KOÇ UNIVERSITY
College of Engineering

COMP 341: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

Midterm 2

Fall 2018, 15/12/2018

Duration: 105 Minutes

Name: Solutions

ID: 00110001 00110000 00110000

- This exam contains 9 pages including this cover page and 5 questions. Check to see if any pages are missing. Put your initials on the top of every page, in case the pages become separated.
- By submitting this exam, you **agree** to fully comply with Koç University Student Code of Conduct, and accept any punishment in case of failure to comply. If you do not submit this exam on time, you will receive 0 credits.
- The exam is **open book** and **open notes** but you are **not** allowed to use any electronic equipment such as computers and mobile phones.
- You are expected to be provide clear and concise answers. Do your best to make sure that your writing is legible (visibly readable).
- Read each question carefully and make sure to follow instructions. The written instructions take precedence over an answer that the instructor might give you during the exam, unless the instructor makes a class wide announcement.
- Do not write in the table below.
- Good luck!

Question:	1	2	3	4	5	Total
Points:	20	18	20	22	20	100
Score:						

1. (20 points) True or False :
BN: Bayesian Network, NB: Naive Bayes, kNN: k-Nearest Neighbor, HMM: Hidden Markov-Model, PF: Particle Filter, ML: Machine learning, MDP: Markov Decision Process

True Toplogy of BNs encode conditional independence.

True Variable elimination may solve an inference problem in linear time even if enumeration takes exponential time on the same problem

False In likelihood weighting for approximate inference, we start with a full sample, chose a non-evidence variable and resample from the BN.

False HMMs are used to model the uncertainty of action outcomes.

True The risk of overfitting is decreased as the number of neighbors is increased in the KNN approach.

False Linear regression always underfits the data since the linear relationship assumption is too simple.

False The NB approach assumes that the input features are the causes of the output.

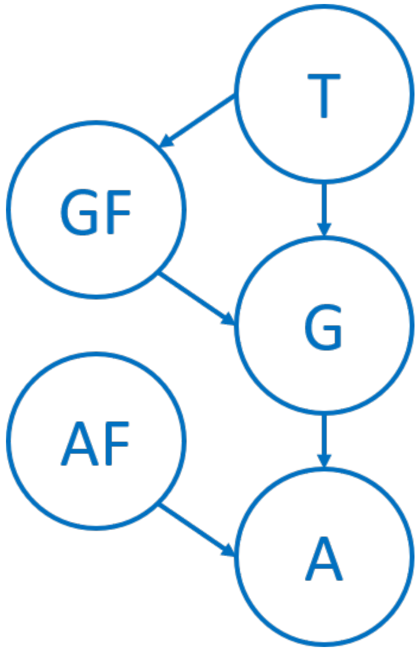
False We always use the training data to tune the hyperparameters of an ML method.

False Values are never calculated in policy iteration.

False It makes sense for Grandpac to use MDP's to hunt ghosts.

2. (18 points) Your local factory has installed an alarm at one of its devices. The alarm (**A**) goes of if the measured temperature (**T**) exceeds a threshold. A gauge (**G**) measures this temperature. The alarm can fail (**AF**). The gauge can also fail (**GF**). The probability of gauge failing increases with temperature.

   (a) (5 points) Draw a **causal** Bayesian Network that fits this description. The nodes are given to you in boldface above. Assume that all the variables are discrete. You do not need to write down any conditional probability tables but write down the form of the conditional probability next to each node (e.g. $P(C|B,D), P(B), \ldots$).



