Math 4637/6637

Review for Test one

1. Tests

**One sample t-test** –

Assumptions – a sample of size n from a normal population with unknown mean and

variance, sample elements are independent

Hypothesis to test :

Test Statistic :

Reference distribution when is true: t-distribution with n-1 degrees of freedom

confidence interval for :

p-value depends on the form of the alternative hypothesis

|  |  |
| --- | --- |
| Alternative hypothesis | p-value |
|  |  |
|  |  |
|  |  |

**Two independent samples t-test**-

Assumptions – two independent samples from normal populations with unknown

means and unknown common variance, variances for the two samples are

equal, sample elements are independent

Hypothesis to test:

Test statistic: where is the pooled variance computed as a weighted

average of the sample variances within each population

Reference distribution when is true: t-distribution with degrees of

Freedom

confidence interval for :

P-value depends on the form of the alternative hypothesis

|  |  |
| --- | --- |
| Alternative hypothesis | p-value |
|  |  |
|  |  |
|  |  |

**Welch’s t-test-**

Assumptions – two independent samples from normal populations with unknown

means and unknown variances. Variances are not assumed equal. Sample

elements are independent.

Hypothesis to test:

Test statistic:

Reference distribution when is true: t-distribution with degrees of freedom, where

is a function of the sample variances and sample sizes within each population.

confidence interval for :

p-value depends on the form of the alternative hypothesis

|  |  |
| --- | --- |
| Alternative hypothesis | p-value |
|  |  |
|  |  |
|  |  |

**Paired Sample t-test**

Assumptions – two samples from normal populations with unknown means and unknown variances. Variances are not assumed equal and populations are not

independent of each other.

Hypothesis to test :

Test statistic: where   for i=1,…,n and is the

sample variance of the ’s.

Reference distribution when is true: t-distribution with n-1 degrees of freedom

confidence interval for :

P-value depends on the form of the alternative hypothesis

|  |  |
| --- | --- |
| Alternative hypothesis | p-value |
|  |  |
|  |  |
|  |  |

**Wilcoxon Tests**

Assumptions – one sample, two independent samples, paired samples with

observations independent within each sample, at least ordinal level of

measurement.

Hypothesis to test: are the same for the t-tests

Test statistic: T is based on the sum of signed ranks

Reference distribution – for small samples exact distributions are known, for

large samples normal approximations are used

p-value – is found as the number of cases more extreme than observed divided

by the total number of all possible cases

**Shapiro-Wilks Test**

Assumption – one sample of independent observations from the same

Distribution

Hypothesis to test: the sample is from a normal distribution

Test statistic: T is a function of the distances of the sample cdf from the cdf of a

normal distribution

**Test for equal variance**

Assumption – two independent samples from normal populations with

unknown variances

Hypothesis to test:

Test statistic:

Reference distribution when is true:

p-value depends on the form of the alternative hypothesis

|  |  |
| --- | --- |
| Alternative Hypothesis | p-value |
|  |  |
|  |  |
|  | 2 times the tail probability depending on where T lies |

**Test for equal proportions**

Assumption – two independent samples from Bernoulli distributions with

unknown probabilities of success, and , sample sizes are large

enough for a good approximation with a normal distribution

Hypothesis to test:

Test Statistic: where is the number of successes in the X

sample divided by the sample size there, and is the number of

successes in the Y sample divided by the sample size there, and is

the pooled estimate of the common probability of success *p* under the

null hypothesis.

Reference distribution when is true: approximately N(0,1)

p-value depends on the form of the alternative hypothesis

|  |  |
| --- | --- |
| Alternative Hypothesis | p-value |
|  |  |
|  |  |
|  |  |

**Goodness of fit test**

Assumption – a sample of counts of independent random variables

Hypothesis to test: a specified pdf provides adequate fit to the data

Test statistic: where *k* is the number of cells, is the

observed number of observation in the *i*th cell, is the expected

number of observations there if is true

Reference distribution when is true: where *k* is the number of cells

in the sum of T and *p* is the number of parameters estimated from the

data. This is only an approximation to the distribution of *T* and may not

be good if cell counts are less than 5

p-value: The null hypothesis is always rejected when *T* is too big

**Test for independence in a contingency table**

Assumption – two-way count data from independent random variables.

Hypothesis to test: : the row classification is independent of the

column classification

Data: given in a contingency table as below

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | Totals |
|  |  |  | … |  |  |
|  |  |  | … |  |  |
|  |  |  | … |  |  |
|  |  |  | … |  |  |
| Totals |  |  | … |  |  |

Test statistic: where is the observed cell count

in the ith row and jth column, is the expected cell count if

rows and columns are independent.

