Lab 12: Gaussian Mixture Models (GMMs)

In lecture, we learned that the Gaussian Mixture Model (GMM) is a more sophisticated unsupervised clustering method than k-means.

The GMM models a dataset $(\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(m)})$ as an i.i.d. sample from the following generative model for each sample $\mathbf{x}^{(i)}$:

1. Sample $z^{(i)}$ from a multinomial distribution over clusters $1..k$ according to probabilities $(\phi_1, \ldots, \phi_k)$.
2. Sample $\mathbf{x}^{(i)}$ from $\mathcal{N}(\mu_{z^{(i)}}, \Sigma_{z^{(i)}})$.

The parameters are estimated using the Expectation Maximization (EM) algorithm, which begins with a guess for parameters $\phi_1, \ldots, \phi_k, \mu_1, \ldots, \mu_k, \Sigma_1, \ldots, \Sigma_k$ then iteratively alternates between computing a soft assignment of data to clusters then updating the parameters according to that soft assignment.

First, we'll build a GMM model for a dataset then use the model for anomaly detection.

Example 1: Anomaly detection

Let's generate synthetic data from a mixture of Gaussians, use EM to recover as best possible the ground truth parameters, and then use the model to find "anomalies" (unusually unlikely points according to the model). First, we set up the ground truth parameters and generate a dataset from those ground truth parameters:

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import warnings
warnings.filterwarnings("ignore")

# Ground truth means and covariances for the data we'll generate

means_gt = [ [1,10], [10,1], [10,10] ]
sigmas_gt = [ np.matrix([[1, 0],[0, 1]]), np.matrix([[4,0],[0,
1]]),
              np.matrix([[1,0],[0,4]]) ]

# Ground truth Prior probability (phi_j) for each cluster

phi_gt = [ 0.2, 0.2, 0.6 ]

# For more interesting covariances, you can also try, for exampl
e,
# [[11.31371, -0.70711],[11.31371, 0.70711]] or
# [[11.31371, 0.70711],[-11.31371, 0.70711]].

# Size of dataset

m = 500

# number of variables

n = len(means_gt[0])

# k number of clusters/outcomes

k = len(phi_gt)

# Ground truth indices of cluster identities

Z = [0]*m

# Generate a new k-means dataset

def gen_dataset():
    X = np.zeros((m,n))
    # Generate m samples from multinomial distribution using phi_
gt
    z_vectors = np.random.multinomial(1, phi_gt, size=m)  # Resul
t: binary matrix of size (m x k)
    for i in range(m):
        # Convert one-hot representation z_vectors[i,:] to an ind
ex
        Z[i] = np.where(z_vectors[i,:] == 1)[0][0]
        # Grab ground truth mean mu_{z^i}
        mu = means_gt[Z[i]]
        # Grab ground truth covariance Sigma_{z^i}
        sigma = sigmas_gt[Z[i]]
        # Sample a 2D point from mu, sigma
        X[i,:] = np.random.multivariate_normal(mu,sigma,1)
    return X
```

```
X = gen_dataset()
```

Next, the EM algorithm itself. We have an initialization step and an iterative step.

In [2]:
```
def init_gmm(X, k):
    m = X.shape[0]
    n = X.shape[1]
    Mu = np.zeros((n,k))
    Sigma = np.zeros((k,n,n))
    Phi = np.zeros(k)
    order = np.random.permutation(m)
    for j in range(k):
        # Initially assign equal probability to each cluster/outc
ome
        Phi[j] = 1/k
        # Ramdomly assign mean to one of the data points
        Mu[:,j] = X[order[j],:].T
        # Initial covariance is identity matrix
        Sigma[j,:,:] = np.eye(n)
    return Phi, Mu, Sigma


def Gaussian(X, mean, covariance):
    k = len(mean)
    X = X - mean.T
    p = 1/((2*np.pi)**(k/2)*(np.linalg.det(covariance)**0.5)) * n
p.exp(-0.5 * np.sum(X @ np.linalg.pinv(covariance) * X, axis=1))
    return p

def gaussian(x, mean,covariance):
    k = len(mean)
    X = (x - mean).reshape(-1,1)
    p = 1/((2*np.pi)**(k/2)*(np.linalg.det(covariance)**0.5)) * n
p.exp(-0.5 * (X.T @ np.linalg.pinv(covariance) @ X))
    return p


# Run one iteration of EM

def iterate_em_gmm(X, threshold, Phi, Mu, Sigma):
    m = X.shape[0]
    n = X.shape[1]
    k = len(Phi)
    threshold = np.reshape(np.repeat(threshold, n*k), (n,k))
    pj_arr = np.zeros((m,k))

