COMP341 Introduction to Artificial Intelligence
HW3

- This homework includes logic and bayesian network related problems. These topics will be in the midterm 2.
- By submitting this homework, you **agree** to fully comply with Koç University Student Code of Conduct, and accept any punishment in case of failure to comply.
- You are expected to be able provide clear and concise answers. Gibberish will not receive any credit. Do not overly crowd your answers. Consiceness is a virtue. Write only what is relevant.
- Your answers need to be readable by a human, illegible writing is not gradable hence there is a strong chance that such answers will not get any credit.

Q1 - Propositional Logic (23)

Part 1 (2)

Convert the following into CNF form:

Procedure to convert:

- Eliminate implications
- Move not inwards
- Distribute $\wedge$ over $\vee$

$(A \vee B) \Leftrightarrow ((B \wedge A) \Rightarrow \neg C)$
$= ((A \vee B) \Rightarrow ((B \wedge A) \Rightarrow \neg C)) \wedge (((B \wedge A) \Rightarrow \neg C) \Rightarrow (A \vee B))$
$= (\neg(A \vee B) \vee (\neg(B \wedge A) \vee \neg C)) \wedge (\neg(\neg(B \wedge A) \vee \neg C) \vee (A \vee B))$
$= ((\neg A \wedge \neg B) \vee (\neg B \vee \neg A \vee \neg C)) \wedge ((B \wedge A \wedge C) \vee (A \vee B))$
$= (\neg A \vee \neg B \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg C) \wedge (B \vee A) \wedge (A \vee B) \wedge (C \vee A \vee B)$
$= (\neg A \vee \neg B \vee \neg C) \wedge (A \vee B)$

$\neg(((P \vee \neg Q) \Rightarrow R) \Rightarrow (P \wedge R))$
$(\neg P \vee \neg R) \wedge (\neg P \vee R) \wedge (Q \vee R)$
$(\neg P) \wedge (Q \vee R)$

Part 2 (7)

Show whether A is entailed by the following knowledge base or not using resolution. Do the same for F. You can use the part of one solution in the other.

$(B \wedge C) \Rightarrow A$
$(D \wedge E) \Rightarrow C$
$E \vee F$
$D \wedge \neg F$
B

After converting to CNF, separating out the clauses and adding $\neg\alpha$

C1: $\neg B \vee \neg C \vee A$

C2: $\neg D \vee \neg E \vee C$

C3: $E \vee F$

C4: D

C5: $\neg F$

C6: B

C7: $\neg A$

C8: Resolve C1 and C6. $\neg C \vee A$

C9: Resolve C2 and C4. $\neg E \vee C$

C10: Resolve C3 and C5. E

C11: Resolve C9 and C10. C

C12: Reolve C8 and C11. A

C13: Resolve C7 and C11 to get $\emptyset$ and conclude that the KB entails A

If you add $\neg F$ instead of $\neg A$ and exhaustively resolve everything, you will see that it is not entailed.

Part 3 (7)

Show that J is entailed by the following knowledge base using the forward algorithm. Assume that the agenda is a FIFO que. Trace the steps in the given table. First steps are filled for you. The length of the steps do not indicate solution length. Feel free to take the print out of the tables and submit it with your HW.

P
R
Q
$C1 : P \wedge R \Rightarrow K$
$C2 : R \wedge K \Rightarrow Q$
$C3 : M \Rightarrow L$
$C4 : P \wedge M \wedge L \Rightarrow J$
$C5 : P \wedge Q \Rightarrow M$

You quit the loop before you can add J to agenda! Skipping steps where we pop a previously inferred symbol but putting them in the agenda

counts	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7
$C1$	2	1	0	0	0	0	0
$C2$	2	2	1	1	0	0	0
$C3$	1	1	1	1	1	0	0
$C4$	3	2	2	2	2	1	0
$C5$	2	1	1	0	0	0	0

