Convolution Method

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$$\mathcal{L}^{-1}\left\{\left(\frac{5}{s+2}\right)\left(\frac{4}{s+1}\right)\right\} = ?$$

① The usual method:

Partial fraction expansion

$$\frac{20}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1} = \frac{20}{s+2} - \frac{20}{s+1}$$

$$As + A + Bs + 2B = 20$$

$$A + B = 0$$

$$\underline{A + 2B = 20}$$

$$B = -20 \longrightarrow A = 20$$

$$\mathcal{L}^{-1}\left(\frac{20}{(s+2)(s+1)}\right) = \mathcal{L}^{-1}\frac{20}{s+2} - \mathcal{L}^{-1}\frac{20}{s+1}$$

$$= 20e^{-2t} - 20e^{-t}$$

② Convolution method

$$\mathcal{L}^{-1}\left\{\frac{5}{s+2}\right\} * \mathcal{L}^{-1}\left\{\frac{4}{s+1}\right\} = 5e^{-2t} * 4e^{-t}$$

$$5e^{-2t} * 4e^{-t} = \int_0^t \left(5e^{-2\tau}\, 4e^{-(t-\tau)}\right) d\tau$$

$$= \int_0^t \left(20\, e^{-2\tau}\, e^{-t} \cdot e^{\tau}\right) d\tau$$

$$= 20\int_0^t \left(e^{-\tau} e^{-t}\right) d\tau$$

$$= 20e^{-t}\int_0^t e^{-\tau}\,d\tau$$

$$= 20e^{-t}\left(-e^{-\tau}\right)_0^t$$

$$= 20e^{-t}\left(-e^{-t}+1\right)$$

$$= -20e^{-2t}+20e^{-t}$$