Di kiri dengan Neumann dan di kanan dengan Dirichlet

Problem 6(m)

$u_{[x]}(0, t) = 2, u_{[x]}(6, t) = 12, f(x) = 0$

$$u_x(0, t) = 2$$

$$u(L, t) = 12$$

Kondisi awal:

$$u(x, 0) = 0$$

$$dimana, L = 6$$

$$dan, \alpha = 0.05830951895$$

Solusi umum:

$$u = (J\cos(\kappa x) + K\sin(\kappa x))\, e^{-\kappa^2 \alpha^2 t} + H + I x$$

Masukkan batas pertama:

$$u_x = K\kappa\, e^{-\kappa^2 \alpha^2 t} + I$$

$$u_x(L, t) = 2$$

$$K\kappa\, e^{-\kappa^2 \alpha^2 t} + I = 2$$

Sehingga

$$I = 2$$

$$K = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = J\cos(\kappa x)\, e^{-\kappa^2 \alpha^2 t} + H + 2x$$

Masukkan batas kedua:

$$u = J\cos(\kappa L)\, e^{-\kappa^2 \alpha^2 t} + H + 2L$$

$$u(L, t) = 12$$

$$J\cos(\kappa L)\, e^{-\kappa^2 \alpha^2 t} + H + 2L = 12$$

Sehingga

$$H = -2L + 12$$

$$\kappa = \frac{n\pi}{2L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u = \left(\sum_n J_n \cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha^2 t}{4L^2}}\right) - 2L + 12 + 2x$$

Masukkan initial condition:

$$u(x, 0) = 0$$

$$\sum_{n} J_n \cos\left(\frac{n\,\pi\,x}{2\,L}\right) = -2\,x + 2\,L - 12$$

$$F(x) = -2\,x + 2\,L - 12$$

$$F(x) = -2\,x$$

Hitung, J_n

$$J_n = \frac{2\left(\int_0^L -2\,x\cos\left(\frac{n\,\pi\,x}{2\,L}\right)\mathrm{d}x\right)}{L}$$

$$J_n = -\frac{48\left(\pi\,n\sin\left(\frac{\pi\,n}{2}\right) + 2\cos\left(\frac{\pi\,n}{2}\right) - 2\right)}{\pi^2\,n^2}$$

$$J_n = \frac{-48\,\pi\,n\sin\left(\frac{\pi\,n}{2}\right) - 96\cos\left(\frac{\pi\,n}{2}\right) + 96}{\pi^2\,n^2}$$

$$J_n = \frac{-48\,\pi\,n\sin\left(\frac{\pi\,n}{2}\right) + 96}{\pi^2\,n^2}$$

Jadi, solusi khususnya adalah:

$$u = \left(\sum_{n} \frac{\left(-48\,\pi\,n\sin\left(\frac{\pi\,n}{2}\right) + 96\right)\cos\left(\frac{n\,\pi\,x}{12}\right)\mathrm{e}^{-\frac{n^2\,\pi^2\,\alpha^2\,t}{144}}}{\pi^2\,n^2}\right) + 2\,x$$

di mana n adalah ganjil

Untuk keperluan simulasi, diubah menjadi:

$$u = \left(\sum_{n=1}^{1000} \frac{1}{\pi^2\,(2\,n-1)^2}\left(\left(-48\,\pi\,(2\,n-1)\sin\left(\frac{\pi\,(2\,n-1)}{2}\right) + 96\right)\cos\left(\frac{(2\,n-1)\,\pi\,x}{12}\right)\mathrm{e}^{-0.0002330323262\,(2\,n-1)^2\,t}\right)\right) - 2\,L + 12 + 2\,x$$

di mana n=1,2,3,...

Problem 6(n)

$u_{[x]}(0, t) = 0,\ u_{[x]}(6, t) = 0, f(x) = \sin(x)$

$$u_x(0, t) = 0$$

$$u(L, t) = 0$$

Kondisi awal:

$$u(x, 0) = \sin(x)$$

$$dimana,\ L = 6$$
$$dan,\ \alpha = 0.05830951895$$

Solusi umum:

$$u = (J\cos(\kappa x) + K\sin(\kappa x))\,e^{-\kappa^2\alpha^2 t} + H + I x$$

Masukkan batas pertama:

$$u_x = K\kappa\,e^{-\kappa^2\alpha^2 t} + I$$
$$u_x(L, t) = 0$$
$$K\kappa\,e^{-\kappa^2\alpha^2 t} + I = 0$$

Sehingga

$$I = 0$$
$$K = 0$$

Pada tahap ini, solusinya sudah menjadi:

$$u = J\cos(\kappa x)\,e^{-\kappa^2\alpha^2 t} + H$$

Masukkan batas kedua:

$$u = J\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t} + H$$
$$u(L, t) = 0$$
$$J\cos(\kappa L)\,e^{-\kappa^2\alpha^2 t} + H = 0$$

Sehingga

$$H = 0$$
$$\kappa = \frac{n\pi}{2L}$$

di mana n=1,3,...

Pada tahap ini, solusinya menjadi:

$$u = \sum_n J_n \cos\left(\frac{n\pi x}{2L}\right) e^{-\frac{n^2\pi^2\alpha^2 t}{4L^2}}$$

Masukkan initial condition:

$$u(x, 0) = \sin(x)$$
$$\sum_n J_n \cos\left(\frac{n\pi x}{2L}\right) = \sin(x)$$
$$F(x) = \sin(x)$$
$$F(x) = \sin(x)$$

$$Hitung,\ J_n$$

$$J_n = \frac{2\left(\int_0^L \sin(x)\cos\left(\frac{n\pi x}{2L}\right)\,\mathrm{d}x\right)}{L}$$

$$J_n = \frac{4\left(\pi n \sin(6)\sin\left(\frac{\pi n}{2}\right) + 12\cos(6)\cos\left(\frac{\pi n}{2}\right) - 12\right)}{\pi^2 n^2 - 144}$$

$$J_n = \frac{4\pi n \sin(6)\sin\left(\frac{\pi n}{2}\right) + 48\cos(6)\cos\left(\frac{\pi n}{2}\right) - 48}{\pi^2 n^2 - 144}$$

$$J_n = \frac{4\pi n \sin(6)\sin\left(\frac{\pi n}{2}\right) - 48}{\pi^2 n^2 - 144}$$

Jadi, solusi khususnya adalah:

$$u = \sum_n \frac{\left(4\pi n \sin(6)\sin\left(\frac{\pi n}{2}\right) - 48\right)\cos\left(\frac{n\pi x}{12}\right) e^{-\frac{n^2\pi^2\alpha^2 t}{144}}}{\pi^2 n^2 - 144}$$

di mana n adalah ganjil

Untuk keperluan simulasi, diubah menjadi:

$$u = \sum_{n=1}^{1000} \frac{1}{\pi^2 (2n-1)^2 - 144}\left(\left(4\pi(2n-1)\sin(6)\sin\left(\frac{\pi(2n-1)}{2}\right) - 48\right)\cos\left(\frac{(2n-1)\pi x}{12}\right) e^{-0.0002330323262\,(2n-1)^2 t}\right)$$

di mana n=1,2,3,...