A Mass with a Reaction Wheel

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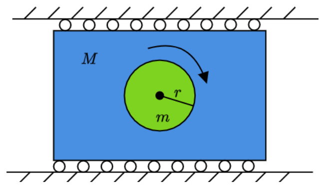

J is the moment inertia off the wheel.

```
> restart
> with(LinearAlgebra):
  with(Physics):
  with(plots):
  with(plottools):
  with(DEtools):
  with(Typesetting):
  interface(typesetting=extended):
  interface(showassumed=0):
> T_m:=1/2*J*diff(theta(t),t)^2 + 1/2*m*diff(x(t),t)^2:
  V_m:=0:

  printf("\n");

  print(`Kinetic energy of wheel m:`);
  T:=simplify(T_m):
  print('T__m'=T_m);

  printf("\n");

  print(`Potential energy of wheel m:`);
  print('V__m'=V_m);

  printf("\n");
```

Kinetic energy of mass m:

$$T_m = \frac{J\,\dot{\theta}(t)^2}{2} + \frac{m\,\dot{x}(t)^2}{2}$$

Potential energy of mass m:

$$V_m = 0$$

```
> T_M:=1/2*M*((diff(x(t),t))^2):
  V_M:=0:

  printf("\n");
```

```
print(`Kinetic energy of box M:`);
print('T__M'=T_M);

printf("\n");

print(`Potential energy of mass M:`);
print('V__M'=V_M);
```

Kinetic energy of box M:

$$T_M = \frac{M \dot{x}(t)^2}{2}$$

Potential energy of mass M:

$$V_M = 0$$

```
> T:=T_m+T_M:
V:=V_m+V_M:
L:=T-V:
print('L'=L);

eq1:=simplify(<diff(L,x(t)), diff(L,theta(t))>):
eq2:=simplify(<diff(L,diff(x(t),t)), diff(L,diff(theta(t),t))>):
eq3:=simplify(<diff(eq2[1],t), diff(eq2[2],t)>):

print('diff(L,x)'=eq1);
print('diff(L,diff(x,t))'=eq2);
print('diff(diff(L,diff(x,t)),t)'=simplify(eq3));

printf("\n");
print(`Thus,`);
print('diff(diff(L,diff(x,t)),t)=diff(L,x)');
eq4:=simplify({(eq3[1]-eq1[1]=0), (eq3[2]-eq1[2]=0)/(m*l)}):
print(<eq4[1],eq4[2]>);
eq5:=simplify(solve(eq4,{diff(x(t),t,t), diff(theta(t),t,t)})):
print(<eq5[1],eq5[2]>);
```

$$L = \frac{J \dot{\theta}(t)^2}{2} + \frac{m \dot{x}(t)^2}{2} + \frac{M \dot{x}(t)^2}{2}$$

$$\frac{\partial}{\partial x} L = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial}{\partial \frac{\partial}{\partial t} x} L = \begin{bmatrix} \dot{x}(t)\ (M+m) \\ J \dot{\theta}(t) \end{bmatrix}$$

$$\frac{\partial^2}{\partial \frac{\partial}{\partial t} x \partial t} L = \begin{bmatrix} \ddot{x}(t)\ (M+m) \\ J \ddot{\theta}(t) \end{bmatrix}$$

Thus,

$$\frac{\partial^2}{\partial \frac{\partial}{\partial t} x \partial t} L = \frac{\partial}{\partial x} L$$

$$\left[\ddot{x}(t)\,(M+m) = 0, \quad \frac{J\,\ddot{\theta}(t)}{m\,l} = 0 \right]$$

$$\left[\ddot{\theta}(t) = 0, \quad \ddot{x}(t) = 0 \right]$$

It is clear that the reaction wheel does not cause any forces to the mass. The two systems are completely decoupled.