Technical Aspect of the Paper: A new model for control of systems with friction[1]

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Given a LuGre friction model:

$$\dot{z} = \frac{dz}{dt} = v - \frac{|v|}{g(v)}z \tag{1}$$

and:

$$g(v) = \frac{F_C + (F_S - F_C)e^{-(v/v_s)^2}}{\sigma_0} \tag{2}$$

Therefore:

$$\dot{z} = v - \frac{z|v|\sigma_0}{F_C + (F_S - F_C)e^{-(v/v_s)^2}} \tag{3}$$

As for a system of mass:

$$\ddot{x} = \frac{u - F}{M} \tag{4}$$

where

$$F = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v \tag{5}$$

and

$$v = \dot{x} \tag{6}$$

Substituting Eq. (5) and Eq. (6) to Eq. (4) gives us:

$$\ddot{x} = \frac{u - \sigma_0 z - \sigma_1 \dot{z} - \sigma_2 v}{M} \tag{7}$$

where:

$$\dot{z} = \dot{x} - \frac{z|\dot{x}|\sigma_0}{F_C + (F_S - F_C)e^{-(\dot{x}/v_s)^2}} \tag{8}$$

Now, let us take:

$$\begin{aligned} q_1 = x &\rightarrow \dot{q}_1 = \dot{x} \\ q_2 = \dot{x} &\rightarrow \dot{q}_2 = \ddot{x} \\ q_3 = z &\rightarrow \dot{q}_3 = \dot{z} \end{aligned} \tag{9}$$

Applying Eq. (9) to Eq. (7) and Eq. (8) then gives us:

$$\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= \frac{(u - \sigma_0 q_3 - \sigma_1 \dot{z} - \sigma_2 q_2)}{M} \\ \dot{q}_3 &= \dot{z} \end{aligned} \tag{10}$$

where:

$$\dot{z} = q_2 - \frac{z|q_2|\sigma_0}{F_C + (F_S - F_C)e^{-(q_2/v_s)^2}} \tag{11}$$

We need to be aware that in the right-hand-side of Eq. (10), we do not replace $\dot{z}$ as $\dot{q}_3$ because we will compute $\dot{z}$ independently as in Eq. (11). Eq. (11) merely acts as an additional equation for Eq. (10). Eq. (10) now follows the formal format of: $\dot{q} = f(q)$, thus, we can implement it into MATLAB built-in ODE solver.

For the next steps, we will address $u(t)$. In the selected paper, only two kinds of force input are found: a ramp force input as in the varying break-away force simulation and a piecewise linear force input as in the presliding simulation.

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[1] C. Canudas de Wit, H. Olsson, K. J. Astrom, and P. Lischinsky, "A new model for control of systems with friction," IEEE Trans. Automat. Contr., vol. 40, no. 3, pp. 419–425, Mar. 1995.

Ramp Force Input

```matlab
1   function xdot = sim_mass_with_ramp_force_input(t, q, M, Fs, Fc, sigma_0, sigma_1, sigma_2, vs, force_rate)

2       u = force_rate*t;      % ramped-up force input
3
4       zdot = q(2) - ( (q(3)*abs(q(2))*sigma_0) / (Fc+(Fs-Fc)*exp(-(q(2)/vs)^2)) );
5       F = sigma_0*q(3) + sigma_1 * zdot + sigma_2*q(2);
6
7       qdot_1 = q(2);
8       qdot_2 = (u - F) / M;
9       qdot_3 = zdot;
10      xdot = [qdot_1 ; qdot_2; qdot_3 ];
11  end
```

Piecewise Linear Force Input
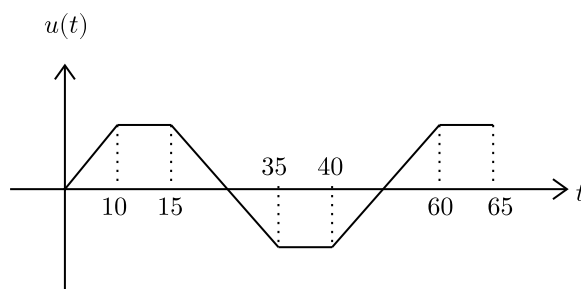

```matlab
function xdot = sim_presliding(t, q, M, Fs, Fc, sigma_0, sigma_1, sigma_2, vs)
    rate = 0.1425;

    if t <= 10
        u = rate*t;
    elseif t > 10 && t <= 15
        u = 1.425;
    elseif t > 15 && t <= 35
        u = 1.425 - rate*(t-15);
    elseif t > 35 && t <= 40
        u = -1.425;
    elseif t > 40 && t <= 60
        u = -1.425 + rate*(t-40);
    elseif t > 60
        u = 1.425;
    end

    zdot = q(2) - ( (q(3)*abs(q(2))*sigma_0) / (Fc+(Fs-Fc)*exp(-(q(2)/vs)^2)) );
    F = sigma_0*q(3) + sigma_1 * zdot + sigma_2*q(2);

    qdot_1 = q(2);
    qdot_2 = (u - F) / M;
    qdot_3 = zdot;
    xdot = [qdot_1 ; qdot_2; qdot_3 ];
end
```

<u>A System with PID</u>

For a system with a PID control, we need to create a new system similar to Eq. (10), which is in a form of $\dot{x} = f(x)$. We will need to introduce a new variable to accommodate the integral term.

Given a system:

$$\ddot{x} = \frac{u - F}{M} \tag{12}$$

where:

$$u = -K_v v - K_p(e) - K_i \int (e)$$

and

$$e = x - x_d \tag{13}$$

Eq. (12) can be rewritten as:

$$\ddot{x} = \frac{\overbrace{-K_v v - K_p(e) - K_i \int(e)}^{u} - \overbrace{(\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v)}^{F}}{M} \tag{14}$$

We introduce a new variables q_4:

$$q_4 = \int e \rightarrow \dot{q}_4 = e \tag{15}$$

Substituting Eq. (15) to Eq. (12) and replacing v with q_2, we now have a new system:

$$\dot{q}_1 = q_2$$
$$\dot{q}_2 = \frac{\left(-K_v q_2 - K_p q_4 - K_i \int(e) - \sigma_0 q_3 - \sigma_1 \dot{z} - \sigma_2 q_2\right)}{M} \tag{16}$$
$$\dot{q}_3 = \dot{z}$$
$$\dot{q}_4 = e$$

```
function xdot = sim_pid(t, q, M, Fs, Fc, sigma_0, sigma_1, sigma_2, vs, xd)
    Kp = 3;
    Ki = 4;
    Kv = 6;

    e = q(1) - xd;

    u = -Kv*q(2)-Kp*e-Ki*q(4);

    zdot = q(2) - ( (q(3)*abs(q(2))*sigma_0) / (Fc+(Fs-Fc)*exp(-(q(2)/vs)^2)) );
    F = sigma_0*q(3) + sigma_1 * zdot + sigma_2*q(2);

    qdot_1 = q(2);
    qdot_2 = (u - F) / M;
    qdot_3 = zdot;
    qdot_4 = e;
    xdot = [qdot_1 ; qdot_2; qdot_3; qdot_4];
end
```