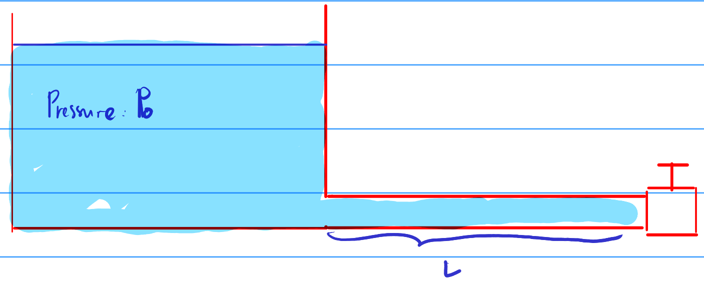
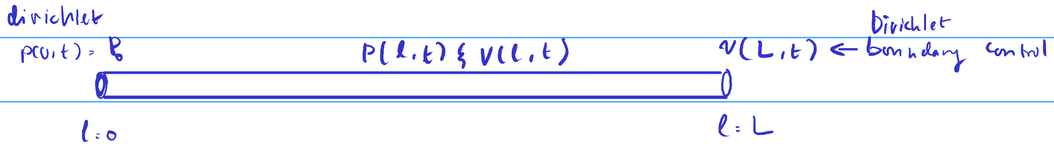
24 Des 2021

Aurelius Manurung, ME - Universitas Pertamina

manurunga@yandex.com

Waterhammer





$$\boxed{\begin{aligned} \frac{\partial v}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial l} - \frac{f}{2D}|v|v \\ \frac{\partial p}{\partial t} &+ \rho c^2 \frac{\partial v}{\partial l} = 0 \end{aligned}}$$

$v(l,t) \Rightarrow$ velocity at location l and time t

$p(l,t) \Rightarrow$ pressure at location l and time t

* Space discretization: $l=0$, $l=\Delta l$, $l = L = m\Delta l$; $i=0$, $i=1$, ..., $i=m$ $\leftarrow$ m equally-spaced segments
* time discretization: $t = k\Delta t$, $k = 0, 1, \ldots$

using central differences method. (Referred paper uses backward/forward difference method. The results are more noisy)

For the velocity:

$$\frac{v(i,k+1) - v(i,k)}{\Delta t} = -\frac{1}{\rho}\frac{p(i+1,k) - p(i-1,k)}{2\Delta l} - \frac{f}{2D}|v(i,k)|v(i,k)$$

$$v(i,k+1) = \frac{\Delta t}{\rho}\frac{p(i+1,t) - p(i-1,k)}{2\Delta l} + \frac{\Delta t f}{2D}|v(i,k)|v(i,k) + v(i,k)$$

For the Pressure

$$\frac{p(i,k+1) - p(i,k)}{\Delta t} = -\rho c^2 \frac{v(i+1,k) - v(i-1,k)}{\Delta l}$$

$$p(i,k+1) = -\rho c^2 \Delta t \frac{v(i+1,k) - v(i-1,k)}{\Delta l} + p(i,k)$$

Initial Conditions

$$p(l,0) = P_0 - \frac{2\rho f}{D} l$$

$$v(l,0) = u_{max}$$

Upstream Boundary Conditions

$$p(0,k+1) = P_0 = p(0,k)$$

$$v(0,k+1) = -\frac{\Delta t}{\rho \Delta l}\left(p(1,k) - P_0\right) - \frac{\Delta t f}{2D}|v(0,k)|v(0,k)$$

Downstream boundary condition

$$v(m,k+1) = \tau(k)\, u_{max}$$

$\boxed{\tau_k \text{ is the command given to the valve, where } 0 \le \tau \le 1}$

$$P(m,k+1) = -\frac{\rho c^2 \Delta t}{\Delta L}\left\{v(m,k) - v(m-1,k)\right\} + p(m,k)$$

Nomenclature

p: pressure (Pa)	ρ: flow density (kg/m^3)
v: velocity (m/s)	f: Darcy-Weisbach friction factor
c: wave velocity (m/s)	D: pipe diameter (m)

Simulation Scenario and parameters (see sec. 4 of the paper)

$P_0 = 2 \cdot 10^5\ Pa$ $\qquad T = 10$ seconds

$L = 200\,m$

$D = 100\,mm$

$\rho = 1000\ kg/m^3$

$C = 1200\ m/s$

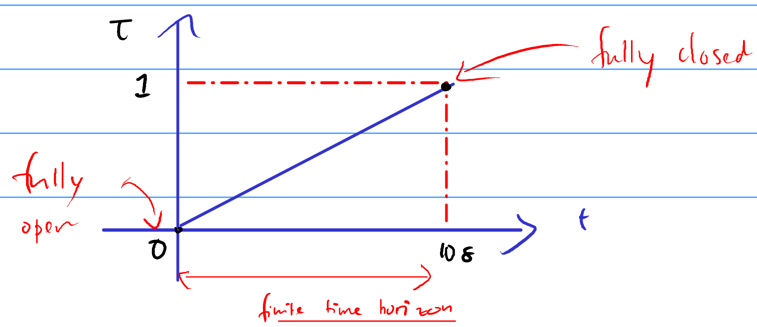
$f = 0.03$

τ: valve closing function of time

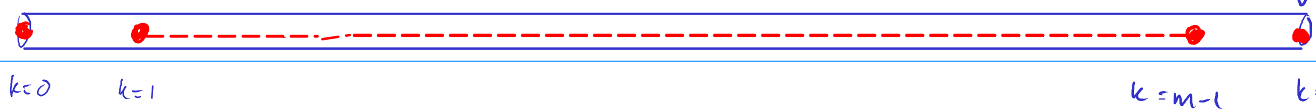
$\tau(t) = 1 \longrightarrow$ valve is fully closed

