Di kiri dengan Dirichlet dan di kanan dengan Neumann

$u(0, t) = 20, u_x(\pi, t) = 3, f(x) = 0$

$$u(0, t) = 20$$

$$u_x(L, t) = 3$$

Kondisi awal:

$$u(x, 0) = 0$$

$$dimana, L = \pi$$

$$dan, \alpha = 0.0034$$

Solusi umum:

$$u = (J\cos(\kappa x) + K\sin(\kappa x))\,e^{-\kappa^2\alpha^2 t} + H + Ix$$

Masukkan batas pertama:

$$u(0, t) = 20$$

$$20 = J\,e^{-\kappa^2\alpha^2 t} + H$$

Sehingga

$$H = 20$$

$$J = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = K\sin(\kappa x)\,e^{-\kappa^2\alpha^2 t} + 20 + Ix$$

Masukkan batas kedua:

$$u_x = K\kappa\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t} + I$$

$$u_x(L, t) = 3$$

$$K\kappa\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t} + I = 3$$

Sehingga

$$I = 3$$

$$\kappa = \frac{n\pi}{2L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u = \left(\sum_n K_n \sin\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha^2 t}{4L^2}}\right) + 20 + 3x$$

Masukkan initial condition:

$$u(x, 0) = 0$$

$$\sum_n K_n \sin\left(\frac{n\pi x}{2L}\right) = -3x - 20$$

$$F(x) = -3\,x - 20$$

Hitung, K_n

$$K_n = \frac{2\left(\int_0^L (-3\,x - 20)\,\sin\left(\frac{n\,\pi\,x}{2\,L}\right)\mathrm{d}x\right)}{L}$$

$$K_n = \frac{4\left(3\,n\cos\left(\frac{\pi\,n}{2}\right)\pi + 20\,n\cos\left(\frac{\pi\,n}{2}\right) - 6\sin\left(\frac{\pi\,n}{2}\right) - 20\,n\right)}{\pi\,n^2}$$

$$K_n = \frac{(12\,\pi + 80)\,n\cos\left(\frac{\pi\,n}{2}\right) - 80\,n - 24\sin\left(\frac{\pi\,n}{2}\right)}{\pi\,n^2}$$

$$K_n = \frac{-80\,n - 24\sin\left(\frac{\pi\,n}{2}\right)}{\pi\,n^2}$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_n \frac{\left(-80\,n - 24\sin\left(\frac{\pi\,n}{2}\right)\right)\sin\left(\frac{n\,x}{2}\right)\mathrm{e}^{-\frac{n^2\,\alpha^2\,t}{4}}}{\pi\,n^2}\right) + 20 + 3\,x$$

di mana n adalah ganjil

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} \frac{1}{\pi\,(2\,n-1)^2}\left(\left(-160\,n + 80 - 24\sin\left(\frac{\pi\,(2\,n-1)}{2}\right)\right)\sin\left(\frac{(2\,n-1)\,x}{2}\right)\mathrm{e}^{-2.890000000\,10^{-6}\,(2\,n-1)^2\,t}\right)\right) + 20 + 3\,x$$

di mana n=1,2,3,...

$$\left(\sum_{n=1}^{1000} \frac{1}{\pi\,(2\,n-1)^2}\left(\left(-160\,n + 80 - 24\sin\left(\frac{\pi\,(2\,n-1)}{2}\right)\right)\sin\left(\frac{(2\,n-1)\,x}{2}\right)\mathrm{e}^{-2.890000000\,10^{-6}\,(2\,n-1)^2\,t}\right)\right)(0,t) + 20 + 3\,x(0,t) = 10,\ u_x(2,t) = -5,\ f(x) = 10$$

$$u(0,t) = 10$$

$$u_x(L,t) = -5$$

Kondisi awal:

$$u(x,0)=10$$
$$dimana,\ L=2$$
$$dan,\ \alpha=0.05830951895$$

Solusi umum:

$$u=(J\cos(\kappa x)+K\sin(\kappa x))\,e^{-\kappa^2\alpha^2 t}+H+Ix$$

Masukkan batas pertama:

$$u(0,t)=10$$
$$10=J\,e^{-\kappa^2\alpha^2 t}+H$$

Sehingga

$$H=10$$
$$J=0$$

Pada tahap ini, solusinya sudah menjadi:

$$u=K\sin(\kappa x)\,e^{-\kappa^2\alpha^2 t}+10+Ix$$

Masukkan batas kedua:

$$u_x=K\kappa\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t}+I$$
$$u_x(L,t)=-5$$
$$K\kappa\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t}+I=-5$$

Sehingga

$$I=-5$$
$$\kappa=\frac{n\pi}{2L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u=\left(\sum_n K_n\sin\left(\frac{n\pi x}{2L}\right)e^{-\frac{n^2\pi^2\alpha^2 t}{4L^2}}\right)+10-5x$$

