COMP 341 Intro to AI
Decision Networks

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Recap

- Uncertainty
  - The real world is uncertain to an agent!
  - Use probabilistic models for representation – Joint Distribution
- Bayesian Networks
  - An intuitive way of representing uncertainty with local conditional distributions
- Inference in BNs: $P(X_q|x_{e_1}, \ldots x_{e_k})$

Stuff you care about → $P(X_q|x_{e_1}, \ldots x_{e_k})$ ← *Stuff you already know*

- Exact Inference: Enumeration
- Approximate Inference: Sampling

Making Decisions

- What do we do with the outcome of an inference query?
- Would you take an umbrella when:
  - No info
  - It is cloudy
  - Forecast says rain
  - It is cloudy and forecast says rain
- Model it as a BN
- Ask the query
- Do inference
- Is the calculated probability enough?

Our Definition of AI

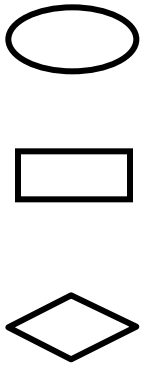
- "Science of making **rational agents**": A **rational** agent selects actions that maximize its (expected) utility
- Utilities in the umbrella example:

Real Weather	Umbrella Decision	Utility
rain	take	
rain	leave	
sunny	take	
sunny	leave	

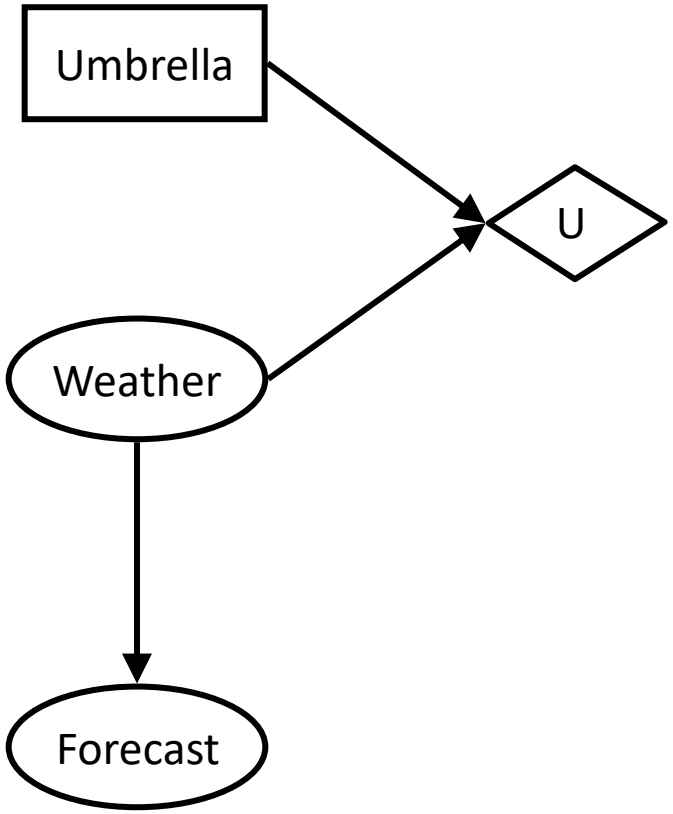
- Expected utility: probability of the outcome X utility of the outcome
- Question: How to put utilities into Bayesian Networks?

Decision Networks: Representation

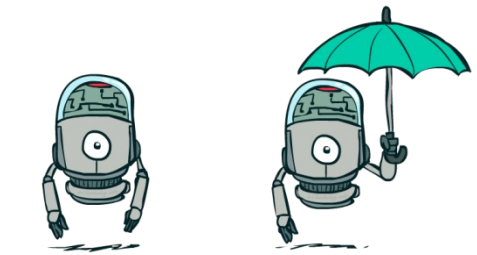
- New node types in the BNs:
  - In addition to regular nodes
  - Actions (rectangles, cannot have parents can be parents, act as observed evidence)
  - Utility node (diamond, depends on action and chance nodes)

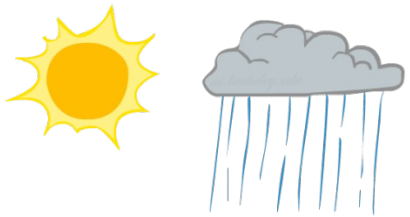


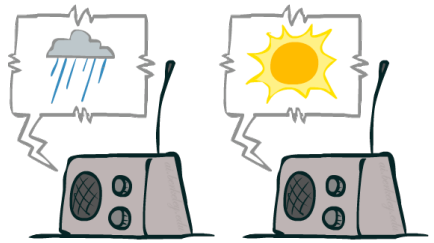
- Info in the nodes:
  - CPTs
  - Available Action List
  - Utility Table

