

1.

Plain text: INFORMATION SECURITY

Caesar cipher table:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W

Ciphertext : FKCL0JXQFLKPBZR0FQV

2.

First, converting those numbers to binary as follows

13 -> 1101

8 -> 1000

4 -> 0100

Therefore,

13 will become 0000 -> 0

8 will become 1101 -> 13

4 will become 1010 -> 10

3.

Because 7 and 23 are relative primes, 7 has its multiplicative inverse.

$$23 = 7 \cdot 3 + 2 \text{ and } 7 = 3 \cdot 2 + 1$$

So $1 = 7 - 3 \cdot 2 = 7 - 3 \cdot (23 - 7 \cdot 3) = 7(10) - 23(3)$ From $23(3) + 1 = 7(10)$,
the multiplicative inverse of 7 is 10.

4.

$$\text{Gcd}(512,240) = \text{gcd}(240, 512 \bmod 240) \Rightarrow \text{gcd}(240, 32)$$

$$= \text{gcd}(32, 240 \bmod 32) \Rightarrow \text{gcd}(32, 16)$$

$$= \text{gcd}(16, 32 \bmod 16) \Rightarrow \text{gcd}(16, 0)$$

$$\text{gcd}(512,240) = 16$$

Therefore, the gcd of 512 and 240 is 16.

5.

$$M=9, k=4, (p,g,y)=(59,2,25)$$

$$a = g^k \bmod p = 2^4 \bmod 59 = 16$$

$$b = My^k \bmod p = (9)(25^4) \bmod 59 = 51$$

$$\text{encryption of } M = (16,51)$$

6.

$$H(x) = (5x + 11) \bmod 19$$

$$H(4) = (20+11) \bmod 19 = 12$$

$$19y+12-11 = 5x \Rightarrow 19y+1 = 5x, 19y+1 \text{ has to be multiples of 5}$$

Search for 19y value where $19y+1 \bmod 5 = 0$

$$19*6=114, 114+1/5 = 23$$

$$19*11=209 209+1/5 = 42$$

$$19*16=304 304+1/5= 61$$

$$19*21=399 399+1/5 = 80$$

$H(23), H(42), H(61), H(80)$ all has same hashed value, the hash function is not weakly collision resistant.

7.

$$20^{10211} \bmod 10401$$

Euler's method

$10401 = 3 * 3467$, (both numbers are prime numbers)

$$10401(1 - 1/3)(1 - 1/3467) = 24040178 \bmod 10401 = 6932$$

$$10211 \bmod 6932 = 3279$$

$$20^{3279} \bmod 10401 = 3299$$

Confirmation:

$$20^{10211} =$$

$$\begin{aligned} & 20^{(2^13)} + 20^{(2^10)} + 20^{(2^9)} + 20^{(2^8)} + 20^{(2^7)} \\ & + 20^{(2^6)} + 20^{(2^5)} + 20^{(2^1)} + 20^{(2^0)} \end{aligned}$$

$$= 1582 * 7255 * 9187 * 3847 * 157 * 2149 * 7249 * 400 * 20 \bmod 10401 = 3299$$