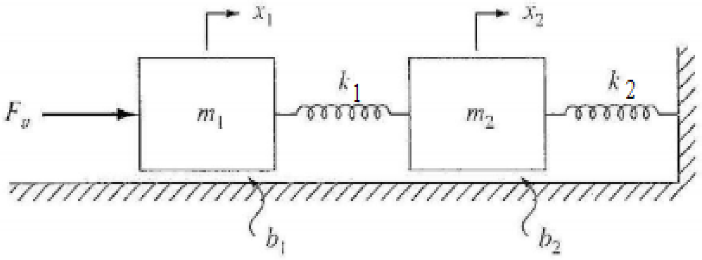
```
restart;
```

```
mass_spring_damper:=proc(n, M, K, B, F)
```

```
in_matrix:=proc(n,eq)
```

Example 1




```
> #setup matrix K and B
  K:=<0,0;k__1,k__2>:
  B:=<b__1,0;0,b__2>:
  M=<m__1, m__2>:
  F:=<F_a(t),0>:

  #write down the equations
  eq:=mass_spring_damper(2, M, K, B, F):
  in_matrix(2,eq);
```

The equations of motion:

$$M_1 \ddot{x}_1(t) + b_1 \dot{x}_1(t) - k_1 \left(x_2(t) - x_1(t)\right) = F\_a(t)$$

$$M_2 \ddot{x}_2(t) + b_2 \dot{x}_2(t) + k_1 \left(x_2(t) - x_1(t)\right) + k_2 x_2(t) = 0$$

when simplified:

$$\ddot{x}_1(t) = -\frac{b_1 \dot{x}_1(t)}{M_1} - \frac{k_1 x_1(t)}{M_1} + \frac{k_1 x_2(t)}{M_1} + \frac{F\_a(t)}{M_1}$$

$$\ddot{x}_2(t) = -\frac{b_2 \dot{x}_2(t)}{M_2} + \frac{k_1 x_1(t)}{M_2} - \frac{(k_1 + k_2)\, x_2(t)}{M_2}$$

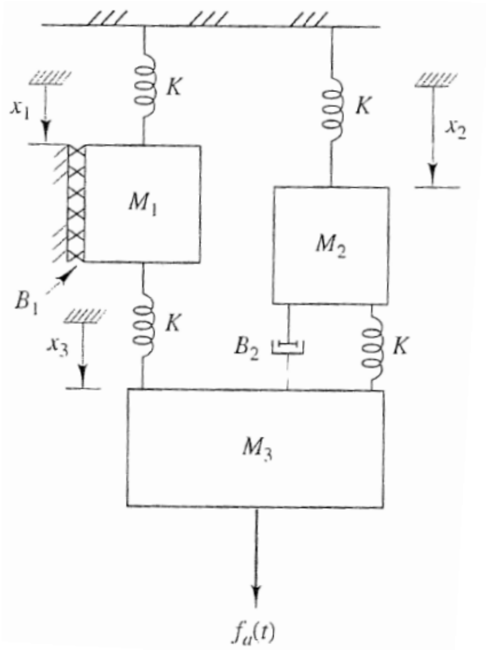
in matrix form:

$$\dot{x}(t) = A\, x(t) + f(t)$$

here:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{M_1} & \frac{k_1}{M_1} & -\frac{b_1}{M_1} & 0 \\ \frac{k_1}{M_2} & -\frac{k_1 + k_2}{M_2} & 0 & -\frac{b_2}{M_2} \end{bmatrix}, x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}, f(t) = \begin{bmatrix} 0 \\ 0 \\ \frac{F\_a(t)}{M_1} \\ 0 \end{bmatrix}$$

Example 2




```
> #clear used variables
K:='K':
M:='M':

#setup matrix K and B
matK:=<K,0,0;0,K,0;K,K,0>:
matB:=<B__1,0,0;0,0,0;0,B__2,0>:
matM:=<M__1,M__2,M__3>:
matF:=<M__1*g,M__2*g,M__3*g+f__a(t)>:

#write down the equations
eq:=mass_spring_damper(3, matM, matK, matB, matF):
in_matrix(3,eq);
```