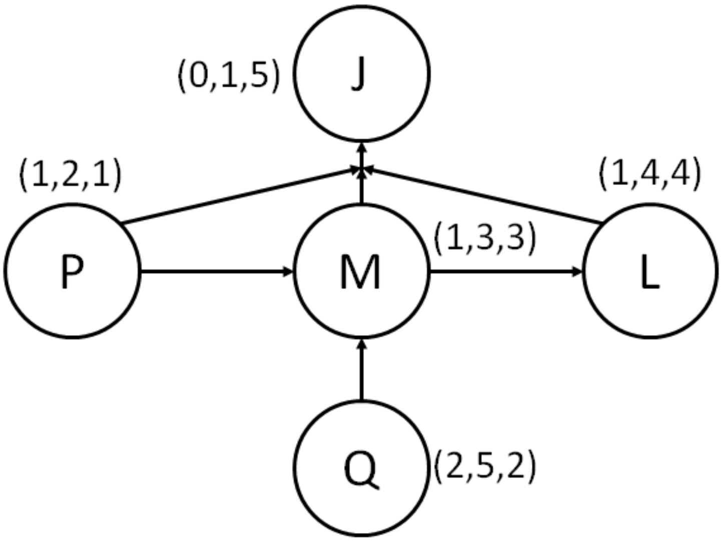
inferred	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7
P	F	T	T	T	T	T	T
R	F	F	T	T	T	T	T
Q	F	F	F	T	T	T	T
K	F	F	F	F	T	T	T
L	F	F	F	F	F	F	T
M	F	F	F	F	F	T	T
J	F	F	F	F	F	F	F

agenda	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7
	P R Q	R Q	Q K	K M	M (Q)	(Q) L	

Part 4 (7)

Solve the same problem using backward chaining. Remember to start from the query, find the clause with this as its head, add the premises of that clause as new subgoals, then repeat. After you reach a point where you have a subgoal as true, propogate back up. Draw a tree, starting from the query. Add an edge between goals and subgoals. Put a number next to the nodes, to mark the n^{th} subgoal that was verified. Hint: There is a solution with P:1, and J:5.

This question was intentionally vague. Recall that we did not go over any backward chaining algorithms for propositional logic. However, we have gone through an example. The important thing is for you to get a sense of how backward chaining would work. I will employ a depth first strategy and will check upward whenever I see a true symbol.



The first number in the parantheses indicate at which step that symbol is added to the subgoal list. The second number indicates when it is expanded and the third one indicates the order it was set to true.

Q2 - First Order Logic: Representation (14)

Consider a vocabulary with the following symbols:
Occupation(p,o): Predicate. Person p has occupation o
Customer(p1,p2): Predicate. Person $p1$ is a customer of person $p2$
Boss(p1,p2): Predicate. Person $p1$ is a boss of person $p2$
Doctor, Surgeon, Lawyer, Actor: Constants denoting occupations
Emily, Joe: Constants denoting people

Use these symbols to write the following assertions in first order logic:

- Joe is either a surgeon or a lawyer
  $Occupation(Joe, Surgeon) \lor Occupation(Joe, Lawyer)$
- Emily is an actor, but she also holds another job
  $\exists x Occupation(Emily, Actor) \land (x \neq Actor) \land Occupation(Emily, x)$
- All surgeons are doctors
  $\forall x Occupation(x, Surgeon) \Rightarrow Occupation(x, Doctor)$
- Emily does not have a lawyer
  $\neg(\exists x Occupation(x, Lawyer) \land Customer(Emily, Lawyer))$
- Joe has a boss who is a lawyer
  $\exists x Boss(x, Joe) \land Occupation(x, Lawyer)$
- There exists a lawyer all of whose customers are doctors
  $\exists x Occupation(x, Lawyer) \land \forall y Customer(y, x) \Rightarrow Occupation(y, Doctor)$
- Every surgeon has a lawyer
  $\forall x Occupation(x, Surgeon) \Rightarrow \exists y Occupation(y, Lawyer) \land Customer(x, y)$

Q3 - First Order Logic: Inference (20)

