Kreyszig (10th Ed.)
Problem Set 1.3 (2–10) No. 2,3,4,5,11,12

No. 2

An ODE:

$$\frac{d}{dx}\ y(x) = -\frac{x^3}{y^3}$$

with:

$$X(x) = -x^3$$
$$Y(y) = \frac{1}{y^3}$$

Therefore

$$\int \frac{1}{Y(y)}\ dy = \int X(x)\ dx$$
$$\int y^3\ dy = \int -x^3\ dx$$
$$\frac{y^4}{4} = -\frac{x^4}{4} + C$$

General solution (implicit form):

$$\frac{y^4}{4} = -\frac{x^4}{4} + C$$

No. 3

An ODE:

$$\frac{d}{dx}\ y(x) = \sec(y)^2$$

with:

$$X(x) = 1$$
$$Y(y) = \sec(y)^2$$

Therefore

$$\int \frac{1}{Y(y)}\ dy = \int X(x)\ dx$$
$$\int \frac{1}{\sec(y)^2}\ dy = \int 1\ dx$$
$$\frac{\cos(y)\ \sin(y)}{2} + \frac{y}{2} = x + C$$

General solution (implicit form):

$$\frac{\cos(y)\ \sin(y)}{2} + \frac{y}{2} = x + C$$

No. 4

An ODE:

$$\frac{d}{dx}\ y(x) = \frac{\pi\cos(2\,\pi\,x)\ y}{\sin(2\,\pi\,x)}$$

with:

$$X(x) = \frac{\pi\cos(2\,\pi\,x)}{\sin(2\,\pi\,x)}$$
$$Y(y) = y$$

Therefore

$$\int \frac{1}{Y(y)}\,\mathrm{d}y = \int X(x)\,\mathrm{d}x$$

$$\int \frac{1}{y}\,\mathrm{d}y = \int \frac{\pi \cos(2\,\pi\,x)}{\sin(2\,\pi\,x)}\,\mathrm{d}x$$

$$\ln(y) = \frac{\ln(\sin(2\,\pi\,x))}{2} + C$$

$$y = \mathrm{e}^{\frac{\ln(\sin(2\,\pi\,x))}{2} + C}$$

No. 5

An ODE:

$$\frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = -\frac{36\,x}{y}$$

with:

$$X(x) = -36\,x$$

$$Y(y) = \frac{1}{y}$$

Therefore

$$\int \frac{1}{Y(y)}\,\mathrm{d}y = \int X(x)\,\mathrm{d}x$$

$$\int y\,\mathrm{d}y = \int -36\,x\,\mathrm{d}x$$

$$\frac{y^2}{2} = -18\,x^2 + C$$

General solution (implicit form):

$$\frac{y^2}{2} = -18\,x^2 + C$$

No. 11

An ODE:

$$\frac{\mathrm{d}}{\mathrm{d}x}\,y(x) = -\frac{y}{x}$$

with:

$$X(x) = -\frac{1}{x}$$

$$Y(y) = y$$

Therefore

$$\int \frac{1}{Y(y)}\,\mathrm{d}y = \int X(x)\,\mathrm{d}x$$

$$\int \frac{1}{y}\,\mathrm{d}y = \int -\frac{1}{x}\,\mathrm{d}x$$

$$\ln(y) = -\ln(x) + C$$

$$y = \frac{\mathrm{e}^C}{x}$$

Particular solution for the given IC:

$$y(4) = 6$$

Thus:

$$y = \frac{\mathrm{e}^C}{x}$$

$$C = \ln(24)$$

If we take $e^C = C$, then $C = 24$.

No. 12

An ODE:

$$\frac{d}{dx} y(x) = 4 y^2 + 1$$

with:

$$X(x) = 1$$

$$Y(y) = 4 y^2 + 1$$

Therefore

$$\int \frac{1}{Y(y)} \, dy = \int X(x) \, dx$$

$$\int \frac{1}{4 y^2 + 1} \, dy = \int 1 \, dx$$

$$\frac{\arctan(2 y)}{2} = x + C$$

$$y = \frac{\tan(2 x + 2 C)}{2}$$

Particular solution for the given IC:

$$y(1) = 0$$

Thus:

$$y = \frac{\tan(2 x + 2 C)}{2}$$

$$C = -1$$