Linear Algebra - Worksheet

Read this article before beginning the exercises: Linear Algebra Explained in 4 Pages

This assignment consists of 3 parts:

- Matrix Dimensions
- Vector Operations
- Matrix Operations

After completing the exercises by hand, use Python to check your work.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 5 & -1 \\ 9 & 1 \\ 6 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

$$u = \begin{bmatrix} 6 & 2 & -3 & 5 \end{bmatrix} \quad v = \begin{bmatrix} 3 & 5 & -1 & 4 \end{bmatrix} \quad w = \begin{bmatrix} 1 \\ 8 \\ 0 \\ 5 \end{bmatrix}$$

1. Matrix Dimensions

Write the dimensions of each matrix.

1.1) A 2×3

1.2) B 2×2

1.3) C 3×2

1.4) D 2×3

1.5) u 1×4

1.6) w 4×1

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2. Vector Operations

Perform the following operations. Assume $\alpha = 6$.

2.1) $\vec{u} + \vec{v} =$ $[6\ 2\ -3\ 5] + [3\ 5\ -1\ 4] = [9\ 7\ -4\ 9]$

2.2) $\vec{u} - \vec{v} =$ $[6\ 2\ -3\ 5] - [3\ 5\ -1\ 4] = [3\ -3\ -2\ 1]$

2.3) $\alpha\vec{u} =$ $6[6\ 2\ -3\ 5] = [36\ 12\ -18\ 30]$

2.4) $\vec{u} \cdot \vec{v} =$ $[6\ 2\ -3\ 5] \cdot [3\ 5\ -1\ 4] = 18 + 10 + 3 + 20 = 51$

2.5) $\|\vec{u}\| =$ $\left\|[6\ 2\ -3\ 5]\right\| = \sqrt{6^2 + 2^2 + 3^2 + 5^2} = \sqrt{36+4+9+25} = \sqrt{74} \approx 8.602$

3. Matrix Operations

Evaluate each of the following expressions, if it is defined; else fill in with "not defined." Do your work by hand on scratch paper.

3.1) $A + C =$ not defined

3.2) $A - C^T = \begin{bmatrix} -4 & -7 & -3 \\ 3 & 6 & 4 \end{bmatrix}$

3.3) $C^T + 3D = \begin{bmatrix} 14 & 3 & 3 \\ 2 & 7 & 9 \end{bmatrix}$

3.4) $BA = \begin{bmatrix} -1 & -5 & -1 \\ 2 & 7 & 4 \end{bmatrix}$

3.5) $BA^T =$ not defined

Optional

3.6) $BC =$ not defined

3.7) $CB = \begin{bmatrix} 5 & -6 \\ 9 & -8 \\ 6 & -6 \end{bmatrix}$

3.8) $B^4 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

3.9) $AA^T = \begin{bmatrix} 14 & 28 \\ 28 & 69 \end{bmatrix}$

3.10) $D^T D = \begin{bmatrix} 10 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 10 \end{bmatrix}$