Given the knowledgebase:
$\forall x Doctor(x) \Rightarrow Rich(x)$
$\forall x \exists y Angry(x) \land Butler(x) \land Boss(x, y) \Rightarrow Kill(x, y)$
$\forall x \exists y \exists z Gardner(x) \land Married(y, z) \land Greedy(y) \land Rich(z) \Rightarrow Kill(x, z)$
$\forall x \exists y \exists z InLove(x, y) \land Married(y, z) \Rightarrow Angry(x)$
$\forall x \exists y Kill(x, y) \Rightarrow Murderer(x)$
$Doctor(John), Doctor(Jane), Greedy(John)$
$Gardner(Jack), Butler(Alfred)$
$Married(John, Jane), Married(Jane, John)$, (could have had a symmetry rule as well)
$InLove(Jack, Jane), InLove(Alfred, Jane)$
$Boss(John, Alfred), Boss(John, Jack)$

Notes:

1. I have removed the inequality from the being *Angry* rule. We did not see how to deal with it for forward/backward chaining in the class. The rule still makes sense with $x = z$.
2. You can assume all the qualifiers are universal for the sake of easyness. Otherwise, to remove a existential quantification use a *Skolem Function*. Then substitute something appropriate for that function when it is time. Incidentally, this question gives you the same answer!
3. It is okay if you just loosely follow the algorithms

Part 1 (10)

Using backward chaining, check if it entails $Murderer(Alfred)$. Feel free to have "smart" picks for your next rules and unification

Standardize apart, handle existential quantification and drop universal quantification:

R1: $Doctor(x1) \Rightarrow Rich(x1)$

R2: $(Angry(x2) \land Butler(x2) \land Boss(x2, F1(x2))) \Rightarrow Kill(x2, F1(x2))$

R3: $(Gardner(x3) \land Married(F2(x3), F3(x3)) \land Greedy(F2(x3)) \land Rich(F3(x3))) \Rightarrow Kill(x3, F3(x3))$

R4: $(InLove(x4, F4(x4)) \land Married(F4(x4), F5(x4))) \Rightarrow Angry(x4)$

R5: $Kill(x5, F6(x5)) \Rightarrow Murderer(x5)$

Solution steps:

1. Start from R5 which has $Murderer(\cdot)$ as its head and $Kill(\cdot, \cdot)$ as its premise. We add $\{x5 \backslash Alfred\}$ to the unification list.

2. Now we have 2 rules, R2 and R3, to add. Let's skip R3 for conciseness. When we move on to R2, we have 3 premises, i.e., 3 subgoals to add; $Angry(Alfred), Butler(Alfred), Boss(Alfred, F6(Alfred))$.

3. We pick $Angry(\cdot)$ and move onto R4, since BC is depth first. At R4 we add 2 new subgoals, $InLove(Alfred, F4(Alfred))$ and $Married(F4(Alfred), F5(Alfred))$

4. We go down to $InLove(Alfred, F4(Alfred))$. We add $\{F4(Alfred) \setminus Jane\}$ to the unification list. Note that we would have needed to check each possible unification at this point if we had other options.

5. Now we move on to $Married(F4(Alfred), F5(Alfred))$ which is now $Married(Jane, F5(Alfred))$. We add $\{F5(Alfred) \setminus John\}$ to the unification list.

6. We are done with the $Angry$ branch and go back to $Butler(Alfred)$. We immediately see that it is satisfied.

7. We then move to the $Boss(F6(Alfred), Alfred)$ branch. We see that we do not have any unification to satisfy this! Since we are out of other possible unifications we conclude that Alfred the Butler is actually not a murderer. There is something funny in the rule R2. It reads as "All angry butlers will kill some of their employees". If it was $Boss(y, x)$, then things would have been different and R2 would have read "All angry butlers will kill some of their bosses", and Alfred would be our murderer.

So our KB does not entail $Murderer(Alfred)$.

