

Achievement Report

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6/22/2021

Math Models

Logistic Growth Models

```
load("~/qmer/projects/ECLSK11_GrowthCurvePaper2022/data/mathlogisModels.Rdata")
```

```
texreg(list(mathlogisA, mathlogisAX, mathlogisAXS),  
caption = "Logistic Growth Models")
```

```
anova(mathlogisA, mathlogisAX, mathlogisAXS)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
mathlogisA	1	5	795876.7	795924.6	-397933.3			
mathlogisAX	2	7	776089.0	776156.1	-388037.5	1 vs 2	19791.677	<.0001
mathlogisAXS	3	10	775581.5	775677.4	-387780.7	2 vs 3	513.511	<.0001

Gompertz Growth Models

```
load("~/qmer/projects/ECLSK11_GrowthCurvePaper2022/data/mathgompModels  
.Rdata")
```

```
texreg(list(mathgompA, mathgompAB, mathgompABB),  
caption = "Gompertz Growth Models")
```

```
anova(mathgompA, mathgompAB, mathgompABB)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
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	Model 1	Model 2	Model 3
Asym	119.63*** (0.19)	120.17*** (0.16)	120.04*** (0.15)
xmid	1.02*** (0.00)	1.07*** (0.00)	1.07*** (0.00)
scal	1.34*** (0.00)	1.29*** (0.00)	1.27*** (0.00)
AIC	795876.68	776089.01	775581.50
BIC	795924.63	776156.14	775677.40
Log Likelihood	-397933.34	-388037.50	-387780.75
Num. obs.	107994	107994	107994
Num. groups: id	17970	17970	17970

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 1: Logistic Growth Models

	Model 1	Model 2	Model 3
Asym	124.72*** (0.21)	126.87*** (0.18)	128.09*** (0.18)
b2	1.22*** (0.00)	1.27*** (0.00)	1.28*** (0.00)
b3	0.59*** (0.00)	0.60*** (0.00)	0.60*** (0.00)
AIC	793477.28	772437.45	770737.48
BIC	793525.23	772504.58	770833.38
Log Likelihood	-396733.64	-386211.73	-385358.74
Num. obs.	107994	107994	107994
Num. groups: id	17970	17970	17970

*** $p < 0.001$; ** $p < 0.01$; * $p < 0.05$

Table 2: Gompertz Growth Models

	Model 1
A	-186.88*** (3.82)
B	132.35*** (0.23)
xmid	-2.02*** (0.05)
scal	2.43*** (0.01)
AIC	785107.94
BIC	785165.48
Log Likelihood	-392547.97
Num. obs.	107994
Num. groups: id	17970

Richard's models with other parameter configurations did not converge

Table 3: Richard Growth Model

```
mathgompA      1  5 793477.3 793525.2 -396733.6
mathgompAB     2  7 772437.5 772504.6 -386211.7 1 vs 2 21043.825 <.0001
mathgompABB    3 10 770737.5 770833.4 -385358.7 2 vs 3 1705.978 <.0001
```

Richards Growth Curves

```
load("~/qmer/projects/ECLSK11_GrowthCurvePaper2022/data/mathrichModel.Rdata")
texreg(mathrichB, caption = "Richard Growth Model",
       custom.note = "Richard's models with other parameter configurations did not converge")
```

Comparing Best Models

```
anova(mathlogisAXS, mathgompABB, mathrichB)
```

	Model	df	AIC	BIC	logLik	Test	L.Ratio	p-value
mathlogisAXS	1	10	775581.5	775677.4	-387780.7			
mathgompABB	2	10	770737.5	770833.4	-385358.7			
mathrichB	3	6	785107.9	785165.5	-392548.0	2 vs 3	14378.47	<.0001

Adding Covariates

```
load("~/qmer/projects/ECLSK11_GrowthCurvePaper2022/data/mathCovariateModels.Rdata")
```

```
summary(mathA_B_RxG)
```

Nonlinear mixed-effects model fit by maximum likelihood

Model: Math ~ SSgompertz(time2, Asym, b2, b3)

Data: eclskmath

AIC	BIC	logLik
687661.6	688012.4	-343793.8

Random effects:

Formula: list(Asym ~ 1, b2 ~ 1)

Level: id

Structure: General positive-definite, Log-Cholesky parametrization

	StdDev	Corr
Asym.(Intercept)	16.3696419	As.(I)
b2.(Intercept)	0.1945613	-0.264
Residual	6.1238307	

Fixed effects: list(Asym ~ Race * SES + Sex, b2 ~ Race + SES + Sex + Race:SES + SES:Sex, b3 ~ Race * SES + Sex)

