KOÇ UNIVERSITY
College of Engineering

COMP 341: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

Midterm 2

Fall 2021, 05/12/2021
Duration: 120 Minutes

Name: Solutions

ID: 00110001 00110000 00110000

- This exam contains 10 pages including this cover page and 5 questions. Check to see if any pages are missing. Put your initials on the top of every page, in case the pages become separated.
- By submitting this exam, you **agree** to fully comply with Koç University Student Code of Conduct, and accept any punishment in case of failure to comply. If you do not submit this exam on time, you will receive 0 credits.
- The exam is **closed book** and **closed notes**. You are **not** allowed to use any additional material including any electronic equipment such as computers and mobile phones.
- You are expected to be able provide clear and concise answers. Gibberish will not receive any credit. Your answers need to be readable by a human, illegible writing is not gradable.
- Read each question carefully and make sure to follow instructions. The written instructions take precedence over an answer that the instructor might give you during the exam, unless the instructor makes a class wide announcement.
- Do not write in the table below.

Question:	1	2	3	4	5	Total
Points:	21	12	24	18	25	100
Score:						

1. (21 points) True or False :

True ______ Expectiminimax and alpha-beta pruning algorithms do not always return the same solution.

False ______ It is enough to keep track of a single utility value for zero-sum games of more than 2 players.

False ______ There cannot be more than one query variable for probabilistic inference.

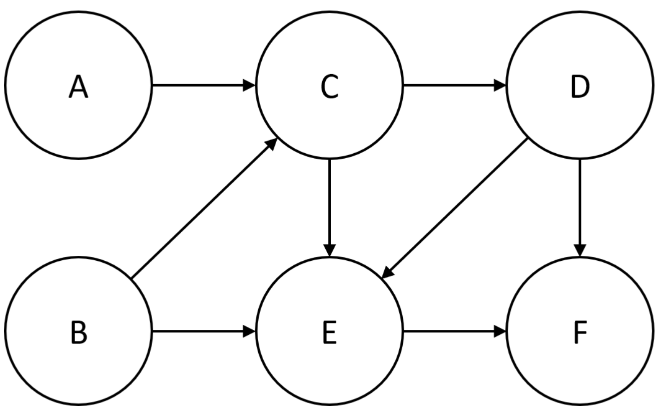
False ______ Probability cannot be used to model deterministic unknowns (e.g. unknown but deterministic ghost behavior in Pacman).

True ______ There are about 2^{16} rows in a joint distribution of 16 boolean variables.

True ______ Specifying only the graph topology of a Bayesian Network is not enough to fully specify a joint distribution.

False ______ Variable ordering does not matter in variable elimination in terms of practical performance.

2. (12 points) Given the Bayesian Network below, answer the questions about conditional independence



True ______ A and B are guaranteed to be independent.

True ______ A and D are guaranteed to be conditionally independent given C.

False ______ A and B are guaranteed to be conditionally independent given D.

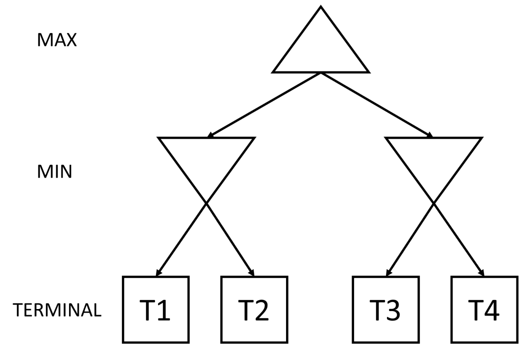
False ______ A and F are guaranteed to be conditionally independent given D.

True ______ A and F are guaranteed to be conditionally independent given D and E.

False ______ A and F are guaranteed to be conditionally independent given C and E.

3. (24 points) Adversarial Search:

   (a) (6 points) Consider the following game tree below



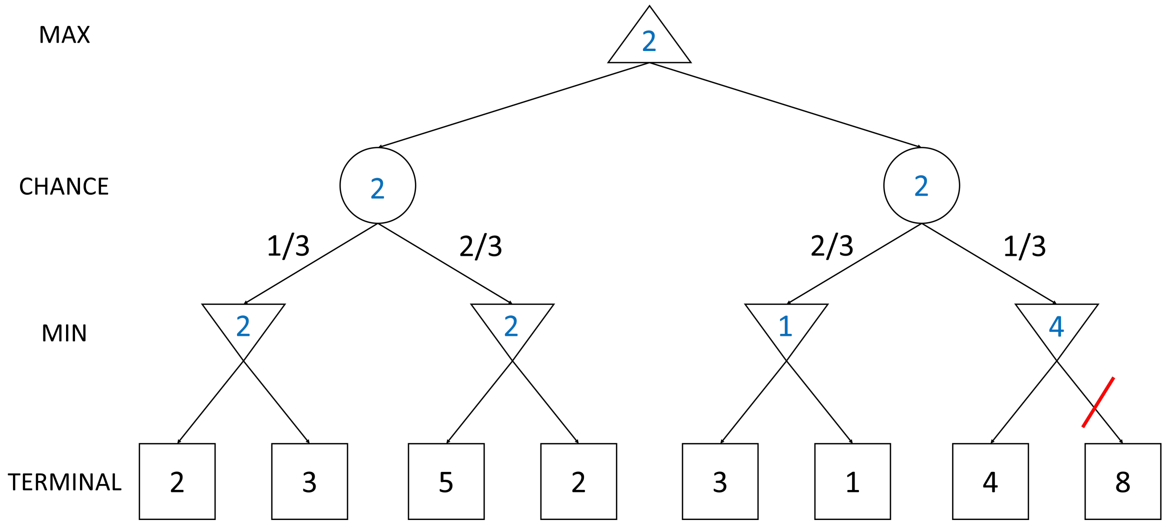
Which terminal nodes (T1,T2,T3,T4) can be pruned with alpha-beta pruning? Give example terminal values for when they can be pruned. If none of them can be pruned, explain why.

- Initially, we have no information and no knowledge on the values of the terminal states. We have to explore T1
- After exploring T1, we still do not know the value of the MIN state (i.e. the best value for min along this path). We have to explore T2 as well
- We would only prune if we knew there is a better option for MIN but we do not know anything about the right node yet so T3 has to be explored as well.
- At this point, if T3 is smaller than the value of the MIN node ($T3 < MIN(T1, T2)$), we know that the right is a better option for MIN so the MAX does not need to explore it further.

   (b) (6 points) Consider the same game tree as part (a). This time, you are also given the information that the **terminal values are guaranteed to be between 0 and 9**. The alpha-beta pruning algorithm needs to be modified to incorporate this. We are leaving it up to you figure out how. (Hint: Modify the value comparison step). Which terminal nodes (T1,T2,T3,T4) can be pruned with alpha-beta pruning with this information? Give example terminal values for when they can be pruned. If none of them can be pruned, explain why.

- Initially, we do not know anything an need to explore T1.
- If $T1 = 0$, then we already know the value of the left MIN state since T2 cannot be smaller than 0. Thus no need to explore it.
- If $T1 = 9$ and $T2 = 9$, we do not need to explore the entirety of the right MIN node (both $T3$ and $T4$) since we already know we can get the maximum outcome when we go left. However, the pseudocode in the slides always explore the first child/successor, in which case T3 is always explored. Thus I will accept both answers.
- If $T3 < MIN(T1, T2)$, we do not need to explore $T4$ which is the same condition as the previous part

(c) (6 points) You are given the game tree with the chance nodes below.



Calculate the values of each state in the game tree (write it next to the nodes)

(d) (6 points) Consider the same game tree as part (c). This time, you are also given the information that the **terminal values are guaranteed to be between 0 and 9**. We cannot prune chance nodes in general. However, we can prune them if we are given a range of state values. The alpha-beta pruning algorithm needs to be modified to incorporate this by testing both values of the range. Show the connections that can be pruned (cross-out the arcs in the figure above)

When we explore the 7th terminal state from the left with the value 4, the worst case for the corresponding parent MIN node becomes 4. With this information, we can say that the parent chance node of this MIN Node would have a value smaller or equal to 2 ($\leq 1 \cdot 2/3 + 4 \cdot 1/3$). The left chance node's value is also 2. Thus, we can say that the right path from the MAX node is either equivalent to the left or better without exploring the last terminal node.

As another example, imagine the left chance node having the value of 4 and that we explored the left MIN node of the right chance node. At this point, even when the two rightmost terminal nodes have the max value of 9, the value of the right chance node would be $2/3 \cdot 1 + 1/3 \cdot 9 < 4$. Thus there would be no need to check the right successor of the right change node.

4. (18 points) Probability: Use the below table to answer the questions

A	**B**	**C**	$P(A, B, C)$
+	+	+	0.11
+	+	-	0.08
+	-	+	0.13
+	-	-	0.07
-	+	+	0.22
-	+	-	0.04
-	-	+	0.14
-	-	-	0.21

(a) (3 points) Calculate $P(B, C)$. Show your work.

