```
> restart
> with(LinearAlgebra):
  with(Physics):
  with(plots):
  with(plottools):
  with(DEtools):
  with(Typesetting):
  interface(typesetting=extended):
  interface(showassumed=0):
> T1:=2*1/2*(J+m*l^2)*diff(theta__1(t),t)^2:
  T2:=2*1/2*(J+m*(l*sin(theta__1(t)))^2)*diff(theta__2(t),t)^2:
  V:=2*m*g*l*(1-cos(theta__1(t))):
  R1:=1/2*b__1*diff(l-l*cos(theta__1(t)),t)^2:
  R2:=1/2*b__2*diff(theta__2(t),t)^2:

  printf("\n");

  print(`Kinetic energy of the bar:`);
  print('T'=T1+T2);

  printf("\n");

  print(`Potential energy of the bar:`);
  print('V'=V);

  printf("\n");

  print(`Translational dissipation along the bar:`);
  print('R__1'=R1);

  printf("\n");

  print(`Rotational dissipation disspation:`);
  print('R__2'=R2);
```

Kinetic energy of the bar:

$$T=\left(m\,l^2+J\right)\,\dot{\theta}_1(t)^2+\left(J+m\,l^2\sin\left(\theta_1(t)\right)^2\right)\,\dot{\theta}_2(t)^2$$

Potential energy of the bar:

$$V=2\,m\,g\,l\,\left(1-\cos\left(\theta_1(t)\right)\right)$$

Translational dissipation along the bar:

$$R_1=\frac{b_1\,l^2\,\dot{\theta}_1(t)^2\,\sin\left(\theta_1(t)\right)^2}{2}$$

Rotational dissipation disspation:

$$R_2=\frac{b_2\,\dot{\theta}_2(t)^2}{2}$$

```
> T:=T1+T2:
  V:=V:
  L:=T-V:
  print('L'=L);

  eq1:=simplify(<diff(L,theta__1(t)), diff(L,theta__2(t))>):
  eq2:=simplify(<diff(L,diff(theta__1(t),t)), diff(L,diff(theta__2(t),t))>):
  eq3:=simplify(<diff(eq2[1],t), diff(eq2[2],t)>):
  eqR1:=simplify(<diff(R1,diff(theta__1(t),t)), diff(R1,diff(theta__2(t),t))>):
  eqR2:=simplify(<diff(R2,diff(theta__1(t),t)), diff(R2,diff(theta__2(t),t))>):

  print('diff(L,theta)'=eq1);
  print('diff(L,diff(theta,t))'=eq2);
  print('diff(R__1,diff(theta,t))'=eqR1);
  print('diff(R__2,diff(theta,t))'=eqR2);
  print('diff(diff(L,diff(theta,t)),t)'=simplify(eq3));
```

$$L = \left(m\,l^2 + J\right)\,\dot{\theta}_1(t)^2 + \left(J + m\,l^2 \sin\left(\theta_1(t)\right)^2\right)\,\dot{\theta}_2(t)^2 - 2\,m\,g\,l\,\left(1 - \cos\left(\theta_1(t)\right)\right)$$

$$\frac{\partial}{\partial \theta}\,L = \begin{bmatrix} -2\,\sin\left(\theta_1(t)\right)\,l\,m\,\left(-\cos\left(\theta_1(t)\right)\,\dot{\theta}_2(t)^2\,l + g\right) \\ 0 \end{bmatrix}$$

$$\frac{\partial}{\partial \frac{\partial}{\partial t}\,\theta}\,L = \begin{bmatrix} 2\,\left(m\,l^2 + J\right)\,\dot{\theta}_1(t) \\ 2\,\dot{\theta}_2(t)\,\left(-\cos\left(\theta_1(t)\right)^2\,l^2\,m + m\,l^2 + J\right) \end{bmatrix}$$

$$\frac{\partial}{\partial \frac{\partial}{\partial t}\,\theta}\,R_1 = \begin{bmatrix} b_1\,l^2\,\dot{\theta}_1(t)\,\sin\left(\theta_1(t)\right)^2 \\ 0 \end{bmatrix}$$

