

Experimental Identification of Stable Nonoscillatory Systems from Step-Responses by Selected Methods

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Abstract: *The paper reviews seven methods for experimental identification based on step-responses applied on stable nonoscillatory systems. This paper presents Matlab developed software for identification purposes. The program provides a useful tool for plotting and comparing the step-responses of the determined system models. The integral square error criterion was used to analyze the method applicability for the step-response approximation.*

Keywords: *Experimental Identification, Stable Nonoscillatory Systems, Step-response, Matlab/Gui*

1. Introduction

One of the paper goals was to summarize the most used methods of the stable nonoscillatory systems experimental identification on the base of the system step-response. Although there are many methods to be found in the literature, just seven of them are reviewed in this paper.

- a) Vítečková's method (using the 1st order approximation model)
- b) Vítečková's method (using the 2nd order approximation model)
- c) Latzel's method
- d) Harriott's method
- e) Smith's method
- f) Strejc's method
- g) Sundaresan's & Krishnaswamy's method

The primary goal was to compare the abilities of above mentioned methods to approximate several types of stable nonoscillatory systems and, on the base of the obtained results, to advise which of these methods approximate which types of systems most efficiently.

To make the method analysis easier, the computing program was developed in MATLAB. The program generates transfer functions of the selected methods, based on the preloaded step-response data. The program can also generate figures, comparing all modeled step-responses with the input system step-response, or comparing just the input system step-response with step-responses of all 1st order models, with all 2nd order models or with all nth order models.

The integral square error criterion is used to analyze the method applicability for the step-response approximation. The decently ordered chart of all methods can be viewed in the program screen.

To enable the simplest operation with the approximating software the computing program was implemented in the graphical interface MATLAB/GUI.

2. Identification Methods

In this chapter the proposals of each identification methods will be introduced.

2.1. Vítečková's Method

As described in [1, 5], the model is computed so that its step-response is equal to the system step-response in 33% and 70% of its final value (Fig. 1).

By the means of the Vítečková's method the system can be approximated either by the 1st order model with the possibility of time delay inclusion or by the 2nd order model with the possibility of time delay inclusion. The 1st order transfer function can be written as follows:

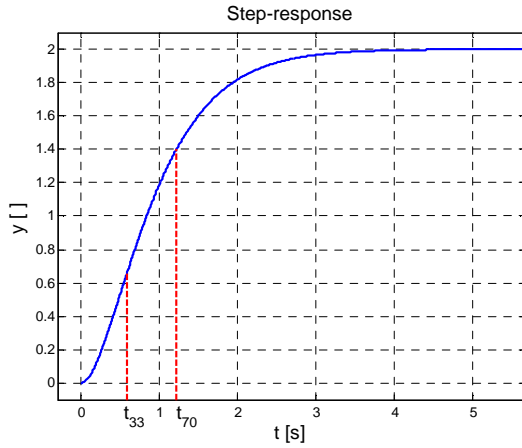


Fig.1 – Step-response of the system with times t_{33} and t_{70} marked

$$G_{V1}(s) = \frac{K}{(\tau_{V1}s + 1)} e^{-T_{dV1}s} \quad (1)$$

where T_{dV1} is a time delay:

$$T_{dV1} = 1,498t_{33} - 0,498t_{70}, \quad (2)$$

τ_{V1} is a time constant:

$$\tau_{V1} = 1,245(t_{70} - t_{33}), \quad (3)$$

where t_{33} and t_{70} are times, in which the step-response reaches 33% respectively 70% of its final value.

The 2nd order transfer function can be written as follows:

$$G_{V2}(s) = \frac{K}{(\tau_{V2}s + 1)^2} e^{-T_{dV2}s} \quad (4)$$

where T_{dV2} is a time delay:

$$T_{dV2} = 1,937t_{33} - 0,937t_{70}$$

and τ_{V1} is a time constant:

$$\tau_{V2} = 0,794(t_{70} - t_{33}). \quad (5)$$

In case of determined $T_{dV2} < 0$ or $T_{dV1} < 0$, respectively the time delay is not included in the transfer function.

