```
> restart:
> CauchyRiemann:=proc(expr::algebraic)
    local u, v, u_x, u_y, v_x, v_y, flag1, flag2;

    u:=evalc(Re(eval(expr, z=x+I*y)));
    v:=evalc(Im(eval(expr, z=x+I*y)));

    u_x:=simplify(diff(u,x));
    u_y:=simplify(diff(u,y));
    v_x:=simplify(diff(v,x));
    v_y:=simplify(diff(v,y));

    print('f(z)'=expr);
    printf("\n");

    print('u(x,y)'=u);
    print('u[x](x,y)'=u_x);
    print('u[y](x,y)'=u_y);
    printf("\n");

    print('v(x,y)'=v);
    print('v[x](x,y)'=v_x);
    print('v[y](x,y)'=v_y);
    printf("\n");

    if u_x=v_y then
      print('u[x]=v[y]');
      print(u_x=v_y);
      flag1:=true;
    else
      print('u[x]<>v[y]');
      print(u_x<>v_y);
      flag1:=false;
    end if;

    if u_y=-v_x then
      print('u[y]=-v[x]');
      print(u_y=-v_x);
      flag2:=true;
    else
      print('u[y]<>-v[x]');
      print(u_y<>-v_x);
      flag2:=false;
    end if;

  printf("\n");
  if flag1=true and flag2=true then
     print(`f(z) meets the Cauchy-Riemann equations at every
  point`);
     print(`The derivative is:`='u[x]+I*v[x]');
     print('diff(f(z),z)'=u_x+I*v_x);
  else
     print(`f(z) DOES NOT meet the Cauchy-Riemann equations at
  every point`);
  end if
```

```
end proc:
```

```
> f(z):=4*exp(7*z)-I*z^2:
CauchyRiemann(f(z))
```

$$f(z) = 4\,e^{7z} - I\,z^2$$

$$u(x, y) = 4\,e^{7x}\cos(7\,y) + 2\,x\,y$$
$$u_x(x, y) = 28\,e^{7x}\cos(7\,y) + 2\,y$$
$$u_y(x, y) = -28\,e^{7x}\sin(7\,y) + 2\,x$$

$$v(x, y) = 4\,e^{7x}\sin(7\,y) - x^2 + y^2$$
$$v_x(x, y) = 28\,e^{7x}\sin(7\,y) - 2\,x$$
$$v_y(x, y) = 28\,e^{7x}\cos(7\,y) + 2\,y$$

$$u_x = v_y$$
$$28\,e^{7x}\cos(7\,y) + 2\,y = 28\,e^{7x}\cos(7\,y) + 2\,y$$
$$u_y = -v_x$$
$$-28\,e^{7x}\sin(7\,y) + 2\,x = -28\,e^{7x}\sin(7\,y) + 2\,x$$

f(z) meets the Cauchy-Riemann equations at every point

The derivative is: $= u_x + I\,v_x$

$$\frac{d}{dz}\,f(z) = 28\,e^{7x}\cos(7\,y) + 2\,y + I\left(28\,e^{7x}\sin(7\,y) - 2\,x\right) \qquad \textbf{(1)}$$

```
> f(z):=1/(z+2):
CauchyRiemann(f(z))
```

$$f(z) = \frac{1}{z+2}$$

$$u(x, y) = \frac{x+2}{(x+2)^2 + y^2}$$
$$u_x(x, y) = \frac{-x^2 + y^2 - 4\,x - 4}{\left(x^2 + y^2 + 4\,x + 4\right)^2}$$
$$u_y(x, y) = -\frac{2\,(x+2)\,y}{\left(x^2 + y^2 + 4\,x + 4\right)^2}$$

$$v(x, y) = -\frac{y}{(x+2)^2 + y^2}$$

$$v_x(x, y) = \frac{2\,(x+2)\,y}{\left(x^2 + y^2 + 4\,x + 4\right)^2}$$

$$v_y(x, y) = \frac{-x^2 + y^2 - 4\,x - 4}{\left(x^2 + y^2 + 4\,x + 4\right)^2}$$

$$u_x = v_y$$

$$\frac{-x^2 + y^2 - 4\,x - 4}{\left(x^2 + y^2 + 4\,x + 4\right)^2} = \frac{-x^2 + y^2 - 4\,x - 4}{\left(x^2 + y^2 + 4\,x + 4\right)^2}$$

$$u_y = -v_x$$

$$-\frac{2\,(x+2)\,y}{\left(x^2 + y^2 + 4\,x + 4\right)^2} = -\frac{2\,(x+2)\,y}{\left(x^2 + y^2 + 4\,x + 4\right)^2}$$

f(z) meets the Cauchy-Riemann equations at every point

The derivative is: $= u_x + \mathrm{I}\,v_x$

$$\frac{d}{dz}\,f(z) = \frac{-x^2 + y^2 - 4\,x - 4}{\left(x^2 + y^2 + 4\,x + 4\right)^2} + \frac{2\,\mathrm{I}\,(x+2)\,y}{\left(x^2 + y^2 + 4\,x + 4\right)^2} \tag{2}$$

```
> f(z):=1/(z):
  CauchyRiemann(f(z))
```

