2^k Factorial Experiments

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2^k Factorial Experiments

k factors are considered each at 2 levels. This creates the 2^k treatment combinations. These experiments are typically useful screening experiments to try to discover which factors have effects on response variables and should be examined more closely.

2^2 design

Suppose you have two factors, A and B each with a low level (-) and a high level (+). We can summarize the results of the experiment and the treatment levels and combinations in a table like

Factor			Response
A	B	AB	Y
-	-	+	10
+	-	-	20
-	+	-	15
+	+	+	35

Note: the interaction effect is labeled AB

It is found by "multiplying" A by B

The design is said to be in standard form written this way.

Estimating Effects

Estimated effects are found as the average response at the high level minus the average response at the low level.

$$Effect(A) = \bar{Y}_{A+} - \bar{Y}_{A-} = \left(\frac{20+35}{2}\right) - \left(\frac{10+15}{2}\right) = 27.5 - 12.5 = 15$$

This can be found by "multiplying" column A by the response column, adding up and dividing by 2

$$Effect(A) = (-10 + 20 - 15 + 35)/2 = 15$$

2^3 Design

The standard form of the 2^3 design is

A	B	C	AB	BC	AC	ABC	Y
-	-	-	+	+	+	-	Y_1
+	-	-	-	+	-	+	Y_2
-	+	-					
+	+	-					
-	-	+					
+	-	+					
-	+	+					
+	+	+					

As before the columns are found by "multiplying"

Estimating Effects

As in the 2^2 design, the effects are each found by "multiplying the column in the table corresponding to the effect by the response variable, summing the results and dividing by four.

Estimating Effects in 2^k

In general to estimate an effect in a 2^k design, "multiply" the response column by the column corresponding to the effect in the design matrix, sum the results and divide by 2^{k-1}.

Are the effects significant?

Once we estimate the main effects and their interactions, we need to discern which are statistically significant, i.e. different from 0. There are two cases

1. When there are repeated measures in each treatment combination, estimate the variance within each combination using the sample variance and take the average of these estimates.
2. When there is no repeated measures, we can assume that higher order interactions (3-way, etc.) are not significant and are merely measuring the variation.

Let $\hat{\sigma}^2$ be the estimated variance from one of the ways given above. A $(1-\alpha)\%$ confidence interval for a given effect is found as

$$\textit{estimated effect} \pm t_{\alpha/2,df}\sqrt{\hat{\sigma}^2}$$

where *df* depends on which method is used to estimate the variance.

Example

A chemical product is produced in a pressure vessel. A factorial experiment is carried out to study the factors thought to influence the filtration rate of this product. Four factors were considered A = temperature, B = pressure, C = Concentration of formaldehyde and D=stirring rate. Interest is in maximizing response. Current yield is about 75 gal/hour. The current process uses a high level concentration of formaldehyde. Interest is in maintaining high yield and reducing the concentration of formaldehyde. The results are shown on the following slide and can be interpreted from this graphical representation. For example the yield of 45 corresponds with low level of A, low level of B, low level of C and low level of D; The yield of 60 corresponds to high level of A, low level of B and high level of C and low level of D.

Example

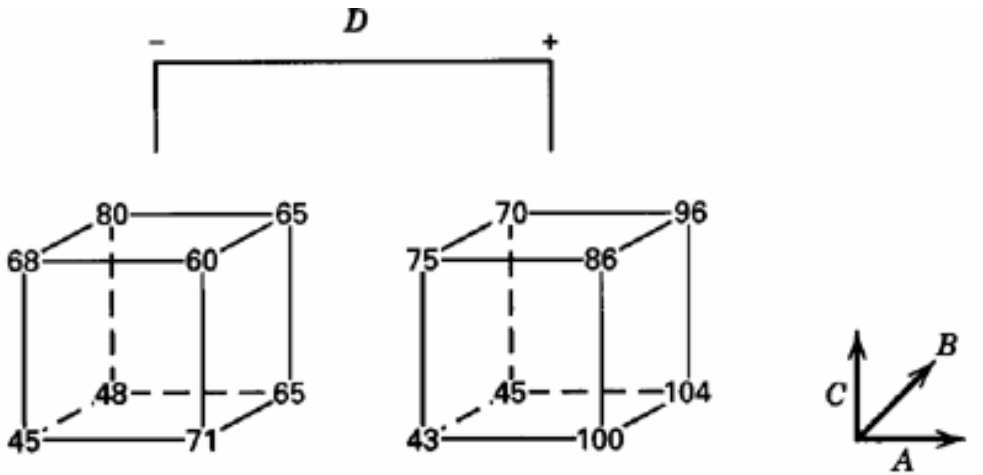

Figure 6-10 Data from the pilot plant filtration rate experiment for

Example

Assume 3 way and higher effects are zero to get an estimate of the variance.

- Effect ABC = 1.875
- Effect ABD = 4.125
- Effect BCD = -2.625
- Effect ABCD = 1.375

If these effects are not significant then they are measuring the variance in the system. We assume they are iid $N(0, \sigma^2)$ and estimate σ^2 with the sample variance of these effects. $S^2 = 7.89$ and estimated standard deviation is 2.81.

Example

The significant effects found are A, C, D, AC and AD, these are temperature, concentration of formaldehyde and stirring rate plus interaction between temperature and formaldehyde and between temperature and stirring. Interactions are the key to solving the problem. Interpret the results in terms of the significant interactions. We will perform the analysis in class.