Part 2 (10)

Using resolution, check if it entails $Murderer(Jack)$. Feel free to have "smart" pick for your nexts rules and unification

We must put the KB in CNF and add $\neg Murderer(Jack)$:

S1: $\neg Doctor(x1) \vee Rich(x1)$

S2: $\neg Angry(x2) \vee \neg Butler(x2) \vee Boss(x2, F1(x2))) \vee Kill(x2, F1(x2))$

S3: $\neg Gardner(x3) \vee \neg Married(F2(x3), F3(x3)) \vee \neg Greedy(F2(x3)) \vee \neg Rich(F3(x3)) \vee Kill(x3, F3(x3))$

S4: $\neg InLove(x4, F4(x4)) \vee \neg Married(F4(x4), F5(x4)) \vee Angry(x4)$

S5: $\neg Kill(x5, F6(x5)) \vee Murderer(x5)$

S6: $Doctor(John)$

S7: $Doctor(Jane)$

S8: $Greedy(John)$

S9: $Gardner(Jack)$

S10: $Butler(Alfred)$

S11: $Married(John, Jane)$

S12: $Married(Jane, John)$

S13: $InLove(Jack, Jane)$

S14: $InLove(Alfred, Jane)$

S15: $Boss(John, Alfred)$

S16: $Boss(John, Jack)$

S17: $\neg Murderer(Jack)$

I will only list out one possible order for the resolution but will not write out the details.

1. S18: Resolve S5 and S6 with $\{x5 \setminus Jack\}$

2. S19: Resolve S18 and S3 with $\{x3 \setminus Jack, F6(\cdot) \setminus F3(\cdot)\}$

3. S20: S19 and S8 with $\{F2(Jack) \setminus John\}$

4. S21: S21 and S8 with $\{F6(Jack) \setminus Jane\}$

5. S22: S21 and S11 (no need for any new substitutions)

6. S23: S22 and S9

7. S24: S23 and S1 with $\{x1 \setminus Jane\}$

8. S25: S24 and S7 which will give us the $\emptyset$

As a result, our KB entails $Murderer(Jack)$.

Q4 - Uncertainty (24)

Part 1 (12)

A	B	C	$P(A, B, C)$
T	T	T	0.19
T	T	F	0.14
T	F	T	0.02
T	F	F	0.21
F	T	T	0.03
F	T	F	0.15
F	F	T	0.17
F	F	F	0.09

Calculate the following:

- $P(A = T)$ $= 0.19 + 0.14 + 0.02 + 0.21 = 0.56$
- $P(A = F, C = T)$ $= 0.03 + 0.17 = 0.2$
- $P(A = T|C = T)$ $= (0.19 + 0.02)/(0.2 + (0.19 + 0.02)) = 0.21/0.41 = 0.51$
- $P(A|B = F)$
  $P(A = F|B = F) = (0.02 + 0.21)/(0.02 + 0.21 + 0.17 + 0.09) = 0.47$
  $P(A = T|B = F) = (0.17 + 0.09)/(0.02 + 0.21 + 0.17 + 0.09) = 0.53$
- $P(C|A = T, B = F)$
  $P(C = F|A = T, B = F) = 0.21/(0.02 + 0.21) = 0.913$
  $P(C = T|A = T, B = F) = 0.02/(0.02 + 0.21) = 0.087$
- $P(B|C)$
  $P(B = F|C = F) = 0.51$
  $P(B = T|C = F) = 0.49$
  $P(B = F|C = T) = 0.46$
  $P(B = T|C = T) = 0.54$

Part 2 (12)

We have a cause C, and 2 effects E_1 and E_2. C can take two values, $\{T, F\}$, and E_i's can take three values, $\{low, medium, high\}$. We run an experiment and measured all 3:

We measured $C = T$, for 100 trials. In this case we had $E_1 = low$ for 30 times, $E_1 = medium$ for 60 times and $E_1 = high$ for 10 times. Similarly we had $E_2 = low$ for 20 times, $E_2 = medium$ for 30 times and $E_2 = high$ for 50 times.

