CS698W: Game Theory and Collective Choice **Jul-Nov 2017**

Project Report: Online marketplace for marginal farmers

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Abstract

Our objective is to tackle the problem of intermediary induced price inflation in the supply chain of food from rural farmers to urban consumers by setting up a theoretical online marketplace for the sale of products. We introduce an extended version of Trade Reduction Mechanism that sets up a double auction in which the farmers directly trade with consumers by submitting their bids. After the allocation is made, farmers sell products to a market in the village which are then transported to another market in the city from where the consumers buy their allocated products. We simulate this mechanism using *SimPy* to verify the underlying properties and demonstrate a live setup of the trade where number of farmers and consumers vary across the trading days. We also show that the ratio of total social welfare and maximum possible social welfare tends to one as the number of participating agents tend to infinity.

1 Introduction to the problem

The biggest problem in rural India is that the suppliers do not directly trade with consumers - rather they are forced to communicate via intermediaries, which gives an unnecessary disadvantage to both the suppliers and consumers. We aim to specifically look at the domestic Indian setting and tackle the problem of intermediary induced price inflation in the supply chain of food from producers to consumers by setting up a theoretical online marketplace for the sale of products.

When designing a mechanism there are several key properties that are desirable to maintain. Some of the more important ones are:

1. Individual rationality (IR) - to make it worthwhile for all players to participate
2. Incentive compatibility (IC) - to give incentive to players to report their true value to the mechanism
3. Budget balance (BB) - not to run the mechanism on a loss.
4. Economic Efficiency (EE) - to ensure that after all trading has completed, the items are in the hands of those that value them the most.

However, it is well known from [2] that any mechanism that maximizes social welfare while maintaining individual rationality and incentive compatibility runs a deficit, i.e., is not budget balanced. Of course, for many applications of practical importance we lack the will and the capability to allow the mechanism to run a deficit and hence one must balance the payments made by the mechanism. To maintain the BB property in an IR and IC mechanism it is necessary to compromise on the optimally of the social welfare.

1.1 Related work

There have been several attempts to design budget balanced mechanisms for particular domains. For instance, for double-sided auctions where both the buyers and sellers are strategic demanding and supplying single unit of a homogeneous good, one of the popular mechanism is the *Trade Reduction Mechanism (TRM)* which gives up a single deal in order to maintain truthfulness. McAfee[3] developed this mechanism that produces an allocation for single unit demand and single unit supply given valuations of buyers and sellers such that the mechanism is IR, IC, and BB while retaining most of the social welfare. In the distributed markets problem, goods are transported between geographic locations while incurring some constant cost for transportation.

Despite the works discussed above, the question of how to design a general mechanism that achieves IR, IC, and BB independently of the problem domain remains open. An interesting example is the open question left by Moshe Babaioff et al[1]; how can one bound the loss in social welfare that is needed to achieve budget balance in an IR and IC distributed market where the transportation edges are strategic.

In this paper, we extend the TRM by McAfee[3] to buyers and sellers demanding and supplying multiple units of a homogeneous product. We will show that these agents have a dominant strategy to bid according to their true type of the product. We will also show that the auction system never runs into deficit *(Weak Budget Balance)* and agents will never suffer a loss from participating *(Individual Rationality)*. We compromise economic efficiency to achieve these other properties but we will show towards the end of this report that the total social welfare tends to the maximum value as the number of participating agents tends to infinity. Cripps et al[5] showed that under some specific criteria, all the nontrivial Bayesian Nash equilibria of the uniform-price double auction are asymptotically efficient. Our convergence result parallels their result.

Simon Loertscher et al[4] had taken up a similar problem of traders with multiple units of demands and supplies and solved it by making one side of the market trade at a single price while the other side trade at VCG prices (with a reserve price). Our method uses single price at both sides and thus is simpler as well as computationally more efficient since VCG requires the allocation function to run $O(n+m)$ times to decide the prices while single price does that in $O(1)$ time. We've also taken the transportation costs into account to make the mechanism more realistic.

