Dirichlet Boundary Conditoin

NO. 6(I)

$u(0, t) = 0, u(10, t) = 100, f(x) = 0$

Kondisi batas:

$$u(0, t) = 0$$

$$u(L, t) = 100$$

Kondisi awal:

$$u(x, 0) = 0$$

$$dimana, L = 10$$

$$dan, \alpha = 0.05830951895$$

Solusi umum:

$$u = (J\cos(\kappa x) + K\sin(\kappa x))\, e^{-\kappa^2 \alpha^2 t} + H + Ix$$

Masukkan batas pertama:

$$u(0, t) = 0$$

$$0 = J\, e^{-\kappa^2 \alpha^2 t} + H$$

Sehingga

$$H = 0$$

$$J = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = K\sin(\kappa x)\, e^{-\kappa^2 \alpha^2 t} + Ix$$

Masukkan batas kedua:

$$u = K\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t} + IL$$

$$u(L, t) = 100$$

$$K\sin(\kappa L)\, e^{-\kappa^2 \alpha^2 t} + IL = 100$$

Sehingga

$$I = \frac{100}{L}$$

$$I = 10$$

$$\kappa = \frac{n\pi}{L}$$

di mana n=1,2,...

Pada tahap ini, solusinya menjadi:

$$u = \left(\sum_n K_n \sin\left(\frac{n\pi x}{L}\right) e^{-\frac{n^2 \pi^2 \alpha^2 t}{L^2}}\right) + 10\,x$$

Masukkan initial condition:

$$u(x, 0) = 0$$

$$\sum_{n} K_n \sin\left(\frac{n \pi x}{L}\right) = -10\,x$$

$$F(x) = -10\,x$$

Hitung, K_n

$$K_n = \frac{2\left(\int_0^L -10\,x \sin\left(\frac{n \pi x}{10}\right) \mathrm{d}x\right)}{L}$$

$$K_n = -\frac{200\,(-n\,\pi \cos(n\,\pi) + \sin(n\,\pi))}{n^2\,\pi^2}$$

$$K_n = \frac{200\,(-1)^n}{n\,\pi}$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_{n} \frac{200\,(-1)^n \sin\left(\frac{n \pi x}{10}\right) \mathrm{e}^{-\frac{n^2 \pi^2 \alpha^2 t}{100}}}{n\,\pi}\right) + 10\,x$$

di mana n=1,2,3,...

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} \frac{200\,(-1)^n \sin\left(\frac{n \pi x}{10}\right) \mathrm{e}^{-0.0003355665497\,n^2\,t}}{n\,\pi}\right) + 10\,x$$

di mana n=1,2,3,...