Neumann Boundary Condition

$u_x(0, t) = u_x(\pi, t) = 0, f(x) = 300$

Kondisi batas:

$$u_x(0, t) = 0$$

$$u_x(L, t) = 0$$

Kondisi awal:

$$u(x, 0) = 300$$

$$dimana, L = \pi$$

$$dan, \alpha = 0.05830951895$$

Solusi umum:

$$u = (J\cos(\kappa x) + K\sin(\kappa x))\, e^{-\kappa^2 \alpha^2 t} + H + I x$$

Masukkan batas pertama:

$$u_x(0, t) = 0$$

$$0 = K\kappa\, e^{-\kappa^2 \alpha^2 t} + I$$

Sehingga

$$I = 0$$

$$K = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = J\cos(\kappa x)\, e^{-\kappa^2 \alpha^2 t} + H$$

Masukkan batas kedua:

$$u_x = -J\kappa\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t}$$

$$u_x(L, t) = 0$$

$$-J\kappa\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t} = 0$$

Sehingga

$$\kappa = \frac{n\pi}{L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u = \left(\sum_n J_n \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{n^2\pi^2\alpha^2 t}{L^2}}\right) + H$$

Masukkan initial condition:

$$u(x, 0) = 300$$

$$\left(\sum_{n} J_n \cos\left(\frac{n\pi x}{L}\right)\right) + H = 300$$

$$F(x) = 300$$

$$\text{Hitung}, J_n$$

$$J_n = \frac{2\left(\int_0^L 300\cos\left(\frac{n\pi x}{L}\right) dx\right)}{L}$$

$$J_n = \frac{600\sin(\pi n)}{\pi n}$$

$$J_n = 0$$

$$\text{Hitung}, H$$

$$H = \frac{\int_0^L 300\, dx}{L}$$

$$H = 300$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_{n} 0\right) + 300$$

di mana n=1,2,3,...

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} 0\right) + 300$$

di mana n=1,2,3,...

$$\left(\sum_{n=1}^{1000} 0\right) + 300$$

$u_x(0,t) = u_x(10,t) = 5, f(x) = 45 + 5x$

Kondisi batas:

$$u_x(0,t) = 5$$

$$u_x(L,t) = 5$$

Kondisi awal:

$$u(x,0) = 45 + 5x$$

$$\text{dimana}, L = 10$$

$$\text{dan}, \alpha = 0.05830951895$$

Solusi umum:

$$u = (J\cos(\kappa x) + K\sin(\kappa x))\, e^{-\kappa^2 \alpha^2 t} + H + I x$$

Masukkan batas pertama:

$$u_x(0, t) = 5$$

$$5 = K\kappa\, e^{-\kappa^2 \alpha^2 t} + I$$

Sehingga

$$I = 5$$

$$K = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = J\cos(\kappa x)\, e^{-\kappa^2 \alpha^2 t} + H + 5\,x$$

Masukkan batas kedua:

$$u_x = -J\kappa\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t} + 5$$

$$u_x(L, t) = 5$$

$$-J\kappa\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t} + 5 = 5$$

Sehingga

$$\kappa = \frac{n\,\pi}{L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u = \left(\sum_n J_n \cos\left(\frac{n\,\pi\,x}{L}\right) e^{-\frac{n^2 \pi^2 \alpha^2 t}{L^2}}\right) + H + 5\,x$$

Masukkan initial condition:

$$u(x, 0) = 45 + 5\,x$$

$$\left(\sum_n J_n \cos\left(\frac{n\,\pi\,x}{L}\right)\right) + H = 45$$

$$F(x) = 45$$

Hitung, J_n

$$J_n = \frac{2\left(\int_0^L 45\cos\left(\frac{n\,\pi\,x}{L}\right) dx\right)}{L}$$

$$J_n = \frac{90\sin(\pi\,n)}{\pi\,n}$$

$$J_n = 0$$

$$Hitung, H$$

$$H = \frac{\int_0^L 45 \, dx}{L}$$

$$H = 45$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_n 0 \right) + 45 + 5\,x$$

di mana n=1,2,3,...

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} 0 \right) + 45 + 5\,x$$

di mana n=1,2,3,...

$$\left(\sum_{n=1}^{1000} 0 \right) + 45 + 5\,x$$

$$u_x(0, t) = u_x(3\,\pi, t) = 0, f(x) = \begin{cases} 0, & 0 < x < 2\,\pi \\ 60, & 2\,\pi < x < 3\,\pi \end{cases}$$

Kondisi batas:

$$u_x(0, t) = 0$$

$$u_x(L, t) = 0$$

Kondisi awal:

$$u(x, 0) = \begin{cases} 0 & 0 < x < 2\,\pi \\ 60 & 2\,\pi < x < 3\,\pi \end{cases}$$

$$dimana, L = 3\,\pi$$

$$dan, \alpha = 0.0034$$

Solusi umum:

$$u = (J \cos(\kappa\, x) + K \sin(\kappa\, x))\, e^{-\kappa^2 \alpha^2 t} + H + I x$$

Masukkan batas pertama:

$$u_x(0, t) = 0$$

$$0 = K\, \kappa\, e^{-\kappa^2 \alpha^2 t} + I$$

Sehingga

$$I = 0$$

$$K = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = J\cos(\kappa x)\, e^{-\kappa^2 \alpha^2 t} + H$$

Masukkan batas kedua:

$$u_x = -J\kappa\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t}$$

$$u_x(L, t) = 0$$

$$-J\kappa\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t} = 0$$

Sehingga

$$\kappa = \frac{n\pi}{L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u = \left(\sum_n J_n \cos\left(\frac{n\pi x}{L}\right) e^{-\frac{n^2\pi^2\alpha^2 t}{L^2}}\right) + H$$

Masukkan initial condition:

$$u(x, 0) = \begin{cases} 0 & 0 < x < 2\pi \\ 60 & 2\pi < x < 3\pi \end{cases}$$

$$\left(\sum_n J_n \cos\left(\frac{n\pi x}{L}\right)\right) + H = \begin{cases} 0 & 0 < x < 2\pi \\ 60 & 2\pi < x < 3\pi \end{cases}$$

$$F(x) = \begin{cases} 0 & 0 < x < 2\pi \\ 60 & 2\pi < x < 3\pi \end{cases}$$

Hitung, J_n

$$J_n = \frac{2\left(\int_0^L \left(\begin{cases} 0 & 0 < x < 2\pi \\ 60 & 2\pi < x < 3\pi \end{cases}\right)\cos\left(\frac{n\pi x}{L}\right) dx\right)}{L}$$

$$J_n = \frac{120\left(-\sin\left(\frac{2\pi n}{3}\right) + \sin(\pi n)\right)}{\pi n}$$

$$J_n = -\frac{120\sin\left(\frac{2\pi n}{3}\right)}{\pi n}$$

Hitung, H

$$H = \frac{\int_0^L \left(\begin{cases} 0 & 0 < x < 2\pi \\ 60 & 2\pi < x < 3\pi \end{cases} \right) dx}{L}$$

$$H = 20$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_n -\frac{120 \sin\left(\frac{2\pi n}{3}\right) \cos\left(\frac{n x}{3}\right) e^{-\frac{n^2 \alpha^2 t}{9}}}{\pi n} \right) + 20$$

di mana n=1,2,3,...

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} -\frac{120 \sin\left(\frac{2\pi n}{3}\right) \cos\left(\frac{n x}{3}\right) e^{-1.284444444\, 10^{-6} n^2 t}}{\pi n} \right) + 20$$

di mana n=1,2,3,...