PROBLEM 9.9, NO. 12
Greenberg's Book

Problem 12 c
Given:

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Find the orthogonal basis!

Step 1:

$$v_1 = u_1$$

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Step 2:

$$v_2 = u_2 - \frac{(u_2 \cdot v_1)\, v_1}{norm(v_1)^2}$$

$$v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Step 3:

$$v_3 = u_3 - \frac{(u_3 \cdot v_1)\, v_1}{norm(v_1)^2} - \frac{(u_3 \cdot v_2)\, v_2}{norm(v_2)^2}$$

$$v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

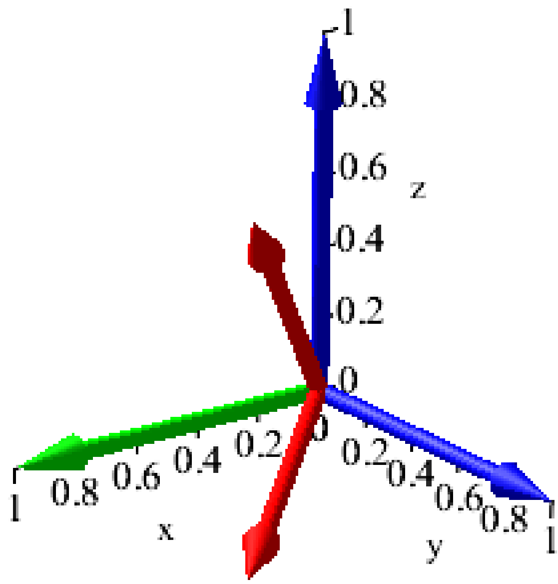
The new orthogonal basis is:

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

which can be normalized to:

$$\frac{v_1}{norm(v_1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \frac{v_2}{norm(v_2)} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \frac{v_3}{norm(v_3)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The plot is as follows (red is shows the original vectors, blue shows the orthogonal vectors, green is the common vector):



Problem 12 d

Given:

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

Find the orthogonal basis!

Step 1:

$$v_1 = u_1$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Step 2:

$$v_2 = u_2 - \frac{(u_2 \cdot v_1)\ v_1}{norm(v_1)^2}$$

$$v_2 = \begin{bmatrix} \frac{3}{2} \\ -\frac{3}{2} \\ 1 \end{bmatrix}$$

Step 3:

$$v_3 = u_3 - \frac{(u_3 \cdot v_1)\ v_1}{norm(v_1)^2} - \frac{(u_3 \cdot v_2)\ v_2}{norm(v_2)^2}$$

$$v_3 = \begin{bmatrix} -\frac{8}{11} \\ \frac{8}{11} \\ \frac{24}{11} \end{bmatrix}$$

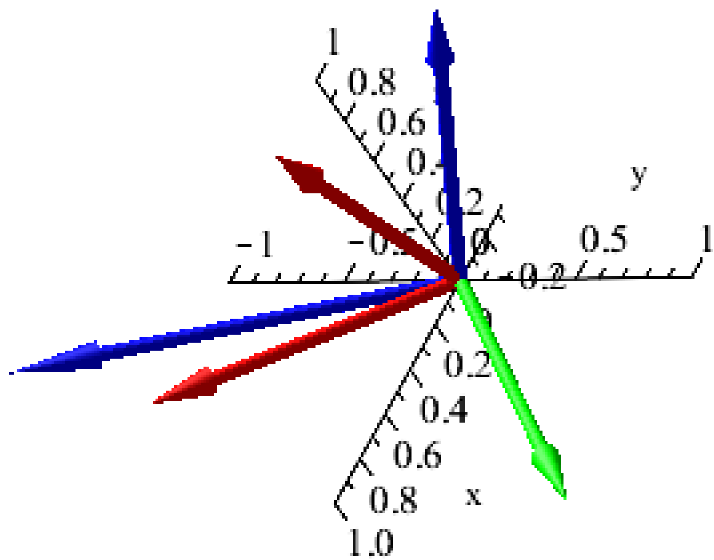
The new orthogonal basis is:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} \frac{3}{2} \\ -\frac{3}{2} \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} -\frac{8}{11} \\ \frac{8}{11} \\ \frac{24}{11} \end{bmatrix}$$

which can be normalized to:

$$\frac{v_1}{norm(v_1)} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{bmatrix}, \frac{v_2}{norm(v_2)} = \begin{bmatrix} \frac{3\sqrt{22}}{22} \\ -\frac{3\sqrt{22}}{22} \\ \frac{\sqrt{22}}{11} \end{bmatrix}, \frac{v_3}{norm(v_3)} = \begin{bmatrix} -\frac{\sqrt{11}}{11} \\ \frac{\sqrt{11}}{11} \\ \frac{3\sqrt{11}}{11} \end{bmatrix}$$

The plot is as follows (red is shows the original vectors, blue shows the orthogonal vectors, green is the common vector):



Problem 12 f

Given:

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Find the orthogonal basis!