$$f(z) = \frac{1}{z}$$

$$u(x, y) = \frac{x}{x^2 + y^2}$$

$$u_x(x, y) = \frac{-x^2 + y^2}{\left(x^2 + y^2\right)^2}$$

$$u_y(x, y) = -\frac{2\,x\,y}{\left(x^2 + y^2\right)^2}$$

$$v(x, y) = -\frac{y}{x^2 + y^2}$$

$$v_x(x, y) = \frac{2\,x\,y}{\left(x^2 + y^2\right)^2}$$

$$v_y(x, y) = \frac{-x^2 + y^2}{(x^2 + y^2)^2}$$

$$u_x = v_y$$

$$\frac{-x^2 + y^2}{(x^2 + y^2)^2} = \frac{-x^2 + y^2}{(x^2 + y^2)^2}$$

$$u_y = -v_x$$

$$-\frac{2\,x\,y}{(x^2 + y^2)^2} = -\frac{2\,x\,y}{(x^2 + y^2)^2}$$

f(z) meets the Cauchy-Riemann equations at every point

The derivative is: $= u_x + \mathrm{I}\, v_x$

$$\frac{\mathrm{d}}{\mathrm{d}z} f(z) = \frac{-x^2 + y^2}{(x^2 + y^2)^2} + \frac{2\,\mathrm{I}\,x\,y}{(x^2 + y^2)^2} \qquad \textbf{(3)}$$

```
> f(z):=z^2:
  CauchyRiemann(f(z))
```

$$f(z) = z^2$$

$$u(x, y) = x^2 - y^2$$

$$u_x(x, y) = 2\,x$$

$$u_y(x, y) = -2\,y$$

$$v(x, y) = 2\,x\,y$$

$$v_x(x, y) = 2\,y$$

$$v_y(x, y) = 2\,x$$

$$u_x = v_y$$

$$2\,x = 2\,x$$

$$u_y = -v_x$$

$$-2\,y = -2\,y$$

f(z) meets the Cauchy-Riemann equations at every point

$$\textit{The derivative is:} = u_x + \mathrm{I}\, v_x$$

$$\frac{d}{dz} f(z) = 2x + 2\,\mathrm{I}\, y \tag{4}$$

```
> f(z):=z^2-i*z-z:
CauchyRiemann(f(z))
```

$$f(z) = -i z + z^2 - z$$

$$u(x, y) = -i x + x^2 - y^2 - x$$

$$u_x(x, y) = -i + 2x - 1$$

$$u_y(x, y) = -2y$$

$$v(x, y) = -i y + 2xy - y$$

$$v_x(x, y) = 2y$$

$$v_y(x, y) = -i + 2x - 1$$

$$u_x = v_y$$

$$-i + 2x - 1 = -i + 2x - 1$$

$$u_y = -v_x$$

$$-2y = -2y$$

f(z) meets the Cauchy-Riemann equations at every point

$$\textit{The derivative is:} = u_x + \mathrm{I}\, v_x$$

$$\frac{d}{dz} f(z) = -i + 2x - 1 + 2\,\mathrm{I}\, y \tag{5}$$

```
> f(z):=exp(conjugate(z)):
CauchyRiemann(f(z))
```

$$f(z) = e^{\overline{z}}$$

$$u(x, y) = e^x \cos(y)$$

$$u_x(x, y) = e^x \cos(y)$$

$$u_y(x, y) = -e^x \sin(y)$$

$$v(x, y) = -e^x \sin(y)$$

$$v_x(x, y) = -e^x \sin(y)$$

$$v_y(x, y) = -e^x \cos(y)$$

$$u_x \neq v_y$$

$$e^x \cos(y) \neq -e^x \cos(y)$$

$$u_y \neq -v_x$$

$$-e^x \sin(y) \neq e^x \sin(y)$$

f(z) DOES NOT meet the Cauchy-Riemann equations at every point **(6)**

```
> f(z):=x*y^2*(x+I*y) / (x^2+y^4):
  CauchyRiemann(f(z))
```

$$f(z) = \frac{x y^2 (x + I y)}{y^4 + x^2}$$

$$u(x, y) = \frac{y^2 x^2}{y^4 + x^2}$$

$$u_x(x, y) = \frac{2 y^6 x}{\left(y^4 + x^2\right)^2}$$

$$u_y(x, y) = \frac{-2 y^5 x^2 + 2 x^4 y}{\left(y^4 + x^2\right)^2}$$

$$v(x, y) = \frac{y^3 x}{y^4 + x^2}$$

$$v_x(x, y) = \frac{y^7 - y^3 x^2}{\left(y^4 + x^2\right)^2}$$

$$v_y(x, y) = \frac{y^2 x \left(-y^4 + 3 x^2\right)}{\left(y^4 + x^2\right)^2}$$

$$u_x \neq v_y$$

$$\frac{2 y^6 x}{\left(y^4 + x^2\right)^2} \neq \frac{y^2 x \left(-y^4 + 3 x^2\right)}{\left(y^4 + x^2\right)^2}$$

$$u_y \neq -v_x$$

$$\frac{-2\,y^5\,x^2+2\,x^4\,y}{\left(y^4+x^2\right)^2} \neq -\frac{y^7-y^3\,x^2}{\left(y^4+x^2\right)^2}$$

f(z) DOES NOT meet the Cauchy-Riemann equations at every point **(7)**

```
> f(z):=x^3+I*y^3:
  CauchyRiemann(f(z))
```

$$f(z)=x^3+\mathrm{I}\,y^3$$

$$u(x,y)=x^3$$
$$u_x(x,y)=3\,x^2$$
$$u_y(x,y)=0$$

$$v(x,y)=y^3$$
$$v_x(x,y)=0$$
$$v_y(x,y)=3\,y^2$$

$$u_x \neq v_y$$
$$3\,x^2 \neq 3\,y^2$$
$$u_y=-v_x$$
$$0=0$$

f(z) DOES NOT meet the Cauchy-Riemann equations at every point **(8)**