2.2. Latzel's Method

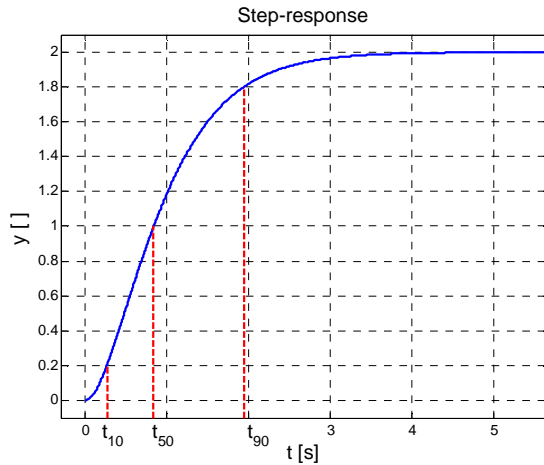


Fig.2 – Step-response of the system with times t_{10} , t_{50} and t_{90} marked

The Latzel's method [2, 5] is based on the system approximation by the model of transfer function:

$$G_L(s) = \frac{K}{(\tau_L s + 1)^n}, \quad (6)$$

where τ_L is a time constant:

$$\tau_L = (\alpha_{10}t_{10} + \alpha_{50}t_{50} + \alpha_{90}t_{90}), \quad (7)$$

where t_{10} , t_{50} and t_{90} are times, in which the step-response reaches 10%, 50% and 70% respectively of its final value (Fig.2.).

The parameter μ can be calculated from the measured times:

$$\mu = \frac{t_{10}}{t_{90}} \quad (8)$$

After that the nearest parameter μ_a to the parameter μ , the order factor n and constants α_{10} , α_{50} and α_{90} are then to be found in the same row in tab. 1.

Then according to the equation (7) the time constant τ_L can be calculated.

Tab.1 – Table of parameters

| μ_a | n | α_{10} | α_{50} | α_{90} | μ_a | n | α_{10} | α_{50} | α_{90} |
|---------|-----|---------------|---------------|---------------|---------|-----|---------------|---------------|---------------|
| 0,137 | 2 | 1,880 | 0,596 | 0,257 | 0,456 | 11 | 0,142 | 0,094 | 0,065 |
| 0,174 | 2,5 | 1,245 | 0,460 | 0,216 | 0,472 | 12 | 0,128 | 0,086 | 0,060 |
| 0,207 | 3 | 0,907 | 0,374 | 0,188 | 0,486 | 13 | 0,116 | 0,079 | 0,056 |
| 0,261 | 4 | 0,573 | 0,272 | 0,150 | 0,499 | 14 | 0,106 | 0,073 | 0,053 |
| 0,304 | 5 | 0,411 | 0,214 | 0,125 | 0,512 | 15 | 0,097 | 0,068 | 0,050 |
| 0,340 | 6 | 0,317 | 0,176 | 0,108 | 0,523 | 16 | 0,090 | 0,064 | 0,047 |
| 0,370 | 7 | 0,257 | 0,150 | 0,095 | 0,533 | 17 | 0,084 | 0,060 | 0,045 |
| 0,396 | 8 | 0,215 | 0,130 | 0,085 | 0,543 | 18 | 0,078 | 0,057 | 0,042 |
| 0,418 | 9 | 0,184 | 0,115 | 0,077 | 0,552 | 19 | 0,073 | 0,054 | 0,040 |
| 0,438 | 10 | 0,161 | 0,103 | 0,070 | 0,561 | 20 | 0,069 | 0,051 | 0,039 |

2.3. Harriott's Method

The Harriott's method [3, 5] approximates the system by a 2nd order model with the possibility of time delay inclusion. The model transfer function can be written as follows:

$$G_H(s) = \frac{K}{(\tau_{H1}s + 1)(\tau_{H2}s + 1)} e^{-T_{dH}s}. \quad (9)$$

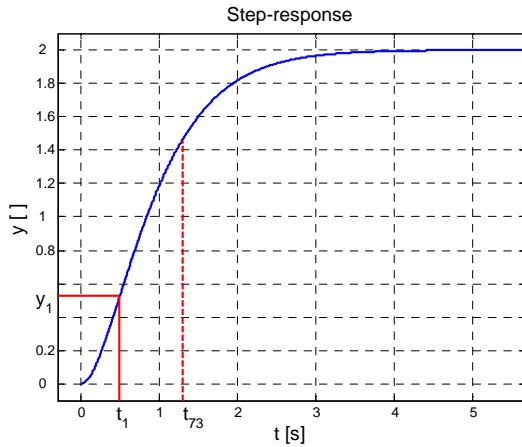


Fig.3 – Step-response of the system with times t_1 and t_{73} marked

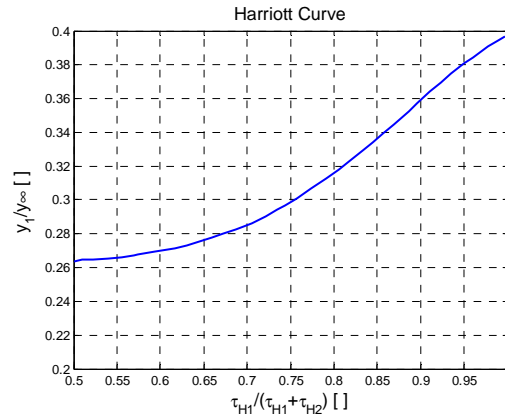


Fig.4 – Smith Curve for parameters τ_{H1} and τ_{H2} estimation

The fractional response of a 2nd order system (without time delay) can be plotted against $t/(\tau_{H1} + \tau_{H2})$ for various ratios of τ_2/τ_1 . All the curves intersect approximately at 73% of the final steady-state value, where $t/(\tau_{H1} + \tau_{H2})$ approximately equals 1,3. Thus by measuring the time required for the system to reach 73% of its final value t_{73} (Fig.3), the sum of the two time constants can be calculated:

$$(\tau_{H1} + \tau_{H2}) = \frac{t_{73}}{1.3}. \quad (10)$$

The value of the fractional response t_1 when $t_1/(\tau_{H1} + \tau_{H2}) = 0,5$ can be determined from the experimental data (Fig.3), and the value of $\tau_{H1}/(\tau_{H1} + \tau_{H2})$ can be read from the fig.4.

If the fractional response is less than 0,26 or greater than 0,39 at this point, the method is not applicable, which generally indicates that the process requires a model that is higher than second order or that is underdamped.

The time delay T_{dH} can be determined as:

$$T_{dH} = 1,937t_{33} - 0,937t_{70} \quad (11)$$

In case of determined $T_{dH} < 0$ the time delay is not included in the transfer function.

2.4. Smith's Method

The Smith's method [3, 5], based on two points of the fractional response of the system at 20% and 60% of its final steady-state final value (Fig.5), approximates the system by a 2nd order model with the possibility of time delay inclusion (12). Smith's method requires the time at which the normalized value of the response reaches 20% and 60%, respectively. Using fig.6, the ratio of t_{20}/t_{60} gives the value of ζ . An estimate of τ can be obtained from the plot of t_{60}/τ vs. t_{20}/t_{60} .

$$G_{SM}(s) = \frac{K}{(\tau_{SM1}s + 1)(\tau_{SM2}s + 1)} \quad (12)$$

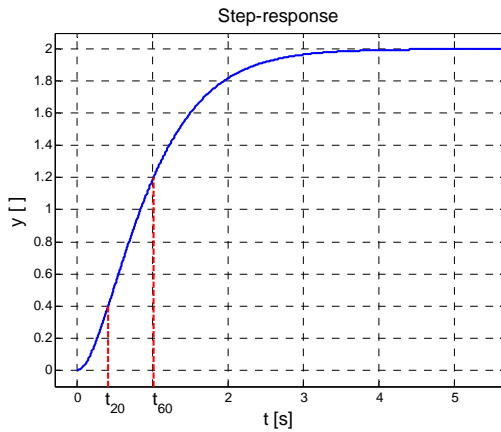


Fig.5 – Step-response of the system with times t_{20} and t_{60} marked

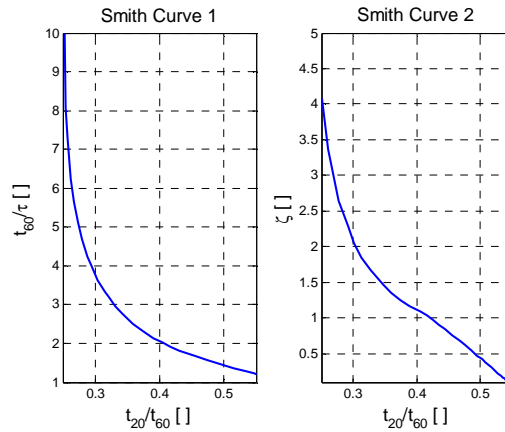


Fig.6 – Curves for τ and ζ assessment

$$\begin{aligned} \tau_{SM1} &= \tau\zeta + \tau\sqrt{(\zeta^2 - 1)} \\ \tau_{SM2} &= \tau\zeta - \tau\sqrt{(\zeta^2 - 1)} \end{aligned} \quad (13)$$