The equations of motion:

$$M_1 \ddot{x}_1(t) + B_1 \dot{x}_1(t) + K x_1(t) - K \left(x_3(t) - x_1(t)\right) = M_1 g$$

$$M_2 \ddot{x}_2(t) - B_2 \left(\dot{x}_3(t) - \dot{x}_2(t)\right) + K x_2(t) - K \left(x_3(t) - x_2(t)\right) = M_2 g$$

$$M_3 \ddot{x}_3(t) + B_2 \left(\dot{x}_3(t) - \dot{x}_2(t)\right) + K \left(x_3(t) - x_1(t)\right) + K \left(x_3(t) - x_2(t)\right) = M_3 g + f_a(t)$$

when simplified:

$$\ddot{x}_1(t) = -\frac{B_1 \dot{x}_1(t)}{M_1} - \frac{2 K x_1(t)}{M_1} + \frac{K x_3(t)}{M_1} + g$$

$$\ddot{x}_2(t) = -\frac{B_2 \dot{x}_2(t)}{M_2} + \frac{B_2 \dot{x}_3(t)}{M_2} - \frac{2 K x_2(t)}{M_2} + \frac{K x_3(t)}{M_2} + g$$

$$\ddot{x}_3(t) = \frac{B_2 \dot{x}_2(t)}{M_3} - \frac{B_2 \dot{x}_3(t)}{M_3} + \frac{K x_1(t)}{M_3} + \frac{K x_2(t)}{M_3} - \frac{2 K x_3(t)}{M_3} - \frac{-M_3 g - f_a(t)}{M_3}$$

in matrix form:

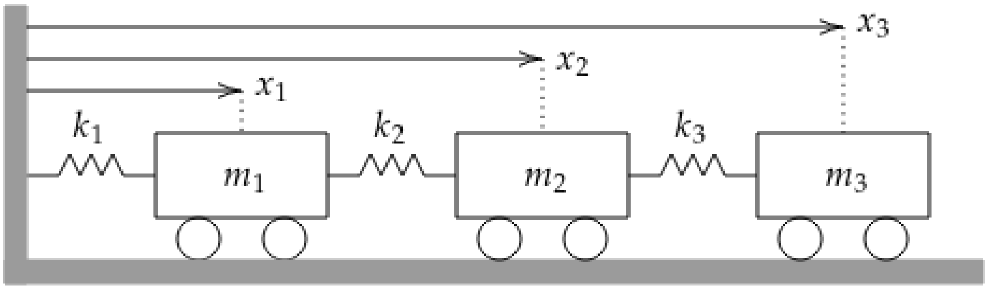
$$\dot{x}(t) = A\, x(t) + f(t)$$

here:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \\ \ddot{x}_3(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{2K}{M_1} & 0 & \frac{K}{M_1} & -\frac{B_1}{M_1} & 0 & 0 \\ 0 & -\frac{2K}{M_2} & \frac{K}{M_2} & 0 & -\frac{B_2}{M_2} & \frac{B_2}{M_2} \\ \frac{K}{M_3} & \frac{K}{M_3} & -\frac{2K}{M_3} & 0 & \frac{B_2}{M_3} & -\frac{B_2}{M_3} \end{bmatrix}, x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix}, f(t)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ g \\ g \\ -\frac{-M_3\, g - f_a(t)}{M_3} \end{bmatrix}$$

Example 3




```
> #clear used variables
  K:='K':
  M:='M':

  #setup matrix K and B
  matK:=<k__1,0,0;k__2,0,0;0,k__3,0>:
  matB:=<0,0,0;0,0,0;0,0,0>:
  matM:=<m__1,m__2,m__3>:
  matF:=<0,0,0>:

  #write down the equations
  eq:=mass_spring_damper(3, matM, matK, matB, matF):
  in_matrix(3,eq);
```