   (b) (1 point) Write down the joint probability distribution of the network you have drawn.
   $P(T, A, G, GF, GA) = P(T)P(GF|T)P(G|GF, T)P(AF)P(A|G, AF)$

   (c) (4 points) Suppose the alarm works correctly unless it is faulty. If it is faulty, it never sounds. Give the conditional probability table associated with **A**.
   There is a missing piece of information here and that is the gauge measurement levels. The next part assumes that there are just two so we will go with that. I will keep this in mind while grading. An apparent approach is to marginalize out G assuming the threshold divides its range evenly (or based on a percentage). (AF = True means the alarm is faulty)

G	AF	A	$P(A\|G, AF)$
Normal	False	OFF	1
Normal	False	ON	0
Normal	True	OFF	1
Normal	True	ON	0
High	False	OFF	0
High	False	ON	1
High	True	OFF	1
High	True	ON	0

(d) (4 points) Suppose there are just two possible actual and measured temperatures, normal and high. The probability that the gauge gives the correct temperature is x when it is working, but y when it is faulty. Give the conditional probability table associated with **G**.

T	GF	G	$P(G\|T, GF)$
Normal	False	Normal	x
Normal	False	High	$1-x$
Normal	True	Normal	y
Normal	True	High	$1-y$
High	False	Normal	$1-x$
High	False	High	x
High	True	Normal	$1-y$
High	True	High	y

(e) (4 points) Assume that all the variables are boolean. Alarm can be on $(+a)$ or off $(-a)$, the devices can be faulty $(+af, +gf)$ or not $(-af, -gf)$, the measured and the real temperatures can be high $(+t, +g)$ or low $(-t, -g)$. You are given the following samples drawn from prior sampling:

- $+a, +t, +g, -af, -gf$
- $-a, +t, +g, +af, -gf$
- $+a, +t, +g, -af, +gf$
- $+a, -t, +g, -af, -gf$
- $-a, -t, +g, -af, +gf$
- $-a, -t, -g, -af, -gf$
- $-a, +t, -g, -af, +gf$
- $-a, +t, -g, +af, +gf$
- $+a, -t, +g, -af, -gf$
- $-a, -t, -g, -af, -gf$

Calculate the following or state why we cannot compute it:

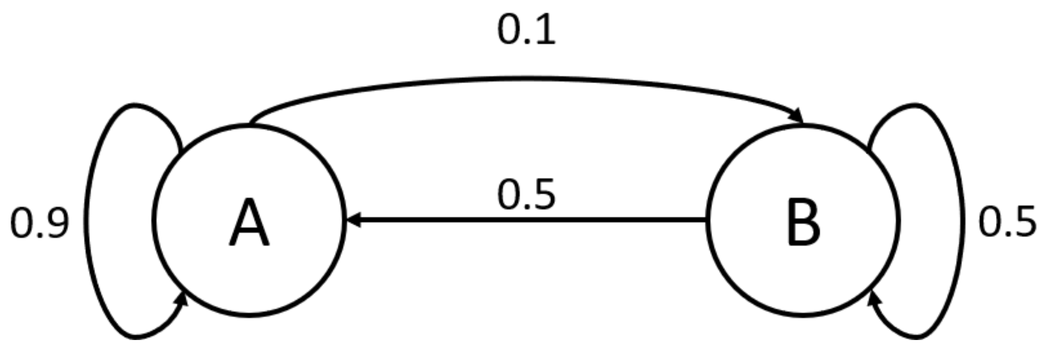
$P(+a|+t)$
$= 2/5 = 0.4$

$P(+gf|+t)$
$= 3/5 = 0.6$

$P(+t|+g, +a)$
$= 2/4 = 0.5$

$P(+a|-g)$
$= 0/4 = 0$

3. (20 points) Consider the two state Markov Model below. Let X denote the current state and t the current time step, then $X_t = A$ or $X_t = B$



(a) (3 points) The prior distribution is uniform, i.e., $P(X_0 = A) = P(X_0 = B) = 0.5$. After one step, what is the state probability distribution (i.e. $P(X_1) =?$)
$P(X_{t+1} = s) = \sum_{s'} P(X_{t+1} = s|X_t = s')P(X_t = s')$