2.5. Strejc's Method

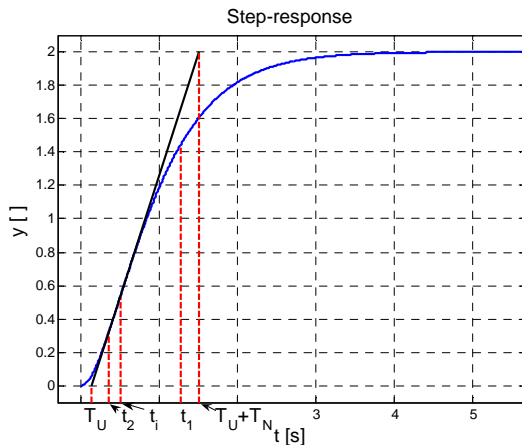


Fig.7 – Step-response of the system with times t_1 , t_2 , t_b , T_U and T_N marked

The Strejc's method [1, 4, 5] approximates the nonoscillatory system without the time delay with the help of the measured times T_U and T_N (Fig.7). The ratio of these times gives the parameter τ :

$$\tau = \frac{T_U}{T_N}. \quad (14)$$

According to the value of τ the right approximation approach is determined.

- a) In case of: $\tau < 0$ the system is approximated by a 2nd order model:

$$G_{ST1}(s) = \frac{K}{(\tau_{ST1}s + 1)(\tau_{ST2}s + 1)}. \quad (15)$$

The constants τ_{ST1} and τ_{ST2} are determined

as follows:

- 1) Find the time t_1 related to the value $0.72y_\infty$ in the system step-response graph (Fig.7) determine the sum of constants τ_{ST1} a τ_{ST2} according to equation (16.):

$$\tau_{ST1} + \tau_{ST2} = \frac{t_1}{1,2564}. \quad (16)$$

2) Determine the time t_2 :

$$t_2 = 0,3574(\tau_{ST1} + \tau_{ST2}). \quad (17)$$

3) Find the value $y(t_2)$ in the system step-response graph (Fig.7).

4) Determine the T ratio from the tab. 2.

$$T = \frac{\tau_{ST1}}{\tau_{ST2}}. \quad (18)$$

5) Determine the unknown values τ_{ST1} and τ_{ST2} from their ratio (18) and sum (16).

b) In case of: $\tau \geq 0$ the system is approximated by an n^{th} order model:

$$G_{ST2}(s) = \frac{K}{(\tau_{ST}s + 1)^n}. \quad (19)$$

The constant τ_{ST} is determined as follows:

1) Design a tangent, which intersects the inflexion point, find the values of T_U and T_N and determine the ratio τ :

$$\tau = \frac{T_U}{T_N}. \quad (20)$$

2) According to the value of τ find the appropriate order of the approximation model and the inflexion point coordinate y_i .

Tab. 3 – Assessment of the order factor n and precision of the inflexion point coordinate

| n | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| τ | 0,104 | 0,218 | 0,319 | 0,41 | 0,493 | 0,57 | 0,642 | 0,709 | 0,773 |
| y_i | 0,264 | 0,327 | 0,359 | 0,371 | 0,384 | 0,394 | 0,401 | 0,407 | 0,413 |

3) Based on the pre assessed coordinate y_i find in the system step-response graph (Fig.7).

4) Determine the time constant τ_{ST} from the equation (21):

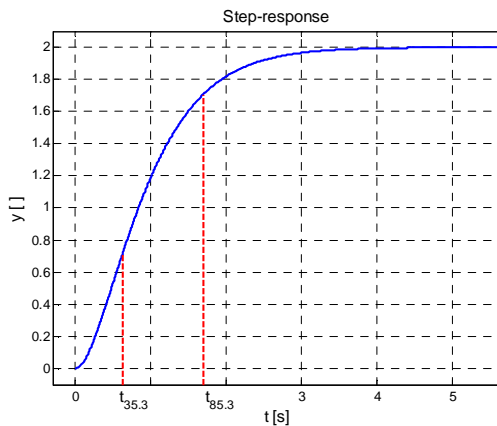


Fig.8 – Step-response of the system with times $t_{35,3}$ and $t_{85,3}$ marked

$$\tau_{ST} = \frac{t_i}{n-1}. \quad (21)$$

2.6. Sundaresan's & Krishnaswamy's Method

The Sundaresan's & Krishnaswamy's method [3, 5] approximates the system by a 1st order time delayed model with a transform function:

$$G_{SK}(s) = \frac{K}{(\tau_{SK}s + 1)} e^{-T_{dSK}s}. \quad (22)$$

where T_{dSK} is a time delay:

$$T_{dSK} = 1,3t_{35,3} - 0,29t_{85,3}, \quad (23)$$

τ_{SK} is a time constant:

$$\tau_{SK} = 0,67(t_{85,3} - t_{35,3}), \quad (24)$$

where $t_{35,3}$ and $t_{85,3}$ are times, in which the value of the response reaches the 35,3% and 85,3% of the final steady-state value (Fig.8).

