**Problem 5.4 #1**
**Greeberg's book**

Note that: $\dot{x}(0)$ is written as $\mathrm{D}(x)(0)$.

### Problem 1a

$$\frac{d}{dt}x(t)+2x(t)=4t^2$$

*The intial conditions(s):*

$$\{x(0)=A\}$$

$$sX(s)-x(0)+2X(s)=\frac{8}{s^3}$$

$$sX(s)-A+2X(s)=\frac{8}{s^3}$$

$$X(s)=\frac{As^3+8}{s^3(s+2)}$$

*By partial fracton expansion:*

$$X(s)=\frac{4}{s^3}-\frac{2}{s^2}+\frac{1}{s}+\frac{A-1}{s+2}$$

$$X(s)=\frac{4}{s^3}-\frac{2}{s^2}+\frac{1}{s}+\frac{A}{s+2}-\frac{1}{s+2}$$

$$\textit{Laplace inverse of},\ \frac{4}{s^3},\ \textit{is},\ 2t^2$$

$$\textit{Laplace inverse of},\ -\frac{2}{s^2},\ \textit{is},\ -2t$$

$$\textit{Laplace inverse of},\ \frac{1}{s},\ \textit{is},\ 1$$

$$\textit{Laplace inverse of},\ \frac{A}{s+2},\ \textit{is},\ A\,e^{-2t}$$

$$\textit{Laplace inverse of},\ -\frac{1}{s+2},\ \textit{is},\ -e^{-2t}$$

$$\textit{Finally, the solution is},\ x(t)=2t^2-2t+1+(A-1)\,e^{-2t}$$

### Problem 1b

$$3\frac{d}{dt}x(t)+x(t)=6e^{2t}$$

*The intial conditions(s):*

$$\{x(0)=0\}$$

$$3sX(s)-3x(0)+X(s)=\frac{6}{s-2}$$

$$3sX(s)+X(s)=\frac{6}{s-2}$$

$$X(s)=\frac{6}{(s-2)(3s+1)}$$

By partial fracton expansion:

$$X(s) = -\frac{18}{7\,(3\,s+1)} + \frac{6}{7\,(s-2)}$$

$$Laplace\ inverse\ of,\ -\frac{18}{7\,(3\,s+1)},\ is\ ,\ -\frac{6\,e^{-\frac{t}{3}}}{7}$$

$$Laplace\ inverse\ of,\ \frac{6}{7\,(s-2)},\ is\ ,\ \frac{6\,e^{2\,t}}{7}$$

$$Finally,\ the\ solution\ is\ ,\ x(t) = -\frac{6\,e^{-\frac{t}{3}}}{7} + \frac{6\,e^{2\,t}}{7}$$

**Problem 1c**

$$\frac{d}{dt}\,x(t) - 6\,x(t) = e^{-t}$$

The intial conditions(s):

$$\{x(0) = 4\}$$

$$s\,X(s) - x(0) - 6\,X(s) = \frac{1}{1+s}$$

$$s\,X(s) - 4 - 6\,X(s) = \frac{1}{1+s}$$

$$X(s) = \frac{5+4\,s}{(1+s)\,(s-6)}$$

By partial fracton expansion:

$$X(s) = -\frac{1}{7\,(1+s)} + \frac{29}{7\,(s-6)}$$

$$Laplace\ inverse\ of,\ -\frac{1}{7\,(1+s)},\ is\ ,\ -\frac{e^{-t}}{7}$$

$$Laplace\ inverse\ of,\ \frac{29}{7\,(s-6)},\ is\ ,\ \frac{29\,e^{6\,t}}{7}$$

$$Finally,\ the\ solution\ is\ ,\ x(t) = -\frac{e^{-t}}{7} + \frac{29\,e^{6\,t}}{7}$$

**Problem 1d**

$$\frac{d^2}{dt^2}\,x(t) = 6\,t$$

The intial conditions(s):

