Exercises 3.4
Problem 4
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Problem 4a

$$\frac{d^2}{dx^2} y(x) + 5 \frac{d}{dx} y(x) = 0$$

$$\mathit{Since}, y(x) = e^{\lambda x}, \ \mathit{and}, \ \frac{d}{dx} y(x) = \lambda e^{\lambda x}, \ \mathit{thus:}$$

$$\lambda^2 e^{\lambda x} + 5 \lambda e^{\lambda x} = 0$$

$$(\lambda^2 + 5 \lambda) e^{\lambda x} = 0$$

$$\mathit{Solving\ for}, \lambda$$

$$\lambda_1 = -5$$

$$\lambda_2 = 0$$

Thus, the general solution is:

$$y(x) = C_1 e^{-5x} + C_2$$

Problem 4b

$$\frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) = 0$$

$$\mathit{Since}, y(x) = e^{\lambda x}, \ \mathit{and}, \ \frac{d}{dx} y(x) = \lambda e^{\lambda x}, \ \mathit{thus:}$$

$$\lambda^2 e^{\lambda x} - \lambda e^{\lambda x} = 0$$

$$(\lambda^2 - \lambda) e^{\lambda x} = 0$$

$$\mathit{Solving\ for}, \lambda$$

$$\lambda_1 = 0$$

$$\lambda_2 = 1$$

Thus, the general solution is:

$$y(x) = C_1 + C_2 e^{x}$$

Problem 4c

$$\frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = 0$$

$$\mathit{Since}, y(x) = e^{\lambda x}, \ \mathit{and}, \ \frac{d}{dx} y(x) = \lambda e^{\lambda x}, \ \mathit{thus:}$$

$$\lambda^2 e^{\lambda x} + \lambda e^{\lambda x} = 0$$

$$(\lambda^2 + \lambda) e^{\lambda x} = 0$$

$$\mathit{Solving\ for}, \lambda$$

$$\lambda_1 = -1$$

$$\lambda_2 = 0$$

Thus, the general solution is:

$$y(x) = C_1 \mathrm{e}^{-x} + C_2$$

and the first derivative is:

$$\frac{\mathrm{d}}{\mathrm{d}x} y(x) = -C_1 \mathrm{e}^{-x}$$

Finding the particular solution for the given initial condition:

$$y(0) = 3$$

$$\frac{\mathrm{d}}{\mathrm{d}x} y(0) = 0$$

Plug the IC into the equations above:

$$3 = C_1 \mathrm{e}^{0} + C_2$$

$$0 = -C_1 \mathrm{e}^{0}$$

Finding the constants:

$$C_1 = 0$$

$$C_2 = 3$$

Therefore, the particular solution is:

$$y(x) = 3$$

Problem 4d

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} y(x) - 3 \frac{\mathrm{d}}{\mathrm{d}x} y(x) + 2\, y(x) = 0$$

$$\textit{Since}, y(x) = \mathrm{e}^{\lambda x}, \ \textit{and}, \ \frac{\mathrm{d}}{\mathrm{d}x} y(x) = \lambda \mathrm{e}^{\lambda x}, \ \textit{thus:}$$

$$\lambda^2 \mathrm{e}^{\lambda x} - 3 \lambda \mathrm{e}^{\lambda x} + 2 \mathrm{e}^{\lambda x} = 0$$

$$\left(\lambda^2 - 3 \lambda + 2\right) \mathrm{e}^{\lambda x} = 0$$

$$\textit{Solving for}, \lambda$$

$$\lambda_1 = 2$$

$$\lambda_2 = 1$$

Thus, the general solution is:

$$y(x) = C_1 \mathrm{e}^{2x} + C_2 \mathrm{e}^{x}$$

and the first derivative is:

$$\frac{\mathrm{d}}{\mathrm{d}x} y(x) = 2\, C_1 \mathrm{e}^{2x} + C_2 \mathrm{e}^{x}$$

Finding the particular solution for the given initial condition:

$$y(1) = 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x} y(1) = 0$$

Plug the IC into the equations above:

$$1 = C_1 \mathrm{e}^{2} + C_2 \mathrm{e}$$

$$0 = 2\, C_1 \mathrm{e}^{2} + C_2 \mathrm{e}$$

Finding the constants:

$$C_1 = -\mathrm{e}^{-2}$$

$$C_2 = 2\,e^{-1}$$

Therefore, the particular solution is:

$$y(x) = -e^{-2+2x} + 2\,e^{-1+x}$$

Problem 4e

$$\frac{d^2}{dx^2}\,y(x) - 4\,\frac{d}{dx}\,y(x) - 5\,y(x) = 0$$

$$Since,\ y(x) = e^{\lambda x},\ and,\ \frac{d}{dx}\,y(x) = \lambda\,e^{\lambda x},\ thus:$$

$$\lambda^2\,e^{\lambda x} - 4\,\lambda\,e^{\lambda x} - 5\,e^{\lambda x} = 0$$

$$\left(\lambda^2 - 4\,\lambda - 5\right)\,e^{\lambda x} = 0$$

$$Solving\ for,\ \lambda$$

$$\lambda_1 = 5$$

$$\lambda_2 = -1$$

Thus, the general solution is:

$$y(x) = C_1\,e^{5x} + C_2\,e^{-x}$$

and the first derivative is:

$$\frac{d}{dx}\,y(x) = 5\,C_1\,e^{5x} - C_2\,e^{-x}$$

Finding the particular solution for the given initial condition:

$$y(1) = 1$$

$$\frac{d}{dx}\,y(1) = 0$$

Plug the IC into the equations above:

$$1 = C_1\,e^5 + C_2\,e^{-1}$$

$$0 = 5\,C_1\,e^5 - C_2\,e^{-1}$$

Finding the constants:

$$C_1 = \frac{e^{-5}}{6}$$

$$C_2 = \frac{5\,e}{6}$$

Therefore, the particular solution is:

$$y(x) = \frac{e^{-5+5x}}{6} + \frac{5\,e^{1-x}}{6}$$

Problem 4f

$$\frac{d^2}{dx^2}\,y(x) + \frac{d}{dx}\,y(x) - 12\,y(x) = 0$$

$$Since,\ y(x) = e^{\lambda x},\ and,\ \frac{d}{dx}\,y(x) = \lambda\,e^{\lambda x},\ thus:$$

$$\lambda^2\,e^{\lambda x} + \lambda\,e^{\lambda x} - 12\,e^{\lambda x} = 0$$

$$\left(\lambda^2 + \lambda - 12\right)\,e^{\lambda x} = 0$$

$$Solving\ for,\ \lambda$$

$$\lambda_1 = 3$$

$$\lambda_2 = -4$$

Thus, the general solution is:

$$y(x) = C_1 e^{3x} + C_2 e^{-4x}$$

and the first derivative is:

$$\frac{d}{dx} y(x) = 3 C_1 e^{3x} - 4 C_2 e^{-4x}$$

Finding the particular solution for the given initial condition:

$$y(-1) = 2$$

$$\frac{d}{dx} y(-1) = 5$$

Plug the IC into the equations above:

$$2 = C_1 e^{-3} + C_2 e^{4}$$

$$5 = 3 C_1 e^{-3} - 4 C_2 e^{4}$$

Finding the constants:

$$C_1 = \frac{13 e^3}{7}$$

$$C_2 = \frac{e^{-4}}{7}$$

Therefore, the particular solution is:

$$y(x) = \frac{13 e^{3+3x}}{7} + \frac{e^{-4-4x}}{7}$$

Problem 4g

$$\frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 5 y(x) = 0$$

$$Since,\ y(x) = e^{\lambda x},\ and,\ \frac{d}{dx} y(x) = \lambda e^{\lambda x},\ thus:$$

$$\lambda^2 e^{\lambda x} - 4 \lambda e^{\lambda x} + 5 e^{\lambda x} = 0$$

$$(\lambda^2 - 4\lambda + 5) e^{\lambda x} = 0$$

$$Solving\ for,\ \lambda$$

$$\lambda_1 = 2 + I$$

$$\lambda_2 = 2 - I$$

Thus, the general solution is:

$$y(x) = C_1 e^{(2+I)x} + C_2 e^{(2-I)x}$$

and the first derivative is:

$$\frac{d}{dx} y(x) = (2+I) C_1 e^{(2+I)x} + (2-I) C_2 e^{(2-I)x}$$

Finding the particular solution for the given initial condition:

$$y(0) = 2$$

$$\frac{d}{dx} y(0) = 5$$

Plug the IC into the equations above:

$$2 = C_1 e^0 + C_2 e^0$$

$$5 = (2 + I)\ C_1 e^0 + (2 - I)\ C_2 e^0$$

Finding the constants:

$$C_1 = 1 - \frac{I}{2}$$

$$C_2 = 1 + \frac{I}{2}$$

Therefore, the particular solution is:

$$y(x) = \left(1 + \frac{I}{2}\right) e^{(2 - I)\,x} + \left(1 - \frac{I}{2}\right) e^{(2 + I)\,x}$$

Problem 4h

$$\frac{d^2}{dx^2} y(x) - 2 \frac{d}{dx} y(x) + 3\,y(x) = 0$$

$$\mathit{Since}, y(x) = e^{\lambda x},\ \mathit{and},\ \frac{d}{dx} y(x) = \lambda e^{\lambda x},\ \mathit{thus:}$$

$$\lambda^2 e^{\lambda x} - 2\,\lambda e^{\lambda x} + 3\,e^{\lambda x} = 0$$

$$\left(\lambda^2 - 2\,\lambda + 3\right) e^{\lambda x} = 0$$

$$\mathit{Solving\ for},\ \lambda$$

$$\lambda_1 = 1 + I\sqrt{2}$$

$$\lambda_2 = 1 - I\sqrt{2}$$

Thus, the general solution is:

$$y(x) = C_1 e^{\left(1 + I\sqrt{2}\right) x} + C_2 e^{\left(1 - I\sqrt{2}\right) x}$$

and the first derivative is:

$$\frac{d}{dx} y(x) = C_1 \left(1 + I\sqrt{2}\right) e^{\left(1 + I\sqrt{2}\right) x} + C_2 \left(1 - I\sqrt{2}\right) e^{\left(1 - I\sqrt{2}\right) x}$$