$P(X_1 = A) = P(X_1 = A|X_0 = A)P(X_0 = A) + P(X_1 = A|X_0 = B)P(X_0 = B)) =$
$0.9 \cdot 0.5 + 0.5 \cdot 0.5 = 0.45 + 0.25 = 0.7$
$P(X_1 = B) = P(X_1 = B|X_0 = A)P(X_0 = A) + P(X_1 = B|X_0 = B)P(X_0 = B)) =$
$0.1 \cdot 0.5 + 0.5 \cdot 0.5 = 0.05 + 0.25 = 0.3$

Unfortunately, we cannot directly observe the system states and we have to rely on a sensor. The emission model, $P(E_t|X_t)$, where E_t represents the current sensor measurement, is given below:

X_t	E_t	$P(E_t\|X_t)$
A	A	0.9
A	B	0.1
B	A	0.1
B	B	0.9

(b) (3 points) At $t = 1$, we get $E_1 = A$. Use your answer to part (a) to compute the curent belief distribution $B(X_1)$ (i.e. $B(X_t) = P(X_t|E_{1...t})$).
$B(X_t = s) = P(X_t = s|E_{1...t} = e_{1...t}) \propto P(E_T = e_t|X_t = s)P(X_t = s|E_{1...t-1} = e_{1...t-1})$
$= P(E_T = e_t|X_t = s)B'(X_t = s)$
In this case $B'(X_1) = P(X_1)$ since the first emission we observe is at $t = 1$
$B(X_t = A) = \alpha P(E_1 = A|X_1 = A)P(X_1 = A) = \alpha 0.9 \cdot 0.7 = \alpha 0.63$
$B(X_t = B) = \alpha P(E_1 = A|X_1 = B)P(X_1 = B) = \alpha 0.1 \cdot 0.3 = \alpha 0.03$
Normalize to get:
$B(X_t = A) = 63/66, B(X_t = B) = 3/66$

(c) (3 points) At $t = 2$, we get $E_2 = B$. Use your answer to part (b) to compute the curent belief distribution $B(X_2)$. $B'(X_{t+1} = s) = \sum_{s'} P(X_{t+1} = s|X_t = s')B(X_t = s')$
$B'(X_2 = A) = P(X_2 = A|X_1 = A)B(X_1 = A) + P(X_2 = A|X_1 = B)B(X_1 = B) = 0.9 \cdot 63/66 + 0.5\dot{3}/66 = 97/110$
$B'(X_2 = B) = P(X_2 = B|X_1 = A)B(X_1 = A) + P(X_2 = B|X_1 = B)B(X_1 = B) = 0.1 \cdot 63/66 + 0.5\dot{3}/66 = 13/110$
$B(X_2 = A) = P(E_2 = B|X_2 = A)B'(X_2 = A) = 0.1 \cdot 97/110 = 97/1100$
$B(X_2 = B) = P(E_2 = B|X_2 = B)B'(X_2 = B) = 0.9 \cdot 13/110 = 117/1100$
Normalize to get:
$B(X_t = A) = 97/214, B(X_t = B) = 117/214$

(d) (5 points) With a uniform prior as in part (a), $E_1 = A$ as in part (b), $E_2 = B$ as in part (c)) and $E_3 = A$, what is the most likely state sequence for $t = 1 \dots 3$?

Let $m(X_t = s) = P(E_t = e_t|X_t = s)max_{s'}(P(X_t = S|X_{t-1} = s')m(X_{t-1} = s'))andm(X_0) = P(X_0)$
$m(X_1 = A) = P(E_1 = A|X_1 = A)max(P(X_1 = A|X_0 = A)m(X_0 = A),$
$P(X_1 = A|X_0 = B)m(X_0 = B)) = 0.9max(0.9 \cdot 0.5, 0.5 \cdot 0.5) = 0.9 \cdot 0.45$ (argmax: $X_0 = A$)
$m(X_1 = B) = P(E_1 = A|X_1 = B)max(P(X_1 = B|X_0 = A)m(X_0 = A),$
$P(X_1 = B|X_0 = B)m(X_0 = B)) = 0.1max(0.5 \cdot 0.5, 0.5 \cdot 0.5) = 0.1 \cdot 0.25$ (both are are equally valid)