Reference distribution when is true: where r is the number of

rows and c is the number of columns. This is only an approximation to

the distribution of *T* and may not be good if cell counts are less than 5

p-value: The null hypothesis is always rejected when *T* is too

big

1. Choosing what test to use and setting up null hypothesis

**Example 1**: A study was conducted to compare the smoking cessation rates for smokers randomly assigned to use a nicotine patch versus a placebo patch. At the end of an 8 week period, the proportion of smokers who had quit smoking was recorded for each group. Interest was in determining whether the nicotine patch increased the chances of quitting.

**State the null and alternative hypothesis:**

: The proportion of smokers who quit in the treatment group is the same as the proportion of smokers who quit in the placebo group.

: The proportion of smokers who quit in the treatment groups is greater than the proportion of smokers who quit in the placebo group.

**State any assumptions you need to make in order to test these hypotheses**

The sample sizes are large enough to use the normal approximation to the binomial distribution and that the two samples are independent.

**Write the test statistic you should use to test these hypotheses**

**If the following results were obtained:**

**with**

**What is the value of the test statistic for this data?**

The pooled estimate of *p* is found as

The test statistic is then

**Write an equation that can be used to find the p-value**

p-value =

**If the computed p-value is found to be .0367, what is your conclusion about the effectiveness of the nicotine patch?**

Since the p-value is relatively small the null hypothesis may be rejected and we can conclude that the proportion of smokers who quit with the patch is greater than the proportion of smokers who quit with the placebo.

**Example 2**. A study is conducted to compare the mean weight loss for men who dieted only and for men who exercised only for a period of one year. Interest is in determining whether there is a difference in weight loss between the two groups.

**State the null and alternative hypotheses:**

: There is no difference in average weight loss for the two groups

: there is a difference in average weight loss for the two groups

**What assumptions should you make in order to test the hypotheses with the method you will use?**

Two independent normal populations with equal variances

**What test statistic could you use to test the hypothese**s?

**If the results of the study are summarized in the table below, what is the value of your test statistic?**

|  |  |
| --- | --- |
| **Diet Only** | **Exercise Only** |
| Sample mean = 5.9 kg | Sample mean = 4.1kg |
| Sample variance = 16.81 | Sample variance = 13.69 |
| Sample size = 42 | Sample size = 47 |

We need to get the pooled variance as and thus our test statistic is

**Write a statement in R that would give the p-value for this test.**

2\*(1-pt(2.177, 42+47-2)) OR 2\*(pt(-2.177,42+47-2))

**If the resulting p-value is .0322, what are your conclusions regarding the two groups?**

The p-value is pretty small so we can reject the null hypothesis that the mean weight loss is the same in both groups. The data in this case suggests that the mean weight loss in the diet only group may be greater than the mean weight loss in the exercise only group.

1. Be able to construct a confidence interval if the formula for the confidence interval is one of those given above.

**Example**

In the situation above, construct a 95% confidence interval for the mean weight loss of the diet only group given and

For the 95% confidence interval for the mean we have

And for this data we have

= (4.622,7.178)

1. Be able to conduct a goodness of fit test.

**Example**

A few years ago the residents of a certain city were represented by four major ethnic groups according to the 8:2:2:1 ratio. A recent random sample of size n=1000 resulted in the observed numbers 410, 218, 297, and 75 for the four ethnic groups. Do these recent observed values agree with the previous ratio?

The null hypothesis is

And the alternative is that at least one of these is not equal . We compute expected numbers as

|  |  |  |
| --- | --- | --- |
| Ethnic Group | Observed Frequency | Expected Frequency |
| 1 | 410 | 615 |
| 2 | 218 | 154 |
| 3 | 297 | 154 |
| 4 | 75 | 77 |
|  | 1000 | 1000 |

The test statistic is

The p-value is found as

We reject the null hypothesis and conclude that the current ethnic ratios are different than the previous ones.

1. Be able to test for independence of rows and columns in a contingency table or to test for homogeneity (the test is the same)

Example –

A survey of 300 workers at ABC Corporation were surveyed to determine whether they felt the current leave program should be modified or not. The following data were collected. It is desired to determine whether the opinion on the modification is independent of gender. (Or I could say it is desired to determine whether the proportion of those who approve the modification is the same for each gender this speaks of homogeneity).

|  |  |  |  |
| --- | --- | --- | --- |
| Gender | Opinion | | Totals |
| Modify | Do Not Modify |
| Male | 157(159) | 30(28) | 187 |
| Female | 98(96) | 15(17) | 113 |
| Total | 255 | 45 | 300 |

The expected cell counts shown in red are found as

The test statistic

We compare this test statistic to a and get the p-value as