Finding the particular solution for the given initial condition:

$$y(0) = 4$$

$$\frac{d}{dx} y(0) = -1$$

Plug the IC into the equations above:

$$4 = C_1 e^0 + C_2 e^0$$

$$-1 = C_1 \left(1 + I\sqrt{2}\right) e^0 + C_2 \left(1 - I\sqrt{2}\right) e^0$$

Finding the constants:

$$C_1 = \frac{\left(4\sqrt{2} + 5\,I\right)\sqrt{2}}{4}$$

$$C_2 = 2 - \frac{5\,I\sqrt{2}}{4}$$

Therefore, the particular solution is:

$$y(x) = \frac{5\left(\left(-I + \frac{4\sqrt{2}}{5}\right) e^{-\left(I\sqrt{2}-1\right)x} + e^{\left(1+I\sqrt{2}\right)x}\left(I + \frac{4\sqrt{2}}{5}\right)\right)\sqrt{2}}{4}$$

Problem 4i

$$\frac{d^2}{dx^2} y(x) - 2\frac{d}{dx} y(x) + 2\,y(x) = 0$$

$$\mathit{Since}, y(x) = e^{\lambda x},\ \mathit{and},\ \frac{d}{dx} y(x) = \lambda e^{\lambda x},\ \mathit{thus:}$$

$$\lambda^2 e^{\lambda x} - 2\lambda e^{\lambda x} + 2 e^{\lambda x} = 0$$

$$\left(\lambda^2 - 2\lambda + 2\right) e^{\lambda x} = 0$$

$$\mathit{Solving\ for},\ \lambda$$

$$\lambda_1 = 1 + I$$

$$\lambda_2 = 1 - I$$

Thus, the general solution is:

$$y(x) = C_1 e^{(1+I)x} + C_2 e^{(1-I)x}$$

and the first derivative is:

$$\frac{d}{dx} y(x) = (1+I)\,C_1 e^{(1+I)x} + (1-I)\,C_2 e^{(1-I)x}$$

Finding the particular solution for the given initial condition:

$$y(0) = 0$$

$$\frac{d}{dx} y(0) = -5$$

Plug the IC into the equations above:

$$0 = C_1 e^0 + C_2 e^0$$

$$-5 = (1+I)\,C_1 e^0 + (1-I)\,C_2 e^0$$

Finding the constants:

$$C_1 = \frac{5I}{2}$$

$$C_2 = -\frac{5I}{2}$$

Therefore, the particular solution is:

$$y(x) = \frac{5I}{2}\left(e^{(1+I)x} - e^{(1-I)x}\right)$$

Problem 4j

$$\frac{d^2}{dx^2} y(x) + 2\frac{d}{dx} y(x) + 3\,y(x) = 0$$

$$\mathit{Since}, y(x) = e^{\lambda x},\ \mathit{and},\ \frac{d}{dx} y(x) = \lambda e^{\lambda x},\ \mathit{thus:}$$

$$\lambda^2 e^{\lambda x} + 2\lambda e^{\lambda x} + 3 e^{\lambda x} = 0$$

$$\left(\lambda^2 + 2\lambda + 3\right) e^{\lambda x} = 0$$

$$\mathit{Solving\ for},\ \lambda$$

$$\lambda_1 = I\sqrt{2} - 1$$

$$\lambda_2 = -1 - I\sqrt{2}$$

Thus, the general solution is:

$$y(x) = C_1\, e^{(I\sqrt{2} - 1)\, x} + C_2\, e^{(-1 - I\sqrt{2})\, x}$$

and the first derivative is:

$$\frac{d}{dx}\, y(x) = C_1\, (I\sqrt{2} - 1)\, e^{(I\sqrt{2} - 1)\, x} + C_2\, (-1 - I\sqrt{2})\, e^{(-1 - I\sqrt{2})\, x}$$

Finding the particular solution for the given initial condition:

$$y(0) = 0$$

$$\frac{d}{dx}\, y(0) = 3$$

Plug the IC into the equations above:

$$0 = C_1\, e^0 + C_2\, e^0$$

$$3 = C_1\, (I\sqrt{2} - 1)\, e^0 + C_2\, (-1 - I\sqrt{2})\, e^0$$

Finding the constants:

$$C_1 = -\frac{3\, I}{4}\, \sqrt{2}$$

$$C_2 = \frac{3\, I}{4}\, \sqrt{2}$$

Therefore, the particular solution is:

$$y(x) = -\frac{3\, I}{4}\, \sqrt{2}\, \left(e^{(I\sqrt{2} - 1)\, x} - e^{-(1 + I\sqrt{2})\, x}\right)$$