$$\{x(0) = 2,\ D(x)(0) = -1\}$$

$$s^2\,X(s) - D(x)(0) - s\,x(0) = \frac{6}{s^2}$$

$$s^2\,X(s) + 1 - 2\,s = \frac{6}{s^2}$$

$$X(s) = \frac{2\,s^3 - s^2 + 6}{s^4}$$

*By partial fracton expansion:*

$$X(s) = \frac{6}{s^4} - \frac{1}{s^2} + \frac{2}{s}$$

$$\textit{Laplace inverse of}, \frac{6}{s^4}, \textit{is}, t^3$$

$$\textit{Laplace inverse of}, -\frac{1}{s^2}, \textit{is}, -t$$

$$\textit{Laplace inverse of}, \frac{2}{s}, \textit{is}, 2$$

$$\textit{Finally, the solution is}, x(t) = t^3 - t + 2$$

**Problem 1e**

$$\frac{d^2}{dt^2} x(t) + 5 \frac{d}{dt} x(t) = 10$$

*The intial conditions(s):*

$$\{x(0) = A, D(x)(0) = B\}$$

$$s^2 X(s) - D(x)(0) - s\,x(0) + 5\,s\,X(s) - 5\,x(0) = \frac{10}{s}$$

$$s^2 X(s) - B - s\,A + 5\,s\,X(s) - 5\,A = \frac{10}{s}$$

$$X(s) = \frac{s^2 A + 5\,s\,A + B\,s + 10}{s^2 (s+5)}$$

*By partial fracton expansion:*

$$X(s) = \frac{-B+2}{5\,(s+5)} + \frac{2}{s^2} + \frac{5\,A + B - 2}{5\,s}$$

$$X(s) = -\frac{B}{5\,(s+5)} + \frac{2}{5\,(s+5)} + \frac{2}{s^2} + \frac{A}{s} + \frac{B}{5\,s} - \frac{2}{5\,s}$$

$$\textit{Laplace inverse of}, -\frac{B}{5\,(s+5)}, \textit{is}, -\frac{B\,e^{-5\,t}}{5}$$

$$\textit{Laplace inverse of}, \frac{2}{5\,(s+5)}, \textit{is}, \frac{2\,e^{-5\,t}}{5}$$

$$\textit{Laplace inverse of}, \frac{2}{s^2}, \textit{is}, 2\,t$$

$$\textit{Laplace inverse of}, \frac{A}{s}, \textit{is}, A$$

$$\textit{Laplace inverse of}, \frac{B}{5\,s}, \textit{is}, \frac{B}{5}$$

$$\textit{Laplace inverse of}, -\frac{2}{5\,s}, \textit{is}, -\frac{2}{5}$$

$$\textit{Finally, the solution is}, x(t) = -\frac{2}{5} + 2\,t + A + \frac{B}{5} - \frac{e^{-5\,t}(B-2)}{5}$$

**Problem 1f**

$$\frac{d^2}{dt^2}x(t) - \frac{d}{dt}x(t) = t^2 + t + 1$$

*The intial conditions(s):*

$$\{x(0) = A, D(x)(0) = B\}$$

$$s^2 X(s) - D(x)(0) - s\,x(0) - s\,X(s) + x(0) = \frac{s^2 + s + 2}{s^3}$$

$$s^2 X(s) - B - s\,A - s\,X(s) + A = \frac{s^2 + s + 2}{s^3}$$

$$X(s) = \frac{s^4 A - A\,s^3 + B\,s^3 + s^2 + s + 2}{s^4 (s-1)}$$

*By partial fracton expansion:*

$$X(s) = -\frac{2}{s^4} + \frac{B+4}{s-1} - \frac{3}{s^3} - \frac{4}{s^2} + \frac{A-B-4}{s}$$

$$X(s) = -\frac{2}{s^4} + \frac{B}{s-1} + \frac{4}{s-1} - \frac{3}{s^3} - \frac{4}{s^2} + \frac{A}{s} - \frac{B}{s} - \frac{4}{s}$$

$$\mathit{Laplace\ inverse\ of}, -\frac{2}{s^4}, \mathit{is}, -\frac{t^3}{3}$$

$$\mathit{Laplace\ inverse\ of}, \frac{B}{s-1}, \mathit{is}, B\,e^t$$

$$\mathit{Laplace\ inverse\ of}, \frac{4}{s-1}, \mathit{is}, 4\,e^t$$

$$\mathit{Laplace\ inverse\ of}, -\frac{3}{s^3}, \mathit{is}, -\frac{3\,t^2}{2}$$

$$\mathit{Laplace\ inverse\ of}, -\frac{4}{s^2}, \mathit{is}, -4\,t$$

$$\mathit{Laplace\ inverse\ of}, \frac{A}{s}, \mathit{is}, A$$

$$\mathit{Laplace\ inverse\ of}, -\frac{B}{s}, \mathit{is}, -B$$

$$\mathit{Laplace\ inverse\ of}, -\frac{4}{s}, \mathit{is}, -4$$

$$\mathit{Finally,\ the\ solution\ is}, x(t) = -\frac{t^3}{3} + (B+4)\,e^t - \frac{3\,t^2}{2} - 4\,t + A - B - 4$$