1.2 Brief overview of the report

In Section 2, we introduce our model of Extended Trade Reduction Mechanism. In Section 3, we prove that this model is DSIC, IR, and WBB, and almost efficient for large number of trades. Our simulations and experiments are described in section 4. We conclude this report in Section 5.

2 Formal model of the problem

This section sets up the notation and definitions you will be using in the rest of the report. Note that a better organized report should have all notation and definitions at one place so that someone navigating through the report knows where to find the definition of a term when found in some place in the paper. Only the definitions/explanatory terms that are very specific to a section, e.g., if a new abbreviation or term is introduced in the experiment section, should be introduced in that section.}

There are two types of players in this system:

1. **Farmers** (F) who produces and sells items. $F = \{1, 2, ..., n\}$
2. **Customers** (C) who buys items produced by farmers. $C = \{1, 2, ..., m\}$

We consider the case of a single product for ease of computation. Note that it can be easily extended to multiple products by bringing in a new dimension for more types for products.

Each farmer i has a private type $t_{f,i} \in T_{f,i}$ and each customer j has a private type $t_{c,j} \in T_{c,j}$ for the product. Let $T_{f,i} \subseteq Z, \forall i \in F$ and $T_{c,j} \subseteq Z, \forall j \in C$, where $Z \subset \mathbb{R}^+$. Let $T_f = \bigtimes_{i \in F} T_{f,i}$ and $T_c = \bigtimes_{j \in C} T_{c,j}$.

Assume that all the farmers reside in a single village and all the customers in a single town. The village has a single market M_v and town has a single market M_t. The goods can only be transported from a farmer to a customer through these markets. If the system allocates a farmer i to do a trade with customer j, the good is first transported from the farmer i to the market M_v for a cost of $c_{f,i}$ bare by the farmer, then to the market M_t for a fixed cost c_M bare by the markets, and from there to the customer for a cost of $c_{c,j}$ bare by the customer.

In the start of the trading day, each of the farmers and customers submit their bids $b_{f,i} \in B_{f,i} \subseteq Z$ and $b_{c,j} \in B_{c,j} \subseteq Z$ respectively. The bid of a farmer implies the minimum pay he expects for the product per unit and the bid of a customer implies the maximum amount she is ready to pay for the product per unit. Note that when a farmer bids truthfully, $b_{f,i} = t_{f,i} + c_{f,i}$ and when a customer bids truthfully, $b_{c,i} = t_{c,i} - c_{c,i}$.

Each farmer has $q_{f,i} \in Q_{f,i}$ units of product to sell and each customer wants to buy $q_{c,i} \in Q_{c,i}$ units in a given trading day. We assume that these units are publicly known. The total quantity a farmer sells in a day is $a_{f,i} \in Q_{f,i}$ and the quantity a customer buys in a day is $a_{c,i} \in Q_{c,i}$. $a_{ij} \in Q_{fc,ij} = Q_{f,i} \cap Q_{c,j}$ is the quantity i sells to j. Define $Q = \bigtimes_{i \in F} \left(\bigtimes_{j \in C} Q_{fc,ij} \right)$ as the set denoting all possible trades. Note that even though $\sum_{i \in F} a_{f,i} = \sum_{j \in C} a_{c,j}$ in general, there might be situations when this doesn't hold. Refer to **Appendix** for details.

The allocation function $f : Z^{n+m} \to Q$ decides how much quantity is traded between each farmer and each customer by taking in the bids of all the farmers and customers as the input. It then decides the payment vectors $p_f = \left(p_{f,1}, p_{f,2}, ..., p_{f,n}\right)$ and $p_c = \left(p_{c,1}, p_{c,2}, ..., p_{c,m}\right)$ where $p_{f,i}$ is the amount to paid to the farmer i and $p_{c,j}$ is the amount paid by the customer j. The amount $\sum_{j \in C} p_{c,j} - \sum_{i \in F} p_{f,i} \geqslant 0$ is the net revenue generated.