$\tau(t) = 0 \longrightarrow$ valve is fully open

Constant closure rate as "control" simulation



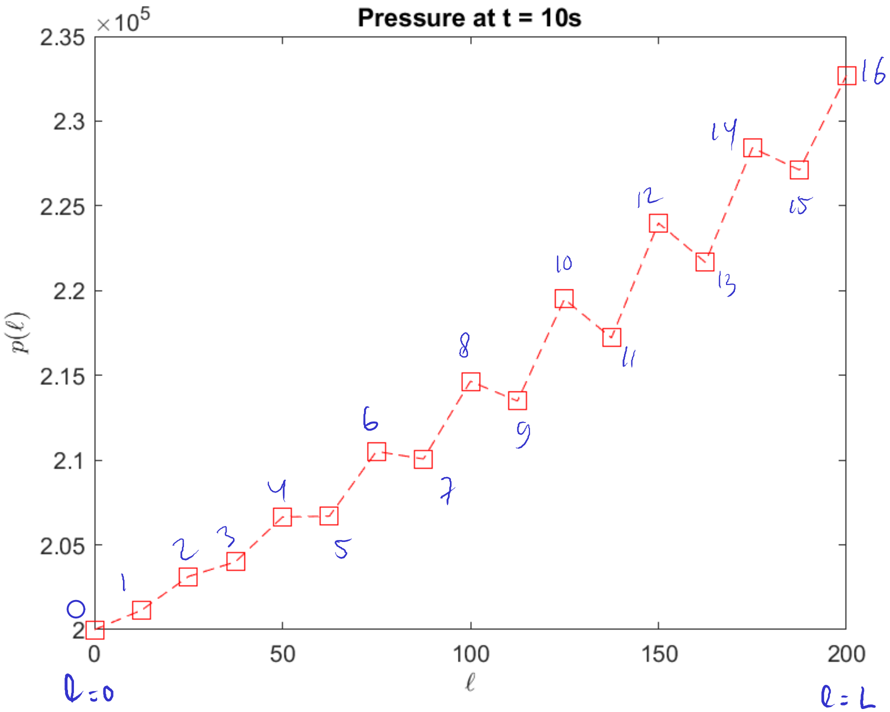
valve is slowly closed with a constant closure-rate

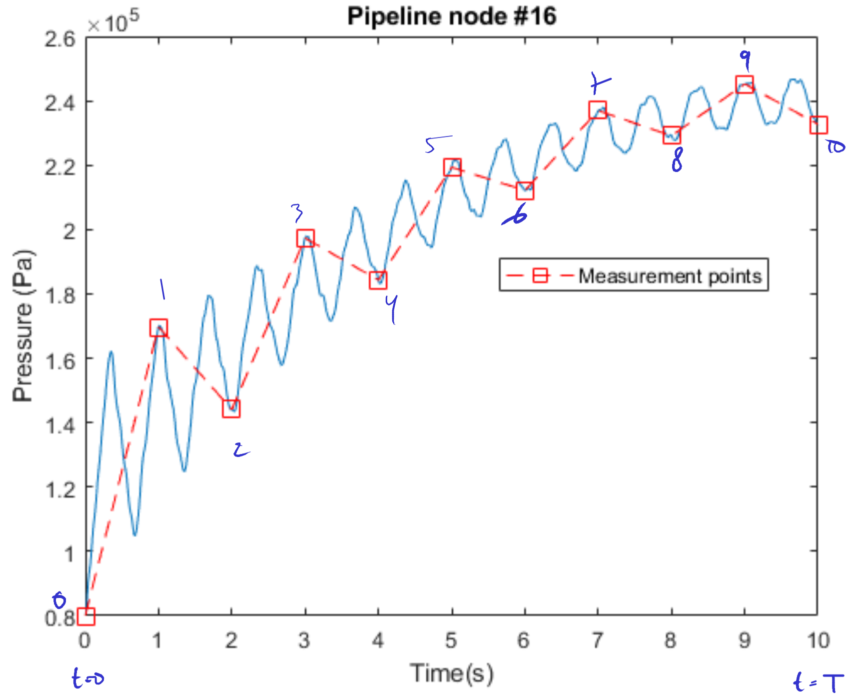


Let us take $m = 16$, and simulation time $T = 10$ seconds

For the optimization process, we will record the pressure data every 1 second!

Later, this pressure data will be used as basis to the cost function.