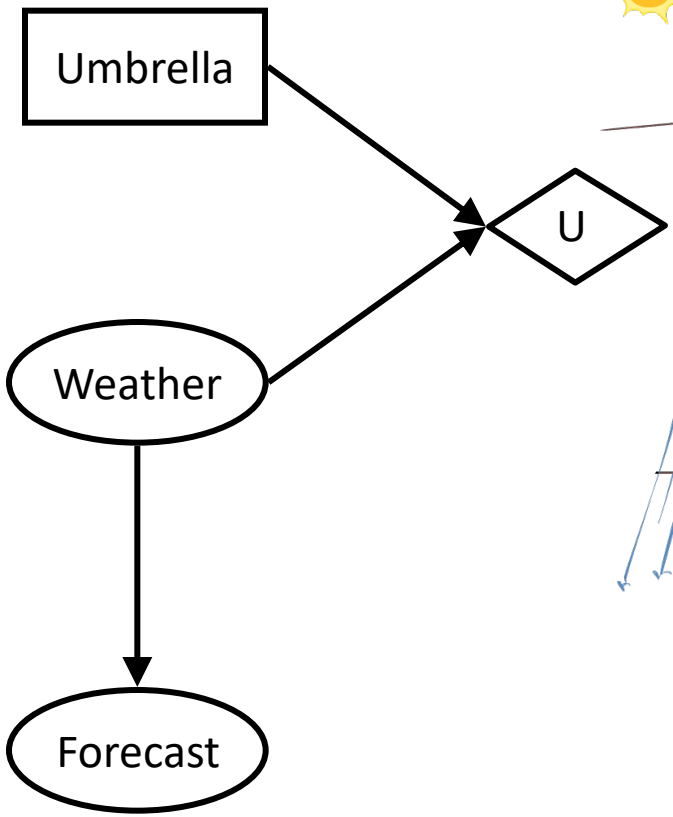


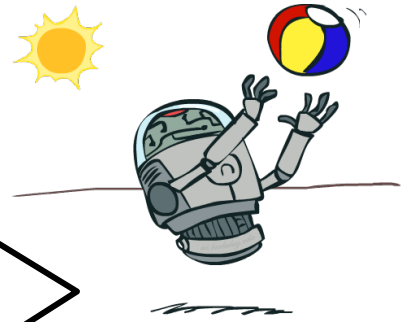
Decision Networks

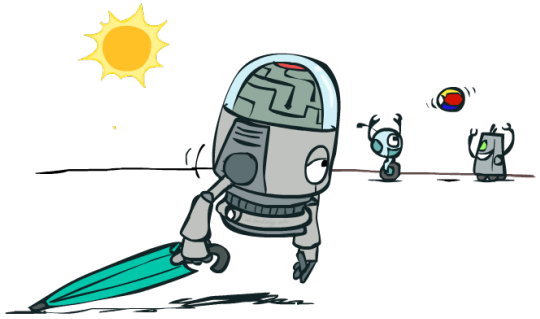


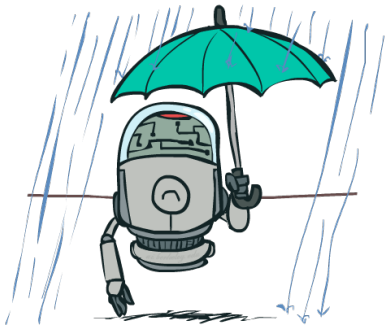












Decision Networks: Expected Utility

Chose actions to "Maximize Expected Utility"

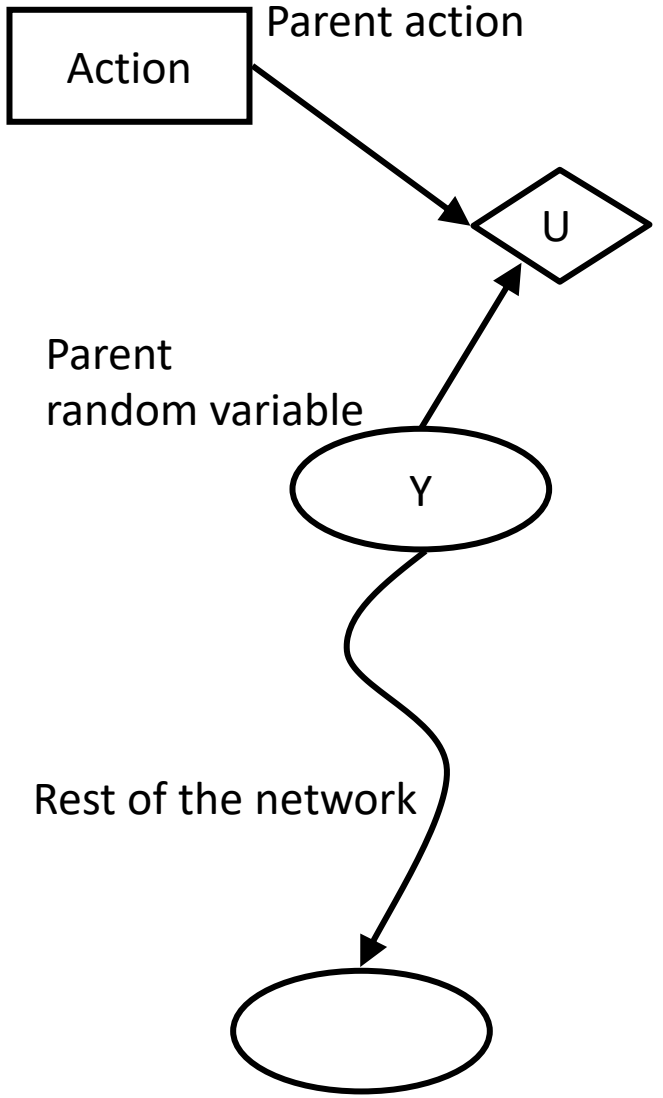
- Expected Utility (EU) of an action

$$EU(action|evidence) = \sum_{y \in Y} P(y_i|evidence)U(y_i, action)$$

- How to calculate $P(y_i|evidence)$?
  - Any inference method we have seen so far!
- Maximum Expected Utility (MEU)

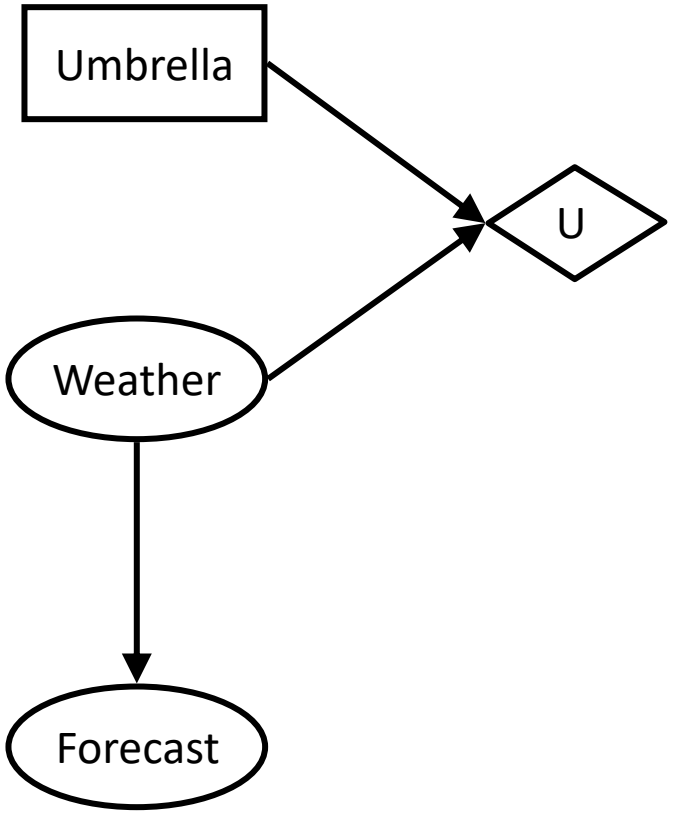
$$MEU(evidence) = \max_a(EU(a|evidence))$$

- Selected Action is $\operatorname{argmax}_a(EU(a|evidence))$



Decision Networks

- Action selection
  - Instantiate all evidence
  - Calculate posterior for all parents of utility node, given the evidence (inference part)
  - Set action node(s) each possible way
  - Calculate expected utility for each action
  - Choose the maximizing action



Decision Networks

EU(action|evidence): Expected Utility of the action given evidence
MEU(evidence): Maximum Expected Utility with the given evidence

Umbrella = leave

$$\mathrm{EU}(\text{leave}) = \sum_w P(w)U(\text{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

