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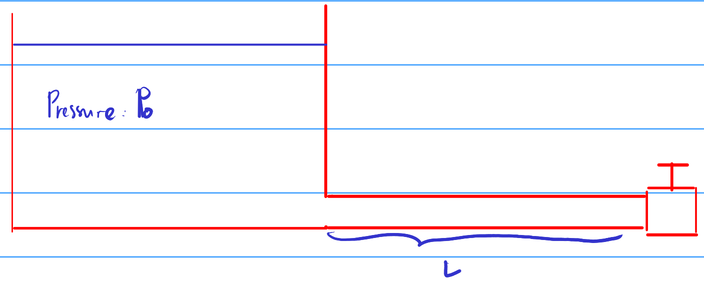
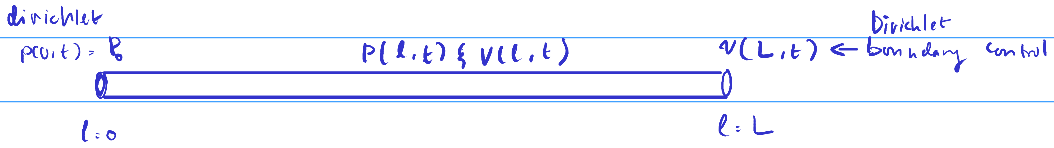
Waterhammer





$$\boxed{\begin{aligned} \frac{\partial v}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial l} - \frac{f}{2D}|v|v \\ \frac{\partial p}{\partial t} &+ \rho c^2 \frac{\partial v}{\partial l} = 0 \end{aligned}}$$

$v(l,t)$ ⇒ velocity at location l and time t

$p(l,t)$ ⇒ pressure at location l and time t

* Space discretization: $l=0$, $l=\Delta l$, …, $l = L = m\Delta l$; $i=0$, $i=1$, …, $i=m$ ← m equally-spaced segments
* time discretization: $t = k\Delta t$, $k = 0, 1, \dots$

using central differences method. (Referred paper uses backward/forward difference method. The results are more noisy)

For the velocity:

$$\frac{v(i,k+1) - v(i,k)}{\Delta t} = -\frac{1}{\rho}\,\frac{p(i+1,k) - p(i-1,k)}{2\Delta l} - \frac{f}{2D}|v(i,k)|v(i,k)$$

$$v(i,k+1) = \frac{\Delta t}{\rho}\,\frac{p(i+1,t) - p(i-1,k)}{2\Delta l} + \frac{\Delta t f}{2D}|v(i,k)|\,v(i,k) + v(i,k)$$

For the Pressure

$$\frac{p(i,k+1) - p(i,k)}{\Delta t} = -\rho c^2\,\frac{v(i+1,k) - v(i-1,k)}{\Delta l}$$

$$p(i,k+1) = -\rho c^2 \Delta t\,\frac{v(i+1,k) - v(i-1,k)}{\Delta l} + p(i,k)$$

Initial Conditions

$$p(l,0) = P_0 - \frac{2\rho f}{D}\,l$$

$$v(l,0) = u_{max}$$

Upstream Boundary Conditions

$$p(0,k+1) = P_0 = p(0,k)$$

$$v(0,k+1) = -\frac{\Delta t}{\rho\Delta l}\left(p(1,k) - P_0\right) - \frac{\Delta t f}{2D}|v(0,k)|\,v(0,k)$$

Downstream boundary condition

$$v(m, k+1) = \tau(k)\, u_{max}$$

τ_k is the command given to the valve, where $0 \leq \tau \leq 1$

$$P(m, k+1) = -\frac{\rho c^2 \Delta t}{\Delta l}\left\{ v(m,k) - v(m-1,k) \right\} + p(m,k)$$

Nomenclature

p : pressure (Pa)

v : velocity (m/s)

c : wave velocity (m/s)

ρ : flow density (kg/m^3)

f : Darcy-Weisbach friction factor (S)

D : pipe diameter (m)