## Introduction:

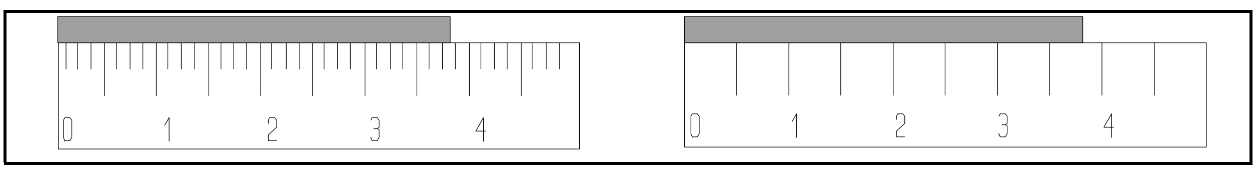
***Significant Figures***
***Calculation of Errors***

Physics is largely a quantitative science; that is, a science that is concerned with measurement. Anytime we take data in an experiment we would like to know both how accurate our numbers are and what useful information may be gleaned from them. To do this we must determine several things about our data: the degree of precision in our measurements; the difference between our measurements and other measurements obtained in similar experiments; what predictions could be based on our measurements; what statistical patterns our measurements follow. Each of these criteria is important in answering the question, "how good is our data." This is a question that you will have to answer many times throughout this course. The following introduction will acquaint you with the fundamentals of data collection and analysis.

## Part 1. *Significant Figures*

Whenever we take any measurement in an experiment, we must first decide upon the *precision* involved with the measurement. The value recorded for a particular measurement generally includes all of the digits that we are sure of, plus **one** additional digit that we *estimate* (assuming the use of analog equipment requiring an estimate of greatest precision as opposed to digital devices that give a discrete numeric reading). As an example, consider the figure below:

The measurement taken from the scale on the left might be recorded as 3.73 centimeters. The measurement taken from the scale on the right might be recorded as 3.7 centimeters. In the first measurement we are *sure* of the first and second digits (3 and 7) but have to *estimate* the third (3). The recorded result contains a single estimated digit. This result has 3 significant figures.




In the second measurement we are *sure* of only the first digit (3), and must *estimate* the second (7). This result contains a single estimated digit, like the first, but has 2 significant figures.

When experimental measurements are used in calculations (e.g., addition, multiplication, etc.) the result of the operation should be written so that it contains all of the digits that we are sure of and one estimated digit. The obvious difficulty is determining exactly how many digits we are sure of. How, exactly, does one go about determining this? Unfortunately, there is no blanket rule that covers every case that you are likely to encounter. The "rules of thumb" given in the following examples, however, may be used as general guidelines:

**Addition and Subtraction:** Let's use the numbers obtained in the measurement above. These are added below. The estimated digits in each number are highlighted.

$$\begin{array}{r} 3.7\mathbf{3} \\ +\ 3.\mathbf{7} \\ \hline 7.\mathbf{43} \end{array}$$

Notice that if there is an estimated digit in a column of *any* of the numbers being added (as in the tenths column above), the same column in the sum also contains an estimated digit. Think about it, if there is uncertainty in any one of a series of digits that are being summed, then the sum of these digits must also be subject to the same uncertainty.

The answer obtained in this example contains 2 estimated digits. As before, we wish to express this value in terms of all the digits that we are sure of and 1 estimated digit. In this case, we are sure of only the first digit. After rounding, the sum in this example becomes 7.4, and the number of significant figures in the answer is 2.

In general:

***The result of an addition or subtraction should not be carried beyond the first column that contains an estimated digit.***

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**Multiplication and Division:** To illustrate the rules for these operations, consider the following example:

$$\begin{array}{r} 3.7\mathbf{3} \\ \times\ 3.\mathbf{7} \\ \hline \mathbf{2611} \\ 111\mathbf{9}\phantom{0} \\ \hline 1\mathbf{3.801} \end{array}$$

The first number in this operation (3.73) contains <u>3 significant figures</u>, the second (3.7) contains <u>2 significant figures</u>. After rounding, the result of this operation is 14. This answer contains <u>2 significant figures</u>, the same number of significant figures as are contained in the second multiplier. Notice that this is *not* the same as 14.0, which contains 3 significant figures, and implies that we are sure of the digit in the units place, which we are not.