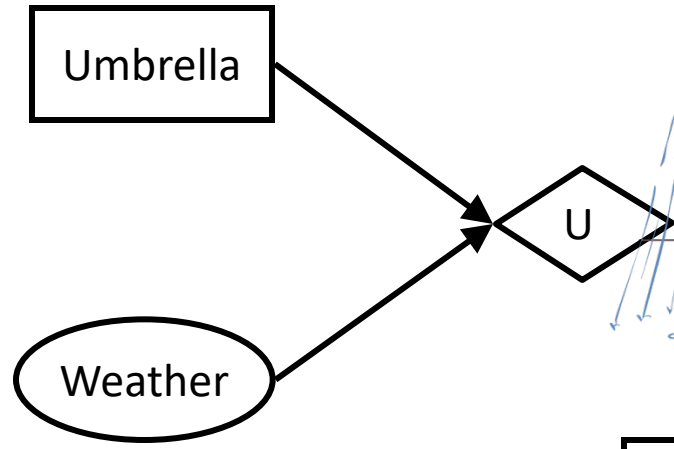
Umbrella = take

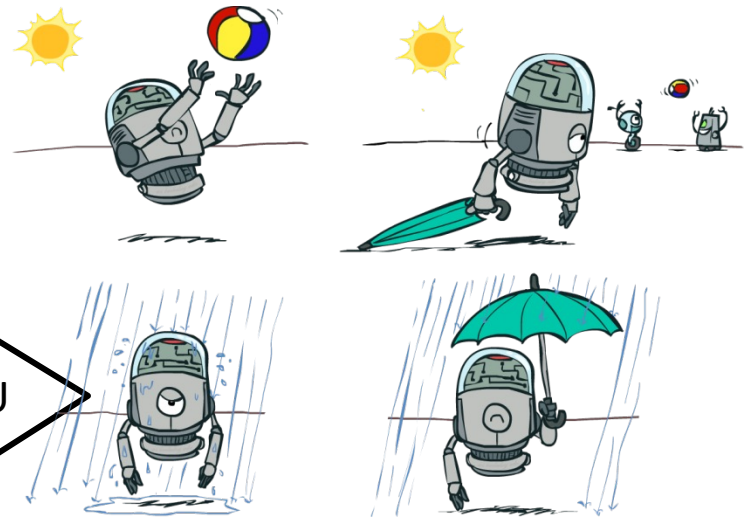
$$\mathrm{EU}(\text{take}) = \sum_w P(w)U(\text{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

