Data Handling and Interpretation
The Bouncing Ball

Objective: **Σ**, **δ**, **σ***#! W*hat exactly is all of this mess and what does it mean?* This reaction is typical of many students upon receiving their first real dose of experimental statistics. This exercise is to help you become better acquainted and more comfortable with the methods of data analysis used by scientists. We will explore a rudimentary graphing technique, do some simple "number crunching" with data that you will collect and gain some experience with the statistical methods used elsewhere in this course. As you proceed with this exercise you will see how *mean, standard deviation, number of trials* and *confidence*, and *graphical analysis* all play a part in determining how scientists evaluate the outcome of an experiment.

This exercise consists of two parts: In the first part you will conduct an experiment then collect and analyze the data. In the second part you will use your experimentally gathered data to determine the acceleration due to gravity close to the earth's surface, *g*.

Remember to record what you do in this exercise in your lab notebook. List the equipment that you use and make a sketch of each experimental setup. Draw a simple schematic of any circuitry required. Write, in sentences and paragraphs, what you are doing and why you are doing it at each step of the way. Be sure that you are aware of what your lab instructor expects in your lab notebook before beginning.

Procedure: In this exercise you will examine an extremely simple system: a bouncing ball. You will drop the ball from a predetermined height (h_0) and measure the time interval between the first and second bounces (Δt_1). Figure 1 shows the notation convention that we will use for this procedure.

Experimental: It will be left up to you to devise a scheme to drop a golf ball 100 times from a set height of at least 1.5 meters (any convenient height above 1.5 meters will do). Notice that this height, h_0, is measured from the *bottom* of the golf ball. Since this distance is set, you need to record it only once. You will measure the time interval, using a digital stopwatch, between the 1st and 2nd bounce for each trial. As you will see, 100 trials should be enough to insure a high degree of confidence in your experimental findings.

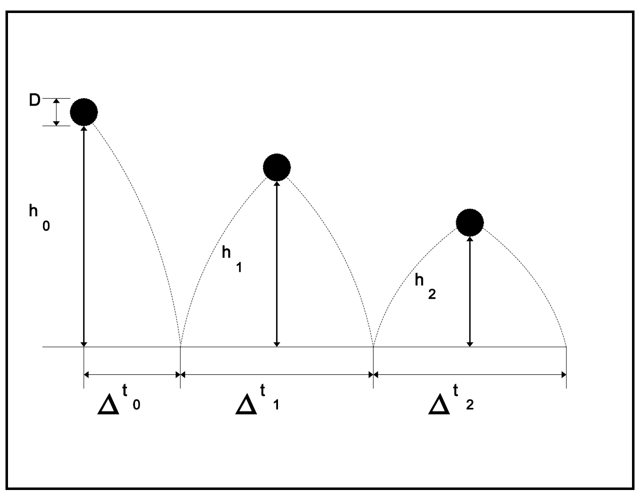

Figure 1. Notation convention for the bouncing ball experiment.

Some coordination is necessary to carry out this experiment and you may encounter an occasional mishap. Suppose that you elect to drop the ball for the first 50 trials while your lab partner operates the digital stopwatch. Your lab partner might start the stopwatch too early or too late a few times during the course of 50 drops of the ball. Feel free to reject such data as experimental *blunders* and start the measurement over. Be aware, however, that you should obtain slightly different numbers even for measurements that have been made correctly as a result of *random error*. After 50 trials you would probably want to change places with your lab partner to avoid any *systematic error*.

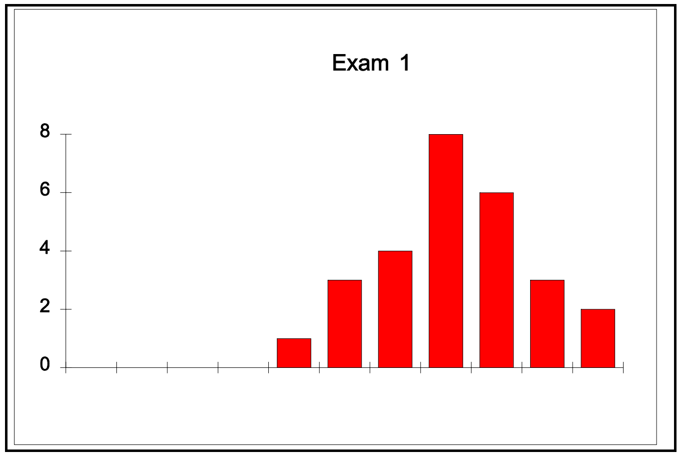

Figure 2. A histogram.