    # E-step: calculate w_j^i
    W = np.zeros((m, k))
    for j in range(k):
        pj = Gaussian(X, Mu[:,j], Sigma[j])
        pj_arr[:,j] = pj
        W[:,j] = Phi[j] * pj

    # W tells us what is the relative weight of each cluster for
each data point
    W[:,:] = W * np.tile(1/np.sum(W,1),(k,1)).T

    # M-step: adjust mean and sigma
    Phi[:] = sum(W) / m
    Mu_previous = Mu.copy()
    for j in range(k):
```

```
        # Split cluster specific W for each dimension
        Wj = np.tile(W[:,j],(2,1)).T
        # Compute Mu for each variable for each cluster
        Mu[:,j] = sum(X * Wj)/sum(Wj)
        Muj = np.tile(Mu[:,j],(m,1))
        Sigma[j,:,:] = np.matmul((X - Muj).T, (X - Muj) * Wj) / s
um(W[:,j])

    if (abs(Mu-Mu_previous) <= threshold).all():
        converged = True
    else:
        converged = False

    labels = np.argmax(pj_arr, axis = 1)
    pj = np.max(pj_arr,axis=1)
    X_label = np.insert(X, 2, labels, axis=1)
    return converged, pj, X_label
```

Let's run the model to convergence:

In [3]:
```
threshold = np.matrix(.01)

Phi, Mu, Sigma = init_gmm(X, k)

converged = False
while not converged:
    converged, pj, X_label = iterate_em_gmm(X, threshold, Phi, M
u, Sigma)
```

In [4]:
```
print(Phi)
print(phi_gt)

phi_gt = np.array(phi_gt).reshape(-1,1)
phi_mse = np.mean(np.min((Phi-phi_gt)**2,axis=1))

print(phi_mse)
```

```
[0.62384633 0.182      0.19415367]
[0.2, 0.2, 0.6]
0.00021233546828076333
```

In [5]:
```
print(Mu)
print(np.array(means_gt).T)
```

```
[[10.03435519  0.98512683  9.54585509]
 [10.06840846 10.08312929  0.89186272]]
[[ 1 10 10]
 [10  1 10]]
```

In [6]:
```
print(Sigma)
print(sigmas_gt)
```

```
[[[1.05273899 0.19871418]
  [0.19871418 3.88084216]]

 [[0.94665083 0.04365856]
  [0.04365856 0.97465652]]

 [[3.41835778 0.08736788]
  [0.08736788 0.90378729]]]
[matrix([[1, 0],
        [0, 1]]), matrix([[4, 0],
        [0, 1]]), matrix([[1, 0],
        [0, 4]])]
```

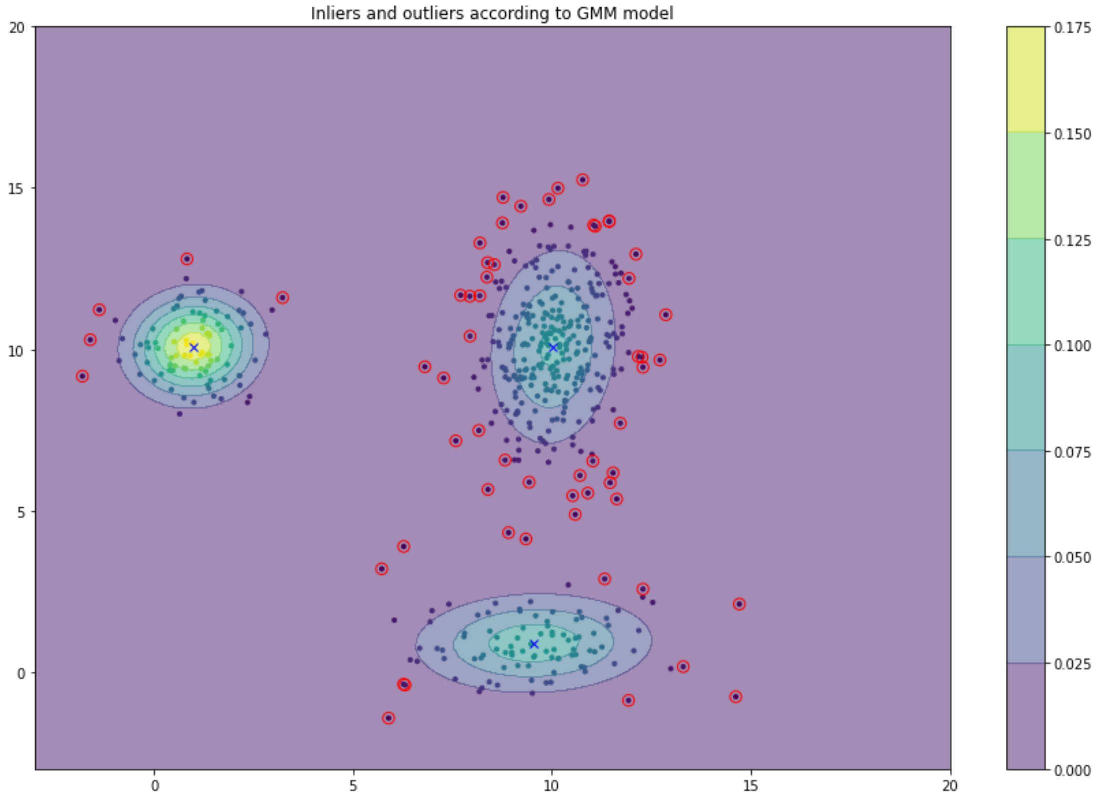
In-class exercise

Determine how close the estimated parmeters Phi, Mu, and Sigma are to the ground trouth values set up at the beginning of the experiment. Report your results and briefly discuss in your lab report.

Next, we continue to find outliers:

In [7]:
```
outlier_prob = .01
outliers = np.nonzero(pj < outlier_prob)[0]
```

In [8]:
```
fig1 = plt.figure(figsize=(15,10))