**Problem 1g**

$$\frac{d^2}{dt^2}x(t) - 3\frac{d}{dt}x(t) + 2\,x(t) = 0$$

*The intial conditions(s):*

$$\{x(0) = 3, D(x)(0) = 1\}$$

$$s^2 X(s) - D(x)(0) - s\,x(0) - 3\,s\,X(s) + 3\,x(0) + 2\,X(s) = 0$$

$$s^2 X(s) + 8 - 3\,s - 3\,s\,X(s) + 2\,X(s) = 0$$

$$X(s) = \frac{-8+3\,s}{s^2-3\,s+2}$$

*By partial fracton expansion:*

$$X(s) = \frac{5}{s-1} - \frac{2}{s-2}$$

$$\textit{Laplace inverse of}, \frac{5}{s-1}, \textit{is}, 5\,\mathrm{e}^{t}$$

$$\textit{Laplace inverse of}, -\frac{2}{s-2}, \textit{is}, -2\,\mathrm{e}^{2\,t}$$

$$\textit{Finally, the solution is}, x(t) = 5\,\mathrm{e}^{t} - 2\,\mathrm{e}^{2\,t}$$

**Problem 1h**

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,x(t) - 4\,\frac{\mathrm{d}}{\mathrm{d}t}\,x(t) - 5\,x(t) = 2 + \mathrm{e}^{-t}$$

*The intial conditions(s):*

$$\{x(0) = 0, \mathrm{D}(x)(0) = 0\}$$

$$s^2\,X(s) - \mathrm{D}(x)(0) - s\,x(0) - 4\,s\,X(s) + 4\,x(0) - 5\,X(s) = \frac{2+3\,s}{s\,(1+s)}$$

$$s^2\,X(s) - 4\,s\,X(s) - 5\,X(s) = \frac{2+3\,s}{s\,(1+s)}$$

$$X(s) = \frac{2+3\,s}{s\,(1+s)\,\left(s^2-4\,s-5\right)}$$

*By partial fracton expansion:*

$$X(s) = \frac{11}{36\,(1+s)} - \frac{1}{6\,(1+s)^2} + \frac{17}{180\,(s-5)} - \frac{2}{5\,s}$$

$$\textit{Laplace inverse of}, \frac{11}{36\,(1+s)}, \textit{is}, \frac{11\,\mathrm{e}^{-t}}{36}$$

$$\textit{Laplace inverse of}, -\frac{1}{6\,(1+s)^2}, \textit{is}, -\frac{t\,\mathrm{e}^{-t}}{6}$$

$$\textit{Laplace inverse of}, \frac{17}{180\,(s-5)}, \textit{is}, \frac{17\,\mathrm{e}^{5\,t}}{180}$$

$$\textit{Laplace inverse of}, -\frac{2}{5\,s}, \textit{is}, -\frac{2}{5}$$

$$\textit{Finally, the solution is}, x(t) = -\frac{2}{5} + \frac{17\,\mathrm{e}^{5\,t}}{180} - \frac{\mathrm{e}^{-t}\,(-11+6\,t)}{36}$$

**Problem 1i**

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\,x(t) - \frac{\mathrm{d}}{\mathrm{d}t}\,x(t) - 12\,x(t) = t$$

*The intial conditions(s):*

$$\{x(0) = -1, \mathrm{D}(x)(0) = 0\}$$

$$s^2\,X(s) - \mathrm{D}(x)(0) - s\,x(0) - s\,X(s) + x(0) - 12\,X(s) = \frac{1}{s^2}$$

$$s^2 X(s) - 1 + s - s X(s) - 12 X(s) = \frac{1}{s^2}$$

$$X(s) = -\frac{s^3 - s^2 - 1}{s^2 (s^2 - s - 12)}$$

*By partial fracton expansion:*

$$X(s) = -\frac{1}{12 s^2} + \frac{1}{144 s} - \frac{37}{63 (s+3)} - \frac{47}{112 (s-4)}$$

$$\textit{Laplace inverse of}, -\frac{1}{12 s^2}, \textit{is}, -\frac{t}{12}$$

$$\textit{Laplace inverse of}, \frac{1}{144 s}, \textit{is}, \frac{1}{144}$$

$$\textit{Laplace inverse of}, -\frac{37}{63 (s+3)}, \textit{is}, -\frac{37 e^{-3t}}{63}$$

$$\textit{Laplace inverse of}, -\frac{47}{112 (s-4)}, \textit{is}, -\frac{47 e^{4t}}{112}$$

$$\textit{Finally, the solution is}, x(t) = -\frac{t}{12} + \frac{1}{144} - \frac{37 e^{-3t}}{63} - \frac{47 e^{4t}}{112}$$

