MANOVA

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One-way MANOVA

Often measures are made on several dependent variables on each experimental unit instead of just one variable.

Group	Y_1	Y_2	Y_3
1	1.11	2.57	3.58
1	1.13	3.06	3.63
⋮	⋮	⋮	⋮
2	0.75	0.85	3.14
2	0.99	2.20	3.27
⋮	⋮	⋮	⋮
4	1.05	1.95	3.34
4	1.20	3.60	4.27

Model

The model for each response vector is

$$Y_{ijr} = \mu_r + \alpha_{ir} + \epsilon_{ijr} \; i = 1, \ldots, k \; j = 1, \ldots, n \; r = 1, \ldots, p$$

- i is the index for the group
- j is the index for the sample
- r is the index for the response variable

Hypothesis

The null hypothesis to be tested is that the means of each of the response variables is the same for each group.

$$H_0 : \begin{pmatrix} \mu_{11} \\ \mu_{12} \\ \vdots \\ \mu_{1p} \end{pmatrix} = \begin{pmatrix} \mu_{21} \\ \mu_{22} \\ \vdots \\ \mu_{2p} \end{pmatrix} = \cdots = \begin{pmatrix} \mu_{k1} \\ \mu_{k2} \\ \vdots \\ \mu_{kp} \end{pmatrix}$$

Tests following rejection of overall MANOVA test

Suppose $k = 3$, $p = 2$ the MANOVA null hypothesis is

$$\mu_1 = \begin{pmatrix} \mu_{11} \\ \mu_{12} \end{pmatrix} = \mu_2 = \begin{pmatrix} \mu_{21} \\ \mu_{22} \end{pmatrix} = \mu_3 = \begin{pmatrix} \mu_{31} \\ \mu_{32} \end{pmatrix}$$

Which is equivalent to testing

$$H_{01} : \mu_{11} = \mu_{21} = \mu_{31}$$

and

$$H_{02} : \mu_{12} = \mu_{22} = \mu_{32}$$

Tests following rejection of overall MANOVA test

- If a MANOVA test rejects the null hypothesis, you can conduct one-way ANOVA tests to see which variable(s) have different means.
- Once you determine which response variables have different means, you can do multiple comparisons to see which groups have different means.