In case of determined $T_{dSK} < 0$ the time delay is not included in the transfer function.

3. Methods Application on System Examples and Result Comparison

It will be shown by the plotted step-responses, which models describes the system most appropriately and vice versa which of them don't suit for system approximation. For the approximation quality quantification the integral square error criterion J (25) was used.

$$J = \int_0^t e(\vartheta)^2 d\vartheta, \quad (25)$$

where $e(\vartheta)$ is the deviation of the model response from the system response in time ϑ .

Based on this criterion the methods can be sorted according to the quality of system approximation.

Approximation methods were applied on systems:

- 1st order system
- Nonoscillatory 2nd order system with equal time constants
- Nonoscillatory 2nd order system with different time constants
- Nonoscillatory 5th order system with equal time constants
- Nonoscillatory 5th order system with different time constants

3.1. 1st Order System

1st order system transfer function:

$$G_S(s) = \frac{2}{(0,5s + 1)}. \quad (26)$$

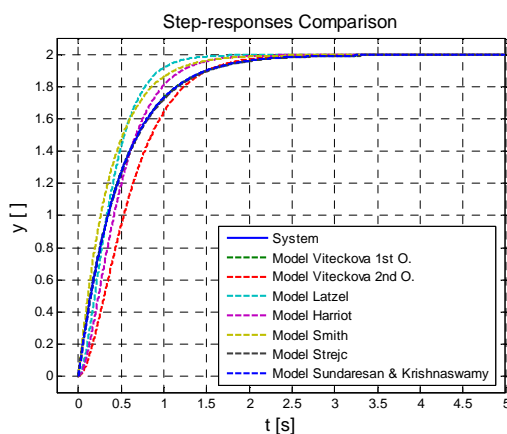


Fig.9 – Comparison of model and system step-responses for 1st order system

Tab. 4 – Arrangement of the methods according to criterion J

| Method | Transfer Function | $J []$ |
|------------------|---|----------------------|
| Vítečková 1st o. | $G_{V1}(s) = \frac{2}{(0,499s + 1)} e^{-0,0013s}$ | $3,32 \cdot 10^{-6}$ |
| Sund. & Krish. | $G_{SK}(s) = \frac{2}{(0,496s + 1)} e^{-0,0053s}$ | $5,89 \cdot 10^{-5}$ |
| Strejc | $G_{ST}(s) = \frac{2}{(0,507s + 1)}$ | $8,59 \cdot 10^{-5}$ |
| Harriott | $G_H(s) = \frac{2}{(0,2s + 1)(0,304s + 1)}$ | $2,61 \cdot 10^{-2}$ |
| Smith | $G_{SM}(s) = \frac{2}{(0,373s + 1)(0,0057s + 1)}$ | $3,43 \cdot 10^{-2}$ |
| Latzel | $G_L(s) = \frac{2}{(0,201s + 1)^2}$ | $4,05 \cdot 10^{-2}$ |
| Vítečková 2nd o. | $G_{V2}(s) = \frac{2}{(0,318s + 1)^2}$ | $8,49 \cdot 10^{-2}$ |

3.2. Nonoscillatory 2nd Order System with Equal Time Constants

Nonoscillatory 2nd order system with equal time constants transfer function:

$$G_s(s) = \frac{2}{(0,5s + 1)^2} \quad (27)$$

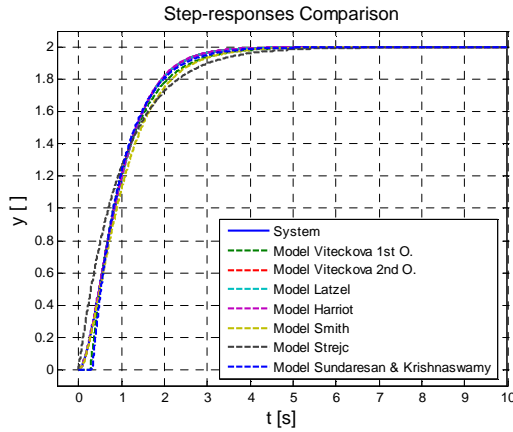


Fig.10 – Comparison of model and system step-responses

Tab. 5 – Arrangement of the methods according to criterion J

| Method | Transfer Function | J [] |
|------------------|---|----------------------|
| Latzel | $G_L(s) = \frac{2}{(0,5s + 1)^2}$ | $7,92 \cdot 10^{-7}$ |
| Vítečková 2nd o. | $G_{V_2}(s) = \frac{2}{(0,5s + 1)^2}$ | $9,74 \cdot 10^{-7}$ |
| Harriott | $G_H(s) = \frac{2}{(0,487s + 1)(0,509s + 1)}$ | $4,85 \cdot 10^{-5}$ |
| Vítečková 1st o. | $G_{V_1}(s) = \frac{2}{(0,784s + 1)} e^{-0,276s}$ | $8,46 \cdot 10^{-3}$ |
| Smith | $G_{SM}(s) = \frac{2}{(0,743s + 1)(0,350s + 1)}$ | $1,02 \cdot 10^{-2}$ |
| Sund. & Krish. | $G_{SK}(s) = \frac{2}{(0,722s + 1)} e^{-0,316s}$ | $1,16 \cdot 10^{-2}$ |
| Strejc | $G_{ST}(s) = \frac{2}{(1,01s + 1)}$ | $5,50 \cdot 10^{-2}$ |

3.3. Nonoscillatory 2nd Order System with Different Time Constants

Nonoscillatory 2nd order system with different time constants transfer function:

$$G_s(s) = \frac{2}{(0,1s + 1)(s + 1)} \quad (28)$$

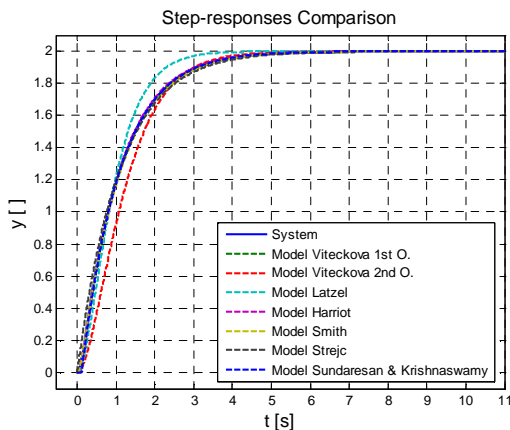


Fig.11 – Comparison of model and system step-responses

Tab. 6 – Arrangement of the methods according to the criterion J

| Method | Transfer Function | J [] |
|------------------|--|----------------------|
| Harriott | $G_H(s) = \frac{2}{(0,975s + 1)(0,115s + 1)}$ | $2,38 \cdot 10^{-4}$ |
| Vítečková 1st o. | $G_{V_1}(s) = \frac{2}{(0,1s + 1)} e^{-0,105s}$ | $4,38 \cdot 10^{-4}$ |
| Smith | $G_{SM}(s) = \frac{2}{(1,03s + 1)(0,0932s + 1)}$ | $6,13 \cdot 10^{-4}$ |
| Sund. & Krish. | $G_{SK}(s) = \frac{2}{(0,993s + 1)} e^{-0,117s}$ | $8,64 \cdot 10^{-4}$ |
| Strejc | $G_{ST}(s) = \frac{2}{(1,1s + 1)}$ | $7,26 \cdot 10^{-3}$ |
| Latzel | $G_L(s) = \frac{2}{(0,486s + 1)^2}$ | $3,49 \cdot 10^{-2}$ |
| Vítečková 2nd o. | $G_{V_2}(s) = \frac{2}{(0,639s + 1)^2}$ | $8,42 \cdot 10^{-2}$ |

3.4. Nonoscillatory 5th Order System with Equal Time Constants

Nonoscillatory 5th order system with equal time constants transfer function:

