Kreyszig 10ed 14.2 no. 9–19

Hitung ulang dengan teorema integral Cauchy!
If $f(z)$ is analytic in a simply connected domain made by a closed path C, then:

$$\int_C f(z)\,\mathrm{d}z = 0$$

No. 9

Carilah integral dari:

$$f(z) = \mathrm{e}^{-z^2}$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = \mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\mathrm{d}r = \int_0^{2\,\pi} \mathrm{I}\,\mathrm{e}^{-(\mathrm{e}^{\mathrm{I}r})^2}\,\mathrm{e}^{\mathrm{I}r}\,\mathrm{d}r$$

Hasilnya adalah:

$$0$$

No. 10

Carilah integral dari:

$$f(z) = \tan\left(\frac{z}{4}\right)$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = \mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\mathrm{d}r = \int_0^{2\,\pi} \mathrm{I}\tan\left(\frac{\mathrm{e}^{\mathrm{I}r}}{4}\right)\mathrm{e}^{\mathrm{I}r}\,\mathrm{d}r$$

Hasilnya adalah:

$$0$$

No. 11

Carilah integral dari:

$$f(z) = \frac{1}{2\,z - 1}$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\pi} f(z) \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\pi} \frac{I\, e^{I r}}{2\, e^{I r} - 1}\, dr$$

Hasilnya adalah:

$$I\,\pi$$

No. 12

Carilah integral dari:

$$f(z) = \overline{z}^3$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\pi} f(z) \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\pi} I \left(e^{-I \overline{r}} \right)^3 e^{I r}\, dr$$

Hasilnya adalah:

$$0$$

No. 13

Carilah integral dari:

$$f(z) = \frac{1}{z^4 - 1.1}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\pi} f(z) \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\pi} \frac{I\, e^{I r}}{\left(e^{I r} \right)^4 - 1.1}\, dr$$

Hasilnya adalah:

$$0.$$

No. 14

Carilah integral dari:

$$f(z) = \frac{1}{\overline{z}}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\,z(r) = I\,e^{I r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{d}{dr}\,z(r)\right)\,dr = \int_0^{2\,\pi} \frac{I\,e^{I r}}{e^{-I\,r}}\,dr$$

Hasilnya adalah:

$$0$$

No. 15

Carilah integral dari:

$$f(z) = \Im(z)$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\,z(r) = I\,e^{I r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{d}{dr}\,z(r)\right)\,dr = \int_0^{2\,\pi} I\,\Im\left(e^{I r}\right)\,e^{I r}\,dr$$

Hasilnya adalah:

$$-\pi$$

No. 16

Carilah integral dari:

$$f(z) = \frac{1}{\pi\,z - 1}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\,z(r) = I\,e^{I r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{d}{dr}\,z(r)\right)\,dr = \int_0^{2\,\pi} \frac{I\,e^{I r}}{\pi\,e^{I r} - 1}\,dr$$

Hasilnya adalah:

$$2\,I$$

No. 17

Carilah integral dari:

$$f(z) = \frac{1}{|z|^2}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\,\pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\,\pi} \frac{I\, e^{I r}}{\left(e^{-\Im(r)} \right)^2}\, dr$$

Hasilnya adalah:

$$0$$

No. 18

Carilah integral dari:

$$f(z) = \frac{1}{4\,z - 3}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\,\pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\,\pi} \frac{I\, e^{I r}}{4\, e^{I r} - 3}\, dr$$

Hasilnya adalah:

$$\frac{I}{2}\, \pi$$

No. 19

Carilah integral dari:

$$f(z) = z^3 \cot(z)$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\,\pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\,\pi} I\, \left(e^{I r} \right)^4 \cot\left(e^{I r} \right)\, dr$$

Hasilnya adalah:

0