Problem 8.3 N0. 1
Greenberg's Book

Problem 1a
The matrix:

$$\begin{bmatrix} 2 & -3 & 1 \\ 5 & 1 & 2 \end{bmatrix}$$

Set row 2 to 2.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & -3 & 1 \\ 0 & \dfrac{17}{2} & -\dfrac{1}{2} \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & -\dfrac{3}{2} & \dfrac{1}{2} \\ 0 & \dfrac{17}{2} & -\dfrac{1}{2} \end{bmatrix}$$

Divide row 2 by 8.50

$$\begin{bmatrix} 1 & -\dfrac{3}{2} & \dfrac{1}{2} \\ 0 & 1 & -\dfrac{1}{17} \end{bmatrix}$$

-->Unique/single solution

Problem 1b
The matrix:

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Set row 2 to 1.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & -\dfrac{7}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 \\ 0 & -\dfrac{7}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by -3.50

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

Problem 1c
The matrix:

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

Problem 1d
The matrix:

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -1 & -1 & 8 \end{bmatrix}$$

Set row 2 to 2.00 times row 1 minus row 2

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -3 & 6 \end{bmatrix}$$

-->Unique/single solution

Problem 1f
The matrix:

$$\begin{bmatrix} 2 & -1 & -1 & 3 & 0 \\ 1 & -1 & 0 & 4 & 2 \end{bmatrix}$$

Set row 2 to 0.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & -1 & -1 & 3 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} & 2 \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} & 2 \end{bmatrix}$$

Divide row 2 by -0.50

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} & \frac{3}{2} & 0 \\ 0 & 1 & -1 & -5 & -4 \end{bmatrix}$$

-->Unique/single solution

Problem 1g
The matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

Set row 2 to 5.00 times row 1 minus row 2

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

Set row 3 to 9.00 times row 1 minus row 3

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \end{bmatrix}$$

Set row 3 to 2.00 times row 2 minus row 3

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by -4.00

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

Problem 1h
The matrix:

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 1 & -1 & -3 & 1 \\ 1 & -3 & -4 & -1 \end{bmatrix}$$

Set row 2 to 1.00 times row 1 minus row 2

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -2 & -1 & -2 \\ 1 & -3 & -4 & -1 \end{bmatrix}$$

Set row 3 to 1.00 times row 1 minus row 3

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -2 & -1 & -2 \\ 0 & -4 & -2 & -4 \end{bmatrix}$$

Set row 3 to 2.00 times row 2 minus row 3

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & -2 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by -2.00

$$\begin{bmatrix} 1 & 1 & -2 & 3 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

Problem 1i
The matrix:

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 6 \\ 3 & 2 & 0 & 0 & 4 \\ 1 & 10 & 0 & 0 & -12 \\ 6 & 11 & 0 & 0 & -2 \end{bmatrix}$$

Set row 2 to 1.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 6 \\ 0 & \dfrac{7}{2} & 0 & 0 & -5 \\ 1 & 10 & 0 & 0 & -12 \\ 6 & 11 & 0 & 0 & -2 \end{bmatrix}$$

Set row 3 to 0.50 times row 1 minus row 3

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 6 \\ 0 & \dfrac{7}{2} & 0 & 0 & -5 \\ 0 & \dfrac{21}{2} & 0 & 0 & -15 \\ 6 & 11 & 0 & 0 & -2 \end{bmatrix}$$

Set row 4 to 3.00 times row 1 minus row 4

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 6 \\ 0 & \dfrac{7}{2} & 0 & 0 & -5 \\ 0 & \dfrac{21}{2} & 0 & 0 & -15 \\ 0 & 14 & 0 & 0 & -20 \end{bmatrix}$$

Set row 3 to 3.00 times row 2 minus row 3

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 6 \\ 0 & \dfrac{7}{2} & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 14 & 0 & 0 & -20 \end{bmatrix}$$

Set row 4 to 4.00 times row 2 minus row 4

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 6 \\ 0 & \dfrac{7}{2} & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & -\dfrac{1}{2} & 0 & 0 & 3 \\ 0 & \dfrac{7}{2} & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by 3.50

$$\begin{bmatrix} 1 & -\dfrac{1}{2} & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -\dfrac{10}{7} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

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Problem 1j
The matrix:

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
2 & 1 & 1 & -1 & 4 \\
1 & 2 & -1 & -2 & 5 \\
1 & 0 & 1 & 0 & 1
\end{bmatrix}
$$

Set row 2 to 2.00 times row 1 minus row 2

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 3 & -3 & -9 & 6 \\
1 & 2 & -1 & -2 & 5 \\
1 & 0 & 1 & 0 & 1
\end{bmatrix}
$$

Set row 3 to 1.00 times row 1 minus row 3

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 3 & -3 & -9 & 6 \\
0 & 3 & -3 & -6 & 6 \\
1 & 0 & 1 & 0 & 1
\end{bmatrix}
$$

Set row 4 to 1.00 times row 1 minus row 4

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 3 & -3 & -9 & 6 \\
0 & 3 & -3 & -6 & 6 \\
0 & 1 & -1 & -4 & 2
\end{bmatrix}
$$

Set row 3 to 1.00 times row 2 minus row 3

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 3 & -3 & -9 & 6 \\
0 & 0 & 0 & 3 & 0 \\
0 & 1 & -1 & -4 & 2
\end{bmatrix}
$$

Set row 4 to 0.33 times row 2 minus row 4

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 3 & -3 & -9 & 6 \\
0 & 0 & 0 & 3 & 0 \\
0 & 0 & 0 & -1 & 0
\end{bmatrix}
$$

Swap row 3 and 4

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 3 & -3 & -9 & 6 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 3 & 0
\end{bmatrix}
$$

Divide row 2 by 3.00

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 1 & -1 & -3 & 2 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 3 & 0
\end{bmatrix}
$$

Divide row 3 by -1.00

$$
\begin{bmatrix}
1 & -1 & 2 & 4 & -1 \\
0 & 1 & -1 & -3 & 2 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 3 & 0
\end{bmatrix}
$$

Divide row 4 by 3.00

$$\begin{bmatrix} 1 & -1 & 2 & 4 & -1 \\ 0 & 1 & -1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

-->Unique/single solution

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Problem 1k
The matrix:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 1 & 2 & -1 & -2 & 5 \\ 1 & -1 & 2 & 1 & 0 \\ 2 & 1 & 1 & -1 & 4 \end{bmatrix}$$

Set row 2 to 1.00 times row 1 minus row 2

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -2 & -2 & 4 \\ 1 & -1 & 2 & 1 & 0 \\ 2 & 1 & 1 & -1 & 4 \end{bmatrix}$$

Set row 3 to 1.00 times row 1 minus row 3

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -2 & -2 & 4 \\ 0 & -1 & 1 & 1 & -1 \\ 2 & 1 & 1 & -1 & 4 \end{bmatrix}$$

Set row 4 to 2.00 times row 1 minus row 4

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -2 & -2 & 4 \\ 0 & -1 & 1 & 1 & -1 \\ 0 & 1 & -1 & -1 & 2 \end{bmatrix}$$

Set row 3 to -0.50 times row 2 minus row 3

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -2 & -2 & 4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & 2 \end{bmatrix}$$

Set row 4 to 0.50 times row 2 minus row 4

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 2 & -2 & -2 & 4 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by 2.00

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Inconsistent solutions

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Problem 1l
The matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Swap row 1 and 2

$$\begin{bmatrix} 0 & 4 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & -1 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Swap row 1 and 3

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 4 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Swap row 2 and 3

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 4 \\ 0 & 4 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by 4.00

$$\begin{bmatrix} 1 & -1 & 2 & 1 & 4 \\ 0 & 1 & -\dfrac{1}{4} & \dfrac{1}{4} & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

Problem 1m - 1
The matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 8 \\ 3 & 4 & 5 & 10 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 2 to 2.00 times row 1 minus row 2

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 3 & 4 & 5 & 10 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 3 to 3.00 times row 1 minus row 3

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -4 & -5 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 4 to 1.00 times row 1 minus row 4

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -4 & -5 \\ 0 & -4 & -7 & -15 \end{bmatrix}$$

Set row 3 to 2.00 times row 2 minus row 3

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 0 & -1 \\ 0 & -4 & -7 & -15 \end{bmatrix}$$

