Problem 1a

The line can be calculated as follows:

$$z(t) = x_0 + (x_1 - x_0)\, t + \mathrm{I}\,(y_0 + (y_1 - y_0)\, t)$$

where

$$z_0 = 0$$

$$x_0 = \Re(z_0), y_0 = \Im(z_0)$$

and

$$z_1 = 1 + \mathrm{I}$$

$$x_1 = \Re(z_1), y_1 = \Im(z_1)$$

Thus, the line parametric equation is:

$$C{:}z(t) = t + \mathrm{I}\, t$$

And the derivative is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\, z(t) = 1 + \mathrm{I}$$

$$Because\,,f(z) = |z|^2,\ \ thus$$

$$f(z(t)) = 2\,t^2$$

$$f(t) = 2\,t^2$$

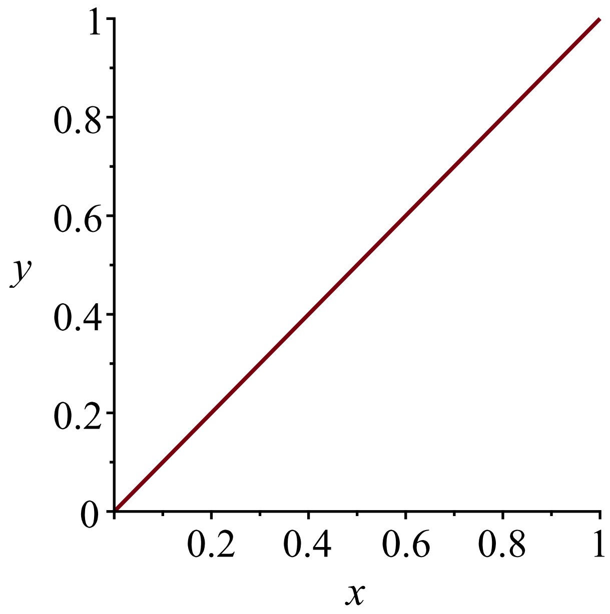
Therefore:

$$\int_C f(z)\,\mathrm{d}z = \int_0^1 f(z(t))\left(\frac{\mathrm{d}}{\mathrm{d}t}\, z(t)\right)\mathrm{d}t$$

$$\int_C f(z)\,\mathrm{d}z = \int_0^1 (2 + 2\,\mathrm{I})\, t^2\,\mathrm{d}t$$

$$\int_C f(z)\,\mathrm{d}z = \int_0^1 2\,t^2\,\mathrm{d}t + \mathrm{I}\left(\int_0^1 2\,t^2\,\mathrm{d}t\right)$$

$$\int_C f(z)\,\mathrm{d}z = \frac{2}{3} + \frac{2}{3}\,\mathrm{I}$$



Problem 1b

$$F := t \to f(z(t))$$

The line can be calculated as follows:

$$z(t) = x_0 + (x_1 - x_0)\, t + \mathrm{I}\,(y_0 + (y_1 - y_0)\, t)$$

where

$$z_0 = 0$$

$$x_0 = \Re(z_0), y_0 = \Im(z_0)$$

and

$$z_1 = 1 + \mathrm{I}$$

$$x_1 = \Re(z_1), y_1 = \Im(z_1)$$

Thus, the line parametric equation is:

$$C{:}z(t) = t + \mathrm{I}\, t$$

And the derivative is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\, z(t) = 1 + \mathrm{I}$$

$$Because\,, f(z) = \overline{z},\ \ thus$$

$$f(z(t)) = t - \mathrm{I}\, t$$

$$f(t) = t - \mathrm{I}\, t$$

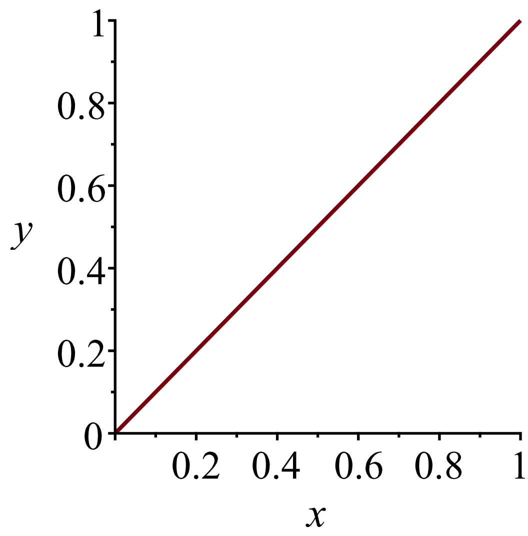
Therefore:

$$\int_C f(z)\, \mathrm{d}z = \int_0^1 f(z(t)) \left(\frac{\mathrm{d}}{\mathrm{d}t}\, z(t)\right) \mathrm{d}t$$

$$\int_C f(z)\, dz = \int_0^1 (1+I)\,(t - I\,t)\, dt$$

$$\int_C f(z)\, dz = \int_0^1 2\,t\, dt$$

$$\int_C f(z)\, dz = 1$$



Problem 1c

The path is a circle centered at 0 with radius of 2 form z=2 to z=-2 (clockwise):

$$C{:}z(t) = 2\, e^{I\,t}$$

And the derivative is:

$$\frac{d}{dt}\, z(t) = 2\, I\, e^{I\,t}$$

$$Because\,,\, f(z) = \overline{z},\ \ thus$$

$$f(z(t)) = \frac{2}{e^{I\,t}}$$

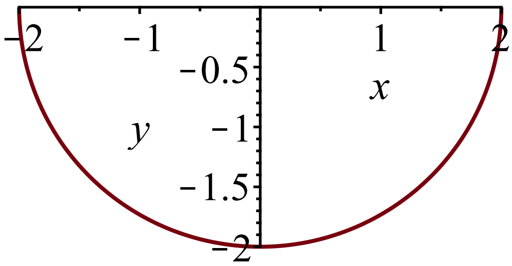
Therefore:

$$\int_C f(z)\, dz = \int_0^1 f(z(t))\, \left(\frac{d}{dt}\, z(t)\right) dt$$

$$\int_C f(z)\, dz = \int_0^1 4\, I\, dt$$

$$\int_C f(z)\, dz = I\left(\int_0^1 4\, dt\right)$$

$$\int_C f(z)\,\mathrm{d}z = -4\,\mathrm{I}\,\pi$$



Problem 1d

The line can be calculated as follows:

$$z(t) = x_0 + (x_1 - x_0)\,t + \mathrm{I}\,(y_0 + (y_1 - y_0)\,t)$$

where

$$z_0 = 0$$

$$x_0 = \Re(z_0),\, y_0 = \Im(z_0)$$

and

$$z_1 = 1 + \mathrm{I}$$

$$x_1 = \Re(z_1),\, y_1 = \Im(z_1)$$

Thus, the line parametric equation is:

$$C{:}z(t) = t + \mathrm{I}\,t$$

And the derivative is:

$$\frac{\mathrm{d}}{\mathrm{d}t}\,z(t) = 1 + \mathrm{I}$$

$$\textit{Because},\, f(z) = |z|^2,\ \textit{thus},\, f(z(t)) = |t + \mathrm{I}\,t|^2$$

$$\textit{Therefore}, \int_C f(z)\,\mathrm{d}z, \textit{ can be calculated as:}$$

$$\int_0^1 f(z(t))\left(\frac{\mathrm{d}}{\mathrm{d}t}\,z(t)\right)\mathrm{d}t = \frac{2}{3} + \frac{2}{3}\,\mathrm{I}$$