We measured $C = F$, for 400 trials. In this case we had $E_1 = low$ for 200 times, $E_1 = medium$ for 100 times and $E_1 = high$ for 100 times. Similarly we had $E_2 = low$ for 200 times, $E_2 = medium$ for 120 times and $E_2 = high$ for 80 times.

Calculate the following, assume that E_1 and E_2 are conditionally independent.

- $P(C)$ $.P(C = T) = 100/(100 + 400) = 0.2, P(C = F) = 400/(100 + 400) = 0.8$
- $P(E_1 = low|C = T)$ $= 30/100 = 0.3$
- $P(E_2 = medium|C = F)$ $= 120/400 = 0.3$
- $P(E_2 = high|C = T)$ $= 50/100 = 0.5$
- $P(C = T|E_1 = low, E_2 = high)$
  $= P(E_1 = low, E_2 = high|C = T)P(C = T)/(P(E_1 = low, E_2 = high)$
  $= P(E_1 = low|C = T)P(E_2 = high|C = T)P(C = T)$ (conditional independence)
  $/\,(P(E_1 = low, E_2 = high|C = T)P(C = T) + P(E_1 = low, E_2 = high|C = F)P(C = F))$
  $= 0.3 \cdot 0.5 \cdot 0.2/(0.3 \cdot 0.5 \cdot 0.2 + 0.5 \cdot 0.2 \cdot 0.8) = 0.2\bar{7}$
- $P(C = F|E_1 = medium, E_2 = medium)$ $= 0.25 \cdot 0.3 \cdot 0.8/(0.25 \cdot 0.3 \cdot 0.8 + 0.2 \cdot 0.6 \cdot 0.3) = 0.625$

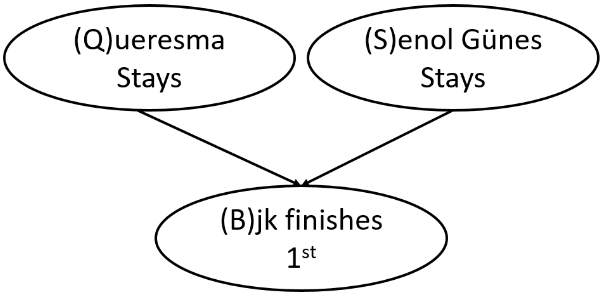
Q5 - Bayesian Networks (19)

Part 1 (11)

Given the following BN, write out the expression for the joint distribution, $P(S, Q, B)$, and fill out the table. Then, calculate the following:

Q	P(Q)
+q	0.7
-q	0.3

S	P(S)
+s	0.85
-s	0.15



S	Q	B	P(B|S,Q)
+s	+q	+b	0.8
		-b	0.2
	-q	+b	0.75
		-b	0.25
-s	+q	+b	0.65
		-b	0.35
	-q	+b	0.4
		-b	0.6

$P(S, Q, B) = P(S)P(Q)P(B|S, Q)$

S	Q	B	$P(S, Q, B)$
+s	+q	+b	0.476
+s	+q	-b	0.119
+s	-q	+b	0.19125
+s	-q	-b	0.06375
-s	+q	+b	0.06825
-s	+q	-b	0.03675
-s	-q	+b	0.018
-s	-q	-b	0.027

- $P(+b)$ Sum out s and q for +b
  $= 0.476 + 0.19125 + 0.06825 + 0.018 = 0.7535$
- $P(+s| + b)$ Fix +b, sum out q, then normalize for +s
  $= (0.476 + 0.19125)/(0.476 + 0.19125 + 0.06825 + 0.018) = 0.8855$
- $P(-q| - b)$ $= (0.06375 + 0.027)/(0.06375 + 0.027 + 0.119 + 0.03675) = 0.3682$