Another example:

$$
\begin{array}{r}
20.55 \\
\times \underline{\quad 5.55} \\
10275 \\
10275\phantom{0} \\
10275\phantom{00} \\
\hline
114.0525
\end{array}
$$

After rounding, the result of this operation is 114. This answer contains 3 significant figures, the same number of significant figures as are contained in the second multiplier (i.e., the multiplier with the fewest number of significant figures).

In general:

***The result of multiplication or division should not be carried beyond the first column that contains an estimated digit.***

Notice that this is the same rule that applies to addition/subtraction.

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**Rounding Answers:** As seen in the previous examples, it is usually necessary to round off answers in order to preserve the correct number of significant figures. We will adopt the following convention for rounding:

***The last digit retained is rounded up if the first digit dropped is equal to 5 or greater.***

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## Part 2. *Calculation of Errors*

Every scientific experiment is subject to a certain amount of experimental error. Researchers in the real world devote a significant portion of their time to data handling and error analysis. You will have to learn the techniques of error analysis to perform the experiments in this lab.

There are two types of errors that you will be concerned with in this lab: *random errors* and *systematic errors*. Random errors are unavoidable errors that are always present when one measures to the limit of available accuracy. In one of the first experiments that we will be doing in this lab you will use a stopwatch to time the interval between the first and second bounce of a golf ball. A typical digital stopwatch measures intervals of time to a hundredth of a second. As you collect your data, you will notice that the times you record for the interval between the first and second bounce vary a small amount from trial to trial. This is largely due to your inability to start and stop the watch at precisely the right times. Over a large number of trials the times recorded are equally likely to be too high as too low (i.e., random). Hence, the use of the term random errors.

Systematic errors are errors that cause measurements to be skewed, that is, consistently off in one direction. Systematic errors are often caused by improperly calibrated instruments. If the stopwatch that you use to measure the interval between the first and second bounce of the golf ball is running slow, for example, your measurements of the time interval will be consistently too short.

A type of error commonly encountered by students in physics laboratories is known as a *blunder*. A blunder is the result of either careless execution of an experiment, incorrect recording of data, a mistake in experimental calculations, a mistake in data interpretation, or all of the above. ***A blunder is not an experimental error***. If you measure the interval between the second and third bounce of the golf ball instead of the first and second, you have committed a blunder, not an experimental error.

In this laboratory you will usually be trying to obtain one of two types of data: a) measurements of quantities that you will compare against known values. b) consistent sets of measurements for quantities whose values are not available. In either case it is possible to obtain a rough estimate of the reliability of your data with a few simple calculations.

**Percentage Error**: If an accepted value is available for the quantity being measured, an estimate of the accuracy of the measured data may be obtained by calculating a percentage error, i.e., the percent difference between the measured and accepted values:

$$\%\ Error = \left(\frac{Measured\ value - Accepted\ value}{Accepted\ value}\right) \times 100\%$$

The measured value used in this calculation may actually be (and most often is) the mean of a number of measurements. Notice that the result of this calculation will be either positive or negative depending upon whether the measured value was greater or less than the accepted value.

**Percentage Variation**: If one measures the same quantity twice some variation is generally obtained between the two measurements due to random errors. If the accepted value of the quantity being measured is not available, there is no reason to believe that either of the values obtained is better than the other. In such a situation one normally calculates the average of the two quantities as the best estimate of their value. This, however, casts little light on the confidence that we should have in this data. One measure of reliability between two measurements is percentage variation:

$$\% \ Variation = \left(\frac{Difference \ of \ the \ two \ measurements}{Average \ of \ the \ two \ measurements}\right) \times 100\%$$

This calculation yields the percentage of the average value that the two measurements differ by. Notice that this is an inherently positive quantity. As the percentage variation between two measurements grows larger, our confidence in them normally diminishes.

