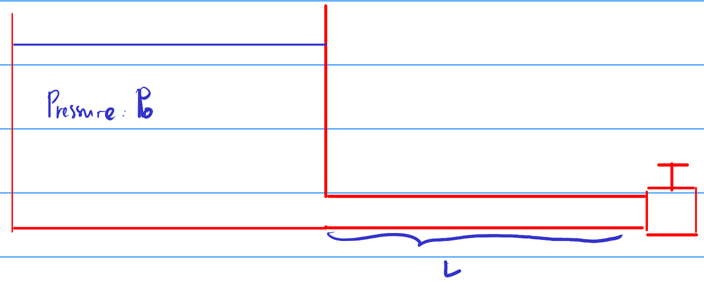
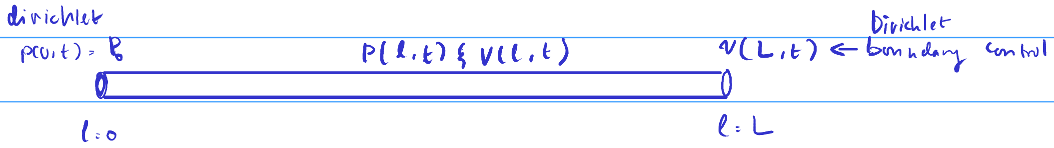
Waterhammer





$$\boxed{\begin{aligned} \frac{\partial v}{\partial t} &= -\frac{1}{\rho}\frac{\partial p}{\partial l} - \frac{f}{2D}|v|v \\ \frac{\partial p}{\partial t} &+ \rho c^2 \frac{\partial v}{\partial l} = 0 \end{aligned}}$$

$v(l,t) \Rightarrow$ velocity at location l and time t

$p(l,t) \Rightarrow$ pressure at location l and time t

$l=0$ $l=\Delta l$ $l=L=m\Delta l$

* Space discretization: $i=0$ $i=1$ ... $i=m$ ← m equally-spaced segments

* time discretization: $t = k\Delta t$, $k=0,1,\ldots$

using central differences method. (Referred paper uses backward/forward difference method. The results are more noisy)

For the velocity:

$$\frac{V(i,k+1)-V(i,k)}{\Delta t} = -\frac{1}{\rho}\frac{p(i+1,k)-p(i-1,k)}{2\Delta l} - \frac{f}{2D}|v(i,k)|v(i,k)$$

$$v(i,k+1) = \frac{\Delta t}{\rho}\frac{p(i+1,t)-p(i-1,k)}{2\Delta l} + \frac{\Delta t f}{2D}|v(i,k)|\,v(i,k) + v(i,k)$$

For the pressure

$$\frac{p(i,k+1)-p(i,k)}{\Delta t} = -\rho c^2 \frac{v(i+1,k)-v(i-1,k)}{\Delta l}$$

$$p(i,k+1) = -\rho c^2 \Delta t\frac{v(i+1,k)-v(i-1,k)}{\Delta l} + p(i,k)$$

Initial Conditions

$$p(l,0) = P_0 - \frac{2\rho f}{D} l$$

$$v(l,0) = u_{max}$$

Upstream Boundary Conditions

$$p(0,k+1) = P_0 = p(0,k)$$

$$v(0,k+1) = -\frac{\Delta t}{\rho\Delta l}\left(p(1,k)-P_0\right) - \frac{\Delta t f}{2D}|v(0,k)|\,v(0,k)$$

Downstream boundary condition

$v(m, k+1) = \tau(k)\, u_{max}$ τ_k is the command given to the valve, where $0 \leq \tau \leq 1$

$$p(m, k+1) = -\frac{\rho c^2 \Delta t}{\Delta l}\left\{v(m,k) - v(m-1,k)\right\} + p(m,k)$$

Nomenclature

p: pressure (Pa)

v: velocity (m/s)

c: wave velocity (m/s)

ρ: flow density

f: Darcy-Weisbach friction factor