The equations of motion:

$$m_1 \ddot{x}_1(t) + k_1 x_1(t) - k_2 \left(x_2(t) - x_1(t)\right) = 0$$

$$m_2 \ddot{x}_2(t) + k_2 \left(x_2(t) - x_1(t)\right) - k_3 \left(x_3(t) - x_2(t)\right) = 0$$

$$m_3 \ddot{x}_3(t) + k_3 \left(x_3(t) - x_2(t)\right) = 0$$

when simplified:

$$\ddot{x}_1(t) = \frac{(-k_1 - k_2)\, x_1(t)}{m_1} + \frac{k_2 x_2(t)}{m_1}$$

$$\ddot{x}_2(t) = \frac{k_2 x_1(t)}{m_2} + \frac{(-k_2 - k_3)\, x_2(t)}{m_2} + \frac{k_3 x_3(t)}{m_2}$$

$$\ddot{x}_3(t) = \frac{k_3 x_2(t)}{m_3} - \frac{k_3 x_3(t)}{m_3}$$

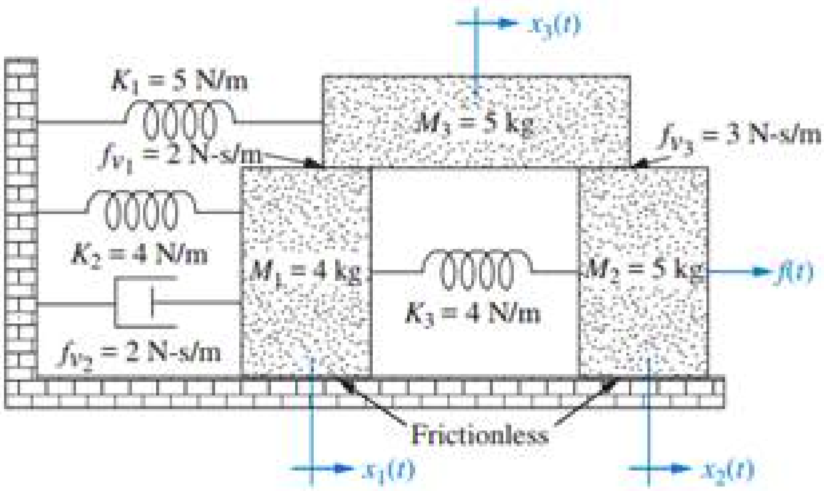
in matrix form:

$$\dot{x}(t) = A\, x(t) + f(t)$$

here:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \\ \ddot{x}_3(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1 - k_2}{m_1} & \frac{k_2}{m_1} & 0 & 0 & 0 & 0 \\ \frac{k_2}{m_2} & \frac{-k_2 - k_3}{m_2} & \frac{k_3}{m_2} & 0 & 0 & 0 \\ 0 & \frac{k_3}{m_3} & -\frac{k_3}{m_3} & 0 & 0 & 0 \end{bmatrix}, x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix}, f(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Example 4




```
> #clear used variables
  K:='K':
  M:='M':

  #setup matrix K and B
  matK:=<1,0,0;4,0,0;0,0,5>:
  matB:=<2,0,0;0,0,0;2,3,0>:
  matM:=<4,5,5>:
  matF:=<0,0,0>:

  #write down the equations
  eq:=mass_spring_damper(3, matM, matK, matB, matF):
  in_matrix(3,eq);
```