B	**C**	$P(B, C)$
+	+	$0.11 + 0.22 = 0.33$
+	-	$0.08 + 0.04 = 0.12$
-	+	$0.13 + 0.14 = 0.27$
-	-	$0.07 + 0.21 = 0.28$

(b) (3 points) Calculate $P(B)$. Show your work.

B	$P(B)$
+	$0.33 + 0.12 = 0.45$
-	$0.27 + 0.28 = 0.55$

(c) (3 points) Calculate $P(C)$. Show your work.

C	$P(C)$
+	$0.33 + 0.27 = 0.6$
-	$0.12 + 0.28 = 0.4$

(d) (3 points) Are B and C independent? Justify your answer

If independent, then $P(B, C) = P(B)P(C)$. Let's test this (for full points, a single comparison is enough)

B	**C**	$P(B,C)$	$P(B)P(C)$	$P(B,C) \approx P(B)P(C)$
+	+	0.33	$0.45 \cdot 0.6 = 0.27$	No
+	-	0.12	$0.45 \cdot 0.4 = 0.18$	No
-	+	0.27	$0.55 \cdot 0.6 = 0.33$	No
-	-	0.28	$0.55 \cdot 0.4 = 0.22$	No

We can conclude that B and C are not independent.

Table repeated here:

A	**B**	**C**	$P(A,B,C)$
+	+	+	0.11
+	+	-	0.08
+	-	+	0.13
+	-	-	0.07
-	+	+	0.22
-	+	-	0.04
-	-	+	0.14
-	-	-	0.21

(e) (6 points) Calculate $P(A|C)$. Show your work.

A	**C**	$P(A\|C)$
+	+	$(0.11 + 0.13)/0.6 = 0.4$
+	-	$(0.08 + 0.07)/0.4 = 0.375$ (will accept 3/8)
-	+	$(0.22 + 0.14)/0.6 = 0.6$
-	-	$(0.04 + 0.21)/0.4)0.625$ (will accept 5/8)

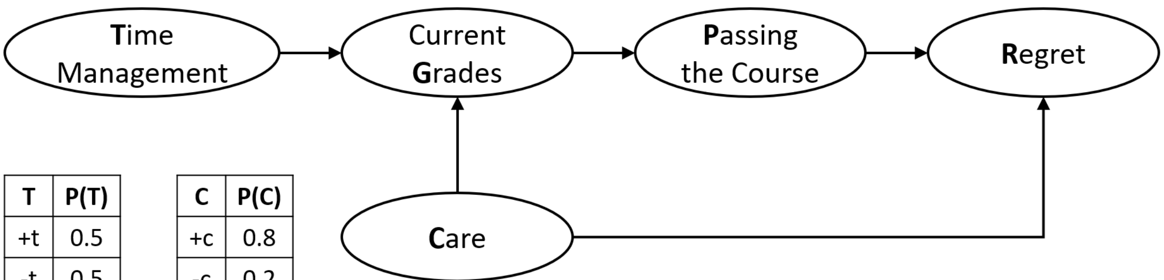
There are two ways of doing this: $P(A|C) = P(A, C)/P(C)$ or normalizing (for A's values to sum up to 1 for each C assignment) $P(A, C)$. We did the former since we already have $P(C)$. Also the placement of the C column maybe misleading, the preffered way to do it is to put all the conditioned variables (in this case A) last.

5. (25 points) BN Exact Inference: You are given the following Bayesian Network (BN) below.

T	C	G	P(G|T,C)
+t	+c	+g	0.9
+t	+c	-g	0.1
+t	-c	+g	0.5
+t	-c	-g	0.5
-t	+c	+g	0.7
-t	+c	-g	0.3
-t	-c	+g	0.2
-t	-c	-g	0.8

G	P	P(P|G)
+g	+p	0.8
+g	-p	0.2
-g	+p	0.3
-g	-p	0.7

C	P	R	P(R|C,P)
+c	+p	+r	0.0
+c	+p	-r	1.0
+c	-p	+r	1.0
+c	-p	-r	0.0
-c	+p	+r	0.0
-c	+p	-r	1.0
-c	-p	+r	0.5
-c	-p	-r	0.5



T	P(T)
+t	0.5
-t	0.5

C	P(C)
+c	0.8
-c	0.2

This BN models whether a student will pass an unnamed course or not, and the associated regret. The variables, which are all binary, are:

1. **T**ime Management: Whether (+t) or not (-t) the student has good time management skills
2. **C**are: Whether the student cares about the course (+c) or not (-c)
3. Current **G**rades: The grades that the student has received so far into the semester. They are modelled as either good (+g) or bad (-g)
4. **P**assing the Course: Whether the student will pass (+p) the course or not (-p).
5. **R**egret: Whether the student feels regret (+r) or not (-r) after the semester ends

Answer the questions below based on this BN.