xlist = np.linspace(-3, 20, 100)
ylist = np.linspace(-3, 20, 100)
XX, YY = np.meshgrid(xlist, ylist)
ZZ = np.zeros(XX.shape)

for c in np.arange(0,k):
    X_class = X[np.where(X_label[:,2] == c)[0],:]

    Z = np.zeros(XX.shape)
    i = 0
    while i < XX.shape[0]:
        j = 0
        while j < XX.shape[0]:
            pt = np.array([[XX[i,j], YY[i,j]]])
            Z[i,j] = Gaussian(pt, Mu[:,c], Sigma[c])[0]
            j = j + 1
        i = i + 1
    ZZ = np.maximum(ZZ,Z)
cp = plt.contourf(XX, YY, ZZ,alpha=0.5)
cbar = fig1.colorbar(cp)

plt.scatter(X[:,0],X[:,1],marker=".",c=pj,cmap='viridis');
plt.scatter(X[outliers,0],X[outliers,1],marker="o",facecolor="non
e",edgecolor="r",s=70);
plt.plot(Mu[0,0], Mu[1,0],'bx',Mu[0,1], Mu[1,1],'bx', Mu[0,2], Mu
[1,2],'bx')

plt.title('Inliers and outliers according to GMM model')
plt.show()
```



In-class exercise

Notice that using a hard threshold for each cluster gives us more outliers for a broad cluster than a tight cluster. First, understand why, and explain in your report. Second, read about Mahalanobis distance of a point to the mean of a multivariate Gaussian distribution and see if you can use Mahalanobis distance to get a better notion of outliers in this dataset.

Exercise 1.1 (10 points)

Notice that using a hard threshold for each cluster gives us more outliers for a broad cluster than a tight cluster. Understand why, and explain in your report.

In [9]:
```
# You may need code to explain
```

Decribe here

Exercise 1.2 (15 points)

Read about Mahalanobis distance of a point to the mean of a multivariate Gaussian distribution and see if you can use Mahalanobis distance to get a better notion of outliers in this dataset.

1. Explain what is Mahalanobis (5 points)
2. Write code Mahalanobis (10 points)

Explain what is Mahalanobis (5 points)

Explain here!

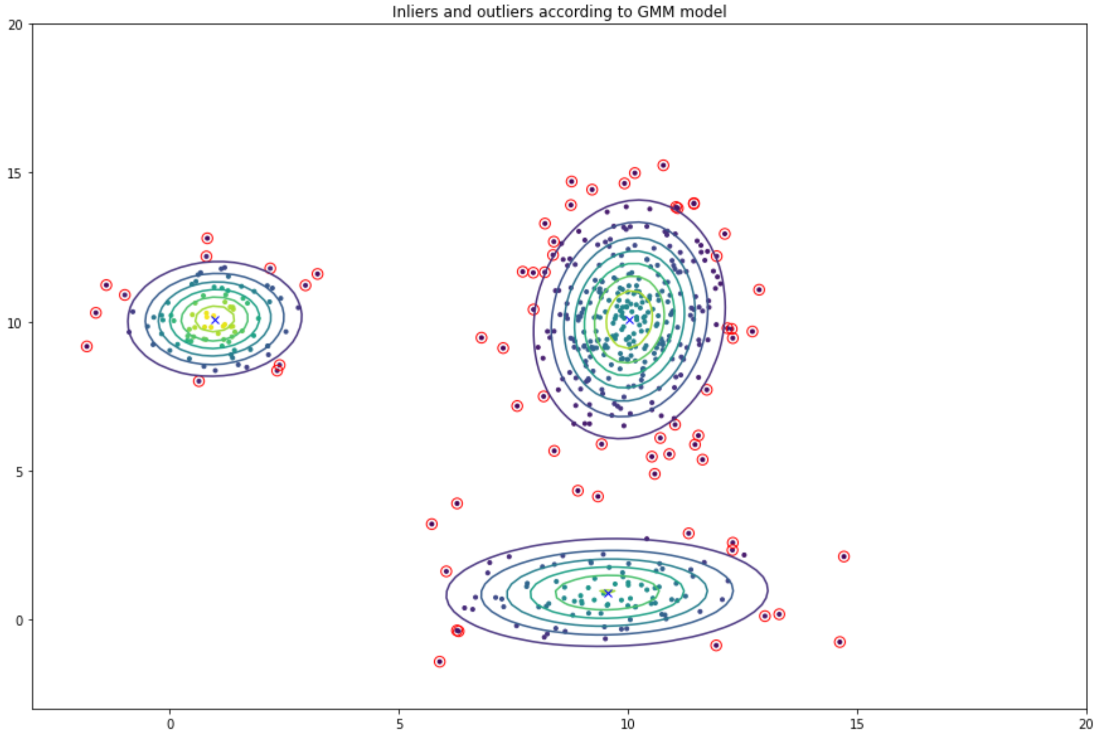
Write code Mahalanobis (10 points)

In [10]:
```
import sys
#np.set_printoptions(threshold=sys.maxsize)