To make calculations easier, I am going to multiply each m value with the same constant, α per step. We can start with $\alpha_0 = 1/200$. Then we get: $m(X_1 = A) = \alpha_0 81, m(X_1 = B) = \alpha_0 5$

$m(X_2 = A) = P(E_2 = A|X_1 = A)max(P(X_2 = A|X_1 = A)m(X_1 = A),$
$P(X_1 = A|X_1 = B)m(X_1 = B)) = 0.1max(0.9 \cdot \alpha_0 81, 0.5 \cdot \alpha_0 5) = \alpha_1 9 \cdot 81$ (argmax: $X_1 = A$)
$m(X_2 = B) = P(E_2 = A|X_1 = B)max(P(X_2 = B|X_1 = A)m(X_1 = A),$
$P(X_1 = B|X_1 = B)m(X_1 = B)) = 0.9max(0.1 \cdot \alpha_0 81, 0.5 \cdot \alpha_0 5) = \alpha_1 9 \cdot 81$ (argmax: $X_1 = A$)

$m(X_3 = A) = P(E_3 = A|X_1 = A)max(P(X_3 = A|X_2 = A)P(X_2 = A),$
$P(X_3 = A|X_2 = B)m(X_2 = B)) = 0.9max(0.9 \cdot \alpha_2, 0.5 \cdot \alpha_2) = \alpha_3 81$ (argmax: $X_2 = A$)
$m(X_3 = B) = P(E_3 = A|X_1 = B)max(P(X_3 = B|X_2 = A)P(X_2 = A),$
$P(X_3 = B|X_2 = B)m(X_2 = B)) = 0.1max(0.1 \cdot \alpha_2, 0.5 \cdot \alpha_2) = \alpha_3 5$ (argmax: $X_2 = B$)

When we go back from $t = 3$ by picking the maximum staets, we can see that $X_1 = A, X_2 = A, X_3 = A$ is the most likely sequence

(e) (6 points) Let's go back to $t = 0$. This time we want to do particle filtering. We start with 4 particles as follows $[A, A, B, B]$. The model evolves for 1 step and we get $E_1 = A$. Apply one iteration of particle filtering. Go over the particles with the given order. Show all your work. Use the following uniform random variables whenever you need to for sampling: 0.35, 0.83, 0.67, 0.41, 0.18, 0.81, 0.99, 0.54.

Current State	Time Lapsed State	Observed w	Normalized w	Resampled State
A	A	0.9	9/28	A
A	A	0.9	9/28	A
B	A	0.9	9/28	B
B	B	0.1	1/28	A

First step is to let the time pass, i.e., sample from the transition function. For the first two particles, we have A to A since the firt two uniform samples are below 0.9. Then one of the next B particles transition to A, and the other one to B since one of the next uniform samples is samller than 0.5 and the other one is larger than 0.5. This is followed by updating the weights with the emission probabilities and normalizing them. Based on the particle filtering we have seen in the class, next we re-sample the weights from the cumulative distribution that is described by the normalized weights. With the given weights, 0.99 resamples B, the rest resample A. If you have the order of the normalized weights differently, then there is a chance that you would have sampled all A, which is fine.

4. (20 points) You are given a dataset with two boolean input variables (X_1, X_2) and a boolean output variable (Y). The counts of observations are given below:

X_1	X_2	Y	Counts
1	1	1	4
1	0	1	10
0	1	1	6
0	1	0	3
0	0	0	7

(a) (6 points) Learn a Naive Bayes (NB) Model for this problem. Clearly specify and compute all the required parameters.

