Kreyszig (10th Ed.)
Problem Set 1.3 (2–10) No. 2,3,4,5,11,12

No. 2

An ODE:

$$\frac{d}{dx}\, y(x) = -\frac{x^3}{y^3}$$

with:

$$X(x) = -x^3$$
$$Y(y) = \frac{1}{y^3}$$

Therefore

$$\int \frac{1}{Y(y)}\, dy = \int X(x)\, dx$$
$$\int y^3\, dy = \int -x^3\, dx$$
$$\frac{y^4}{4} = -\frac{x^4}{4} + C$$

General solution (implicit form):

$$\frac{y^4}{4} = -\frac{x^4}{4} + C$$

No. 3

An ODE:

$$\frac{d}{dx}\, y(x) = \sec(y)^2$$

with:

$$X(x) = 1$$
$$Y(y) = \sec(y)^2$$

Therefore

$$\int \frac{1}{Y(y)}\, dy = \int X(x)\, dx$$
$$\int \frac{1}{\sec(y)^2}\, dy = \int 1\, dx$$
$$\frac{\cos(y)\,\sin(y)}{2} + \frac{y}{2} = x + C$$

General solution (implicit form):

$$\frac{\cos(y)\,\sin(y)}{2} + \frac{y}{2} = x + C$$

No. 4

An ODE:

$$\frac{d}{dx}\, y(x) = \frac{\pi\cos(2\,\pi\,x)\; y}{\sin(2\,\pi\,x)}$$

with:

$$X(x) = \frac{\pi\cos(2\,\pi\,x)}{\sin(2\,\pi\,x)}$$
$$Y(y) = y$$

Therefore

$$\int \frac{1}{Y(y)}\, dy = \int X(x)\, dx$$

$$\int \frac{1}{y}\, dy = \int \frac{\pi \cos(2\pi x)}{\sin(2\pi x)}\, dx$$

$$\ln(y) = \frac{\ln(\sin(2\pi x))}{2} + C$$

General solution (implicit form):

$$\ln(y) = \frac{\ln(\sin(2\pi x))}{2} + C$$

No. 5

An ODE:

$$\frac{d}{dx}\, y(x) = -\frac{36\, x}{y}$$

with:

$$X(x) = -36\, x$$

$$Y(y) = \frac{1}{y}$$

Therefore

$$\int \frac{1}{Y(y)}\, dy = \int X(x)\, dx$$

$$\int y\, dy = \int -36\, x\, dx$$

$$\frac{y^2}{2} = -18\, x^2 + C$$

General solution (implicit form):

$$\frac{y^2}{2} = -18\, x^2 + C$$

No. 11

An ODE:

$$\frac{d}{dx}\, y(x) = -\frac{y}{x}$$

with:

$$X(x) = \frac{1}{x}$$

$$Y(y) = -y$$

Therefore

$$\int \frac{1}{Y(y)}\, dy = \int X(x)\, dx$$

$$\int -\frac{1}{y}\, dy = \int \frac{1}{x}\, dx$$

$$-\ln(y) = \ln(x) + C$$

General solution (implicit form):

$$-\ln(y) = \ln(x) + C$$

Particular solution for the given IC:

$$y(4) = 6$$

Thus:

$$-\ln(6) = 2\ln(2) + C$$

$$C = -2\ln(2) - \ln(6)$$

No. 12

An ODE:

$$\frac{d}{dx}\, y(x) = 4\, y^2 + 1$$

with:

$$X(x) = 1$$
$$Y(y) = 4\, y^2 + 1$$

Therefore

$$\int \frac{1}{Y(y)}\, dy = \int X(x)\, dx$$
$$\int \frac{1}{4\, y^2 + 1}\, dy = \int 1\, dx$$
$$\frac{\arctan(2\, y)}{2} = x + C$$

General solution (implicit form):

$$\frac{\arctan(2\, y)}{2} = x + C$$

Particular solution for the given IC:

$$y(1) = 0$$

Thus:

$$0 = 1 + C$$
$$C = -1$$