PROBLEM 9.10, NO. 2, 3, 4
Greenberg's Book

Problem 2

Problem 2a:
Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 5 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, \; c_1 = \frac{3\sqrt{5}}{5}$$

$$c_2 = u \cdot \hat{e}_2, \; c_2 = \frac{11\sqrt{6}}{6}$$

$$c_3 = u \cdot \hat{e}_3, \; c_3 = 0$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} -\frac{19}{15} \\ -2 \\ \frac{19}{30} \\ 0 \\ \frac{19}{6} \end{bmatrix}$$

$$norm(E) = \frac{\sqrt{14430}}{30}$$

Problem 2b:
Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = 0$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \frac{\sqrt{6}}{6}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 2$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} -\frac{1}{3} \\ 0 \\ \frac{1}{6} \\ 0 \\ \frac{5}{6} \end{bmatrix}$$

$$norm(E) = \frac{\sqrt{30}}{6}$$

Problem 2c:
Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 4 \\ 1 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = \sqrt{5}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \sqrt{6}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 4$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E) = 0$$

Problem 2d:
Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = \frac{\sqrt{5}}{5}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \frac{\sqrt{6}}{2}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 1$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} -\frac{1}{5} \\ 1 \\ \frac{1}{10} \\ 0 \\ \frac{1}{2} \end{bmatrix}$$

$$norm(E) = \frac{\sqrt{130}}{10}$$

Problem 2e:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = 0$$

$$c_2 = u \cdot \hat{e}_2, c_2 = 0$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 0$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E) = 2$$

Problem 2f:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 3 \\ 1 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = -\sqrt{5}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \sqrt{6}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 3$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E) = 0$$

Problem 2g:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 0 \\ 7 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

We define the best approximation as:

$$\underset{\sim}{u} = c_1\,\hat{e}_1 + c_2\,\hat{e}_2 + c_3\,\hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1,\ c_1 = 0$$

$$c_2 = u \cdot \hat{e}_2,\ c_2 = 0$$

$$c_3 = u \cdot \hat{e}_3,\ c_3 = 3$$

The error is:

$$E = u - \underset{\sim}{u}$$

$$E = \begin{bmatrix} 0 \\ 7 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E) = 7$$

Problem 2h:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = \frac{7\sqrt{5}}{5}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \frac{2\sqrt{6}}{3}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 4$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} -\frac{26}{15} \\ 2 \\ \frac{13}{15} \\ 0 \\ \frac{13}{3} \end{bmatrix}$$

$$norm(E) = \frac{\sqrt{5970}}{15}$$

Problem 2i:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ 0 \\ \frac{2\sqrt{5}}{5} \\ 0 \\ 0 \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ 0 \\ -\frac{\sqrt{6}}{6} \\ 0 \\ \frac{\sqrt{6}}{6} \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Find best approximation for:

$$u = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

We define the best approximation as:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = \frac{11\sqrt{5}}{5}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \frac{4\sqrt{6}}{3}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 2$$

The error is:

$$E = u - \tilde{u}$$

$$E = \begin{bmatrix} \frac{2}{15} \\ 4 \\ -\frac{1}{15} \\ 0 \\ -\frac{1}{3} \end{bmatrix}$$

$$norm(E) = \frac{11\sqrt{30}}{15}$$

Problem 3 & 4

Problem 3:
Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 4 \\ -2 \\ 1 \\ 6 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1 \hat{e}_1$$

$$\tilde{u}_2 = c_1 \hat{e}_1 + c_2 \hat{e}_2$$

$$\tilde{u}_3 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

$$\tilde{u}_4 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = -\frac{4\sqrt{3}}{3}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \frac{5\sqrt{3}}{3}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = \frac{11\sqrt{3}}{3}$$

$$c_4 = u \cdot \hat{e}_4, c_4 = \sqrt{3}$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$

$$E_2 = u - \tilde{u}_2$$

$$E_3 = u - \tilde{u}_3$$

$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} \frac{16}{3} \\ -\frac{2}{3} \\ 1 \\ \frac{14}{3} \end{bmatrix}, E_2 = \begin{bmatrix} \frac{11}{3} \\ 1 \\ \frac{8}{3} \\ \frac{14}{3} \end{bmatrix}, E_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \frac{\sqrt{465}}{3}, norm(E_2) = \frac{\sqrt{390}}{3}, norm(E_3) = \sqrt{3}, norm(E_4) = 0$$

Problem 4a:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 4 \\ 1 \\ 0 \\ -1 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1 \hat{e}_1$$

$$\tilde{u}_2 = c_1 \hat{e}_1 + c_2 \hat{e}_2$$

$$\tilde{u}_3 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

$$\tilde{u}_4 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = 2\sqrt{3}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \sqrt{3}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = \sqrt{3}$$

$$c_4 = u \cdot \hat{e}_4, c_4 = 0$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$
$$E_2 = u - \tilde{u}_2$$
$$E_3 = u - \tilde{u}_3$$
$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, E_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \sqrt{6}, norm(E_2) = \sqrt{3}, norm(E_3) = 0, norm(E_4) = 0$$

Problem 4b:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 2 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1 \hat{e}_1$$
$$\tilde{u}_2 = c_1 \hat{e}_1 + c_2 \hat{e}_2$$
$$\tilde{u}_3 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$
$$\tilde{u}_4 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = 0$$
$$c_2 = u \cdot \hat{e}_2, c_2 = \sqrt{3}$$
$$c_3 = u \cdot \hat{e}_3, c_3 = 2\sqrt{3}$$

$$c_4 = u \cdot \hat{e}_4,\ c_4 = 0$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$

$$E_2 = u - \tilde{u}_2$$

$$E_3 = u - \tilde{u}_3$$

$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 2 \end{bmatrix},\ E_2 = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 2 \end{bmatrix},\ E_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\ E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \sqrt{15},\ norm(E_2) = 2\sqrt{3},\ norm(E_3) = 0,\ norm(E_4) = 0$$

