Excercises 10.8
No. 2 (a,b,e,f)

> $F = \begin{bmatrix} x_1^2 \\ x_1 + x_2 \end{bmatrix}$

$$T := x \mapsto \langle x_1^2, x_1 + 2\,x_2 \rangle$$

$$T(\alpha\,u + \beta\,v) = \begin{bmatrix} (\alpha\,u_1 + \beta\,v_1)^2 \\ \alpha\,u_1 + 2\,\alpha\,u_2 + \beta\,v_1 + 2\,\beta\,v_2 \end{bmatrix}$$

$$T(\alpha\,u) = \begin{bmatrix} \alpha\,u_1^2 \\ \alpha\,(u_1 + 2\,u_2) \end{bmatrix}$$

$$T(\beta\,v) = \begin{bmatrix} \beta\,v_1^2 \\ \beta\,(v_1 + 2\,v_2) \end{bmatrix}$$

$$\Delta = \begin{bmatrix} (\alpha^2 - \alpha)\,u_1^2 + 2\,\alpha\,\beta\,u_1\,v_1 + \beta\,v_1^2\,(\beta - 1) \\ 0 \end{bmatrix}$$

> $F = \begin{bmatrix} 3\,x_1 \\ x_1 + x_2 \end{bmatrix}$

$$T := x \mapsto \langle 3\,x_1, x_1 + 2\,x_2 \rangle$$

$$T(\alpha\,u + \beta\,v) = \begin{bmatrix} 3\,\alpha\,u_1 + 3\,\beta\,v_1 \\ \alpha\,u_1 + 2\,\alpha\,u_2 + \beta\,v_1 + 2\,\beta\,v_2 \end{bmatrix}$$

$$T(\alpha\,u) = \begin{bmatrix} 3\,\alpha\,u_1 \\ \alpha\,(u_1 + 2\,u_2) \end{bmatrix}$$

$$T(\beta\,v) = \begin{bmatrix} 3\,\beta\,v_1 \\ \beta\,(v_1 + 2\,v_2) \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

> $F = \begin{bmatrix} 4\,x_1 \\ \sin(x_1) \end{bmatrix}$

$$T := x \mapsto \langle 4\,x_1, \sin(x_2) \rangle$$

$$T(\alpha u + \beta v) = \begin{bmatrix} 4\alpha u_1 + 4\beta v_1 \\ \sin(\alpha u_2 + \beta v_2) \end{bmatrix}$$

$$T(\alpha u) = \begin{bmatrix} 4\alpha u_1 \\ \alpha \sin(u_2) \end{bmatrix}$$

$$T(\beta v) = \begin{bmatrix} 4\beta v_1 \\ \beta \sin(v_2) \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0 \\ \sin(\alpha u_2 + \beta v_2) - \alpha \sin(u_2) - \beta \sin(v_2) \end{bmatrix}$$

> $F = \begin{bmatrix} x_1 + 1 \\ x_2 + 1 \end{bmatrix}$

$$T := x \mapsto \langle x_1 + 1, x_2 + 1 \rangle$$

$$T(\alpha u + \beta v) = \begin{bmatrix} \alpha u_1 + \beta v_1 + 1 \\ \alpha u_2 + \beta v_2 + 1 \end{bmatrix}$$

$$T(\alpha u) = \begin{bmatrix} \alpha (u_1 + 1) \\ \alpha (u_2 + 1) \end{bmatrix}$$

$$T(\beta v) = \begin{bmatrix} \beta (v_1 + 1) \\ \beta (v_2 + 1) \end{bmatrix}$$

$$\Delta = \begin{bmatrix} -\alpha - \beta + 1 \\ -\alpha - \beta + 1 \end{bmatrix}$$