print(Sigma.shape)
print(Mu.shape)
print(X.shape)

m_distance = np.zeros((X.shape[0],Mu.shape[1]))

for kk in range(Mu.shape[1]):
    for i, x in enumerate(X):
        # get all row data from target column
        mu = None
        # get target sigma
        sig = None
        # inverse matrix of sigma
        sig_inv = None
        # find difference of mu and x and reshape it (if need)
        diff = None
        # calculate distance from diff and sigma
        distance = None
        ### BEGIN SOLUTION
        mu = Mu[:,kk]
        sig = Sigma[kk]
        sig_inv = np.linalg.inv(sig)
        diff = (mu-x).reshape(-1,1)
        distance = np.sqrt(diff.T@sig_inv@diff)
        ###END SOLUTION
        # keep distance
        m_distance[i,kk] = distance

# find unique of minimum m_distance and count
# hint: use np.unique and np.argmin
(unique, counts) = None, None

max_z_score = 2.05
# find minimum distance
# hint: use np.min
min_distance = None
# find outlier from min_distance
outlier = None
### BEGIN SOLUTION
a = np.argmin(m_distance,axis=1)
(unique, counts) = np.unique(a, return_counts=True)

max_z_score = 2.05
min_distance = np.min(m_distance,axis=1)
outlier = min_distance > max_z_score
outlier = np.nonzero(outlier)[0]
### END SOLUTION
```

```
(3, 2, 2)
(2, 3)
(500, 2)
```

In [11]:
```
# Test function: Do not remove
print('outlier', outlier)

fig1 = plt.figure(figsize=(15,10))

xlist = np.linspace(-3, 20, 100)
ylist = np.linspace(-3, 20, 100)
XX, YY = np.meshgrid(xlist, ylist)
ZZ = np.zeros(XX.shape)

for c in np.arange(0,k):
    X_class = X[np.where(X_label[:,2] == c)[0],:]

    Z = np.zeros(XX.shape)
    i = 0
    while i < XX.shape[0]:
        j = 0
        while j < XX.shape[0]:
            pt = np.array([[XX[i,j], YY[i,j]]])
            Z[i,j] = Gaussian(pt, Mu[:,c], Sigma[c])[0]
            j = j + 1
        i = i + 1
    cp = plt.contour(XX,YY,Z)

plt.scatter(X[:,0],X[:,1],marker=".",c=pj,cmap='viridis');
plt.scatter(X[outlier,0],X[outlier,1],marker="o",facecolor="non
e",edgecolor="r",s=70);
plt.plot(Mu[0,0], Mu[1,0],'bx',Mu[0,1], Mu[1,1],'bx', Mu[0,2], Mu
[1,2],'bx')

plt.title('Inliers and outliers according to GMM model')
plt.show()
print('success!')
# End test function
```

```
outlier [  6   8  19  27  34  37  51  55  62  64  97  98 103 109
116 117 137 146
 151 160 164 173 174 187 191 194 196 200 214 232 233 248 252 254
257 260
 262 266 269 270 278 283 305 308 349 358 363 365 367 368 379 380
381 398
 405 406 417 419 424 434 439 449 474 477 482 485 499]
```