$$G_s(s) = \frac{2}{(0,5s + 1)^5}. \quad (29)$$

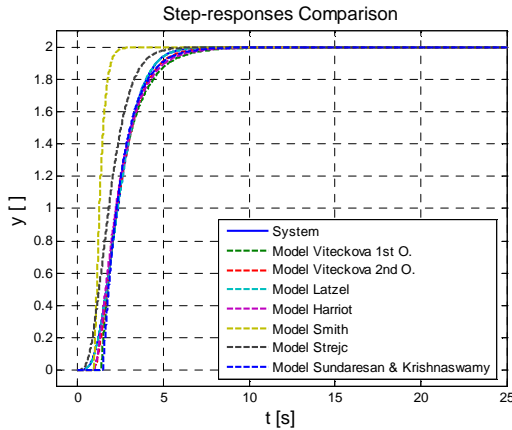


Fig.12 – Comparison of model and system step-responses

Tab. 7 – Arrangement of the methods according to criterion J

| Method | Transfer Function | J [] |
|------------------|--|----------------------|
| Latzel | $G_L(s) = \frac{2}{(0,501s + 1)^5}$ | $1,23 \cdot 10^{-5}$ |
| Vítečková 2nd o. | $G_{V_2}(s) = \frac{2}{(0,838s + 1)^2} e^{-0,906s}$ | $7,83 \cdot 10^{-3}$ |
| Harriott | $G_H(s) = \frac{2}{(0,437s + 1)(1,22s + 1)} e^{-0,906s}$ | $1,48 \cdot 10^{-2}$ |
| Vítečková 1st o. | $G_{V_1}(s) = \frac{2}{(1,31s + 1)} e^{-1,37s}$ | $3,89 \cdot 10^{-2}$ |
| Sund. & Krish. | $G_{SK}(s) = \frac{2}{(1,14s + 1)} e^{-1,48s}$ | $4,92 \cdot 10^{-2}$ |
| Strejc | $G_{ST}(s) = \frac{2}{(1,14s + 1)^4}$ | $2,99 \cdot 10^{-1}$ |
| Smith | The Method doesn't suit for the system approximation. | |

3.5. Nonoscillatory 5th Order System with Different Time Constants

Nonoscillatory 5th order system with different time constants transfer function:

$$G_s(s) = \frac{2}{(0,2s + 1)(0,4s + 1)(0,6s + 1)(0,8s + 1)(1s + 1)}. \quad (30)$$

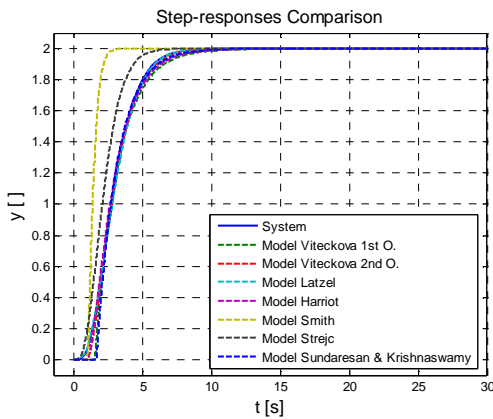


Fig.13 – Comparison of model and system step-responses

Tab. 8 – Arrangement of the methods according to criterion J

| Method | Transfer Function | J [] |
|------------------|--|----------------------|
| Latzel | $G_L(s) = \frac{2}{(0,758s + 1)^4}$ | $1,55 \cdot 10^{-3}$ |
| Vítečková 2nd o. | $G_{V_2}(s) = \frac{2}{(1,08s + 1)^2} e^{-0,913s}$ | $5,08 \cdot 10^{-3}$ |
| Harriott | $G_H(s) = \frac{2}{(0,563s + 1)(1,57s + 1)} e^{-0,913s}$ | $1,28 \cdot 10^{-2}$ |
| Vítečková 1st o. | $G_{V_1}(s) = \frac{2}{(1,69s + 1)} e^{-1,51s}$ | $3,85 \cdot 10^{-2}$ |
| Sund. & Krish. | $G_{SK}(s) = \frac{2}{(1,5s + 1)} e^{-1,64s}$ | $5,13 \cdot 10^{-2}$ |
| Strejc | $G_{ST}(s) = \frac{2}{(1,5s + 1)^4}$ | $4,77 \cdot 10^{-1}$ |
| Smith | The Method doesn't suit for the system approximation. | |

4. Software

The software *Prechar* was designed to enable the system approximation and the comparison of system step-response and generated models step-responses.

The software was implemented in the graphical interface MATLAB/GUI, to make its operation more users friendly and to provide its tools also to those, who don't have wide

MATLAB experience. All tools can be operated simply by clicking the mouse on each operation dedicated button. The main software window can be seen on the fig. 14.

