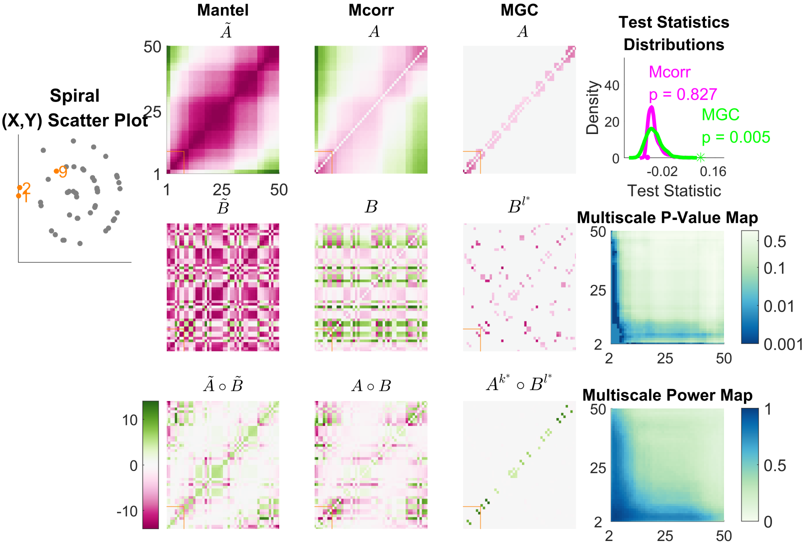
Theorem 3

There exists f_{xy} *and* n *such that* MGC *is better than its global counterpart in testing power.*

Illustration of Utilizing locality



Advantage of Utilizing Locality

	Mantel	Mcorr	MGC
$\delta_x(1,2)$	-2.42	-5.21	-5.07
$\delta_y(1,2)$	-1.58	-0.91	-0.12
$\delta_x \times \delta_y$	3.82	4.74	0.61
$\delta_x(2,9)$	0.70	0.61	0.14
$\delta_y(2,9)$	-0.91	-0.28	0.12
$\delta_x \times \delta_y$	-0.63	-0.17	0.02
$\sum \delta_x \times \delta_y$	-162.14	-93.04	116.41
$\sum \delta_x \times \delta_y / \sum \delta_x^2 \sum \delta_y^2$	-0.02	-0.02	0.16

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Our MGC implementation is based on modified distance correlation.

Visualizations of Simulation Settings

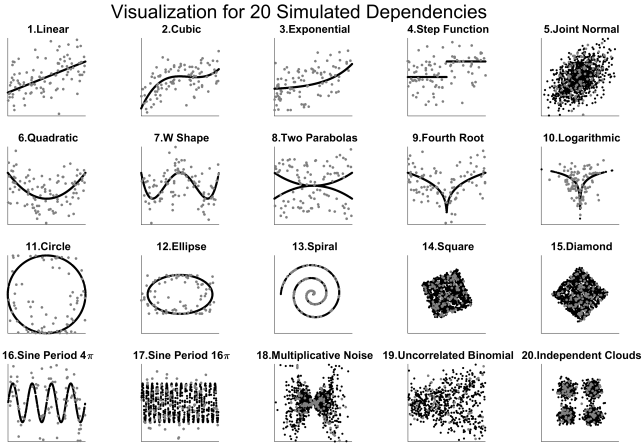

Figure: Visualization of the 20 dependencies for one-dimensional simulations.