Masukkan initial condition:

$$u(x,0)=10$$
$$\sum_n K_n\sin\left(\frac{n\pi x}{2L}\right)=5x$$
$$F(x)=5x$$

$$Hitung,\ K_n$$

$$K_n = \frac{2\left(\int_0^L 5\,x \sin\left(\frac{n\,\pi\,x}{2\,L}\right) \mathrm{d}x\right)}{L}$$

$$K_n = -\frac{40\left(n\cos\left(\frac{\pi\,n}{2}\right)\pi - 2\sin\left(\frac{\pi\,n}{2}\right)\right)}{n^2\,\pi^2}$$

$$K_n = \frac{-40\,n\cos\left(\frac{\pi\,n}{2}\right)\pi + 80\sin\left(\frac{\pi\,n}{2}\right)}{n^2\,\pi^2}$$

$$K_n = \frac{80\sin\left(\frac{\pi\,n}{2}\right)}{n^2\,\pi^2}$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_n \frac{80\sin\left(\frac{\pi\,n}{2}\right)\sin\left(\frac{n\,\pi\,x}{4}\right)\mathrm{e}^{-\frac{n^2\pi^2\alpha^2 t}{16}}}{n^2\,\pi^2}\right) + 10 - 5\,x$$

di mana n adalah ganjil

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} \frac{80\sin\left(\frac{\pi\,(2\,n-1)}{2}\right)\sin\left(\frac{(2\,n-1)\,\pi\,x}{4}\right)\mathrm{e}^{-0.002097290936\,(2\,n-1)^2\,t}}{(2\,n-1)^2\,\pi^2}\right) + 10 - 5\,x$$

di mana n=1,2,3,...

$u(0, t) = 0, u_x(2, t) = 0, f(x) = 50\sin\left(\frac{\pi x}{2}\right)$

$$u(0, t) = 0$$

$$u_x(L, t) = 0$$

Kondisi awal:

$$u(x, 0) = 50\sin\left(\frac{\pi\,x}{2}\right)$$

$$dimana,\ L = 2$$

$$dan,\ \alpha = 0.05830951895$$

Solusi umum:

$$u = \left(J\cos(\kappa\,x) + K\sin(\kappa\,x)\right)\mathrm{e}^{-\kappa^2\alpha^2 t} + H + I\,x$$

Masukkan batas pertama:

$$u(0,t)=0$$

$$0=J\,e^{-\kappa^2\alpha^2 t}+H$$

Sehingga

$$H=0$$

$$J=0$$

Pada tahap ini, solusinya sudah menjadi:

$$u=K\sin(\kappa x)\,e^{-\kappa^2\alpha^2 t}+I x$$

Masukkan batas kedua:

$$u_x=K\,\kappa\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t}+I$$

$$u_x(L,t)=0$$

$$K\,\kappa\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t}+I=0$$

Sehingga

$$I=0$$

$$\kappa=\frac{n\,\pi}{2\,L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u=\sum_n K_n\sin\left(\frac{n\,\pi\,x}{2\,L}\right)e^{-\frac{n^2\pi^2\alpha^2 t}{4\,L^2}}$$

Masukkan initial condition:

$$u(x,0)=50\sin\left(\frac{\pi\,x}{2}\right)$$

$$\sum_n K_n\sin\left(\frac{n\,\pi\,x}{2\,L}\right)=50\sin\left(\frac{\pi\,x}{2}\right)$$

$$F(x)=50\sin\left(\frac{\pi\,x}{2}\right)$$

Hitung, K_n

$$K_n=\frac{2\left(\int_0^L 50\sin\left(\frac{\pi\,x}{2}\right)\sin\left(\frac{n\,\pi\,x}{2\,L}\right)\mathrm{d}x\right)}{L}$$

$$K_n=-\frac{400\sin\left(\frac{\pi\,n}{2}\right)}{\pi\,(n^2-4)}$$

$$K_n=-\frac{400\sin\left(\frac{\pi\,n}{2}\right)}{\pi\,(n^2-4)}$$

$$K_n = -\frac{400 \sin\left(\frac{\pi n}{2}\right)}{\pi (n^2 - 4)}$$

Jadi, solusi khususnya adalah:

$$u = \sum_{n} -\frac{400 \sin\left(\frac{\pi n}{2}\right) \sin\left(\frac{n \pi x}{4}\right) e^{-\frac{n^2 \pi^2 \alpha^2 t}{16}}}{\pi (n^2 - 4)}$$

di mana n adalah ganjil

Untuk keperluan simulasi, diubah menjadi:

$$u = \sum_{n=1}^{1000} -\frac{400 \sin\left(\frac{\pi (2n-1)}{2}\right) \sin\left(\frac{(2n-1)\pi x}{4}\right) e^{-0.002097290936 (2n-1)^2 t}}{\pi ((2n-1)^2 - 4)}$$

di mana n=1,2,3,...