	Value	Std.Error	DF	t-value	p-value
Asym.(Intercept)	130.37334	0.3038426	80993	429.0818	0.0000
Asym.RaceBlack	-11.56104	0.6484886	80993	-17.8277	0.0000
Asym.RaceHispanic	-3.89602	0.4860982	80993	-8.0149	0.0000
Asym.RaceAsian	6.42181	0.6972319	80993	9.2104	0.0000
Asym.Raceother	-1.93950	0.7735190	80993	-2.5074	0.0122
Asym.SES	7.39406	0.2869088	80993	25.7715	0.0000
Asym.SexFemale	-0.93927	0.3532235	80993	-2.6591	0.0078
Asym.RaceBlack:SES	1.34876	0.6802504	80993	1.9827	0.0474
Asym.RaceHispanic:SES	-0.02420	0.4994607	80993	-0.0484	0.9614
Asym.RaceAsian:SES	-2.85154	0.6440298	80993	-4.4277	0.0000
Asym.Raceother:SES	1.87734	0.8224309	80993	2.2827	0.0225
b2.(Intercept)	1.25642	0.0034496	80993	364.2270	0.0000
b2.RaceBlack	-0.00183	0.0071760	80993	-0.2550	0.7987
b2.RaceHispanic	0.04659	0.0056194	80993	8.2903	0.0000
b2.RaceAsian	0.00937	0.0078650	80993	1.1917	0.2334
b2.Raceother	0.00096	0.0084952	80993	0.1133	0.9098
b2.SES	-0.09252	0.0043990	80993	-21.0328	0.0000
b2.SexFemale	-0.02036	0.0039105	80993	-5.2074	0.0000
b2.RaceBlack:SES	0.02593	0.0090632	80993	2.8606	0.0042
b2.RaceHispanic:SES	-0.00938	0.0066614	80993	-1.4074	0.1593
b2.RaceAsian:SES	-0.00371	0.0084087	80993	-0.4418	0.6587
b2.Raceother:SES	-0.01477	0.0106870	80993	-1.3824	0.1668
b2.SES:SexFemale	0.02834	0.0047699	80993	5.9413	0.0000
b3.(Intercept)	0.58391	0.0013102	80993	445.6589	0.0000
b3.RaceBlack	0.02000	0.0032939	80993	6.0725	0.0000
b3.RaceHispanic	0.01662	0.0021888	80993	7.5918	0.0000
b3.RaceAsian	0.00138	0.0034806	80993	0.3967	0.6916
b3.Raceother	0.00425	0.0040073	80993	1.0601	0.2891
b3.SexFemale	0.02127	0.0018398	80993	11.5613	0.0000
b3.RaceBlack:SexFemale	-0.01377	0.0042467	80993	-3.2417	0.0012
b3.RaceHispanic:SexFemale	-0.01228	0.0028864	80993	-4.2555	0.0000