Simulation Powers

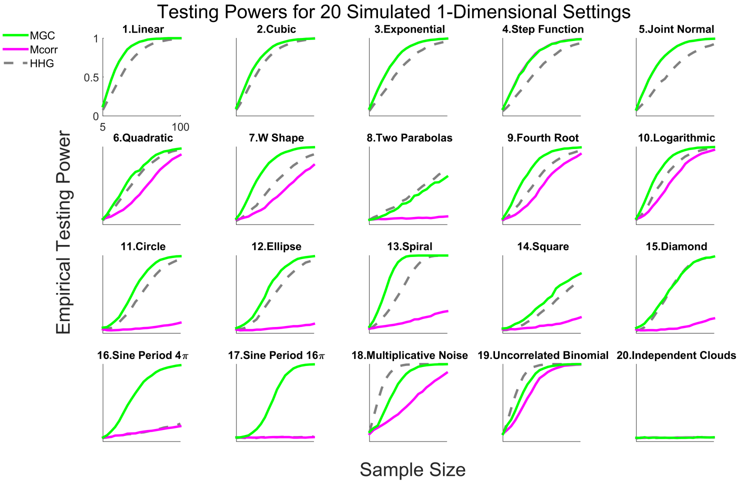

Figure: Powers of different methods for 20 different one-dimensional dependence structures.

Simulation Power Map

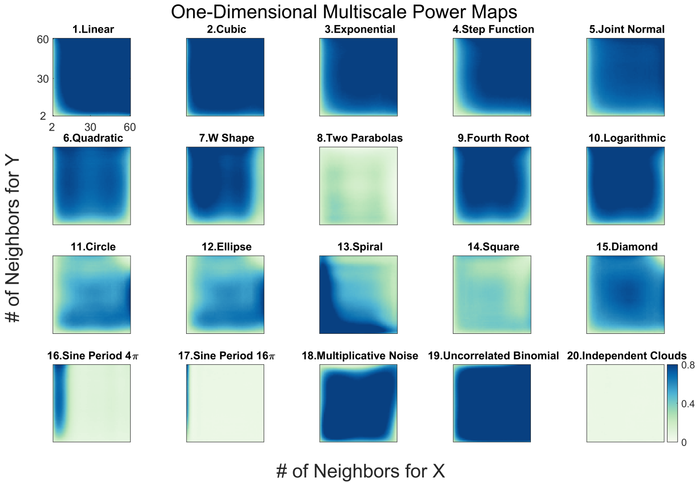

Figure: Multiscale Power Maps indicating the influence of neighborhood size on MGC testing power.

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For the five-factor personality data, we use the Euclidean distance.

Permutation Test P-value map

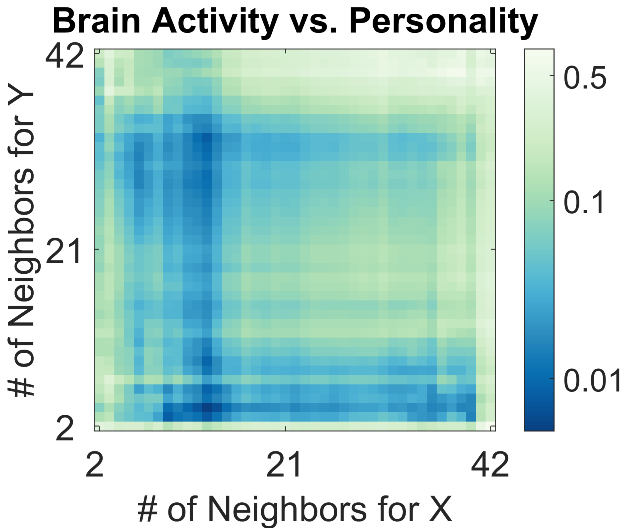

Figure: P-value map (log scale) for brain fMRI scan vs five-factor personality.

Section 3

Conclusion

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- 5) MGC not only yields a p-value in real data testing, but also provides useful information on the local scale where the dependency is the strongest, which implies the geometry of the underlying dependency.

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- Our MGC implementation is based on an equivalent but different mcorr version. In particular, we used single-centering rather than double centering, which is equivalent for the global correlation in testing power and p-value, but preserves the rank information better for local correlations.
- When applying MGC to sample data without known model or training data, we provide a heuristic algorithm to accurately select the optimal scale by finding consecutive regions of significant p-values.

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