**Problem 1j**

$$\frac{d^2}{dt^2} x(t) + 6 \frac{d}{dt} x(t) + 9 x(t) = 1$$

*The intial conditions(s):*

$$\{x(0) = 0, D(x)(0) = -2\}$$

$$s^2 X(s) - D(x)(0) - s x(0) + 6 s X(s) - 6 x(0) + 9 X(s) = \frac{1}{s}$$

$$s^2 X(s) + 2 + 6 s X(s) + 9 X(s) = \frac{1}{s}$$

$$X(s) = -\frac{-1 + 2 s}{s (s^2 + 6 s + 9)}$$

*By partial fracton expansion:*

$$X(s) = -\frac{7}{3 (s+3)^2} + \frac{1}{9 s} - \frac{1}{9 (s+3)}$$

$$\textit{Laplace inverse of}, -\frac{7}{3 (s+3)^2}, \textit{is}, -\frac{7 t e^{-3t}}{3}$$

$$\textit{Laplace inverse of}, \frac{1}{9 s}, \textit{is}, \frac{1}{9}$$

$$\textit{Laplace inverse of}, -\frac{1}{9 (s+3)}, \textit{is}, -\frac{e^{-3t}}{9}$$

$$\textit{Finally, the solution is}, x(t) = \frac{1}{9} - \frac{e^{-3t} (21 t + 1)}{9}$$

**Problem 1k**

$$\frac{d^2}{dt^2} x(t) - 2 \frac{d}{dt} x(t) + 2 x(t) = -2 t$$

*The intial conditions(s):*

$$\{x(0)=0, \mathrm{D}(x)(0)=-5\}$$

$$s^2 X(s) - \mathrm{D}(x)(0) - s\,x(0) - 2\,s\,X(s) + 2\,x(0) + 2\,X(s) = -\frac{2}{s^2}$$

$$s^2 X(s) + 5 - 2\,s\,X(s) + 2\,X(s) = -\frac{2}{s^2}$$

$$X(s) = -\frac{5\,s^2+2}{s^2\,(s^2-2\,s+2)}$$

*By partial fracton expansion:*

$$X(s) = -\frac{1}{s^2} - \frac{1}{s} + \frac{s-6}{s^2-2\,s+2}$$

$$X(s) = -\frac{1}{s^2} - \frac{1}{s} + \frac{s}{s^2-2\,s+2} - \frac{6}{s^2-2\,s+2}$$

$$\textit{Laplace inverse of}, -\frac{1}{s^2}, \textit{is}, -t$$

$$\textit{Laplace inverse of}, -\frac{1}{s}, \textit{is}, -1$$

$$\textit{Laplace inverse of}, \frac{s}{s^2-2\,s+2}, \textit{is}, \mathrm{e}^t\,(\cos(t)+\sin(t))$$

$$\textit{Laplace inverse of}, -\frac{6}{s^2-2\,s+2}, \textit{is}, -6\,\mathrm{e}^t \sin(t)$$

$$\textit{Finally, the solution is}, x(t) = -t - 1 + \mathrm{e}^t\,(\cos(t) - 5\,\sin(t))$$

**Problem 1l**

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t) - 2\,\frac{\mathrm{d}}{\mathrm{d}t} x(t) + 3\,x(t) = 5$$

*The intial conditions(s):*

$$\{x(0)=1, \mathrm{D}(x)(0)=-1\}$$

$$s^2 X(s) - \mathrm{D}(x)(0) - s\,x(0) - 2\,s\,X(s) + 2\,x(0) + 3\,X(s) = \frac{5}{s}$$

$$s^2 X(s) + 3 - s - 2\,s\,X(s) + 3\,X(s) = \frac{5}{s}$$

$$X(s) = \frac{s^2-3\,s+5}{s\,(s^2-2\,s+3)}$$

*By partial fracton expansion:*

$$X(s) = \frac{1-2\,s}{3\,(s^2-2\,s+3)} + \frac{5}{3\,s}$$

$$X(s) = \frac{1}{3\,(s^2-2\,s+3)} - \frac{2\,s}{3\,(s^2-2\,s+3)} + \frac{5}{3\,s}$$

$$\textit{Laplace inverse of}, \frac{1}{3\,(s^2-2\,s+3)}, \textit{is}, \frac{\sqrt{2}\,\sin(\sqrt{2}\,t)\,\mathrm{e}^t}{6}$$

$$\textit{Laplace inverse of},\ -\frac{2\,s}{3\,(s^2-2\,s+3)},\ \textit{is}\ ,\ -\frac{e^t\left(2\cos(\sqrt{2}\,t)+\sqrt{2}\,\sin(\sqrt{2}\,t)\right)}{3}$$

$$\textit{Laplace inverse of},\ \frac{5}{3\,s},\ \textit{is}\ ,\ \frac{5}{3}$$

$$\textit{Finally, the solution is}\ ,\ x(t)=\frac{5}{3}-\frac{e^t\left(\sqrt{2}\,\sin(\sqrt{2}\,t)+4\cos(\sqrt{2}\,t)\right)}{6}$$