The equations of motion:

$$4\,\ddot{x}_1(t)+4\,\dot{x}_1(t)-2\,\dot{x}_3(t)+5\,x_1(t)-4\,x_2(t)=0$$
$$5\,\ddot{x}_2(t)-3\,\dot{x}_3(t)+3\,\dot{x}_2(t)+4\,x_2(t)-4\,x_1(t)=0$$
$$5\,\ddot{x}_3(t)+5\,\dot{x}_3(t)-2\,\dot{x}_1(t)-3\,\dot{x}_2(t)+5\,x_3(t)=0$$

when simplified:

$$\ddot{x}_1(t)=-\dot{x}_1(t)+\frac{\dot{x}_3(t)}{2}-\frac{5\,x_1(t)}{4}+x_2(t)$$
$$\ddot{x}_2(t)=\frac{3\,\dot{x}_3(t)}{5}-\frac{3\,\dot{x}_2(t)}{5}-\frac{4\,x_2(t)}{5}+\frac{4\,x_1(t)}{5}$$
$$\ddot{x}_3(t)=-\dot{x}_3(t)+\frac{2\,\dot{x}_1(t)}{5}+\frac{3\,\dot{x}_2(t)}{5}-x_3(t)$$

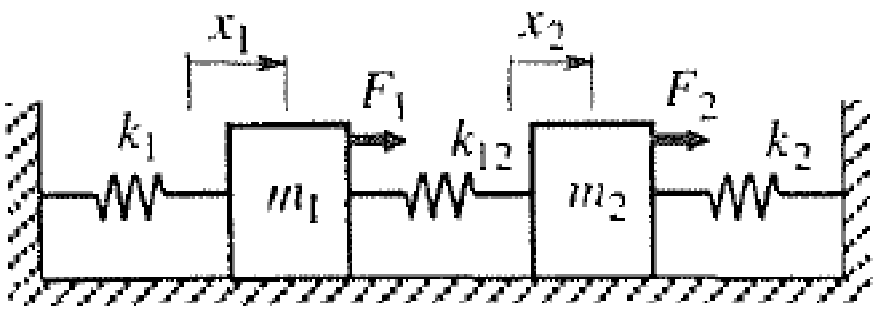
in matrix form:

$$\dot{x}(t)=A\,x(t)+f(t)$$

here:

$$\dot{x}(t)=\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \ddot{x}_1(t)\\ \ddot{x}_2(t)\\ \ddot{x}_3(t)\end{bmatrix},\ A=\begin{bmatrix}0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ -\frac{5}{4}&1&0&-1&0&\frac{1}{2}\\ \frac{4}{5}&-\frac{4}{5}&0&0&-\frac{3}{5}&\frac{3}{5}\\ 0&0&-1&\frac{2}{5}&\frac{3}{5}&-1\end{bmatrix},\ x(t)=\begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\\ \dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix},\ f(t)=\begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

Example 5




```
> #setup matrix K and B
  K:=<k__1,0;k__12,k__2>:
  B:=<0,0;0,0>:
  M=<m__1, m__2>:
  F:=<F__1,F__2>:

  #write down the equations
  eq:=mass_spring_damper(2, M, K, B, F):
  in_matrix(2,eq);
```

The equations of motion:

$$M_1\,\ddot{x}_1(t)+k_1\,x_1(t)-k_{12}\,\left(x_2(t)-x_1(t)\right)=F_1$$

$$M_2\,\ddot{x}_2(t) + k_{12}\,\big(x_2(t) - x_1(t)\big) + k_2\,x_2(t) = F_2$$

when simplified:

$$\ddot{x}_1(t) = \frac{(-k_1 - k_{12})\,x_1(t)}{M_1} + \frac{k_{12}\,x_2(t)}{M_1} + \frac{F_1}{M_1}$$

$$\ddot{x}_2(t) = \frac{k_{12}\,x_1(t)}{M_2} + \frac{(-k_{12} - k_2)\,x_2(t)}{M_2} + \frac{F_2}{M_2}$$

in matrix form:

$$\dot{x}(t) = A\,x(t) + f(t)$$

here:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_1 - k_{12}}{M_1} & \frac{k_{12}}{M_1} & 0 & 0 \\ \frac{k_{12}}{M_2} & \frac{-k_{12} - k_2}{M_2} & 0 & 0 \end{bmatrix}, x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}, f(t) = \begin{bmatrix} 0 \\ 0 \\ \frac{F_1}{M_1} \\ \frac{F_2}{M_2} \end{bmatrix}$$