$$\frac{\partial}{\partial \frac{\partial}{\partial t}\,\theta}\,R_2 = \begin{bmatrix} 0 \\ b_2\,\dot{\theta}_2(t) \end{bmatrix}$$

$$\frac{\partial^2}{\partial \frac{\partial}{\partial t}\,\theta\,\partial t}\,L$$

$$= \begin{bmatrix} 2\,\left(m\,l^2 + J\right)\,\ddot{\theta}_1(t) \\ \left(-2\,\cos\left(\theta_1(t)\right)^2\,l^2\,m + 2\,m\,l^2 + 2\,J\right)\,\ddot{\theta}_2(t) + 4\,\dot{\theta}_2(t)\,\cos\left(\theta_1(t)\right)\,l^2\,m\,\dot{\theta}_1(t)\,\sin\left(\theta_1(t)\right) \end{bmatrix}$$

```
>
  printf("\n");
  print(`Thus,`);
  print('diff(diff(L,diff(theta,t)),t)-diff(L,theta)+diff(R__1,diff(theta(t),t))+diff
  (R__2,diff(theta(t),t))'=<0, tau(t)>);
  eq4:=simplify({(eq3[1]-eq1[1]+eqR1[1]+eqR2[1]=0), (eq3[2]-eq1[2]+eqR1[2]+eqR2[2]=tau(t)
  )}):
  print(<eq4[1],eq4[2]>);
  eq5:=simplify(solve(eq4,{diff(theta__1(t),t,t), diff(theta__2(t),t,t)})):
  print(<eq5[1],eq5[2]>);
```

Thus,

$$\frac{\partial^2}{\partial \frac{\partial}{\partial t}\theta \partial t} L - \frac{\partial}{\partial \theta} L + \frac{\partial}{\partial \dot{\theta}(t)} R_1 + \frac{\partial}{\partial \dot{\theta}(t)} R_2 = \begin{bmatrix} 0 \\ \tau(t) \end{bmatrix}$$

$$\left[\left[\left(-2\cos(\theta_1(t))^2 l^2 m + 2 m l^2 + 2 J\right)\ddot{\theta}_2(t) + 4\dot{\theta}_2(t)\left(\cos(\theta_1(t))\, l^2 m\, \dot{\theta}_1(t)\sin(\theta_1(t)) + \frac{b_2}{4}\right) = \tau(t)\right],\right.$$

$$\left.\left[\left(2 m l^2 + 2 J\right)\ddot{\theta}_1(t) - 2 l\left(\frac{l\, b_1 \left(\cos(\theta_1(t)) - 1\right)\left(\cos(\theta_1(t)) + 1\right)\dot{\theta}_1(t)}{2} + \sin(\theta_1(t))\, m\left(\cos(\theta_1(t))\,\dot{\theta}_2(t)^2 l - g\right)\right) = 0\right]\right]$$

$$\left[\left[\ddot{\theta}_1(t) = \frac{1}{2 m l^2 + 2 J}\left(2 l\left(\frac{l\, b_1\left(\cos(\theta_1(t)) - 1\right)\left(\cos(\theta_1(t)) + 1\right)\dot{\theta}_1(t)}{2} + \sin(\theta_1(t))\, m\left(\cos(\theta_1(t))\,\dot{\theta}_2(t)^2 l - g\right)\right)\right)\right],\right.$$

$$\left.\left[\ddot{\theta}_2(t) = \frac{\left(-4\cos(\theta_1(t))\, l^2 m\,\dot{\theta}_1(t)\sin(\theta_1(t)) - b_2\right)\dot{\theta}_2(t) + \tau(t)}{-2\cos(\theta_1(t))^2 l^2 m + 2 m l^2 + 2 J}\right]\right]$$

```
> m:=0.1:
  J:=0.01:
  l:=0.2:
  g:=9.8:
  T:=1.2:
  b__1:=1:
  b__2:=0.1:
```