Set row 4 to 4.00 times row 2 minus row 4

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

Swap row 3 and 4

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Divide row 2 by -1.00

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Divide row 4 by -1.00

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

-->Inconsistent solutions

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Problem 1m - 2
The matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 8 \\ 3 & 4 & 5 & 11 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 2 to 2.00 times row 1 minus row 2

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 3 & 4 & 5 & 11 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 3 to 3.00 times row 1 minus row 3

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -4 & -4 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 4 to 1.00 times row 1 minus row 4

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -4 & -4 \\ 0 & -4 & -7 & -15 \end{bmatrix}$$

Set row 3 to 2.00 times row 2 minus row 3

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & -7 & -15 \end{bmatrix}$$

Set row 4 to 4.00 times row 2 minus row 4

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

Swap row 3 and 4

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & -1 & -2 & -2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by -1.00

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

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Problem 1n
The matrix:

$$\begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 1 & -1 & 6 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 2 to 1.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & -\dfrac{1}{2} & -\dfrac{5}{2} & -9 \\ 1 & -2 & -4 & -10 \end{bmatrix}$$

Set row 3 to 0.50 times row 1 minus row 3

$$\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & -\dfrac{1}{2} & -\dfrac{5}{2} & -9 \\ 0 & -\dfrac{5}{2} & -\dfrac{9}{2} & -15 \end{bmatrix}$$

Set row 3 to 5.00 times row 2 minus row 3

$$\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & -\dfrac{1}{2} & -\dfrac{5}{2} & -9 \\ 0 & 0 & 8 & 30 \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & \dfrac{1}{2} & \dfrac{1}{2} & 5 \\ 0 & -\dfrac{1}{2} & -\dfrac{5}{2} & -9 \\ 0 & 0 & 8 & 30 \end{bmatrix}$$

Divide row 2 by -0.50

$$\begin{bmatrix} 1 & \dfrac{1}{2} & \dfrac{1}{2} & 5 \\ 0 & 1 & 5 & 18 \\ 0 & 0 & 8 & 30 \end{bmatrix}$$

Divide row 3 by 8.00

$$\begin{bmatrix} 1 & \dfrac{1}{2} & \dfrac{1}{2} & 5 \\ 0 & 1 & 5 & 18 \\ 0 & 0 & 1 & \dfrac{15}{4} \end{bmatrix}$$

-->Unique/single solution

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Problem 1o
The matrix:

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 1 \\ 1 & 2 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 \end{bmatrix}$$

Set row 2 to 0.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 1 \\ 0 & \dfrac{3}{2} & 1 & 0 & \dfrac{1}{2} \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 2 & 1 \end{bmatrix}$$

Set row 3 to 0.67 times row 2 minus row 3

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 1 \\ 0 & \dfrac{3}{2} & 1 & 0 & \dfrac{1}{2} \\ 0 & 0 & \dfrac{4}{3} & 1 & \dfrac{2}{3} \\ 0 & 0 & 1 & 2 & 1 \end{bmatrix}$$

Set row 4 to 0.75 times row 3 minus row 4

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 1 \\ 0 & \dfrac{3}{2} & 1 & 0 & \dfrac{1}{2} \\ 0 & 0 & \dfrac{4}{3} & 1 & \dfrac{2}{3} \\ 0 & 0 & 0 & \dfrac{5}{4} & \dfrac{1}{2} \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & 0 & \dfrac{1}{2} \\ 0 & \dfrac{3}{2} & 1 & 0 & \dfrac{1}{2} \\ 0 & 0 & \dfrac{4}{3} & 1 & \dfrac{2}{3} \\ 0 & 0 & 0 & \dfrac{5}{4} & \dfrac{1}{2} \end{bmatrix}$$

Divide row 2 by 1.50

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & 0 & \dfrac{1}{2} \\ 0 & 1 & \dfrac{2}{3} & 0 & \dfrac{1}{3} \\ 0 & 0 & \dfrac{4}{3} & 1 & \dfrac{2}{3} \\ 0 & 0 & 0 & \dfrac{5}{4} & \dfrac{1}{2} \end{bmatrix}$$

Divide row 3 by 1.33

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & 0 & \dfrac{1}{2} \\ 0 & 1 & \dfrac{2}{3} & 0 & \dfrac{1}{3} \\ 0 & 0 & 1 & \dfrac{3}{4} & \dfrac{1}{2} \\ 0 & 0 & 0 & \dfrac{5}{4} & \dfrac{1}{2} \end{bmatrix}$$