```
success!
```

Example 2: Customer segmentation

In this example we will use the Kaggle customer segmentation from last week dataset [Mall_Customers.csv (https://www.kaggle.com/vjchoudhary7/customer-segmentation-tutorial-in-python)](https://www.kaggle.com/vjchoudhary7/customer-segmentation-tutorial-in-python).

Let's stick to just two dimensions in the dataset:

In [17]:
```
data = pd.read_csv('Mall_Customers.csv')
data = data.drop(['CustomerID', 'Gender', 'Age'], axis = 1)
print(data.head())
```

```
   Annual Income (k$)  Spending Score (1-100)
0                  15                      39
1                  15                      81
2                  16                       6
3                  16                      77
4                  17                      40
```

In [18]:
```
X  = np.array(data, dtype=float)

n = X.shape[1]
m = X.shape[0]
k = 3

threshold = np.matrix(.01)

# Slightly different version of init_gmm due to the data format a
nd spread
def init_gmm(X, k):
    Mu = np.zeros((n,k))
    Sigma = np.zeros((k,n,n))
    Phi = np.zeros(k)
    order = np.random.permutation(m)
    for j in range(k):
        Phi[j] = 1/k
        Mu[:,j] = X[order[j],:].T
        Sigma[j,:,:] = np.cov(X.T)
    return Phi, Mu, Sigma


Phi, Mu, Sigma = init_gmm(X, k)

converged = False
while not converged:
    converged, pj, X_label = iterate_em_gmm(X, threshold, Phi, M
u, Sigma)
```

In [19]:
```
print(Mu)
```

```
[[51.22197835 57.28747103 95.58290812]
 [53.12645837 54.89122093 27.73437282]]
```

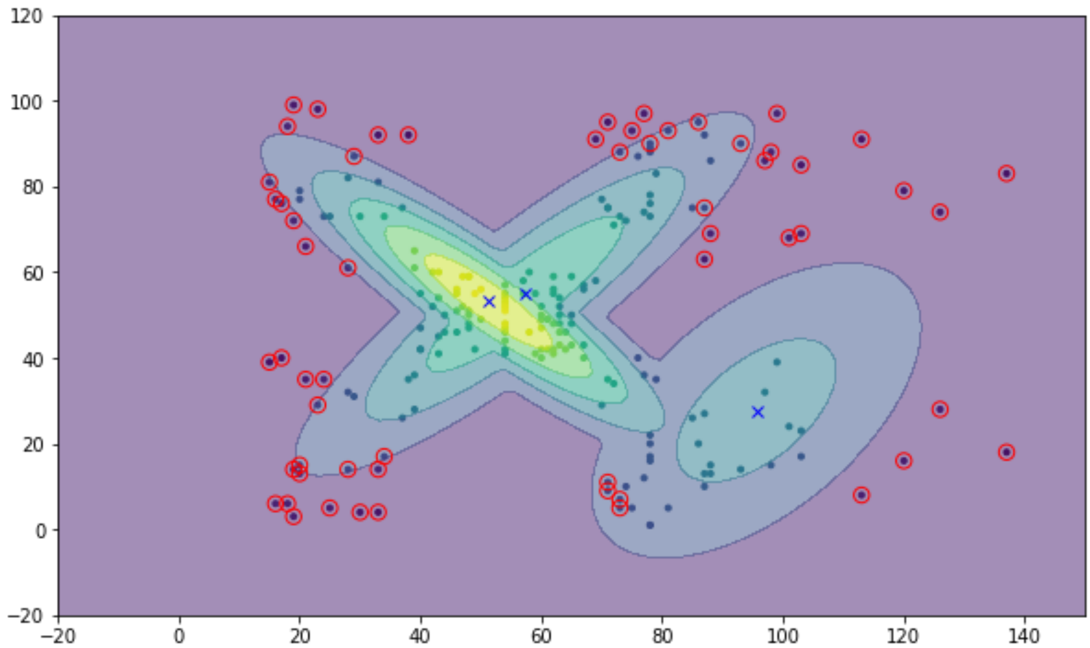
The first row represents annual income, whereas the second row represents the spending score. From what i noticed, these values changes in every iteration, and therefore it is difficult segregate this data into 3 categories.

In [20]:
```
print(Sigma.shape)
print(Mu.shape)
```