Calculating a percentage error or a percentage variation gives us a quick and rough way of assessing the reliability of small amounts of data through simple comparisons. But what should we do if our data consists of many measurements, or if we wish to specify a more precise degree of accuracy for our results? As you might suspect, satisfying either of these conditions requires more sophisticated methods of error analysis.

Whenever a large number of measurements of a quantity are taken, their individual values will always vary, at least by a small amount, due to random errors. In such a case, one is usually justified in assuming that the average or *mean* value, $\bar{x}$, of the measurements is a more accurate estimate of their value than any of the individual measurements. By convention, $\bar{x}$ is considered to be the most probable value of a range of measurements. Formally, the mean or average value, $\bar{x}$, of a range of $n$ measurements is:

$$\bar{x} = \frac{\sum x}{n}$$

Consider the sample spreadsheet shown in Table 1. The data was taken during an experiment using the bouncing golf ball.

The values in the first column are time intervals, in seconds, between the first and second bounce. The second column contains a cumulative total of the mean or average values of these intervals. The mean value of these measurements, $\bar{t}$, is 1.669 seconds.

The third column contains the deviation of each individual measurement from the mean of all the measurements, $\bar{t}$ (1.669 s). This quantity is denoted by the symbol $\delta$. For a large number of measurements, the algebraic sum of the individual deviations is zero because of the random nature of the errors that are responsible for the deviations. Consequently, the average deviation from the mean is defined in terms of the absolute values of the individual deviations. The fourth column contains the absolute values of the individual deviations ($|\delta|$). The fifth column contains the square of each value of $\delta$. This, as will be seen shortly, is a very useful quantity.

**Table 1**.

| t (s) | $\bar{t}$ (s) | $\delta$ (s) | $\lvert\delta\rvert$ (s) | $\delta^2$ ($s^2$) |
|---|---|---|---|---|
| 1.682 | 1.682 | 0.013 | 0.013 | 0.000 |
| 1.683 | 1.683 | 0.014 | 0.014 | 0.000 |
| 1.701 | 1.689 | 0.032 | 0.032 | 0.001 |
| 1.633 | 1.675 | -0.036 | 0.036 | 0.001 |
| 1.631 | 1.666 | -0.038 | 0.038 | 0.001 |
| 1.621 | 1.659 | -0.048 | 0.048 | 0.002 |
| 1.652 | 1.658 | -0.017 | 0.017 | 0.000 |
| 1.663 | 1.658 | -0.006 | 0.006 | 0.000 |
| 1.661 | 1.659 | -0.008 | 0.008 | 0.000 |
| 1.651 | 1.658 | -0.018 | 0.018 | 0.000 |
| 1.681 | 1.660 | 0.012 | 0.012 | 0.000 |
| 1.652 | 1.659 | -0.017 | 0.017 | 0.000 |
| 1.709 | 1.663 | 0.040 | 0.040 | 0.002 |
| 1.661 | 1.663 | -0.008 | 0.008 | 0.000 |
| 1.632 | 1.661 | -0.037 | 0.037 | 0.001 |
| 1.693 | 1.663 | 0.024 | 0.024 | 0.001 |
| 1.665 | 1.663 | -0.004 | 0.004 | 0.000 |
| 1.692 | 1.665 | 0.023 | 0.023 | 0.001 |
| 1.631 | 1.663 | -0.038 | 0.038 | 0.001 |
| 1.714 | 1.665 | 0.045 | 0.045 | 0.002 |
| 1.631 | 1.664 | -0.038 | 0.038 | 0.001 |
| 1.689 | 1.665 | 0.020 | 0.020 | 0.000 |
| 1.678 | 1.666 | 0.009 | 0.009 | 0.000 |
| 1.697 | 1.669 | 0.028 | 0.028 | 0.001 |
| 1.711 | 1.669 | 0.042 | 0.042 | 0.002 |
| 1.681 | 1.669 | 0.012 | 0.012 | 0.000 |
| 1.688 | 1.670 | 0.019 | 0.019 | 0.000 |
| 1.701 | 1.671 | 0.032 | 0.032 | 0.001 |
| 1.631 | 1.670 | -0.038 | 0.038 | 0.001 |
| 1.631 | 1.668 | -0.038 | 0.038 | 0.001 |
| 1.627 | 1.667 | -0.042 | 0.042 | 0.002 |
| 1.656 | 1.667 | -0.013 | 0.013 | 0.000 |
| 1.666 | 1.667 | -0.003 | 0.003 | 0.000 |
| 1.661 | 1.666 | -0.008 | 0.008 | 0.000 |
| 1.654 | 1.666 | -0.015 | 0.015 | 0.000 |
| 1.687 | 1.667 | 0.018 | 0.018 | 0.000 |
| 1.655 | 1.666 | -0.014 | 0.014 | 0.000 |
| 1.701 | 1.667 | 0.032 | 0.032 | 0.001 |
| 1.669 | 1.667 | 0.000 | 0.000 | 0.000 |
| 1.631 | 1.666 | -0.038 | 0.038 | 0.001 |
| 1.692 | 1.667 | 0.023 | 0.023 | 0.001 |
| 1.669 | 1.667 | 0.000 | 0.000 | 0.000 |
| 1.698 | 1.667 | 0.029 | 0.029 | 0.001 |
| 1.631 | 1.667 | -0.038 | 0.038 | 0.001 |
| 1.719 | 1.668 | 0.050 | 0.050 | 0.003 |
| 1.633 | 1.667 | -0.036 | 0.036 | 0.001 |
| 1.681 | 1.668 | 0.012 | 0.012 | 0.000 |
| 1.671 | 1.668 | 0.002 | 0.002 | 0.000 |
| 1.698 | 1.668 | 0.029 | 0.029 | 0.001 |
| 1.701 | 1.669 | 0.032 | 0.032 | 0.001 |
| | $\bar{t}$<br>1.67 | | $\bar{\delta}$<br>0.03 | $\bar{\delta}^2$<br>0.001 |

