Problem Sets 2.2
Undetermined Coefficients
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No. 1

$$\frac{d^2}{dx^2}\, y(x) - 25\, \frac{d}{dx}\, y(x) = 0$$

Obtain the homogenous solution:

$$\frac{d^2}{dx^2}\, y(x) - 25\, \frac{d}{dx}\, y(x) = 0$$

Since:

$$y(x) = e^{\lambda x}$$

$$\frac{d}{dx}\, y(x) = \lambda\, e^{\lambda x}$$

$$\frac{d^2}{dx^2}\, y(x) = \lambda^2\, e^{\lambda x}$$

Thus:

$$\lambda^2\, e^{\lambda x} - 25\, \lambda\, e^{\lambda x} = 0$$

$$\left(\lambda^2 - 25\, \lambda\right) e^{\lambda x} = 0$$

Solving for, λ

$$\lambda_1 = 0$$

$$\lambda_2 = 25$$

Therefore, the homogenous solution is:

$$y(x) = C_1 + C_2\, e^{25\, x}$$

No.2

$$\frac{d^2}{dx^2}\, y(x) + 36\, \frac{d}{dx}\, y(x) = 0$$

Obtain the homogenous solution:

$$\frac{d^2}{dx^2}\, y(x) + 36\, \frac{d}{dx}\, y(x) = 0$$

Since:

$$y(x) = e^{\lambda x}$$

$$\frac{d}{dx}\, y(x) = \lambda\, e^{\lambda x}$$

$$\frac{d^2}{dx^2}\, y(x) = \lambda^2\, e^{\lambda x}$$

Thus:

$$\lambda^2\, e^{\lambda x} + 36\, \lambda\, e^{\lambda x} = 0$$

$$\left(\lambda^2 + 36\, \lambda\right) e^{\lambda x} = 0$$

Solving for, λ

$$\lambda_1 = -36$$

$$\lambda_2 = 0$$

Therefore, the homogenous solution is:

$$y(x) = C_1 \mathrm{e}^{-36\,x} + C_2$$

No.3

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + 6\, \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) + 8.96\, y(x) = 0$$

Obtain the homogenous solution:

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + 6\, \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) + 8.96\, y(x) = 0$$

Since:

$$y(x) = \mathrm{e}^{\lambda x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \lambda\, \mathrm{e}^{\lambda x}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) = \lambda^2\, \mathrm{e}^{\lambda x}$$

Thus:

$$\lambda^2\, \mathrm{e}^{\lambda x} + 6\, \lambda\, \mathrm{e}^{\lambda x} + 8.96\, \mathrm{e}^{\lambda x} = 0$$

$$\left(\lambda^2 + 6.\, \lambda + 8.96\right) \mathrm{e}^{\lambda x} = 0$$

Solving for, λ

$$\lambda_1 = -2.800000000$$

$$\lambda_2 = -3.200000000$$

Therefore, the homogenous solution is:

$$y(x) = C_1 \mathrm{e}^{-2.800000000\, x} + C_2 \mathrm{e}^{-3.200000000\, x}$$

No.4

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + 4\, \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) + \left(\pi^2 + 4\right) y(x) = 0$$

Obtain the homogenous solution:

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\, y(x) + 4\, \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) + \left(\pi^2 + 4\right) y(x) = 0$$

Since:

$$y(x) = \mathrm{e}^{\lambda x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \lambda\, \mathrm{e}^{\lambda x}$$

$$\frac{d^2}{dx^2}\, y(x) = \lambda^2\, e^{\lambda x}$$

Thus:

$$\lambda^2 e^{\lambda x} + 4\,\lambda\, e^{\lambda x} + \left(\pi^2 + 4\right) e^{\lambda x} = 0$$

$$\left(\pi^2 + \lambda^2 + 4\,\lambda + 4\right) e^{\lambda x} = 0$$

Solving for, λ

$$\lambda_1 = -2 + \mathrm{I}\,\pi$$

$$\lambda_2 = -2 - \mathrm{I}\,\pi$$

Therefore, the homogenous solution is:

$$y(x) = C_1\, e^{(-2 + \mathrm{I}\pi)\, x} + C_2\, e^{(-2 - \mathrm{I}\pi)\, x}$$

No. 5

$$\frac{d^2}{dx^2}\, y(x) + 2\,\pi \left(\frac{d}{dx}\, y(x)\right) + \pi^2\, y(x) = 0$$

Obtain the homogenous solution:

$$\frac{d^2}{dx^2}\, y(x) + 2\,\pi \left(\frac{d}{dx}\, y(x)\right) + \pi^2\, y(x) = 0$$

Since:

$$y(x) = e^{\lambda x}$$

$$\frac{d}{dx}\, y(x) = \lambda\, e^{\lambda x}$$

$$\frac{d^2}{dx^2}\, y(x) = \lambda^2\, e^{\lambda x}$$

Thus:

$$\lambda^2 e^{\lambda x} + 2\,\pi\,\lambda\, e^{\lambda x} + \pi^2\, e^{\lambda x} = 0$$

$$\left(\pi^2 + 2\,\pi\,\lambda + \lambda^2\right) e^{\lambda x} = 0$$

Solving for, λ

$$\lambda = -\pi$$

Therefore, the homogenous solution is:

$$y(x) = \left(x\, C_2 + C_1\right) e^{-\pi x}$$

No.6

$$10\, \frac{d^2}{dx^2}\, y(x) - 32\, \frac{d}{dx}\, y(x) + 25.6\, y(x) = 0$$

Obtain the homogenous solution:

$$10\, \frac{d^2}{dx^2}\, y(x) - 32\, \frac{d}{dx}\, y(x) + 25.6\, y(x) = 0$$

Since:

$$y(x) = e^{\lambda x}$$
$$\frac{d}{dx} y(x) = \lambda e^{\lambda x}$$
$$\frac{d^2}{dx^2} y(x) = \lambda^2 e^{\lambda x}$$
Thus:
$$10 \lambda^2 e^{\lambda x} - 32 \lambda e^{\lambda x} + 25.6 e^{\lambda x} = 0$$
$$(10. \lambda^2 - 32. \lambda + 25.6) e^{\lambda x} = 0$$
Solving for, λ
$$\lambda = 1.600000000$$
Therefore, the homogenous solution is:
$$y(x) = (x C_2 + C_1) e^{1.600000000 x}$$

No.7

$$10 \frac{d^2}{dx^2} y(x) - 4.5 \frac{d}{dx} y(x) = 0$$

Obtain the homogenous solution:
$$10 \frac{d^2}{dx^2} y(x) - 4.5 \frac{d}{dx} y(x) = 0$$
Since:
$$y(x) = e^{\lambda x}$$
$$\frac{d}{dx} y(x) = \lambda e^{\lambda x}$$
$$\frac{d^2}{dx^2} y(x) = \lambda^2 e^{\lambda x}$$
Thus:
$$10 \lambda^2 e^{\lambda x} - 4.5 \lambda e^{\lambda x} = 0$$
$$(10. \lambda^2 - 4.5 \lambda) e^{\lambda x} = 0$$
Solving for, λ
$$\lambda_1 = 0.$$
$$\lambda_2 = 0.4500000000$$
Therefore, the homogenous solution is:
$$y(x) = 1. C_1 + C_2 e^{0.4500000000 x}$$