```
(3, 2, 2)
(2, 3)
```

Next, the visualization:

In [21]:
```
plt.figure(figsize=(10,6))
plt.scatter(X[:,0],X[:,1],marker=".",c=pj,cmap='viridis');

outlier_prob = .0002
outliers = np.nonzero(pj<outlier_prob)[0]


xlist = np.linspace(-20, 150, 100)
ylist = np.linspace(-20, 120, 100)
XX, YY = np.meshgrid(xlist, ylist)
ZZ = np.zeros(XX.shape)
for c in np.arange(0,k):
    X_class = X[np.where(X_label[:,2] == c)[0],:]
    Z = np.zeros(XX.shape)
    i = 0
    while i < XX.shape[0]:
        j = 0
        while j < XX.shape[0]:
            pt = np.array([[XX[i,j], YY[i,j]]])
            Z[i,j] = Gaussian(pt, Mu[:,c], Sigma[c])
            j = j + 1
        i = i + 1
    ZZ = np.maximum(ZZ,Z)
cp = plt.contourf(XX, YY, ZZ,alpha=0.5)
plt.scatter(X[outliers,0],X[outliers,1],marker="o",facecolor="none",edgecolor="r",s=70);
plt.plot(Mu[0,0], Mu[1,0],'bx',Mu[0,1], Mu[1,1],'bx', Mu[0,2], Mu[1,2],'bx')

plt.show()
```



In-class exercise (25 points)

Examine the cluster centers and determine whether you can find any reasonable interpretation of them. Discuss in your report (5 points), and compare to last week's results with k-means. (20 points)

Discussion report (5 points)

Describe here!

Do k-mean and compare the result

In [22]:
```
# k-mean code and compare here
```

In-class exercise (10 points)

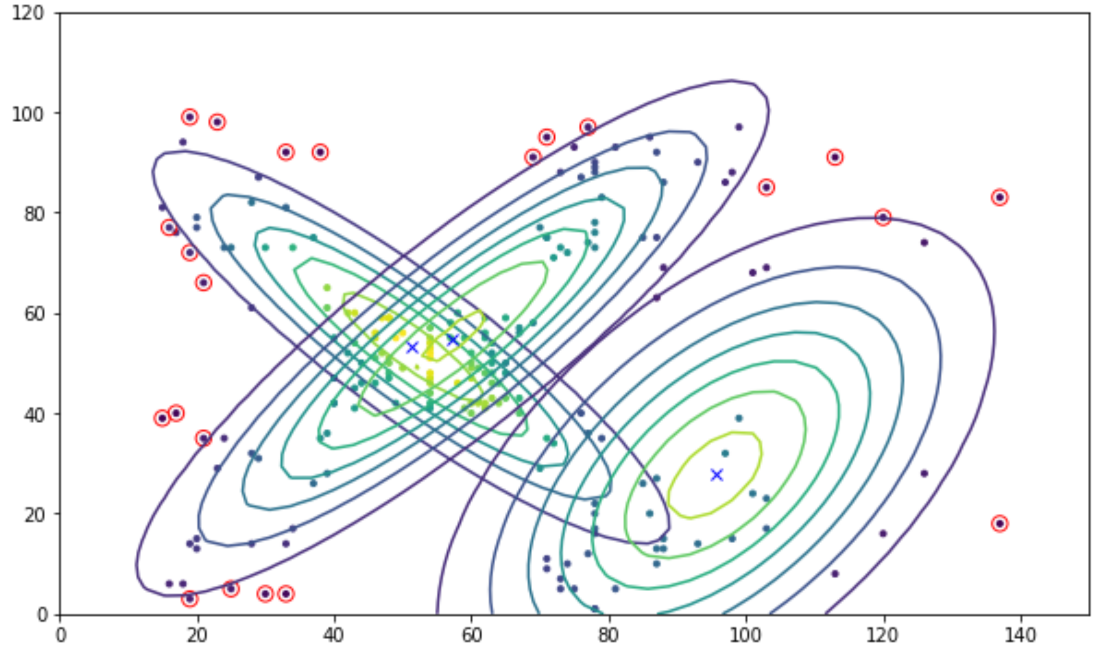
Do the same analysis with Mahalanobis distance as in the first example.

In [23]:
```
outlier = None
### BEGIN SOLUTION
print(Sigma.shape)
print(Mu.shape)
print(X.shape)

m_distance = np.zeros((X.shape[0],Mu.shape[1]))

for kk in range(Mu.shape[1]):
    for i, x in enumerate(X):
        mu = Mu[:,kk]
        sig = Sigma[kk]
        sig_inv = np.linalg.inv(sig)
        diff = (mu-x).reshape(-1,1)
        distance = np.sqrt(diff.T@sig_inv@diff)
        m_distance[i,kk] = distance

print("Mahalanobis Distance")
#print(m_distance)
a = np.argmin(m_distance,axis=1)
(unique, counts) = np.unique(a, return_counts=True)
#print(counts)

max_z_score = 2.0 #99 percent
min_distance = np.min(m_distance,axis=1)
outlier = min_distance > max_z_score
outlier = np.nonzero(outlier)[0]
print(outlier)
#END SOLUTION
```