b3.RaceAsian:SexFemale	0.00491	0.0043690	80993	1.1234	0.2613		
b3.Raceother:SexFemale	-0.00407	0.0051913	80993	-0.7839	0.4331		
Correlation:							
	As.(I)	Asy.RB	Asy.RH	Asy.RA	Asym.R	As.SES	Asy.SF
Asym.RaceBlack	-0.329						
Asym.RaceHispanic	-0.442	0.210					
Asym.RaceAsian	-0.298	0.147	0.196				
Asym.Raceother	-0.277	0.132	0.176	0.123			
Asym.SES	-0.212	0.099	0.132	0.092	0.083		
Asym.SexFemale	-0.532	-0.012	-0.011	-0.026	-0.009	-0.001	
Asym.RaceBlack:SES	0.094	0.289	-0.056	-0.039	-0.035	-0.422	-0.009
Asym.RaceHispanic:SES	0.118	-0.057	0.293	-0.053	-0.048	-0.574	0.007
Asym.RaceAsian:SES	0.097	-0.044	-0.059	-0.271	-0.037	-0.445	-0.004
Asym.Raceother:SES	0.074	-0.035	-0.046	-0.032	-0.052	-0.349	0.000
b2.(Intercept)	0.022	-0.011	-0.015	-0.012	-0.009	-0.038	-0.003
b2.RaceBlack	-0.012	0.122	0.008	0.006	0.005	0.018	-0.002
b2.RaceHispanic	-0.015	0.007	0.010	0.007	0.006	0.023	-0.002
b2.RaceAsian	-0.012	0.005	0.007	-0.005	0.005	0.016	0.001
b2.Raceother	-0.009	0.005	0.007	0.005	0.066	0.015	-0.003
b2.SES	-0.036	0.017	0.020	0.014	0.014	0.015	0.008
b2.SexFemale	-0.003	-0.002	-0.002	0.001	-0.003	0.001	0.027
b2.RaceBlack:SES	0.017	0.012	-0.009	-0.006	-0.006	-0.008	-0.004
b2.RaceHispanic:SES	0.020	-0.009	0.000	-0.009	-0.008	-0.010	0.001
b2.RaceAsian:SES	0.016	-0.007	-0.010	0.020	-0.006	-0.008	-0.001
b2.Raceother:SES	0.014	-0.006	-0.008	-0.005	-0.038	-0.006	-0.004
b2.SES:SexFemale	0.012	-0.007	-0.003	-0.001	-0.004	0.002	-0.015
b3.(Intercept)	0.456	-0.147	-0.198	-0.133	-0.124	-0.007	-0.253
b3.RaceBlack	-0.111	0.424	0.080	0.056	0.050	0.003	-0.031
b3.RaceHispanic	-0.185	0.090	0.371	0.084	0.076	0.004	-0.014
b3.RaceAsian	-0.115	0.057	0.076	0.382	0.048	0.003	-0.011
b3.Raceother	-0.100	0.049	0.066	0.046	0.416	0.002	-0.010
b3.SexFemale	-0.202	-0.013	-0.015	-0.018	-0.011	0.007	0.412
b3.RaceBlack:SexFemale	-0.030	0.024	0.004	0.002	0.003	-0.003	0.042
b3.RaceHispanic:SexFemale	-0.016	0.005	0.019	0.004	0.004	-0.005	0.009
b3.RaceAsian:SexFemale	-0.007	0.003	0.005	0.025	0.003	-0.003	0.000
b3.Raceother:SexFemale	-0.011	0.003	0.004	0.002	0.023	-0.003	0.008
	A.RB:S	A.RH:S	A.RA:S	A.R:SE	b2.(I)	b2.RcB	b2.RcH
Asym.RaceBlack							
Asym.RaceHispanic							
Asym.RaceAsian							
Asym.Raceother							
Asym.SES							
Asym.SexFemale							
Asym.RaceBlack:SES							
Asym.RaceHispanic:SES	0.242						
Asym.RaceAsian:SES	0.188	0.256					
Asym.Raceother:SES	0.147	0.200	0.155				
b2.(Intercept)	0.017	0.021	0.017	0.014			
b2.RaceBlack	0.015	-0.010	-0.008	-0.006	-0.335		
b2.RaceHispanic	-0.009	-0.012	-0.010	-0.008	-0.432	0.208	
b2.RaceAsian	-0.007	-0.009	-0.002	-0.006	-0.294	0.149	0.190
b2.Raceother	-0.006	-0.009	-0.007	-0.048	-0.283	0.138	0.176
b2.SES	-0.007	-0.010	-0.008	-0.004	-0.235	0.115	0.136
b2.SexFemale	-0.004	0.000	-0.001	-0.003	-0.542	-0.008	-0.003

b2.RaceBlack:SES	0.076	0.004	0.003	0.003	0.117	0.291	-0.069
b2.RaceHispanic:SES	0.004	0.008	0.005	0.004	0.149	-0.073	0.302
b2.RaceAsian:SES	0.003	0.005	0.016	0.003	0.123	-0.058	-0.074
b2.Raceother:SES	0.003	0.004	0.003	0.047	0.093	-0.046	-0.058
b2.SES:SexFemale	0.001	0.002	0.003	-0.002	0.007	-0.007	0.012
b3.(Intercept)	0.005	0.002	0.004	0.003	-0.002	0.000	-0.001
b3.RaceBlack	0.004	-0.002	-0.001	-0.001	0.001	0.063	0.001
b3.RaceHispanic	-0.002	0.004	-0.002	-0.001	-0.006	0.001	-0.010
b3.RaceAsian	-0.001	-0.002	-0.010	-0.001	-0.005	0.000	0.000
b3.Raceother	-0.001	-0.001	-0.001	0.004	-0.001	0.000	0.000
b3.SexFemale	-0.007	-0.002	-0.005	-0.003	0.010	-0.009	-0.011
b3.RaceBlack:SexFemale	0.018	0.002	0.001	0.001	-0.009	0.016	0.004
b3.RaceHispanic:SexFemale	0.002	0.001	0.002	0.002	0.000	0.005	0.014
b3.RaceAsian:SexFemale	0.001	0.002	0.015	0.001	-0.002	0.003	0.004
b3.Raceother:SexFemale	0.001	0.002	0.001	-0.005	-0.005	0.003	0.004
	b2.RcA	b2.Rct	b2.SES	b2.SxF	b2.RB:	b2.RH:	b2.RA:

Asym.RaceBlack

Asym.RaceHispanic

Asym.RaceAsian

Asym.Raceother

Asym.SES

Asym.SexFemale

Asym.RaceBlack:SES

Asym.RaceHispanic:SES

Asym.RaceAsian:SES

Asym.Raceother:SES

b2.(Intercept)

b2.RaceBlack

b2.RaceHispanic

b2.RaceAsian

b2.Raceother

0.126

b2.SES

0.106 0.092

b2.SexFemale

-0.029 -0.007 0.005

b2.RaceBlack:SES

-0.049 -0.045 -0.355 -0.010

b2.RaceHispanic:SES

-0.067 -0.062 -0.484 0.007 0.233

b2.RaceAsian:SES

-0.376 -0.049 -0.370 -0.004 0.185 0.251

b2.Raceother:SES

-0.042 -0.117 -0.296 0.004 0.145 0.198 0.157

b2.SES:SexFemale

-0.009 0.003 -0.522 -0.010 0.003 0.007 -0.021

b3.(Intercept)

-0.001 0.000 -0.004 0.002 0.003 0.002 0.002

b3.RaceBlack

0.000 0.000 0.003 -0.004 0.004 -0.001 -0.001

b3.RaceHispanic

0.000 0.000 0.008 0.009 -0.001 0.016 -0.001

b3.RaceAsian

-0.036 0.000 0.003 0.007 -0.001 -0.001 0.036

b3.Raceother

0.000 0.016 0.004 0.000 -0.001 -0.001 0.000

b3.SexFemale

-0.007 -0.008 -0.005 0.021 -0.001 0.002 0.000

b3.RaceBlack:SexFemale

0.003 0.003 0.003 0.004 0.000 -0.001 0.000

b3.RaceHispanic:SexFemale

0.005 0.004 -0.004 -0.019 0.000 0.000 -0.001

b3.RaceAsian:SexFemale

0.030 0.003 -0.001 -0.009 0.000 -0.001 -0.008

b3.Raceother:SexFemale

0.003 0.021 -0.001 -0.002 0.000 0.000 0.000

b2.R:S b2.SES: b3.(I) b3.RcB b3.RcH b3.RcA b3.Rct

Asym.RaceBlack

Asym.RaceHispanic

Asym.RaceAsian

Asym.Raceother

Asym.SES

```

Asym.SexFemale
Asym.RaceBlack:SES
Asym.RaceHispanic:SES
Asym.RaceAsian:SES
Asym.Raceother:SES
b2.(Intercept)
b2.RaceBlack
b2.RaceHispanic
b2.RaceAsian
b2.Raceother
b2.SES
b2.SexFemale
b2.RaceBlack:SES
b2.RaceHispanic:SES
b2.RaceAsian:SES
b2.Raceother:SES
b2.SES:SexFemale      -0.006
b3.(Intercept)        0.002  0.000
b3.RaceBlack          0.000 -0.003  -0.364
b3.RaceHispanic       -0.001 -0.012  -0.557  0.223
b3.RaceAsian          0.000 -0.003  -0.350  0.140  0.211
b3.Raceother          0.008 -0.005  -0.304  0.122  0.183  0.115
b3.SexFemale          -0.001  0.006  -0.654  0.206  0.323  0.202  0.176
b3.RaceBlack:SexFemale 0.000 -0.004   0.227 -0.605 -0.143 -0.090 -0.078
b3.RaceHispanic:SexFemale -0.001  0.010   0.348 -0.140 -0.618 -0.132 -0.115
b3.RaceAsian:SexFemale  0.000  0.004   0.231 -0.092 -0.139 -0.661 -0.076
b3.Raceother:SexFemale -0.008  0.003   0.193 -0.078 -0.117 -0.073 -0.627
b3.SxF b3.RB: b3.RH: b3.RA:

Asym.RaceBlack
Asym.RaceHispanic
Asym.RaceAsian
Asym.Raceother
Asym.SES
Asym.SexFemale
Asym.RaceBlack:SES
Asym.RaceHispanic:SES
Asym.RaceAsian:SES
Asym.Raceother:SES
b2.(Intercept)
b2.RaceBlack
b2.RaceHispanic
b2.RaceAsian
b2.Raceother
b2.SES
b2.SexFemale
b2.RaceBlack:SES
b2.RaceHispanic:SES
b2.RaceAsian:SES
b2.Raceother:SES
b2.SES:SexFemale
b3.(Intercept)
b3.RaceBlack
b3.RaceHispanic
b3.RaceAsian

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```

b3.Raceother
b3.SexFemale
b3.RaceBlack:SexFemale    -0.342
b3.RaceHispanic:SexFemale -0.525  0.230
b3.RaceAsian:SexFemale    -0.350  0.151  0.223
b3.Raceother:SexFemale    -0.291  0.128  0.188  0.124

```

Standardized Within-Group Residuals:

Min	Q1	Med	Q3	Max
-8.51765505	-0.58124470	-0.01316058	0.55365609	6.00705763

Number of Observations: 96858

Number of Groups: 15833