Optimal decision = leave

$$\mathrm{MEU}(\emptyset) = \max_a \mathrm{EU}(a) = 70$$



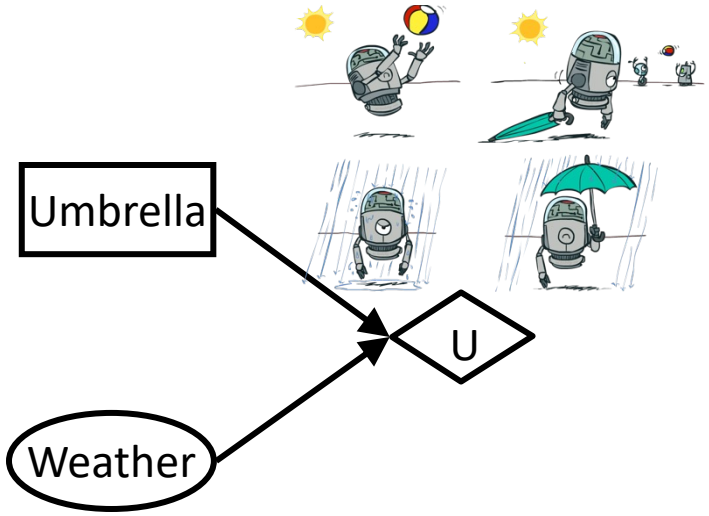


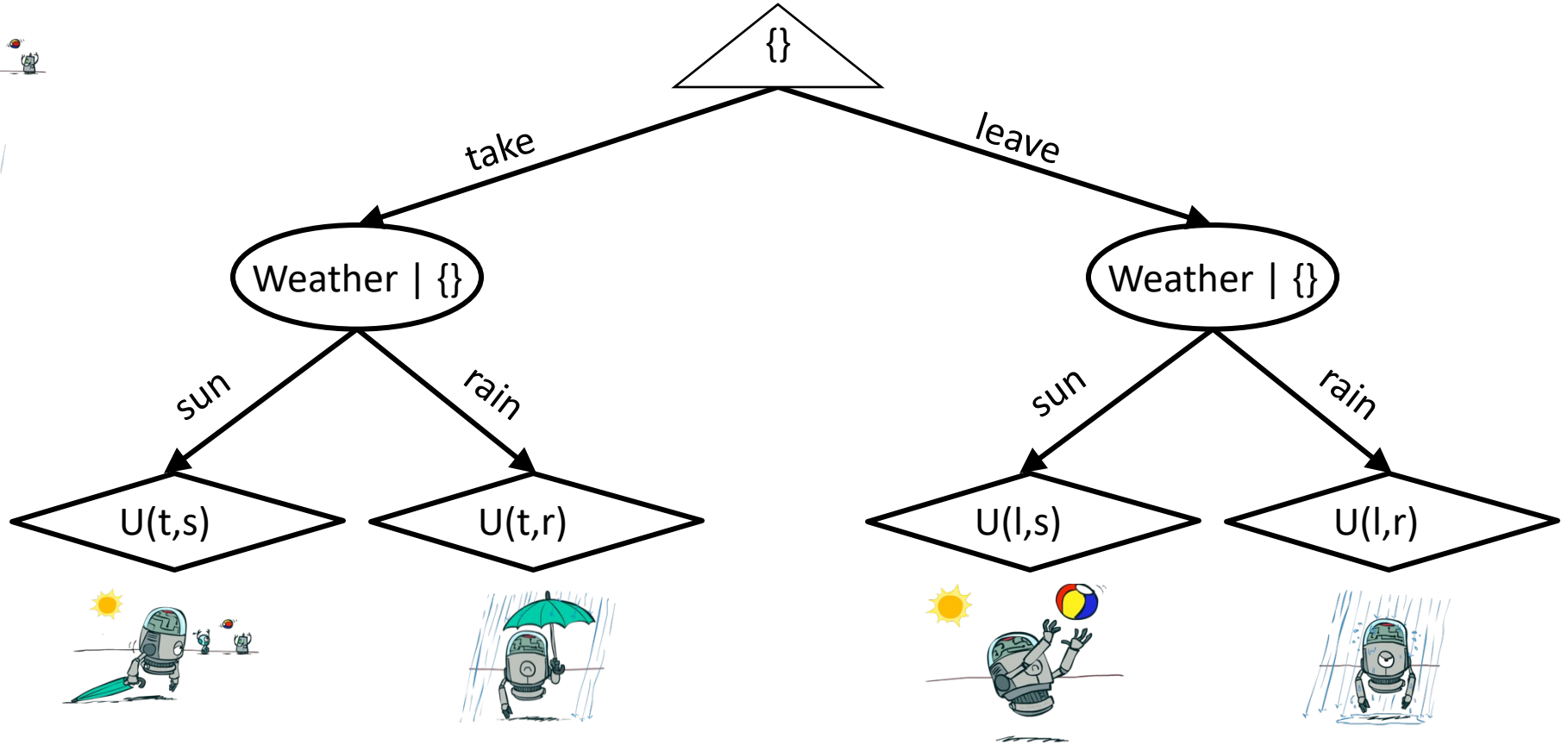
W	P(W)
sun	0.7
rain	0.3

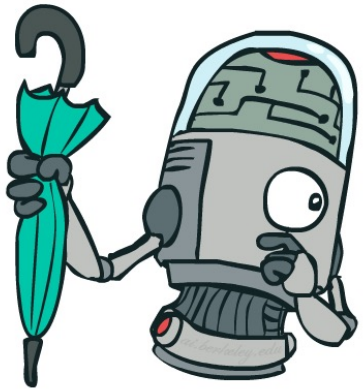
A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

What if W=rain?

Decisions as Outcome Trees







- Almost exactly like expectimax
- What's changed?

Example

EU(action|evidence): Expected Utility of the action given evidence
MEU(evidence): Maximum Expected Utility with the given evidence

Umbrella = leave

$$\mathrm{EU}(\text{leave}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

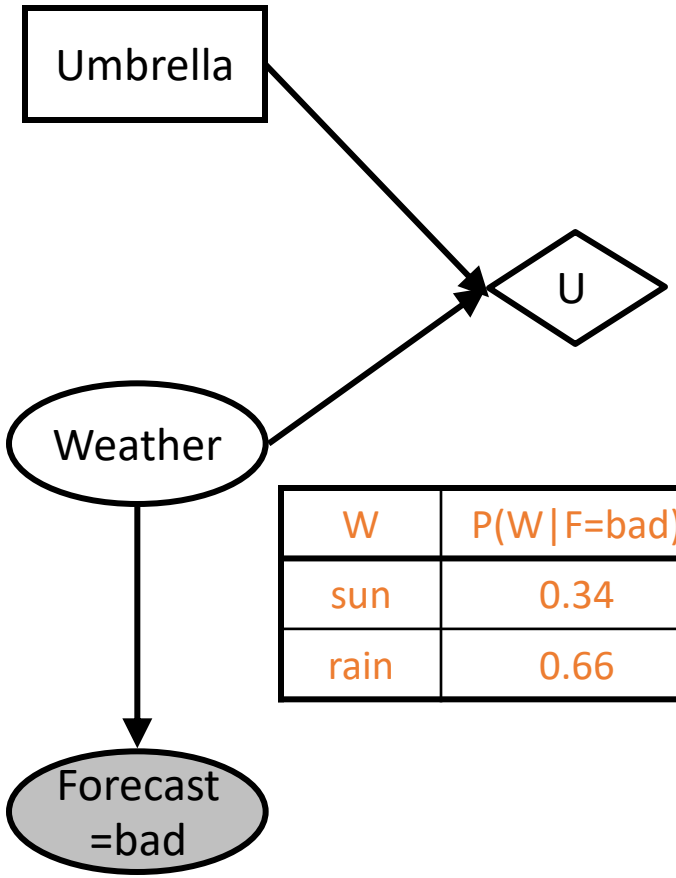
Umbrella = take

$$\mathrm{EU}(\text{take}|\text{bad}) = \sum_w P(w|\text{bad})U(\text{take}, w)$$

$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

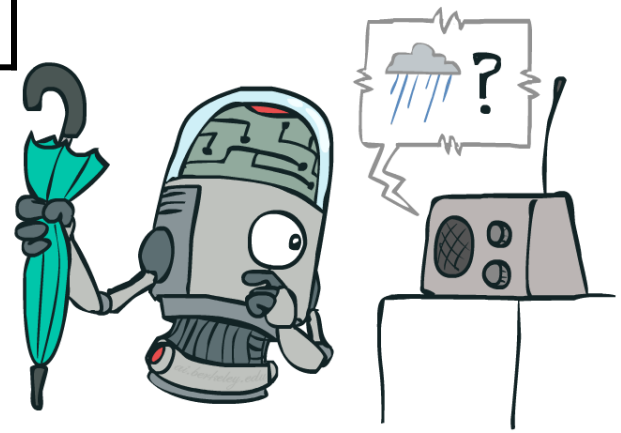
Optimal decision = take

$$\mathrm{MEU}(F = \text{bad}) = \max_a \mathrm{EU}(a|\text{bad}) = 53$$

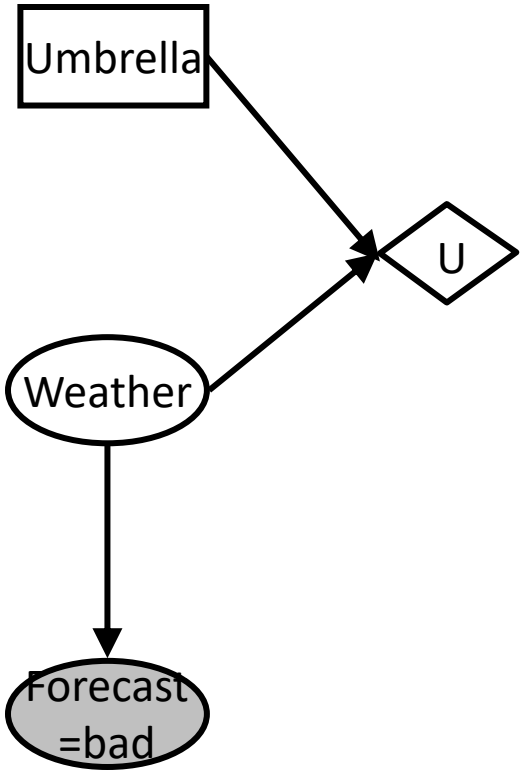


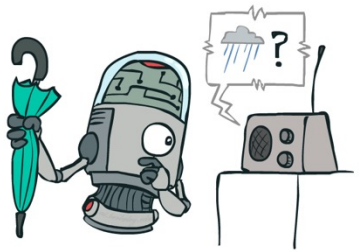
A	W	U(A,W)
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70