```
(3, 2, 2)
(2, 3)
(200, 2)
Mahalanobis Distance
[  0   3   4   8   9  11  16  17  19  22  30  32  33  41 123 127
145 189
 193 195 198 199]
```

In [25]:
```
# Test function: Do not remove
print('outlier', outlier)

plt.figure(figsize=(10,6))
plt.scatter(X[:,0],X[:,1],marker=".",c=pj,cmap='viridis');

plt.scatter(X[outlier,0],X[outlier,1],marker="o",facecolor="non
e",edgecolor="r",s=70);
plt.plot(Mu[0,0], Mu[1,0],'bx',Mu[0,1], Mu[1,1],'bx', Mu[0,2], Mu
[1,2],'bx')

for c in np.arange(0,k):
    X_class = X[np.where(X_label[:,2] == c)[0],:]
    xlist = np.linspace(0, 150, 50)
    ylist = np.linspace(0, 120, 50)

    XX, YY = np.meshgrid(xlist, ylist)
    Z = np.zeros(XX.shape)
    i = 0
    while i < XX.shape[0]:
        j = 0
        while j < XX.shape[0]:
            pt = np.array([[XX[i,j], YY[i,j]]])
            Z[i,j] = Gaussian(pt, Mu[:,c], Sigma[c])
            j = j + 1
        i = i + 1
    cp = plt.contour(XX, YY, Z)

plt.show()
print('success!')
# End test function
```

```
outlier [  0   3   4   8   9  11  16  17  19  22  30  32  33  41
123 127 145 189
 193 195 198 199]
```




```
success!
```

Example 3 Customer segmentation

This example is based on Nguyen Hanh's tutorial on Medium.com (https://medium.com/@nguyenbaha/buiding-customer-segmentation-by-gmm-from-scratch-4ea6adc3da1c). In this example we use the Kaggle OnlineRetail.csv (https://www.kaggle.com/vijayuv/onlineretail) dataset for customer segmentation.

In [26]:
```
data = pd.read_csv('Online_Retail.csv')
data = data.iloc[0:5000,:]
print(data.head())
data = data.drop(['InvoiceNo','Description', 'CustomerID'], axis
= 1)
```

```
  InvoiceNo StockCode                          Description  Quant
ity  \
0    536365    85123A   WHITE HANGING HEART T-LIGHT HOLDER
6
1    536365     71053                  WHITE METAL LANTERN
6
2    536365    84406B       CREAM CUPID HEARTS COAT HANGER
8
3    536365    84029G  KNITTED UNION FLAG HOT WATER BOTTLE
6
4    536365    84029E       RED WOOLLY HOTTIE WHITE HEART.
6

  InvoiceDate  UnitPrice CustomerID        Country  TotalSum
0   12/1/2010       2.55      17850  United Kingdom     15.30
1   12/1/2010       3.39      17850  United Kingdom     20.34
2   12/1/2010       2.75      17850  United Kingdom     22.00
3   12/1/2010       3.39      17850  United Kingdom     20.34
4   12/1/2010       3.39      17850  United Kingdom     20.34
```

In [27]:
```
print(data.dtypes)
```

```
StockCode       object
Quantity         int64
InvoiceDate     object
UnitPrice      float64
Country         object
TotalSum       float64
dtype: object
```

Let's view the categorical and numeric columns:

In [28]:
```
categorical_colmns = data.select_dtypes(include=['object']).colum
ns
print(categorical_colmns)
numerical_colmns = data._get_numeric_data().columns
print(numerical_colmns)
```

```
Index(['StockCode', 'InvoiceDate', 'Country'], dtype='object')
Index(['Quantity', 'UnitPrice', 'TotalSum'], dtype='object')
```

In [29]:
```
def missing_percentage(data):
    """This function takes a DataFrame(df) as input and returns t
wo columns,
     total missing values and total missing values percentage"""
    total = data.isnull().sum().sort_values(ascending = False)
    percent = round(data.isnull().sum().sort_values(ascending = F
alse)/len(data)*100,2)
    return pd.concat([total, percent], axis=1, keys=['Total','Per
cent'])


missing_percentage(data)
```

Out[29]:

	Total	Percent
TotalSum	48	0.96
Country	12	0.24
UnitPrice	0	0.00
InvoiceDate	0	0.00
Quantity	0	0.00
StockCode	0	0.00

Next, let's fill the "na" values with "No information" and 0

In [30]:
```
data[categorical_colmns] = data[categorical_colmns].fillna("No in
formation")
data[numerical_colmns] = data[numerical_colmns].fillna(0)

