Kreyszig 10ed 14.2 no. 9-19

Hitung ulang dengan integral Cauchy!

No. 9

Carilah integral dari:

$$f(z) = e^{-z^2}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\,\pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\,\pi} I\, e^{-(e^{I r})^2}\, e^{I r}\, dr$$

Hasilnya adalah:

0

No. 10

Carilah integral dari:

$$f(z) = \tan\left(\frac{z}{4} \right)$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\,\pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_0^{2\,\pi} I \tan\left(\frac{e^{I r}}{4} \right) e^{I r}\, dr$$

Hasilnya adalah:

0

No. 11

Carilah integral dari:

$$f(z) = \frac{1}{2\,z - 1}$$

Sepanjang kurva:

$$z = e^{I r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{d}{dr}\, z(r) = I\, e^{I r}$$

$$\int_0^{2\pi} f(z)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\mathrm{d}r=\int_0^{2\pi}\frac{\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}}{2\,\mathrm{e}^{\mathrm{I}r}-1}\,\mathrm{d}r$$

Hasilnya adalah:

$$\mathrm{I}\,\pi$$

No. 12

Carilah integral dari:

$$f(z)=\overline{z}^3$$

Sepanjang kurva:

$$z=\mathrm{e}^{\mathrm{I}r}$$

$$\textit{Sepanjang},\ r=0,\ \textit{menuju},\ r=2\,\pi$$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)=\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\pi} f(z)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\mathrm{d}r=\int_0^{2\pi}\mathrm{I}\left(\mathrm{e}^{-\mathrm{I}\overline{r}}\right)^3\mathrm{e}^{\mathrm{I}r}\,\mathrm{d}r$$

Hasilnya adalah:

0

No. 13

Carilah integral dari:

$$f(z)=\frac{1}{z^4-1.1}$$

Sepanjang kurva:

$$z=\mathrm{e}^{\mathrm{I}r}$$

$$\textit{Sepanjang},\ r=0,\ \textit{menuju},\ r=2\,\pi$$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)=\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\pi} f(z)\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\mathrm{d}r=\int_0^{2\pi}\frac{\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}}{\left(\mathrm{e}^{\mathrm{I}r}\right)^4-1.1}\,\mathrm{d}r$$

Hasilnya adalah:

0.

No. 14

Carilah integral dari:

$$f(z)=\frac{1}{\overline{z}}$$

Sepanjang kurva:

$$z=\mathrm{e}^{\mathrm{I}r}$$

$$\textit{Sepanjang},\ r=0,\ \textit{menuju},\ r=2\,\pi$$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\, z(r) = \mathrm{I}\, \mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\pi} f(z) \left(\frac{\mathrm{d}}{\mathrm{d}r}\, z(r) \right) \mathrm{d}r = \int_0^{2\pi} \frac{\mathrm{I}\, \mathrm{e}^{\mathrm{I}r}}{\mathrm{e}^{-\mathrm{I}\overline{r}}}\, \mathrm{d}r$$

Hasilnya adalah:

0

No. 15

Carilah integral dari:

$$f(z) = \Im(z)$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\, z(r) = \mathrm{I}\, \mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\pi} f(z) \left(\frac{\mathrm{d}}{\mathrm{d}r}\, z(r) \right) \mathrm{d}r = \int_0^{2\pi} \mathrm{I}\, \Im\left(\mathrm{e}^{\mathrm{I}r}\right)\, \mathrm{e}^{\mathrm{I}r}\, \mathrm{d}r$$

Hasilnya adalah:

$-\pi$

No. 16

Carilah integral dari:

$$f(z) = \frac{1}{\pi\, z - 1}$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\, z(r) = \mathrm{I}\, \mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\pi} f(z) \left(\frac{\mathrm{d}}{\mathrm{d}r}\, z(r) \right) \mathrm{d}r = \int_0^{2\pi} \frac{\mathrm{I}\, \mathrm{e}^{\mathrm{I}r}}{\pi\, \mathrm{e}^{\mathrm{I}r} - 1}\, \mathrm{d}r$$

Hasilnya adalah:

$2\,\mathrm{I}$

No. 17

Carilah integral dari:

$$f(z) = \frac{1}{|z|^2}$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = \mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\,\mathrm{d}r = \int_0^{2\,\pi} \frac{\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}}{\left(\mathrm{e}^{-\Im(r)}\right)^2}\,\mathrm{d}r$$

Hasilnya adalah:

0

No. 18

Carilah integral dari:

$$f(z) = \frac{1}{4\,z - 3}$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = \mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\,\mathrm{d}r = \int_0^{2\,\pi} \frac{\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}}{4\,\mathrm{e}^{\mathrm{I}r} - 3}\,\mathrm{d}r$$

Hasilnya adalah:

$$\frac{\mathrm{I}}{2}\,\pi$$

No. 19

Carilah integral dari:

$$f(z) = z^3\,\cot(z)$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = \mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\,\mathrm{d}r = \int_0^{2\,\pi} \mathrm{I}\,\left(\mathrm{e}^{\mathrm{I}r}\right)^4\,\cot\left(\mathrm{e}^{\mathrm{I}r}\right)\,\mathrm{d}r$$

Hasilnya adalah:

0