NB model: $P(Y, X_1, X_2) = P(Y)P(X_1|Y)P(X_2|Y)$, thus we need to estimate these probabilities. Note that the total number of observations: 4+10+6+3+7 = 30

Y	$P(Y)$
0	$(3+7)/30 = 1/3$
1	$(4+10+6)/30 = 2/3$

X_1	Y	$P(X_1|Y)$
0	0	7/10
0	1	6/20
1	0	3/10
1	1	14/20

X_2	Y	$P(X_2|Y)$
0	0	7/10
0	1	10/20
1	0	3/10
1	1	10/20

(b) (4 points) Does the Naive Bayes assumption hold for this problem? Justify your answer.

The questions asks whether X_1 and X_2 are independet symptoms of Y or not. Check:
$P(X_1, X_2|Y) \stackrel{?}{=} P(X_1|Y)P(X_2|Y)$

X_1	X_2	Y	$P(X_1, X_2|Y)$	$P(X_1|Y)P(X_2|Y)$
0	0	0	7/30	$7/10 \cdot 7/10 = 49/100$
0	0	1	0	$6/20 \cdot 10/20 = 15/100$
0	1	0	3/30	$7/10 \cdot 3/10 = 21/100$
0	1	1	6/30	$6/20 \cdot 10/20 = 15/100$
1	0	0	0	$3/10 \cdot 7/10 = 21/100$
1	0	1	10/30	$14/20 \cdot 10/20 = 35/100$
1	1	0	0	$3/10 \cdot 3/10 = 9/100$
1	1	1	4/30	$14/20 \cdot 10/20 = 35/100$

From the table, it is obvious that $P(X_1, X_2|Y) \neq P(X_1|Y)P(X_2|Y)$, thus the assumption does not hold.

(c) (2 points) What is the posterior probability of the point $[X_1 = 1, X_2 = 0]$? What would be its label?

The question is asking $P(Y|X_1 = 1, X_2 = 0)$ based on the NB model: $P(Y|X_1 = 1, X_2 = 0) = P(Y, X_1 = 1, X_2 = 0)/P(X_1 = 1, X_2 = 0) = \alpha P(Y, X_1 = 1, X_2 = 0)$
$P(Y = 0, X_1 = 1, X_2 = 0) = P(Y = 0)P(X_1 = 1|Y = 1)P(X_2 = 0|Y = 0) = 1/3 \cdot 3/10 \cdot 7/10 = 21/300$
$P(Y = 1, X_1 = 1, X_2 = 0) = P(Y = 1)P(X_1 = 1|Y = 1)P(X_2 = 0|Y = 1) = 2/3 \cdot 14/20 \cdot 10/20 = 70/300$
Normalize to get the conditionals:
$P(Y = 0|X_1 = 1, X_2 = 0) = 21/91, P(Y = 1|X_1 = 1, X_2 = 0) = 70/91$
Since $P(Y = 1|X_1 = 1, X_2 = 0) > P(Y = 0|X_1 = 1, X_2 = 0)$, we pick the label as $Y = 1$
Note that we have not observed any $Y = 0$ for the given inputs, yet the corresponding probability is not 0!

(d) (6 points) What is the classification accuracy of the learned NB model with the following test set?

X_1	X_2	Y	Counts
0	0	0	12
0	0	1	1
0	1	1	2
1	0	1	6
1	1	0	2
1	1	1	7

Find the label of each point as was done at part c. Then compare them with the given Y.
Let $f(x_1, x_2) = argmax_{y'} P(Y = y'|X_1 = x_1, X_2 = x2)$, i.e., the prediction of the NB model. Do not use the ground truth values to calculate posteriors!

X_1	X_2	$f(X_1, X_2)$
0	0	0
0	1	1
1	0	1
1	1	1

Total number of test points: $12 + 1 + 2 + 6 + 2 + 7 = 30$. Correct guesses $12 + 2 + 6 + 7 = 27$. Then the accuracy is $27/30$.