The **Extended Trade Reduction Mechanism** (ETRM) $M \equiv (f, p)$ is defined as follows:

The farmers are ordered in the increasing order of their bids and customers are ordered in the decreasing order. Then one farmer i and one customer j is considered at a time in the above mentioned order. They do a trade for a quantity which is the minimum of the supply of farmer $\left(q_{f,i}\right)$ and demand of customer $\left(q_{c,j}\right)$. If farmer (customer) i is not left with any supply (demand) after the trade, the next farmer (customer) is considered. This is repeated until $b_{c,j} - b_{f,i} \geqslant c_M$. The last pair $\left(i, j\right)$ that satisfies this constraint is the break even index pair $\left(I_f, I_c\right)$. All the farmers $i < I_f$ and customers $j < I_c$ does complete trade, i.e., they trade all their supplies/demands. Rest of the players does not trade.

Each farmer i is then paid a sum of $a_{f,i}.b_{f,I_f}$ and each customer j pays a sum of $a_{c,j}.b_{c,I_c}$. The payment function for a farmer is $p_{f,i} = a_{f,i}.b_{f,I_f}$
The payment function for a customer is $p_{c,i} = a_{c,i}.b_{c,I_c}$

The utility function for farmer is given by $u_i(f(b_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) = (p_{f,i} - t_{f,i}) \times a_{f,i}$
For the buyer it can be given by the function, $u_i(f(b_{c,i}, \{\hat{b}_{c,-i}, \hat{b}_c\}), t_{c,i}) = (t_{c,i} - p_{c,i}) \times a_{c,i}$

Sometimes it might happen that when the break even index is reached, either farmer or customer but not both, has already participated in some trade *(partial trade hitch)*. **This issue and how to handle it is discussed in detail in the appendix.**

3 Main results/findings

Theorem 1 *Extended Trade Reduction Mechanism (ETRM) is DSIC, IR, and WBB.*

Proof: This is the proof of the first theorem. If proofs are long, place that in appendix.

PART 1 The Extended Trade Reduction Mechanism (ETRM) is DSIC. Lets consider the two types of agents in the mechanism, farmers and buyers. For farmer i , the true type is $t_{f,i}$ and the reported bid is $b_{f,i}$. The farmer can try to change it's reported type to increase its utility in the following ways,
case 1(a) : Farmer's index is less than the break even index. ie. $i < I_f$
In this case all of the farmer's goods are guaranteed to be sold by the definition of the mechanism. Also the payment the farmer receives is fixed according to b_{f,I_f}. If the farmer changes its bid such that the break even index is not altered, the utility of the farmer remains same. If the farmer increases the bid such that $b_{f,i} \geqslant b_{f,I_f}$,then utility of the farmer is zero as none of his goods are traded.
case 1(b) : Farmer's index is less than the break even index. ie. $i \geqslant I_f$
Currently, according to the mechanism, none of the farmer's goods will be traded. It can be trivially seen that if farmer changes his bid such that $b'_{f,i} \geqslant b_{f,I_f}$
$u_i(f(b'_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) = u_i(f(b_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) = 0$ where $b'_{f,i}, b_{f,i} \in B_{f,i}, \forall \hat{b}_{f,-i} \in B_{f,-i}, \forall \hat{b}_c \in B_c$
If the farmer reduces his bid such that, $b'_{f,i} < b_{f,I_f}$
$u(f(b'_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) = (p_{f,i} - t_{f,I_f}) \times a_{f,i} < 0$
Hence we see that $u_i(f(b'_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) \leqslant u_i(f(b_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i})$, $\forall b'_{f,i}, b_{f,i} \in B_{f,i}, \forall \hat{b}_{f,-i} \in B_{f,-i}$
Hence ETRM is DSIC with respect to the farmer.
case 2 : changes in the bid of the consumer
Similar argument can be given for the consumers side to prove the theorem for that case.
Hence ETRM is DSIC.

PART 2 The Extended Trade Reduction Mechanism (ETRM) is IR.
For any given agent we can say that by the definition of utility and the mechanism specifications,
For the farmer, $u_i(f(b_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) = (p_{f,i} - t_{f,i}) \times a_{f,i} = (b_{f,I_f} - t_{f,i}) \times a_{f,i}$
If the farmer bids such that the mechanism doesn't allow him to trade any goods, the utility is 0. If the trade happens then $(b_{f,I_f} - t_{f,i}) > 0$.
Hence for all cases, for the farmer, $u_i(f(b_{f,i}, \{\hat{b}_{f,-i}, \hat{b}_c\}), t_{f,i}) \geqslant 0$

PART 3 The Extended Trade Reduction Mechanism (ETRM) is WBB.
The amount paid to the farmer is always less than or equal to the amount paid by the customer. This is because bid of break even indexed farmer is less than or equal to bid of break even indexed customer which is ensured by the mechanism. Since the number of trades is same for customer and farmer, the total amount received by the market would never go negative.