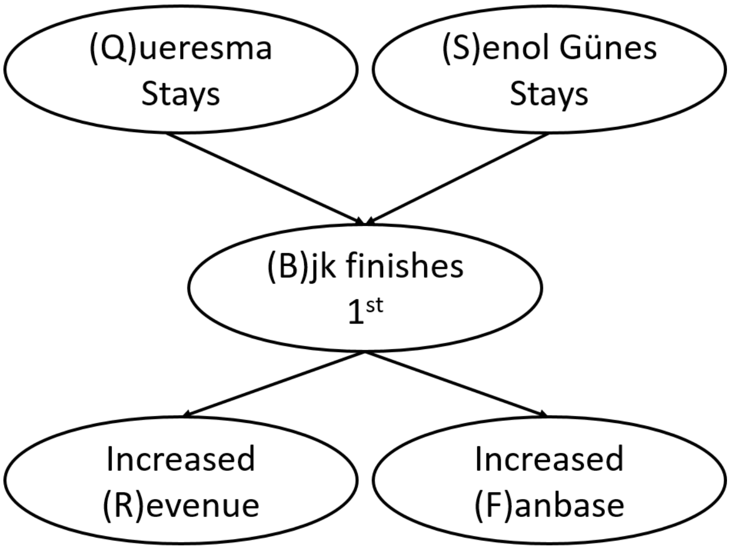
Part 2 (8)

We enlarge the BN. Calculate the following (note you do not need the calculate the joint distribution):

Q	P(Q)
+q	0.7
-q	0.3

S	P(S)
+s	0.85
-s	0.15

S	Q	B	P(B|S,Q)
+s	+q	+b	0.8
		-b	0.2
	-q	+b	0.75
		-b	0.25
-s	+q	+b	0.65
		-b	0.35
	-q	+b	0.4
		-b	0.6



B	R	P(R|B)
+b	+r	0.95
	-r	0.05
-b	+r	0.65
	-r	0.35

B	F	P(F|B)
+b	+f	0.88
	-f	0.12
-b	+f	0.3
	-f	0.7

For reference, the joint distribution is: $P(S,Q,B,R,F) = P(S)P(Q)P(B|S,Q)P(R|B)P(F|B)$

- $P(+b)$ $= 0.7535$, same as before!
- $P(+r|+b)$ $= 0.95$, just look at the $P(R|B)$ table!
- $P(+r|+b,+q)$ $= 0.95$, same as above, conditional independence!!
- $P(+f|+q,+s)$ $= 0.764$, see just below for the computation

$P(+f|+q,+s)$ gives us these relevant factors (there is one more factor that I did not write! Why?):
$P(+s), P(+q), P(B|+s,+q), P(+f|B)$
We need to eliminate the hidden variable B, to get a new factor:
$f_1(+f,+s,+q) = \Sigma_{b=\{+b,-b\}} P(b|+s,+q)P(+f|b) = 0.8 \cdot 0.88 + 0.2 \cdot 0.3 = 0.764$
You can see that this is actually $P(+f|+q,+s)$. However, for the sake of completeness let's keep going.
Join +s and +q to get another factor:
$f_2(+f) = f_1(+f,+s,+q)P(+s)P(+q) = 0.764 \cdot 0.85 \cdot 0.7 = 0.4546$
If we normalize, we are done. However, we need $f_2(-f)$. We can do it by first falculating $f_1(-f,+s,+q) = 0.8 \cdot 0.12 + 0.2 \cdot 0.7 = 0.236$. Then, $f_2(-f) = 0.1404$
Finally we normlize: $P(+f|+s,+q) = f_2(+f)/(f_2(+f)+f_2(-f)) = 0.4546/(0.4546+0.1404) = 0.764$
Not in the homework but as an exercise calculate the following:

- You observe the revenue and the fanbase increase. Did Senol stay?
  $P(+s|+r,+f) = 0.875$
- You observe the revenue increase, fanbase decrease and Queresma leaving. Was BJK not the 1^{st}?
  $P(-b|-q,+r,-f) = 0.634$

Submission

The deadline is December 7^{th} at 9.50AM. You should to hand-in your solutions to me in person **before** class or upload it to blackboard (must be readable). I will leave my office at 9.50AM to get to class. I am giving an extra 10 minutes to people who is showing up to the class, i.e., you can give me your homeworks in the class before I start teaching.
The late policy is in the syllabus.