Data Analysis: Make a histogram of your data. A histogram is a type of graph that shows the number of occurrences of a range of data vs. the value of that range of data. You are probably acquainted with a familiar histogram used to post grades, i.e., the number of students scoring between 61-70, 71-80, 81-90, 91-100, etc. on an exam vs. the score ranges 61-70, etc. In this experiment, the x-axis (abscissa) will be time values (or ranges of time values) and the y-axis (ordinate) will be the number of times that time value, or a number on that range of time values, was recorded. Select a fresh page in your lab notebook. Turning the notebook on its side, plot the time values along the length of the page and fill in a block on the corresponding column for each occurrence of that time value. Examples of histograms are shown in appendix C and Figure 2. When your histogram is complete you should be able to trace a bell-shaped curve which approximately conforms to the shape of the bars on the graph. The shape of this curve is a characteristic of processes involving random error.

Calculate the mean and standard deviation of the first 5 data points recorded, the first 10 data points, the first 50 and finally all 100 points. What do you notice about the mean μ and the standard deviation σ that you obtain compared to your histogram? Just above the histogram draw a horizontal line centered on μ with a length of twice σ. This error bar graphically represents the mean plus or minus the standard deviation ($\mu \pm \sigma$). You will note that about 68% of the filled blocks fall within the range of $\mu \pm \sigma$ (count them!). For a normal random error curve such as this 68% of the data fall within $\pm\ \sigma$, 95% within $\pm\ 2\sigma$, and 99.7% within $\pm\ 3\sigma$.

Next plot the values of $\mu \pm \sigma$ as a function of the number of data points *N* used in the determination. Note that the standard deviation is about the same whether 5 points or all 100 points are used. Does this mean that we could have saved ourselves the trouble of plotting all 100 points? Does our confidence necessarily increase with increasing the number of trials? This is where a quantity known as standard deviation of the mean (σ_m) comes into play.

Add a column to your data table for the number of data points *N*. Note that as *N* increases your mean values tend to converge or get closer together. You should be able to see this in the plot you just created. For any set of data the mean value of that set gets closer to the "true" value for the distribution as the number of data points or measurements increases. The variance in the value of the mean is what is known as σ_m.

For a normal error curve the relationship between the standard deviation σ and the standard deviation of the mean σ_m for N data points is:

$$\sigma_m = \frac{\sigma}{\sqrt{N}} \qquad \textbf{1}$$

Note that σ_m gets smaller as the number of data points increases. Add another column to your data table with the values for σ_m. Add dashed error bars to your plot of $\mu \pm \sigma_m$. Observe that as the number N of data points increases, the mean converges and the standard deviation of the mean σ_m decreases.

Most of the time we think in terms of the standard deviation of a measurement which is equivalent to the standard deviation of the parent distribution error curve σ. For instance, if we asked someone making a measurement with a ruler how accurate his or her measurements were, and they said to ± 5mm, they would be talking about a standard deviation of their measurement and not the standard deviation of the mean. If we make a number of measurements of the same thing, we may have reason to talk about the standard deviation of the mean as representative of our error in measurement. It is important, therefore, to state explicitly whether the error quoted is the standard deviation of the measurement σ or the standard deviation of the mean σ_m.

Determination of the acceleration due to gravity

Newton's laws with a little math yield the following relationship for distance in terms of time and initial velocity for a freely falling body:

$$h - h_0 = v_0 t + \frac{1}{2} g t^2 \qquad \textbf{2}$$

For an object in free fall starting from rest at height $h_0 = 0$, this relationship may be written:

$$h = \frac{1}{2} g t^2 \qquad \textbf{3}$$

where $g = 9.80\text{m/s}^2$ close to the earth's surface, assuming that retarding forces (such as air friction) are not present. Is this a good assumption for all objects?

Consider the exercise you have just completed. The time from the initial release to the ball hitting the floor the first time is given by:

$$\Delta t_0 = \sqrt{\frac{2 h_0}{g}} \qquad \textbf{4}$$

Similarly, for the time between bounce i and bounce $i + 1$ the time interval is twice as long as it would be if the body were dropped a distance h_i:

$$\Delta t_i = 2\left(\sqrt{\frac{2 h_i}{g}}\right) \qquad \textbf{5}$$

For the time interval between the first and second bounce, $i = 1$ and we can solve for g to obtain:

$$g = \frac{8 h_1}{(\Delta t_1)^2} \qquad \textbf{6}$$

In the first part of this experiment you obtained a fairly good value of Δt_1. You now need to obtain a value for h_1. This is not a particularly easy measurement (estimation) to obtain. You are free to choose any method that works. Once you have obtained a value for g, compute a % error using the method discussed in the introduction to this section. Use 9.80m/s^2 for the accepted value of g.