At the bottom of the spreadsheet is a row containing mean values for $t$, $\delta$, and $\delta^2$. These are given by:

$$\bar{t} = \frac{\sum t}{n}, \quad \bar{\delta} = \frac{\sum |\delta|}{n}, \quad \overline{\delta^2} = \frac{\sum \delta^2}{n}$$

The quantity $\bar{\delta}$ is known as the *average deviation*. If $\bar{\delta}$ is small, our deviations from the mean are small and we can be confident in the reliability of our data. If $\bar{\delta}$ is large, then our measurements are probably not very uniform and we would not be as confident in the precision of our technique of measurement.

A more elegant (and more precise) method of determining the degree of error in our measurements is to calculate the *standard deviation*, denoted by the symbol $\sigma$. To calculate the standard deviation, we first compute the square of each $\delta$, an inherently positive number. Next we sum the individual values of $\delta^2$ and calculate their mean value. If we then take the square root of this result we obtain a quantity with the same units as that of the original measurements. This calculation yields the *Root Mean Square Deviation*:

$$\sigma = \sqrt{\frac{\sum \delta^2}{n}}$$

For large numbers of measurements, the rms deviation is equal to the standard deviation. In our example, $\sigma = 0.03$ seconds. For smaller numbers of measurements it is standard practice to use a more conservative expression for the standard deviation:

$$\sigma = \sqrt{\frac{\sum \delta^2}{n-1}}$$

It can be shown that approximately 68% of all the measurements will lie within $\pm\sigma$ (one standard deviation) of the mean of a set of data, 95% within two standard deviations and 99% within three standard deviations. Hence, about two-thirds of our measurements should be within $\pm 0.03$ seconds of our mean, 1.67 seconds. We would write our best estimate of the value of $t$ between the first and second bounce of the ball based on our measurements as: $\underline{1.67 \pm 0.03}$ seconds.

The process of computing a standard deviation from the above equations can be quite cumbersome, especially for large amounts of data. Every scientific calculator will compute means and standard deviations. You should learn how to perform these operations on your calculator. Computation of experimental statistics is a part of nearly every experiment that you will be performing in this course. You should practice using the data in Table 1.