IIT CS536: Science of Programming

Homework 2: State and IMP
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Task 1.1

$$\sigma = \{x = 5, y = 2, z = 1, a = [8; 2; 5]\}$$

a) $\sigma[x \to 3][x \to 5] = \{x = 3, y = 2, z = 1, a = [8; 2; 5]\}[x \to 5] = \{x = 5, y = 2, z = 1, a = [8; 2; 5]\}$
b) $\sigma[w \to 4](w) = \{x = 5, y = 2, z = 1, a = [8; 2; 5], w = 4\}(w) = 4$
c) $\sigma[y \to 7][w \to 8] = \{x = 5, y = 7, z = 1, a = [8; 2; 5]\} \to \{x = 5, y = 7, z = 1, a = [8; 2; 5], w = 8\}$
d) $|\sigma(a)| = |5| = 5$

Task 1.2

a) $\{x = 0\} \vDash \forall y \in \mathbb{Z}. x \leq y^2$ this satisfaction holds for the state $\{x = 0\}$. As square of all integers are greater or equal to 0.
b) $\{x = 2, y = 4\} \vDash \exists x \in \mathbb{Z}. x > y$ this does not hold for the state $\{x = 2, y = 4\}$ as value of x in this state which is 2 is not greater than value of y in this state which is equal to 4.
c) $\{x = 1, y = 2\} \vDash \forall z \in \mathbb{Z}. z > x \to y \cdot z > 0$ this holds for the state $\{x = 1, y = 2\}$. For all integers $z, z > x\ implies\ y \cdot z > 0$. So, in this state $z > 1 \to 2 \cdot z > 0$ if $z > 1$ is false, the implication statement will be true as false implies anything is true. If $z > 1$ is true, then $2 \cdot z$ will always be greater than 0 which means implication is true. So, it holds for the given state.
d) $\{x = 5\} \vDash \exists y \in \mathbb{Z}. 2 \cdot y = x$ this statement does not hold for the given state $\{x = 5\}$. As it suggests that there is y such that $2 \cdot y = 5$, but as we know no integer when multiplied by to is equal to 5 as it is an odd number.

Task 1.3

a) $\vDash \exists x \in \mathbb{Z}. \forall y \in \mathbb{Z}. p$ Holds if for some states σ, it is true that $\sigma[x \to \alpha 1][y \to \alpha 2] \vDash\ p$ for some $\alpha_1 \in \mathbb{Z}$ and all $\alpha_2 \in \mathbb{Z}$.
b) $\vDash \neg(\forall x \in \mathbb{Z}. \exists y \in \mathbb{Z}. q)$ Holds if for is no states σ, it is true that $\sigma[x \to \alpha 1][y \to \alpha 2] \vDash q$ for all $\alpha_1 \in \mathbb{Z}$ and some $\alpha_2 \in \mathbb{Z}$.

Task 2.1

a) $\sigma(x * y) = \sigma(x) * \sigma(y) = 10$
b) $\sigma(if\ x > y\ then\ x - z\ else\ y - z) = \sigma(x - z) = 4$
c) $\sigma(a[z] + x) = \sigma(a[z]) + \sigma(x) = \sigma(a[\sigma(z)]) + \sigma(x) = \sigma(a[1]) + 5 = 2 + 5 = 7$
d) $\sigma(w \vee v) = \sigma(w) * \sigma(v) = T \vee F = T$
e) $\sigma(a[size(a) - z]) = \sigma\big(a\big[\sigma\big(size(a)\big) - \sigma(z)\big]\big) = \sigma(a[2]) = 5$

Task 2.2

$S = x := n; if\ (x \leq 0)\ then\ (x \coloneqq 0)\ else (skip); if\ (size(a) > x)\ then\ \big(while\big(x < size(a)\big)\ do\ a[x] := 0; x \coloneqq x + 1\ od\big)\ else\ (skip)$

Task 2.3

a) $\langle S, \{x = 3, y = 2\}\rangle \to \langle if\ x > y\ then\ x \coloneqq y;\ S\ else\ skip, \{x = 3, y = 2\}\rangle \to \langle x \coloneqq y; S, \{x = 3, y = 2\}\rangle \to \langle skip; S, \{x = 3, y = 2\}[x \mapsto 2]\rangle \to \langle S, \{x = 2, y = 2\}\rangle \to \langle if\ x > y\ then\ x \coloneqq y;\ S\ else\ skip, \{x = 2, y = 2\}\rangle \to \langle skip, \{x = 2; y = 2\}\rangle$

b) $M(S, \sigma) = M(if\ x > y\ then\ x \coloneqq y;\ S\ else\ skip, \{x = 3, y = 2\}) = M(x \coloneqq y; S, \{x = 3, y = 2\}) = \cup_{\sigma^1 \in M(x \coloneqq y, \{x=3, y=2\})} M(S, \sigma^1) = \cup_{\sigma^1 \in \{\{x=3, y=2\}[x \mapsto 2]\}} M(S, \sigma^1) = M(S, \{x = 2, y = 2\}) = M(if\ x > y\ then\ x \coloneqq y;\ S\ else\ skip, \{x = 2, y = 2\}) =< M(skip, \{x = 2, y = 2\}) = \{x = 2, y = 2\}$. So, $M(S, \sigma)$ is the big step semantics for the statement S. It takes statement and state and results in a final state of the program. In the above example I just used recursive version of while statement to show the logic but we can just simply continue evaluating until state does not satisfy the stamen and consider that a final state and the result of the function.

Task 3.1

I have spent around 5 hours for this assignment.