■

Theorem 2 *The efficiency of Extended Trade Reduction Mechanism (ETRM) tends to maximum achievable efficiency when the number of agents tends to infinity.*

Proof: We have provided a general idea for the proof.
According to the mechanism specifications of the ETRM, the market cancels only 1 trade per allocation. This cancellation is the main cause of the reduction of efficiency of the mechanism. For large values of farmer

population and seller population, it becomes clear that the volume traded in a general scenario will be very small when compared to the total volume traded. Hence we say that ETRM achieves close to maximum achievable efficiency for large number of agents. ■

4 Experiments/Simulations

We created a simulation for this mechanism using SimPy Library. The code is written in Python.

Find the code here at FoodMartSim. Suggestions are welcome.

WE conducted few experiments using the code written here. We include some of the plots generated using the data recovered from these experiments.

Here we include 4 plots corresponding to the market profits vs net quantity traded. As it can be seen in the above plots that an equilibrium is achieved. However, that equilibrium may be higher or lower than the initial market profit.

Due to space constraints we cannot include more plots here. However, our developed framework allows us to carry out a whole spectrum of experiments which we will give live demo of in the presentations.

5 Future Work and scope for extensions

This mechanism can be extended to the case of 1 village trading with consumers from multiple cities. This can be achieved by making appropriate changes to the consumers side of bids. We can add/subtract the distance from the village to the city into the consumer's bid to check for weather the mechanism will allow trade to happen or not. As the distance between the village and the city depends on the on the city number, the time complexity of the model Will not be altered.
The same cant be said for the case of multiple cities and multiple villages. The distance measure between a village and a city $d_{(v_i,c_j)}$ is now not linearly separable into the buyer and consumer terms. We can go around this by increasing the time complexity of the mechanism, but this is not desirable. Further more rigorous study is required to tackle this case.

6 Summary and Discussions

In section 1 we introduce the problem of developing an online Marketplace for farmers. The introduction includes various desirable properties that such a market place must have. We also give various related works. In section 2 we give our formal modelling of the problem. And describe the Extended Trade Reduction Mechanism which we have used to perform allocations and payments. In section 3 we give various theorems and proofs related to the mechanism. In section 4 we have given the link for our simulation framework developed on top of SimPy python library. We also include some plots generated with the help of this framework.

We can conclude that Extended trade reduction mechanism is a viable approach for such a marketplace. We have also developed a framework which can be used to test this mechanism or any other mechanism. A good future direction of this work, will be test out a number of mechanisms using the framework and preparing a comparison analysis on various parameters or properties for each of those mechanisms.

References

[1] Moshe Babaioff, Noam Nisan and Elan Pavlov "Mechanisms for a Spatially Distributed Market," *Proceedings of the 5th ACM conference on Electronic commerce*, 2004, pp. 9–20.

[2] Myerson R. B. and Satterthwaite M. A. "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory*, 1983, vol 29, pp. 265–281.

[3] McAfee R. P. "A Dominant Strategy Double Auction." Journal of Economic Theory, vol 56, pp. 434–450, 1992.

[4] Simon Loertscher and Claudio Mezzetti "A Dominant Strategy Double Auction with Multi-Unit Traders" Department of Economics, University of Melbourne, 2013

[5] Cripps, M. W. and J. M. Swinkels "Efficiency of Large Double Auctions", Econometrica, 74 (1), 2006, pp. 47–92.