(e) (4 points) Comment on the test accuracy. What could be a way to improve the test accuracy, without collecting new data?

The test accuracy is very high. It is the same as the training accuracy! Both the training and the testing accuracy imply something interesting: Either the data collection is noisy and/or the features are not enough. The only answer to this question is to extract additional features. This of course requires some domaion knowledge. There is really no way, other than calculating new features, to increase test accuracy for this particular problem.

5. (20 points) Consider a robot that is surveying a very large planet. The goal of the robot is to cover distance as fast as possible. The robot only has two different speeds; slow and fast. There is a chance that the robot's temperature increases if it goes fast and its temperature decreases when it goes slow. The robot can measure two temperatures; cool and warm. The robot has the limitation that if it moves fast, its engine may overheat if it is already warm. The robot breaks when its engine overheats, preventing it from moving again. Even though it is important to go as fast as possible (to aid scientific discovery), it is more important to keep the robot going (very expensive mission)!

(a) (12 points) Model this domain as a Markov Decision Problem (S, A, T, R, γ). Try to assign meaningful rewards and a meaningful discount factor. You can leave probabilities as variables. Give a complete answer while being clear and concise.

The state space is related to the car being cool, warm or disable. $S = \{cool, warm, broken\}$ (2 points)

The action space is made up of the car's speeds. $A = \{slow, fase\}$ (1 point1)

Transition model $T(s, a, s') = P(s'|s, a)$ is as follows: (6 points, important transitions are from cool and warm. Each entry is 0.75 points)

s	a	s'	P(s' | s, a)
$cool$	$slow$	$cool$	1
$cool$	$fast$	$cool$	x
$cool$	$fast$	$warm$	$1 - x$
$warm$	$slow$	$cool$	y
$warm$	$slow$	$warm$	$1 - y$
$warm$	$fast$	$warm$	z
$warm$	$fast$	$broken$	$1 - z$
$broken$	-	$broken$	1.0

0 probability transitions are not shown. x: probability of staying cool while riding fast, y: probability of cooling down while going fast, z: probability of staying warm while going fast.

$R(cool) = R(warm) = 0, R(broken) = -\inf$, I think you get the idea. Giving warm something smaller than cool is also okay but not necessary (2 points)

$\gamma \in (0, 1)$, doesn't really matter with this formulation as long as it is a positive number smaller than 1. (1 point)

(b) (4 points) Based on your MDP, what are the values of your states for going slow all the time? You may leave your result as a mathematical expression, you do not need to calculate a number.

$V^\pi(s) = R(s) + \sum_{s'} P(s'|s, \pi(s))\gamma V^\pi(s')$ Replacing states and actions with their first letters:
$V^\pi(c) = R(c) + P(c|c, s)\gamma V^\pi(c) + P(w|c, s)\gamma V^\pi(w) = R(c) + \gamma V^\pi(c)$
$(1 - \gamma)V^\pi(c) = R(c) \Rightarrow V^\pi(c) = R(c)/(1 - \gamma) = 0$
This policy would never overheat, thus we do not need to calculate it for that state. If you want to, its solution is analogous to the cool case.

$V^\pi(w) = R(w) + P(c|w, s)\gamma V^\pi(c) + P(w|w, s)\gamma V^\pi(w) = R(w) + y\gamma V^\pi(c) + (1 - y)\gamma V^\pi(w)$
It is easy to leave $V^\pi(w)$ alone in this linear equation. For our choice of rewards, it is 0

(c) (4 points) What would be a reasonable policy for your MDP, other than going slow all the time? Justify your answer. You do not need to do any calculations.

We want to avoid the robot to be broken but also want to explore as fast as possible. Since there is no danger of overheating when the robot is cool, it can go fast, the moment it warms up, it should go slow: $\pi(cool) = fast, \pi(warm) = slow$