Problem 4c:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix},\ \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix},\ \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix},\ \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 5 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1 \hat{e}_1$$

$$\tilde{u}_2 = c_1 \hat{e}_1 + c_2 \hat{e}_2$$

$$\tilde{u}_3 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

$$\tilde{u}_4 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1,\ c_1 = -\frac{5\sqrt{3}}{3}$$

$$c_2 = u \cdot \hat{e}_2,\ c_2 = -\frac{2\sqrt{3}}{3}$$

$$c_3 = u \cdot \hat{e}_3,\ c_3 = \frac{7\sqrt{3}}{3}$$

$$c_4 = u \cdot \hat{e}_4,\ c_4 = \sqrt{3}$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$

$$E_2 = u - \tilde{u}_2$$

$$E_3 = u - \tilde{u}_3$$

$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} \frac{5}{3} \\ \frac{5}{3} \\ 2 \\ \frac{10}{3} \end{bmatrix}, E_2 = \begin{bmatrix} \frac{7}{3} \\ 1 \\ \frac{4}{3} \\ \frac{10}{3} \end{bmatrix}, E_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \frac{\sqrt{186}}{3}, norm(E_2) = \frac{\sqrt{174}}{3}, norm(E_3) = \sqrt{3}, norm(E_4) = 0$$

Problem 4d:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 4 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1\,\hat{e}_1$$
$$\tilde{u}_2 = c_1\,\hat{e}_1 + c_2\,\hat{e}_2$$
$$\tilde{u}_3 = c_1\,\hat{e}_1 + c_2\,\hat{e}_2 + c_3\,\hat{e}_3$$
$$\tilde{u}_4 = c_1\,\hat{e}_1 + c_2\,\hat{e}_2 + c_3\,\hat{e}_3 + c_4\,\hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1,\ c_1 = -\frac{\sqrt{3}}{3}$$
$$c_2 = u \cdot \hat{e}_2,\ c_2 = -\frac{5\sqrt{3}}{3}$$
$$c_3 = u \cdot \hat{e}_3,\ c_3 = 3\sqrt{3}$$
$$c_4 = u \cdot \hat{e}_4,\ c_4 = \frac{2\sqrt{3}}{3}$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$
$$E_2 = u - \tilde{u}_2$$
$$E_3 = u - \tilde{u}_3$$
$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} \frac{4}{3} \\ \frac{7}{3} \\ 4 \\ \frac{11}{3} \end{bmatrix},\ E_2 = \begin{bmatrix} 3 \\ \frac{2}{3} \\ \frac{7}{3} \\ \frac{11}{3} \end{bmatrix},\ E_3 = \begin{bmatrix} 0 \\ \frac{2}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{bmatrix},\ E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \frac{\sqrt{330}}{3},\ norm(E_2) = \frac{\sqrt{255}}{3},\ norm(E_3) = \frac{2\sqrt{3}}{3},\ norm(E_4) = 0$$

Problem 4e:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix},\ \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix},\ \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix},\ \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 0 \\ 5 \\ 3 \\ -1 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1 \hat{e}_1$$

$$\tilde{u}_2 = c_1 \hat{e}_1 + c_2 \hat{e}_2$$

$$\tilde{u}_3 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

$$\tilde{u}_4 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = 2\sqrt{3}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = -\frac{8\sqrt{3}}{3}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = \frac{2\sqrt{3}}{3}$$

$$c_4 = u \cdot \hat{e}_4, c_4 = \frac{\sqrt{3}}{3}$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$

$$E_2 = u - \tilde{u}_2$$

$$E_3 = u - \tilde{u}_3$$

$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} -2 \\ 3 \\ 3 \\ 1 \end{bmatrix}, E_2 = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ 1 \end{bmatrix}, E_3 = \begin{bmatrix} 0 \\ \frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix}, E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \sqrt{23}, norm(E_2) = \frac{\sqrt{15}}{3}, norm(E_3) = \frac{\sqrt{3}}{3}, norm(E_4) = 0$$

Problem 4f:

Given:

$$\hat{e}_1 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ 0 \\ -\frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_2 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ 0 \end{bmatrix}, \hat{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ 0 \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}, \hat{e}_4 = \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$$

Find the best approximation for:

$$u = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -1 \end{bmatrix}$$

The best approximation is given by:

$$\tilde{u} = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

However, we are asked to do approximation one-by-one:

$$\tilde{u}_1 = c_1 \hat{e}_1$$

$$\tilde{u}_2 = c_1 \hat{e}_1 + c_2 \hat{e}_2$$

$$\tilde{u}_3 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3$$

$$\tilde{u}_4 = c_1 \hat{e}_1 + c_2 \hat{e}_2 + c_3 \hat{e}_3 + c_4 \hat{e}_4$$

where:

$$c_1 = u \cdot \hat{e}_1, c_1 = \sqrt{3}$$

$$c_2 = u \cdot \hat{e}_2, c_2 = \sqrt{3}$$

$$c_3 = u \cdot \hat{e}_3, c_3 = 0$$

$$c_4 = u \cdot \hat{e}_4, c_4 = 0$$

The error is also calculated one-by-one:

$$E_1 = u - \tilde{u}_1$$

$$E_2 = u - \tilde{u}_2$$

$$E_3 = u - \tilde{u}_3$$

$$E_4 = u - \tilde{u}_4$$

$$E_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, E_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, E_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$norm(E_1) = \sqrt{3}, norm(E_2) = 0, norm(E_3) = 0, norm(E_4) = 0$$