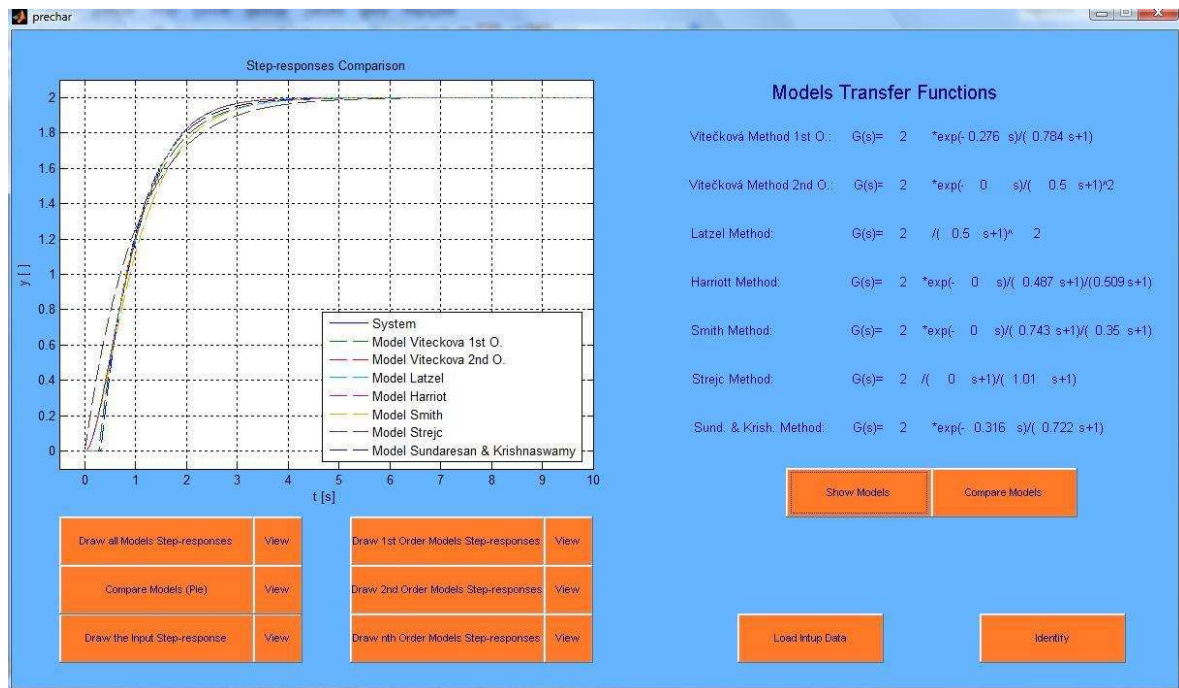


Fig.14 – The Prechar software window with plotted step-responses comparison and viewed models transfer functions

The software can be started by typing the *Prechar* command into the MATLAB's command window. The only active button is the *Load Input Data* button, which is used to load the step-response of the input system, stored in a data file **.mat*. The data file must contain three vectors that define the system step-response. They must be named t , u , y and must be in the incremental form Δt , Δu , Δy . In case of different denomination of input vectors or their absolute form the program won't work properly.

After the input data are loaded the system identification can start by clicking the button *Identify*. When the program is running the window *Computing* is displayed, which informs about the actual software state.

After all computations are finished the software plots the system and models step-responses comparison. By clicking on the appropriate button the plot can be switched to a different one, which enables the of 1st order or 2nd order or n^{th} and model step-responses comparison. The pie plot comparing the methods according to the computed criterion J can be also displayed. By clicking the button *View* the corresponding plot is displayed in the new figure, which enables further operation or saving the plot.

By clicking the button *Show Models* the approximated models transfer function can be displayed. This screen can be switched into the model comparison according to the determined criterion J by clicking the button *Compare Models*. The conclusion about the applicability of the determined models can be made on the base of this table.

5. Conclusion

The overview and approaches of the most applied system approximation methods based on its step-response were given in opening chapters.

The applicability of chosen methods was explored with the help of the software *Prechar*.

On the base of the plotted and compared step-responses of the system and models can be presented that the most suitable methods for approximation of systems, which can be indicated as the 1st order systems, was found the Vítečková's 1st o. method (Fig.9, Tab.4). Other two methods (Strejc's method and Sundaresan's & Krishnaswamy's method), which enable the system approximation by the 1st order model are less accurate, but still applicable. The 1st order system approximation by other chosen methods is not recommended.

System which can be indicated as 2nd order systems can be approximated most successfully by those methods, which enable the system approximation by the 2nd order system. The best applicable methods of them are the Latzel's method and the Vítečková's 2nd o. method (Fig.10, Tab.5). In case of one time constant domination the Harriott's method and Smith's method are the best suitable ones (Fig.11, Tab.6). Thus they enable to approximate the system by 2nd order model with different time constants. Pretty well applicable are also the 1st order model methods (Vítečková's 1st o. method and Sundaresan's & Krishnaswamy's method). Bad results shows the Strejc's method, which is not recommended for such a system approximation.

Systems which can be indicated as higher order system can be best approximated by the Latzel's method (Fig.12, Fig. 13, Tab.7 and Tab.8), which identifies the system by the nth order model with equal time constants. On the other hand bad results were frequently observed in case of the Strejc's method assessment. Other chosen methods usually approximate the nth order system successfully, with an exception of the Smith's method, which wasn't originally designed for such systems approximation.

6. References

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