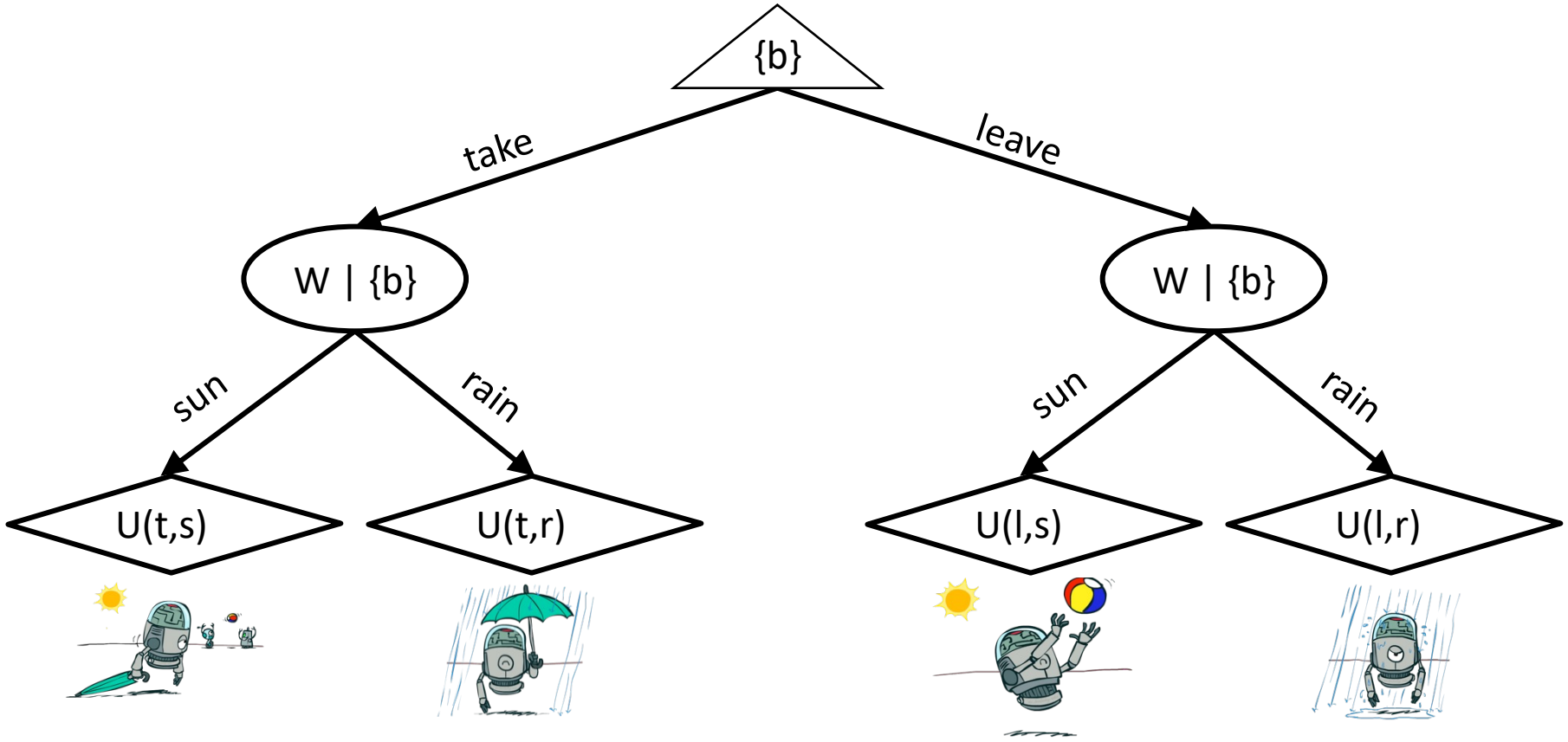
W	P(W|F=bad)
sun	0.34
rain	0.66



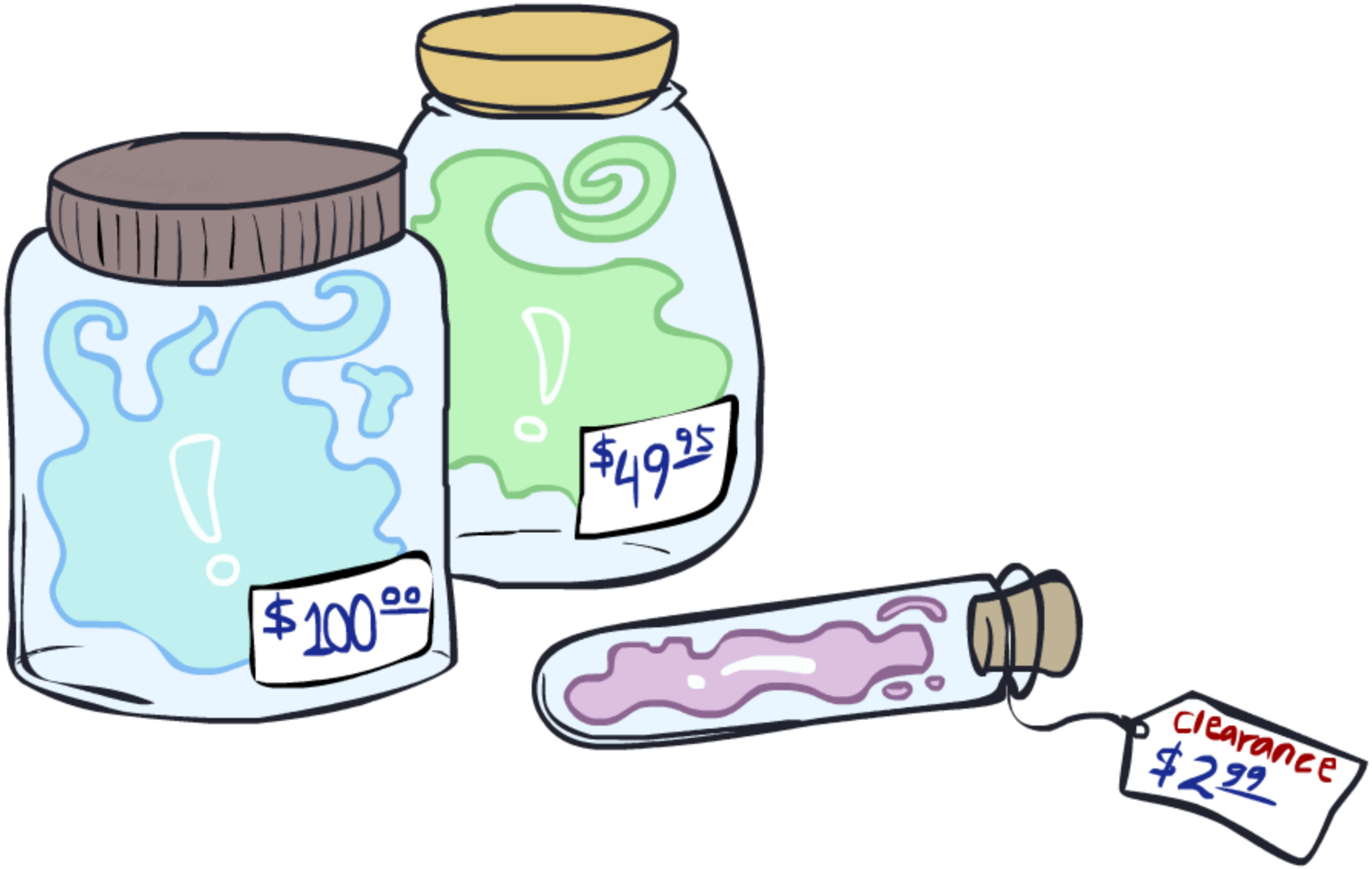
Decisions as Outcome Trees







Value of Information

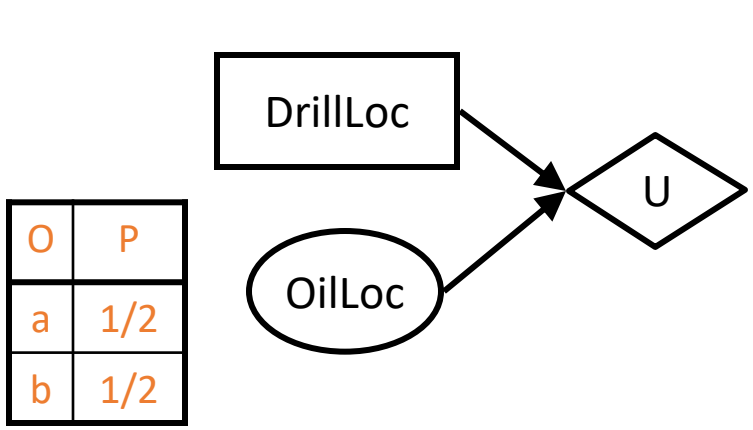


Value of Information

- Idea: compute value of acquiring evidence
  - Can be done directly from decision network

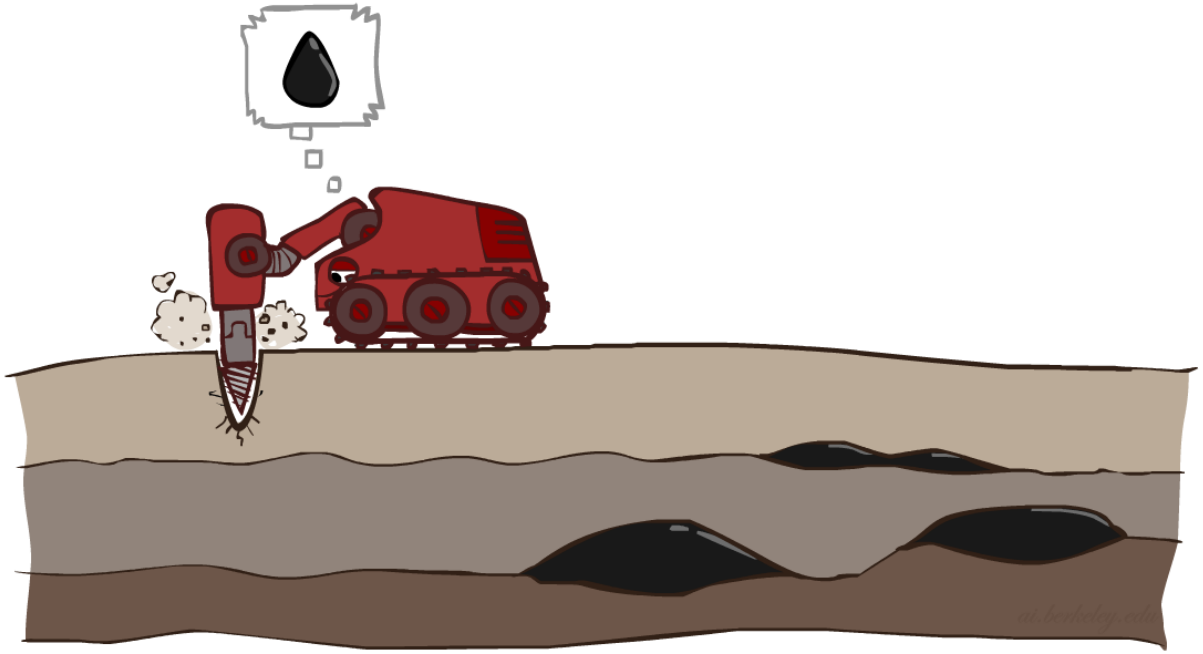
- Example: buying oil drilling rights
  - Two blocks A and B, exactly one has oil, worth k
  - You can drill in one location
  - Prior probabilities 0.5 each, & mutually exclusive
  - Drilling in either A or B has EU = k/2, MEU = k/2

- Question: what's the value of information of O?