Appendices

A Partial Trade Hitch

Sometimes it happens when doing the allocation that either breakeven indexed farmer or customer, but not both, does a partial trade. In such a case, that trade is handled by the market. If it is the farmer who does a partial trade, market gives the goods to those customers whom that farmer was supposed to give and receives the payment. If it is the customer who does a partial trade, market pays to those farmers who that customer was trading with and receives goods from them. This ensures that breakeven indexed players doesnot trade. Note that for this to work, market should have sufficient money and goods initially for the trade to take place. When the trades are done multiple time, this trade by the market gets asymptotically balanced.

Market keep sufficient quantity of the product and sufficient money so that if breakeven index players has a partially allocated trade, they won't trade and the other side is balanced by the market.

B Additional Experiments

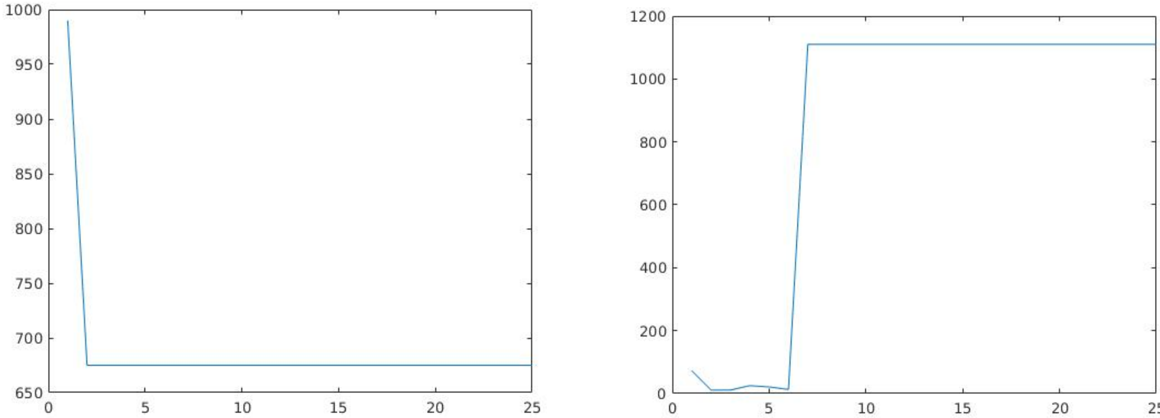

Figure 1: Left: Farmer and Buyer Pop is 5 , Right: Farmer and Buyer Pop is 10

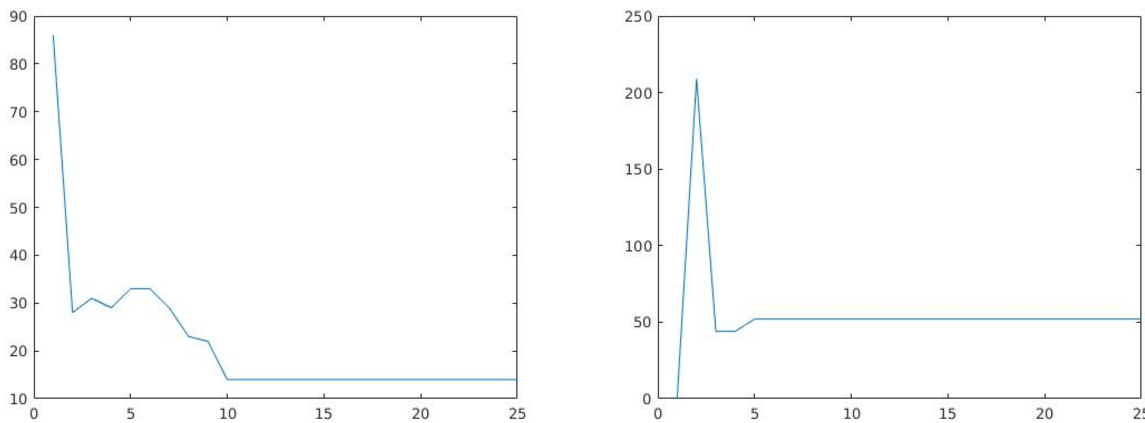

Figure 2: Left: Farmer and Buyer Pop is 15 , Farmer and Buyer Pop is 20

Figure 3: **Plots depicting Market Profit vs Quantity traded. Different Plots are for different populations of buyers and farmers**