Greenberg 23.2

Carilah integral dari:

$$f(z) = \frac{1}{z}$$

Sepanjang kurva:

$$z = \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju,* $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = \mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\,\mathrm{d}r = \int_0^{2\,\pi} \mathrm{I}\,\mathrm{d}r$$

Hasilnya adalah:

$$2\,\mathrm{I}\,\pi$$

Carilah integral dari:

$$f(z) = |z|^2$$

Sepanjang kurva:

$$z = r + \mathrm{I}\,r$$

Sepanjang, $r = 0$, *menuju,* $r = 1$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = 1 + \mathrm{I}$$

$$\int_0^1 f(z)\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\,\mathrm{d}r = \int_0^1 (1 + \mathrm{I})\,|r + \mathrm{I}\,r|^2\,\mathrm{d}r$$

Hasilnya adalah:

$$\frac{2}{3} + \frac{2\,\mathrm{I}}{3}$$

Carilah integral dari:

$$f(z) = \overline{z}$$

Sepanjang kurva:

$$z = r + \mathrm{I}\,r$$

Sepanjang, $r = 0$, *menuju,* $r = 1$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\,z(r) = 1 + \mathrm{I}$$

$$\int_0^1 f(z)\,\left(\frac{\mathrm{d}}{\mathrm{d}r}\,z(r)\right)\,\mathrm{d}r = \int_0^1 (1 + \mathrm{I})\,\overline{(r + \mathrm{I}\,r)}\,\mathrm{d}r$$

Hasilnya adalah:

$$1$$

Carilah integral dari:

$$f(z) = \overline{z}$$

Sepanjang kurva:

$$z = 2\, e^{I r}$$

Sepanjang, $r = \pi$, *menuju*, $r = 0$

Jawab:

$$\frac{d}{dr}\, z(r) = 2\, I\, e^{I r}$$

$$\int_{\pi}^{0} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_{\pi}^{0} 4\, I\, e^{-I \overline{r}}\, e^{I r}\, dr$$

Hasilnya adalah:

$$-4\, I\, \pi$$

Carilah integral dari:

$$f(z) = 4\, z$$

Sepanjang kurva:

$$z = 2\, e^{I r}$$

Sepanjang, $r = 0$, *menuju*, $r = \pi$

Jawab:

$$\frac{d}{dr}\, z(r) = 2\, I\, e^{I r}$$

$$\int_{0}^{\pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_{0}^{\pi} 16\, I\, \left(e^{I r} \right)^2 dr$$

Hasilnya adalah:

$$0$$

Carilah integral dari:

$$f(z) = \frac{1}{(z - 3\, I)\, (z + 5)}$$

Sepanjang kurva:

$$z = 4\, e^{I r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\, \pi$

Jawab:

$$\frac{d}{dr}\, z(r) = 4\, I\, e^{I r}$$

$$\int_{0}^{2\, \pi} f(z)\, \left(\frac{d}{dr}\, z(r) \right) dr = \int_{0}^{2\, \pi} \frac{4\, I\, e^{I r}}{\left(4\, e^{I r} - 3\, I \right)\, \left(4\, e^{I r} + 5 \right)}\, dr$$

Hasilnya adalah:

$$\frac{3\, \pi}{17} + \frac{5\, I\, \pi}{17}$$

Carilah integral dari:

$$f(z) = \frac{1}{(z - 3\,\mathrm{I})\ (z + 5)}$$

Sepanjang kurva:

$$z = 6\ \mathrm{e}^{\mathrm{I}r}$$

Sepanjang, $r = 0$, *menuju*, $r = 2\,\pi$

Jawab:

$$\frac{\mathrm{d}}{\mathrm{d}r}\ z(r) = 6\,\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}$$

$$\int_0^{2\,\pi} f(z)\ \left(\frac{\mathrm{d}}{\mathrm{d}r}\ z(r)\right)\ \mathrm{d}r = \int_0^{2\,\pi} \frac{6\,\mathrm{I}\,\mathrm{e}^{\mathrm{I}r}}{\left(6\,\mathrm{e}^{\mathrm{I}r} - 3\,\mathrm{I}\right)\ \left(6\,\mathrm{e}^{\mathrm{I}r} + 5\right)}\ \mathrm{d}r$$

Hasilnya adalah:

0