  - Value of knowing which of A or B has oil
  - Value is expected gain in MEU from new info
  - Survey may say "oil in a" or "oil in b", prob 0.5 each
  - If we know OilLoc, MEU is k (either way)
  - Gain in MEU from knowing OilLoc?
  - VPI(OilLoc) = k/2
  - Fair price of information: k/2



O	P
a	1/2
b	1/2

D	O	U
a	a	k
a	b	0
b	a	0
b	b	k



VPI Example

MEU with no evidence

$$\mathrm{MEU}(\emptyset) = \max_a \mathrm{EU}(a) = 70$$

MEU if forecast is bad

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

MEU if forecast is good

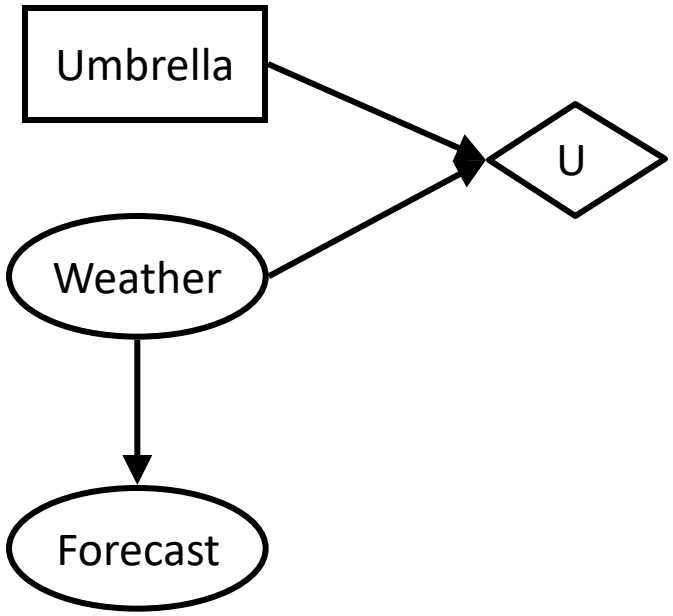
$$\mathrm{MEU}(F = \mathrm{good}) = \max_a \mathrm{EU}(a|\mathrm{good}) = 95$$