Divide row 4 by 1.25

$$
\begin{bmatrix}
1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\
0 & 1 & \frac{2}{3} & 0 & \frac{1}{3} \\
0 & 0 & 1 & \frac{3}{4} & \frac{1}{2} \\
0 & 0 & 0 & 1 & \frac{2}{5}
\end{bmatrix}
$$

-->Unique/single solution

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Problem 1p

The matrix:

$$
\begin{bmatrix}
2 & 1 & 0 & 0 \\
1 & 2 & 1 & -1 \\
0 & 1 & 2 & -4 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

Set row 2 to 0.50 times row 1 minus row 2

$$
\begin{bmatrix}
2 & 1 & 0 & 0 \\
0 & \frac{3}{2} & 1 & -1 \\
0 & 1 & 2 & -4 \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

Set row 3 to 0.67 times row 2 minus row 3

$$
\begin{bmatrix}
2 & 1 & 0 & 0 \\
0 & \frac{3}{2} & 1 & -1 \\
0 & 0 & \frac{4}{3} & -\frac{10}{3} \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

Divide row 1 by 2.00

$$
\begin{bmatrix}
1 & \frac{1}{2} & 0 & 0 \\
0 & \frac{3}{2} & 1 & -1 \\
0 & 0 & \frac{4}{3} & -\frac{10}{3} \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

Divide row 2 by 1.50

$$
\begin{bmatrix}
1 & \frac{1}{2} & 0 & 0 \\
0 & 1 & \frac{2}{3} & -\frac{2}{3} \\
0 & 0 & \frac{4}{3} & -\frac{10}{3} \\
0 & 0 & 0 & 0
\end{bmatrix}
$$

Divide row 3 by 1.33

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -\frac{5}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions

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Problem 1q

The matrix:

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & -3 & 2 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Set row 2 to 0.50 times row 1 minus row 2

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & -1 & 0 \\ 1 & 1 & 1 & -3 & 2 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Set row 3 to 0.50 times row 1 minus row 3

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & -1 & 0 \\ 0 & \frac{1}{2} & 1 & -\frac{7}{2} & 1 & 0 \\ 2 & 2 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Set row 4 to 1.00 times row 1 minus row 4

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & -1 & 0 \\ 0 & \frac{1}{2} & 1 & -\frac{7}{2} & 1 & 0 \\ 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Set row 3 to 1.00 times row 2 minus row 3

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & \frac{1}{2} & -1 & -\frac{1}{2} & -1 & 0 \\ 0 & 0 & 2 & -3 & 2 & 0 \\ 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Set row 4 to 2.00 times row 2 minus row 4

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & \dfrac{1}{2} & -1 & -\dfrac{1}{2} & -1 & 0 \\ 0 & 0 & 2 & -3 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Set row 4 to 0.50 times row 3 minus row 4

$$\begin{bmatrix} 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & \dfrac{1}{2} & -1 & -\dfrac{1}{2} & -1 & 0 \\ 0 & 0 & 2 & -3 & 2 & 0 \\ 0 & 0 & 0 & \dfrac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 1 by 2.00

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & \dfrac{1}{2} & 1 & 0 \\ 0 & \dfrac{1}{2} & -1 & -\dfrac{1}{2} & -1 & 0 \\ 0 & 0 & 2 & -3 & 2 & 0 \\ 0 & 0 & 0 & \dfrac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 2 by 0.50

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & \dfrac{1}{2} & 1 & 0 \\ 0 & 1 & -2 & -1 & -2 & 0 \\ 0 & 0 & 2 & -3 & 2 & 0 \\ 0 & 0 & 0 & \dfrac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 3 by 2.00

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & \dfrac{1}{2} & 1 & 0 \\ 0 & 1 & -2 & -1 & -2 & 0 \\ 0 & 0 & 1 & -\dfrac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & \dfrac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide row 4 by 1.50

$$\begin{bmatrix} 1 & \dfrac{1}{2} & 0 & \dfrac{1}{2} & 1 & 0 \\ 0 & 1 & -2 & -1 & -2 & 0 \\ 0 & 0 & 1 & -\dfrac{3}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

-->Non-unique/many solutions
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