Let us use the fmincon - sqp in MATLAB. The input to the valve is a piecewise - linear function that is updated every 1 second.

Cost function :

$$J = \overbrace{\frac{1}{T}\int_0^T \left(\frac{p(0,t)-\hat{p}(\ell)}{\Delta p}\right) dt}^{\text{pipeline beginning}} + \overbrace{\frac{1}{T}\int_0^T \left(\frac{p(L,t)-\hat{p}(\ell)}{\Delta p}\right) dt}^{\text{pipeline end}} + \underbrace{\frac{1}{LT}\int_0^T\int_0^L \left\{\frac{p(\ell,t)-\hat{p}(\ell)}{\Delta p}\right\} d\ell\, dt}_{\text{pipeline interior}}$$

Δp : Allowable deviation $= 1.10^4$ Pa

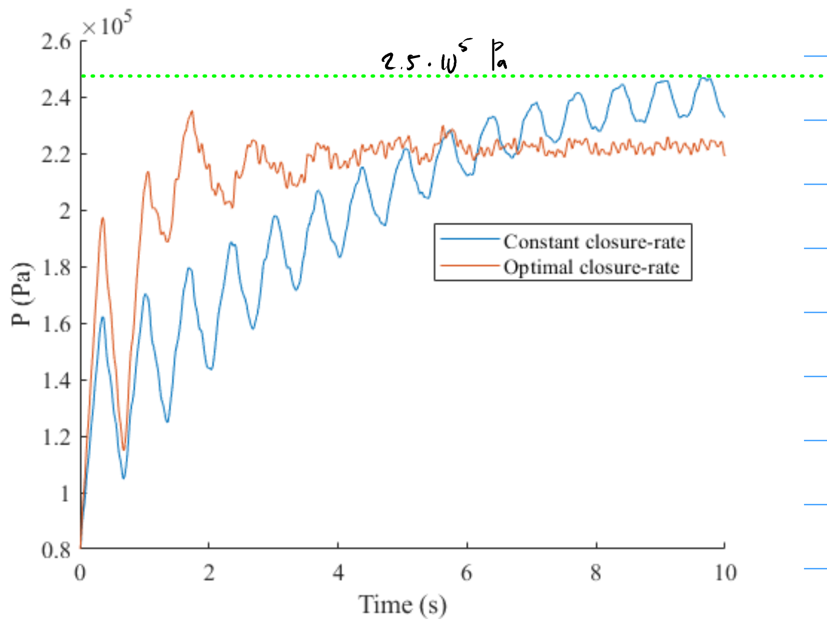
$\hat{p}$: desired / reference pressure along the pipe $= 2.10^5$ Pa

see:

VARIATIONAL PROBLEM OF WATER-LEVEL STABILIZATION IN OPEN CHANNELS
By Gennadii A. Atanov, Elena G. Evseeva and Paul A. Work

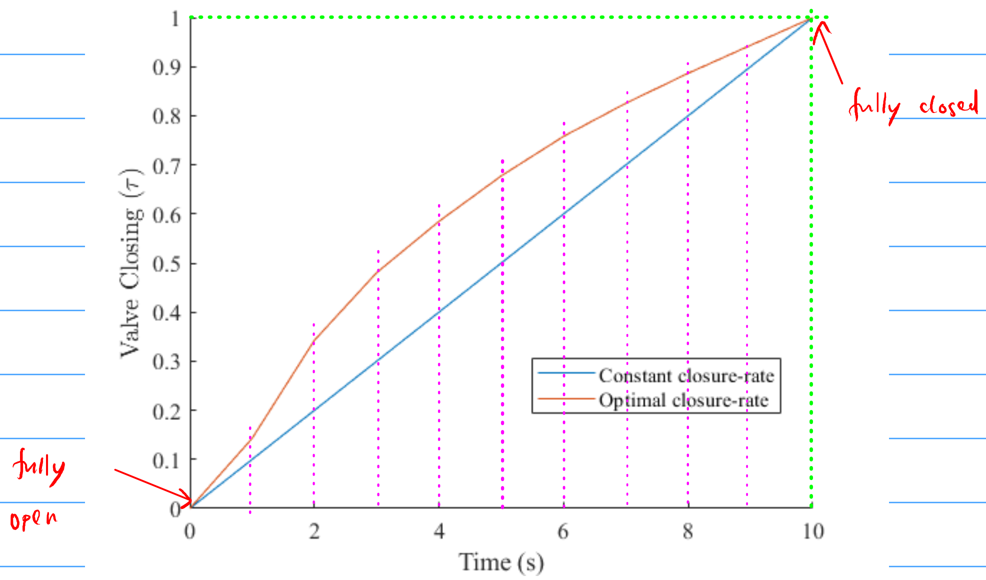
To this point, we ingnore the gradient of the objective function above. MATLAB will compute its gradient numerically. Of course, this means MATLAB will take more iterations to find the optimal solutions.

The Results :



Pressure at the pipeline terminus (node #16)

Optimal closure rate generate less oscillations at the pipeline terminus,



The optimal piecewise linear control, updated every 1 second