Forecast distribution

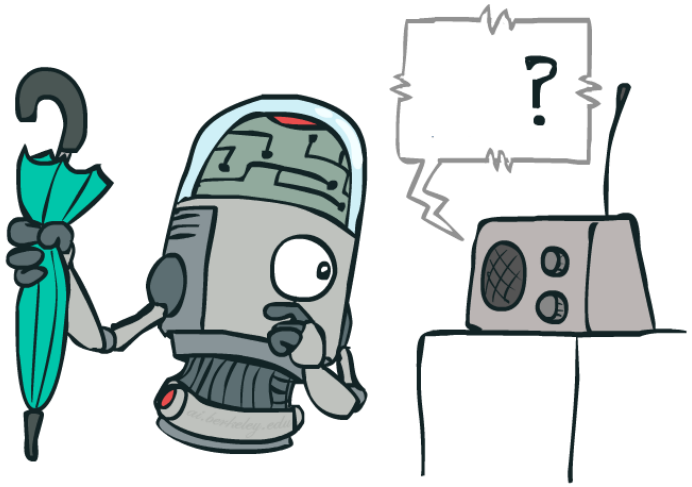
F	P(F)
good	0.59
bad	0.41

$$0.59 \cdot (95) + 0.41 \cdot (53) - 70$$
$$77.8 - 70 = 7.8$$

$$\mathrm{VPI}(E'|e) = \left(\sum_{e'} P(e'|e)\mathrm{MEU}(e, e')\right) - \mathrm{MEU}(e)$$



A	W	U
leave	sun	100
leave	rain	0
take	sun	20
take	rain	70



Value of Information

- Assume we have evidence E=e. Value if we act now:

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

- Assume we see that E' = e'. Value if we act then:

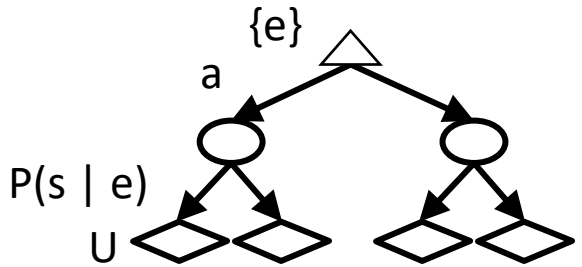
$$\text{MEU}(e,e') = \max_a \sum_s P(s|e,e')\, U(s,a)$$

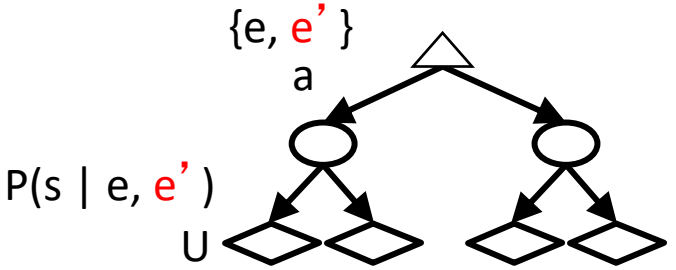
- BUT E' is a random variable whose value is unknown, so we don't know what e' will be
- Expected value if E' is revealed and then we act:

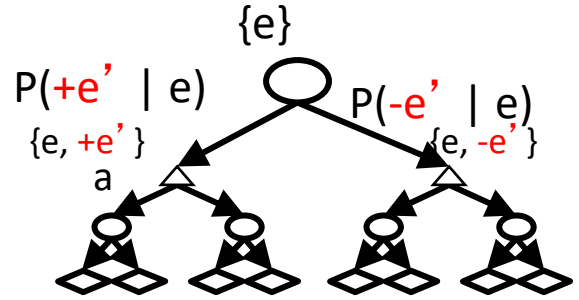
$$\text{MEU}(e,E') = \sum_{e'} P(e'|e)\text{MEU}(e,e')$$

- Value of information: **how much MEU goes up by revealing E' first then acting, over acting now**

$$\text{VPI}(E'|e) = \text{MEU}(e,E') - \text{MEU}(e)$$







What do we need to “Infer”?

$$\text{MEU}(e) = \max_a \sum_s P(s|e)\, U(s,a)$$

$$\text{MEU}(e, e') = \max_a \sum_s P(s|e, e')\, U(s,a)$$

$$\text{MEU}(e, E') = \sum_{e'} P(e'|e)\, \text{MEU}(e, e')$$

$$\text{VPI}(E'|e) = \text{MEU}(e, E') - \text{MEU}(e)$$

Careful, we need all of the highlighted distributions to calculate VPI!

VPI Properties

- Nonnegative

$$\forall E', e : \text{VPI}(E'|e) \geq 0$$

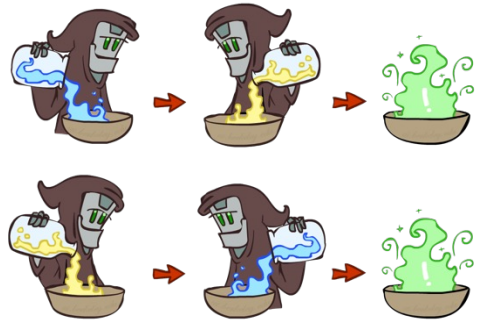
- Nonadditive

(think of observing E_j twice)

$$\text{VPI}(E_j, E_k|e) \neq \text{VPI}(E_j|e) + \text{VPI}(E_k|e)$$

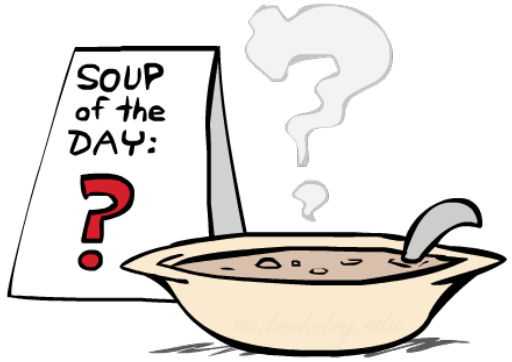
- Order-independent

$$\text{VPI}(E_j, E_k|e) = \text{VPI}(E_j|e) + \text{VPI}(E_k|e, E_j)$$
$$= \text{VPI}(E_k|e) + \text{VPI}(E_j|e, E_k)$$