print(data.head())
```

```
  StockCode  Quantity InvoiceDate  UnitPrice         Country  Tot
alSum
0    85123A         6   12/1/2010       2.55  United Kingdom   
15.30
1     71053         6   12/1/2010       3.39  United Kingdom   
20.34
2    84406B         8   12/1/2010       2.75  United Kingdom   
22.00
3    84029G         6   12/1/2010       3.39  United Kingdom   
20.34
4    84029E         6   12/1/2010       3.39  United Kingdom   
20.34
```

In [31]:
```
def category_to_numeric(categorical_columns):
    i = 0;
    columnname = '';
    while i < len(categorical_colmns):
        col_idx = data.columns.get_loc(categorical_colmns[i])
        distinct_values = data[categorical_colmns[i]].unique()
        j = 0;
        for val in distinct_values:
            idx = np.where(data[categorical_colmns[i]] == val);
            data.iloc[idx[0],col_idx] = j
            j = j + 1;
        i = i + 1;


category_to_numeric(data[categorical_colmns])

data = data.astype('float64')
print(data.head())
```

```
   StockCode  Quantity  InvoiceDate  UnitPrice  Country  TotalSum
0        0.0       6.0          0.0       2.55      0.0     15.30
1        1.0       6.0          0.0       3.39      0.0     20.34
2        2.0       8.0          0.0       2.75      0.0     22.00
3        3.0       6.0          0.0       3.39      0.0     20.34
4        4.0       6.0          0.0       3.39      0.0     20.34
```

In [32]:
```
Mu = np.std(data[numerical_colmns])
Sigma = np.mean(data[numerical_colmns])
print(Mu)
print(Sigma)
```

```
Quantity     146.544560
UnitPrice      5.908027
TotalSum      65.901813
dtype: float64
Quantity     10.941200
UnitPrice     3.252928
TotalSum     22.146162
dtype: float64
```

In [33]:
```
# Check for outliers

def cnt_outlier(data,sigma, mu, inc_cols=[]):
    num_cols = data.select_dtypes(include=[np.number]).columns
    num_cols = [e for e in num_cols if e in inc_cols]
    outlier = (data[numerical_colmns]-mu).abs() > sigma**2
    return outlier.sum()

cnt_outlier(data,Sigma,Mu, numerical_colmns).sort_values(ascendin
g=False)
```

Out[33]:
```
Quantity     4650
UnitPrice      56
TotalSum       12
dtype: int64
```

In [34]:
```
if len(data[data.duplicated()]) > 0:
    print("No. of duplicated entries: ", len(data[data.duplicated
()]))
    print(data[data.duplicated(keep=False)].sort_values(by=list(d
ata.columns)).head())
    data.drop_duplicates(inplace=True)
else:
    print("No duplicated entries found")
```

```
No. of duplicated entries:  987
      StockCode  Quantity  InvoiceDate  UnitPrice  Country  Total
Sum
4099        0.0       2.0          2.0       2.95      0.0
5.9
4131        0.0       2.0          2.0       2.95      0.0
5.9
4190        0.0       2.0          2.0       2.95      0.0
5.9
0           0.0       6.0          0.0       2.55      0.0      1
5.3
49          0.0       6.0          0.0       2.55      0.0      1
5.3
```

In-class and take-home exercise

Use the same GMM code as in the previous two examples on this dataset. Try to interepret the results you get and plot the inliers/outliers with a Mahalanobis distance threshold. Plot likelihood as a function of k and determine whether there is an "elbow" in the plot. How many clusters should you use? Describe your experiments and results in your report.

In [35]:
```
X = data.values

mean = np.mean(X,axis=0)
std = np.std(X,axis=0)

X = (X-mean)/std

print(X)
```

```
[[-1.3238868  -0.03572409 -1.0726451  -0.11163787 -0.26564736 -0.
11949508]
 [-1.32140824 -0.03572409 -1.0726451   0.02039671 -0.26564736 -0.
05026551]
 [-1.31892968 -0.02348829 -1.0726451  -0.08020106 -0.26564736 -0.
0274637 ]
 ...
 [-0.60510319  0.66171654  1.5196613  -0.44643984  6.87883167  0.
36263946]
 [-1.15038731  0.22122772  1.5196613  -0.42600592  6.87883167  0.
03297482]
 [-0.28784697  0.22122772  1.5196613  -0.42600592  6.87883167  0.
03297482]]
```

In [ ]:
```
# Your code here
```

In [ ]:

```

```

In [ ]:

```

```