Step 1:

$$v_1 = u_1$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 2:

$$v_2 = u_2 - \frac{(u_2 \cdot v_1)\ v_1}{norm(v_1)^2}$$

$$v_2 = \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

Step 3:

$$v_3 = u_3 - \frac{(u_3 \cdot v_1)\, v_1}{norm(v_1)^2} - \frac{(u_3 \cdot v_2)\, v_2}{norm(v_2)^2}$$

$$v_3 = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$

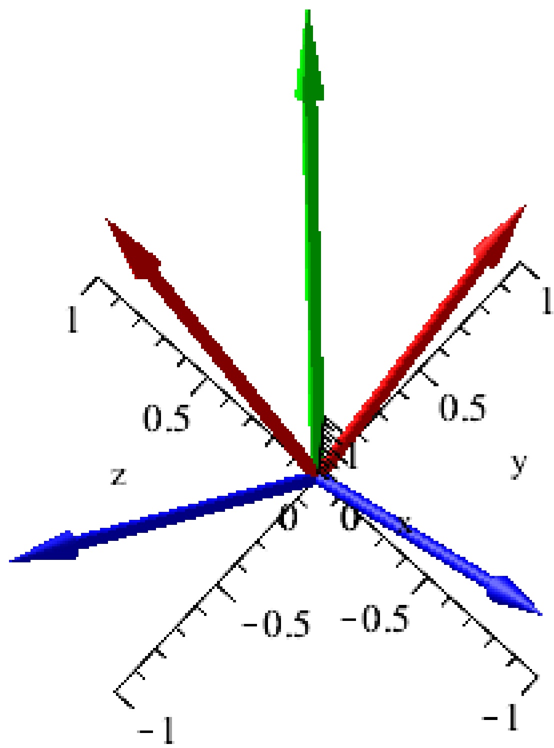
The new orthogonal basis is:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, v_3 = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$

which can be normalized to:

$$\frac{v_1}{norm(v_1)} = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}, \frac{v_2}{norm(v_2)} = \begin{bmatrix} \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \frac{v_3}{norm(v_3)} = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$

The plot is as follows (red is shows the original vectors, blue shows the orthogonal vectors, green is the common vector):



Problem 12 g

Given:

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, u_3 = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

Find the orthogonal basis!

Step 1:

$$v_1 = u_1$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Step 2:

$$v_2 = u_2 - \frac{(u_2 \cdot v_1)\, v_1}{norm(v_1)^2}$$

$$v_2 = \begin{bmatrix} \frac{5}{6} \\ -\frac{4}{3} \\ \frac{11}{6} \end{bmatrix}$$

Step 3:

$$v_3 = u_3 - \frac{(u_3 \cdot v_1)\, v_1}{norm(v_1)^2} - \frac{(u_3 \cdot v_2)\, v_2}{norm(v_2)^2}$$

$$v_3 = \begin{bmatrix} -\frac{11}{7} \\ \frac{11}{35} \\ \frac{33}{35} \end{bmatrix}$$

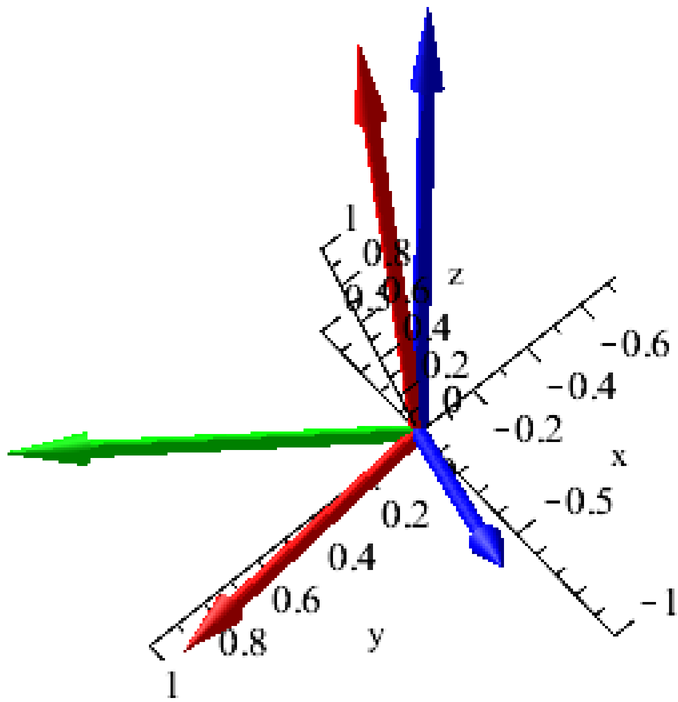
The new orthogonal basis is:

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} \frac{5}{6} \\ -\frac{4}{3} \\ \frac{11}{6} \end{bmatrix}, v_3 = \begin{bmatrix} -\frac{11}{7} \\ \frac{11}{35} \\ \frac{33}{35} \end{bmatrix}$$

which can be normalized to:

$$\frac{v_1}{norm(v_1)} = \begin{bmatrix} \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \frac{v_2}{norm(v_2)} = \begin{bmatrix} \frac{\sqrt{210}}{42} \\ -\frac{4\sqrt{210}}{105} \\ \frac{11\sqrt{210}}{210} \end{bmatrix}, \frac{v_3}{norm(v_3)} = \begin{bmatrix} -\frac{\sqrt{35}}{7} \\ \frac{\sqrt{35}}{35} \\ \frac{3\sqrt{35}}{35} \end{bmatrix}$$

The plot is as follows (red is shows the original vectors, blue shows the orthogonal vectors, green is the common vector):



Problem 12 h

Given:

$$u_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

Find the orthogonal basis!

Step 1:

$$v_1 = u_1$$

$$v_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

Step 2:

$$v_2 = u_2 - \frac{(u_2 \cdot v_1)\, v_1}{norm(v_1)^2}$$

$$v_2 = \begin{bmatrix} -\frac{1}{5} \\ 1 \\ \frac{2}{5} \end{bmatrix}$$

Step 3:

$$v_3 = u_3 - \frac{(u_3 \cdot v_1)\, v_1}{norm(v_1)^2} - \frac{(u_3 \cdot v_2)\, v_2}{norm(v_2)^2}$$

$$v_3 = \begin{bmatrix} -\frac{4}{3} \\ -\frac{4}{3} \\ \frac{8}{3} \end{bmatrix}$$

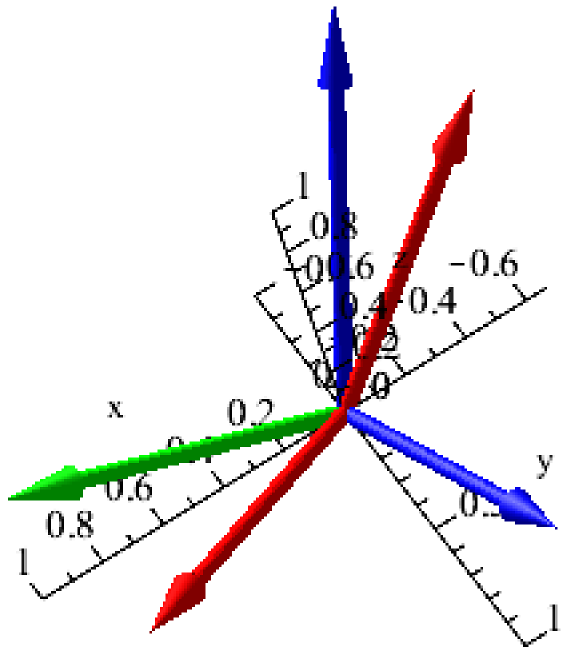
The new orthogonal basis is:

$$v_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -\frac{1}{5} \\ 1 \\ \frac{2}{5} \end{bmatrix}, v_3 = \begin{bmatrix} -\frac{4}{3} \\ -\frac{4}{3} \\ \frac{8}{3} \end{bmatrix}$$

which can be normalized to:

$$\frac{v_1}{norm(v_1)} = \begin{bmatrix} \frac{2\sqrt{5}}{5} \\ 0 \\ \frac{\sqrt{5}}{5} \end{bmatrix}, \frac{v_2}{norm(v_2)} = \begin{bmatrix} -\frac{\sqrt{30}}{30} \\ \frac{\sqrt{30}}{6} \\ \frac{\sqrt{30}}{15} \end{bmatrix}, \frac{v_3}{norm(v_3)} = \begin{bmatrix} -\frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{3} \end{bmatrix}$$

The plot is as follows (red is shows the original vectors, blue shows the orthogonal vectors, green is the common vector):


x
y
z
1
0.8
0.6
0.4
0.2
0
-0.6
-0.4
0.2
0.6
0.8
1
1