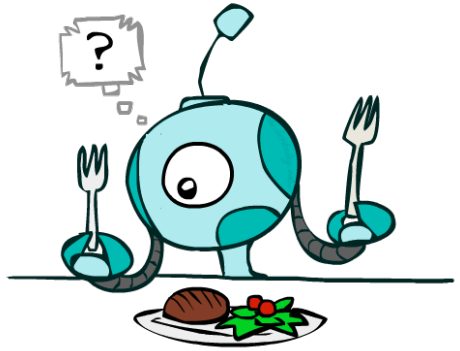


Quick VPI Questions

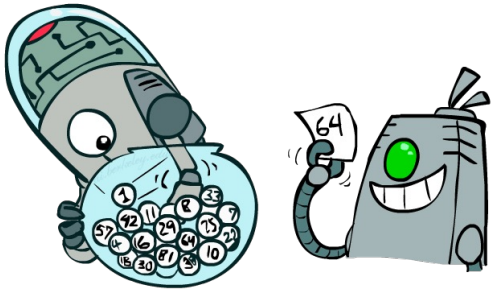
- The soup of the day is either clam chowder or split pea, but you wouldn't order either one. What's the value of knowing which it is?



- There are two kinds of plastic forks at a picnic. One kind is slightly sturdier. What's the value of knowing which?



- You're playing the lottery. The prize will be $0 or $100. You can play any number between 1 and 100 (chance of winning is 1%). What is the value of knowing the winning number?



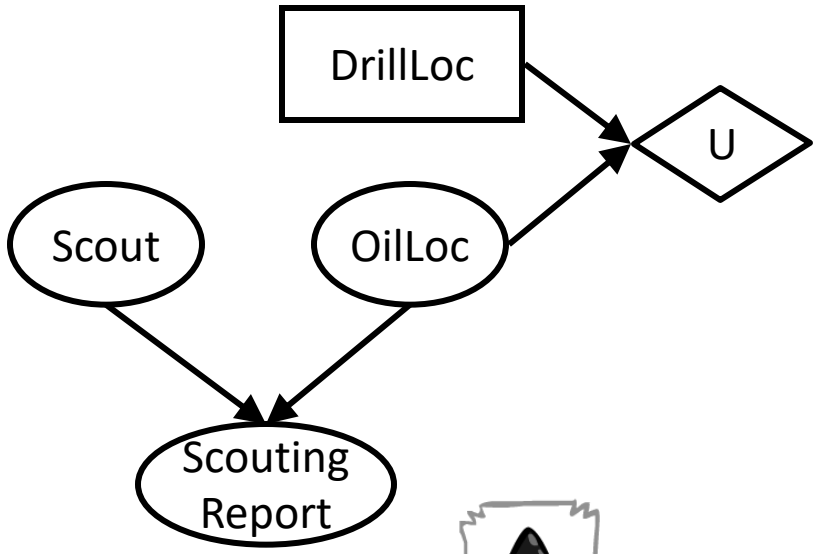
Value of Imperfect Information?

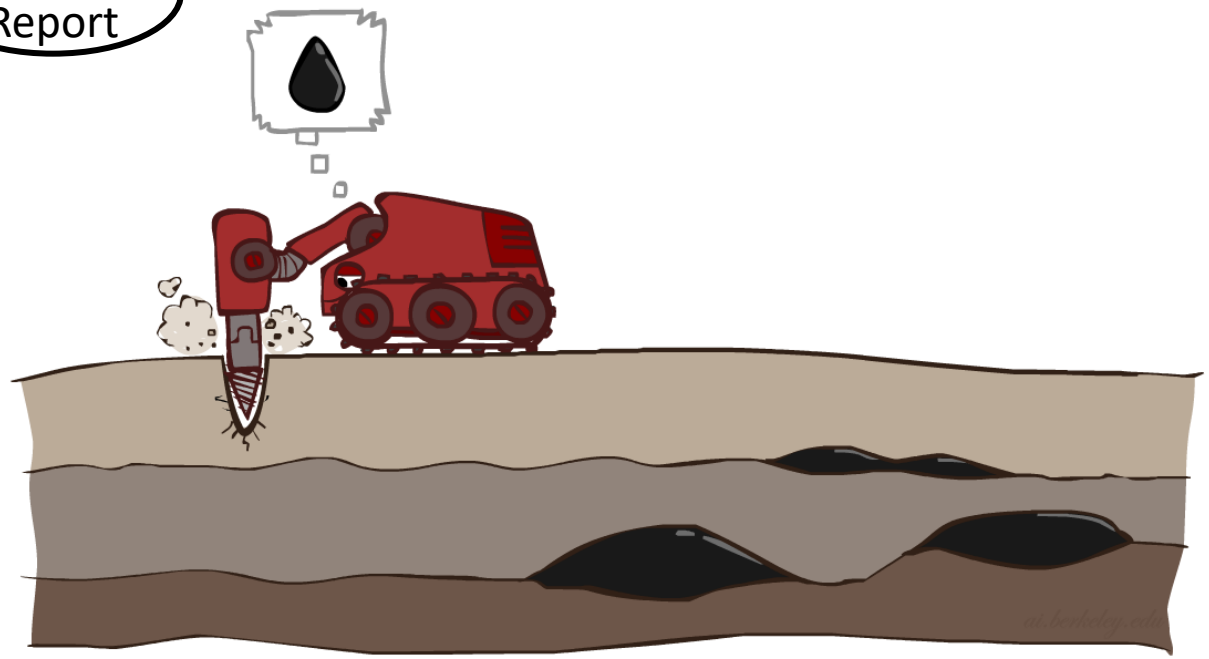
- No such thing (as we formulate it)
- Information corresponds to the observation of a node in the decision network
- If data is “noisy” that just means we don’t observe the original variable, but another variable which is a noisy version of the original one

VPI Question

- VPI(OilLoc) ?
- VPI(ScoutingReport) ?
- VPI(Scout) ?
- VPI(Scout | ScoutingReport) ?
- Generally:

  If Parents(U) $\perp\!\!\!\perp$ Z | CurrentEvidence
  Then VPI(Z | CurrentEvidence) = 0





Additional Notes

- Action nodes as parents to variable nodes
  - Treat as evidence when going over actions
- Utility nodes having multiple random variable parents
  - Calculate posteriors over each parent to calculate the EU
- Utility nodes having multiple action parents
  - Instantiate all possible action combinations and max wrt these combinations
- Multiple utility nodes
  - Separate actions: Treat them individually
  - Overlapping actions: Max over the sum of EUs