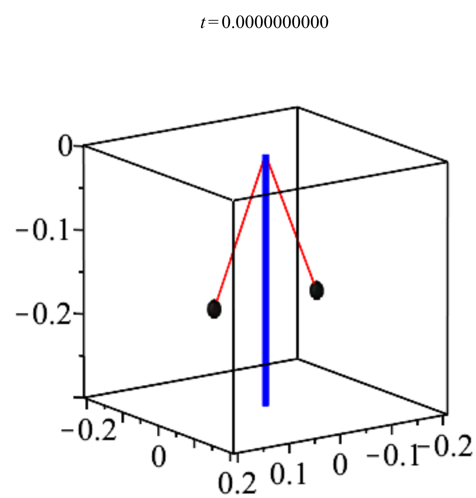
```
> ode:=eval(eq5):
  ode:=subs(tau(t)=T,ode):
```

```
> sol:=dsolve({ode[1], ode[2], theta__1(0)=Pi/6, D(theta__1)(0)=0, theta__2(0)=0, D
  (theta__2)(0)=0}, numeric, method=rkf45, output=listprocedure):
  th1_array:=eval(theta__1(t),sol):
  th2_array:=eval(theta__2(t),sol):
```

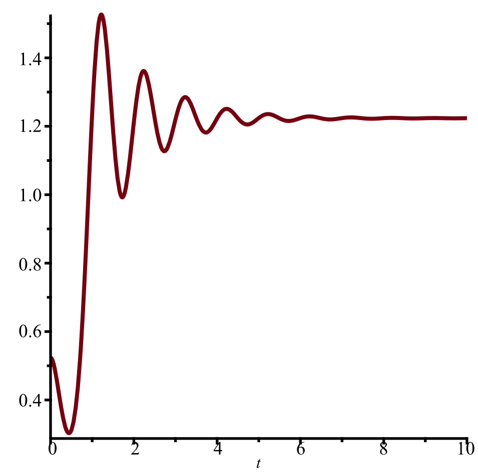
```
> A:=[0,0,0]:
  B:=[-l*sin(th1_array(t))*cos(th2_array(t)), -l*sin(th1_array(t))*sin(th2_array(t)), -l*
  cos(th1_array(t))]:
  C:=[ l*sin(th1_array(t))*cos(th2_array(t)),  l*sin(th1_array(t))*sin(th2_array(t)), -l*
```

```
cos(th1_array(t))]:
E:=[ 0,0, -1.5*l]:
anim1:=animate(line, [A, B, color=red, scaling=constrained], t=0..5,frames=200):
anim2:=animate(line, [A, C, color=red, scaling=constrained], t=0..5,frames=200):
anim3:=animate(line, [A, E, color=blue,thickness=5, scaling=constrained], t=0..5,
frames=200):
anim4:=animate(sphere, [B, 0.01,scaling=constrained], t=0..5,frames=200):
anim5:=animate(sphere, [C, 0.01,scaling=constrained], t=0..5,frames=200):


h1:=display(anim1, anim2, anim3, anim4, anim5);
```




```
> plot(th1_array(t),t=0..10)
```




```
> ball := proc(x,y) plots[pointplot]([[x,y]],color=blue,symbol=solidcircle,symbolsize=
100): end proc:

anim1:=animate(plot,[[0,-l*sin(th1_array(t))],[0, -l*cos(th1_array(t))], color=red,
scaling=constrained], t=0..10,frames=100):
anim2:=animate(plot,[[0, l*sin(th1_array(t))],[0, -l*cos(th1_array(t))], color=red,
scaling=constrained], t=0..10,frames=100):

anim3:=animate(plot,[[0,0],[0, -2*l], color=blue, thickness=5, scaling=constrained], t=
0..10,frames=100):

anim4:=animate(plot,[[-l*sin(th1_array(t)),0],[-l*cos(th1_array(t)),-2*l*cos(th1_array
(t))], color=green, scaling=constrained], t=0..10,frames=100):
anim5:=animate(plot,[[ l*sin(th1_array(t)),0],[-l*cos(th1_array(t)),-2*l*cos(th1_array
```

```
(t))], color=green, scaling=constrained], t=0..10,frames=100):

anim6:=animate(ball, [-l*sin(th1_array(t)), -l*cos(th1_array(t))], t=0..10,scaling=
constrained,frames=100):
anim7:=animate(ball, [l*sin(th1_array(t)), -l*cos(th1_array(t))], t=0..10,scaling=
constrained,frames=100):

h2:=display(anim1,anim2,anim3,anim4,anim5,anim6,anim7);
```