Important: You can leave the final answer as fractions or multiplications and sumations of values!

Hints:

- Be on the look out for simplifications via conditional independence!
- Read all the inference parts before solving them. There may be shared inference steps. However, do not lose time searching for these.

(a) (2 points) What is the joint probability distribution, $P(T, C, G, P, R)$, of the given Bayesian Network?

$P(T, C, G, P, R) = P(T)P(C)P(G|T, C)P(P|G)P(R|P, C)$

(b) (5 points) The instructor has not released all your grades to you yet. You are a student with good time management skills and care about the course. What is the probability of you passing the course? (Calculate $P(P = +p|T = +t, C = +c)$)

This question has a shortcut. Note that probabilistically $P(P|G)P(G|T, C) = P(P, G|T, C)$. It is even simpler since we have $T = +t$, $C = +c$ and $P = +p$. Thus we can easily calculate $P(P = +p, G|T = +t, C = +c)$ and sum out G!

T	C	G	P	$P(P = +p\|G)P(G\|T = +t, C = +c) = P(P = +p, G\|T = +t, C = +c)$
$+t$	$+c$	$+g$	$+p$	$0.9 \cdot 0.8 = 0.72$
$+t$	$+c$	$-g$	$+p$	$0.1 \cdot 0.3 = 0.03$

Summing out G to get $P(P = +p|T = +t, C = +c) = 0.72 + 0.03 = 0.75$

(c) (9 points) At the end of the semester, you see someone who cared about the class fail. You are curious about their time management skills so you want to calculate $P(T|C = +c, P = -p)$.

T is conditionally independent on R given C and P. So we do not need the factors involving R. We are left with factors $P(T), P(C = +c), P(G|T, C = +c), P(P = -p|G)$. We need to get rid of the hidden variable G first. We did some of the work in the previous part. It is easy to see $P(P = -p|T = +t, C = +c) = 1 - 0.75 = 0.25$. Let's calculate the $T = -t$ versions.

T	C	G	P	$P(P = -p\|G)P(G\|T = +t, C = +c) = P(P = -p, G\|T = +t, C = +c)$
$-t$	$+c$	$+g$	$-p$	$0.7 \cdot 0.2 = 0.14$
$-t$	$+c$	$-g$	$-p$	$0.3 \cdot 0.7 = 0.21$

Summing out G to get $P(P = -p|T = -t, C = +c) = 0.35$

We need to join the remaining (no need to join P(C=+c), since it multiplies everything and will not matter after normalization) and normalize

T	C	P	$P(P = -p\|T, C = +c)P(T)$	Normalized
$+t$	$+c$	$-p$	$0.25 \cdot 0.5 = 0.125$	$0.125/(0.125 + 0.175) = 25/60 \approx 0.417$
$-t$	$+c$	$-p$	$0.35 \cdot 0.5 = 0.175$	$0.175/(0.125 + 0.175) = 35/60 \approx 0.583$

(d) (9 points) You are worried about your friend. Your friend does not care about the course and his current grades are low. You want to calculate the probability of him feeling regret at then end of the semester. Calculate $P(R = +r|C = -c, G = -g)$.

R is conditionally independent on T given C and G. So we do not need the factors involving T. We are left with factors $P(R), P(C = -c), P(P|G = -g), P(R|C, P)$ (We are not doing it just for $R = +r$ in case we need to normalize at the end but as you will see this was not necessary). We need to get rid of the hidden variable P first.

We get rid of P by first joining $P(P|G = -g), P(R|C = -c, P)$ and summing out P.

C	G	P	R	$P(P\|G = -g)P(R\|C = -c, P)$
$-c$	$-g$	$+p$	+r	$0.3 \cdot 0. = 0.0$
$-c$	$-g$	$+p$	-r	$0.3 \cdot 1. = 0.3$
$-c$	$-g$	$-p$	+r	$0.7 \cdot 0.5 = 0.35$
$-c$	$-g$	$-p$	-r	$0.7 \cdot 0.5 = 0.35$

Summing out P

C	G	R	Summed Out
$-c$	$-g$	+r	$0.0 + 0.35 = 0.35$
$-c$	$-g$	-r	$0.3 + 0.35 = 0.65$