

COMP 341: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

DURATION: 135 MINUTES

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- [illegible]

1. (10 points) True or False :

False The environment of an agent does not affect its rationality.

True Iterative deepening search is optimal if all transition costs are equal.

False A* search is stopped as soon as the goal is encountered as a child of an expanded node.

False Simulated annealing with $T = 0$ is equivalent to hill-climbing.

False In constraint satisfaction problems, all variables must have the same domain.

False Expectiminimax algorithm is only useful when the game is stochastic.

False Backward chaining has a better worst-case complexity than forward chaining.

True First-order logic without functions can be converted to propositional logic in finite time.

False A Bayesian Network can have an arbitrary graph topology.

True Given the joint distribution of n random variables, we can calculate each of their prior probabilities.

2. (5 points) Given the Bayesian Network, answer (Yes/No) the independence (for all possible CPTs) questions:

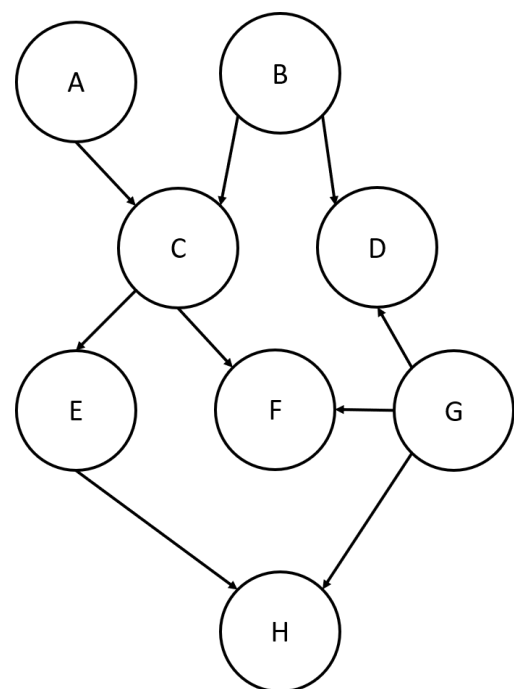
No Are A and B independent given E?

No Are A and H independent?

No Are A and H independent given F?

Yes Are A and F independent given C and H?

No Are A and F independent given C and D?



3. (15 points) **Search:** Consider the graph below. **S** is the initial state and **G** is the goal state. The arcs represent the state transitions, which are directional. The cost of state transitions are written on the arcs. The heuristic values of the states are written in their respective circles. For the given algorithms, write the expansion (i.e. popping from the frontier) order of the nodes, breaking ties **alphabetically**, and the resulting solution path. Use the **graph search** versions of the algorithms.

I will give partial credit to consistent orderings

- (a) (3 points) Breadth First Search

Expanded Nodes:

S-A-B-C-D-G

OR

S-C-B-A-F-E-D-G

Resulting Path:

S-A-G

- (b) (3 points) Depth First Search

Expanded Nodes:

S-A-D-F-G

OR

S-C-F-G

Resulting Path:

S-A-D-F-G

OR

S-C-F-G

- (c) (3 points) Greedy Search

Expanded Nodes:

S-C-F-G

Resulting Path:

S-C-F-G

- (d) (4 points) A* Search

Expanded Nodes:

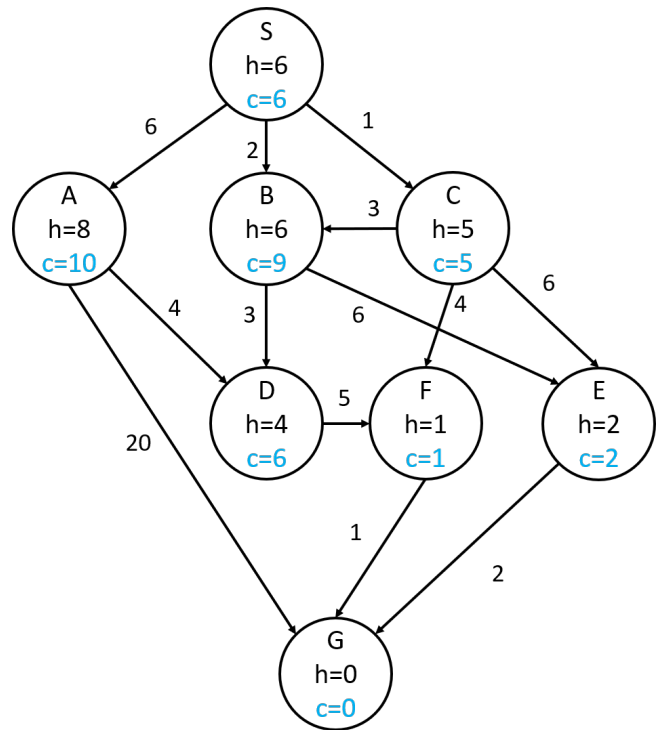
S-C-F-G

Resulting Path:

S-C-F-G

- (e) (2 points) Is this heuristic admissible? Justify your answer by giving the optimal cost for each state.

For all the nodes, $c^*(s) \geq h(s)$, look at the graph. Thus the heuristic is admissible



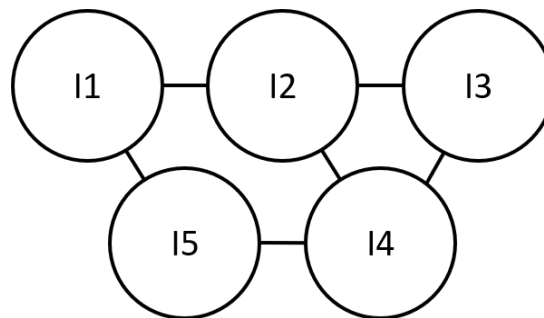
5. (15 points) **Constraint Satisfaction Problems:** You are tasked with scheduling classrooms. There are 5 instructors and 3 rooms and you need to assign rooms to instructors. Beware, these instructors are a little bit fussy. The instructors need the rooms in the following times:

- I1: 9:00 to 11:00
- I2: 10:00 to 13:00
- I3: 12:00 to 14:00
- I4: 12:00 to 16:00
- I5: 15:00 to 17:00

A room can only be assigned to one instructor at a time. Room R1 is too small for instructor I1. Room R3 is too far away from instructor I2 such that he will not use it. Instructor I4 does not like the projector at room R2. Instructor I5 refuses to use the same room as I1, regardless of the time difference.

Solve this CSP by filling in the table below by using the minimum remaining value heuristic, degree heuristic, optionally least constraining value heuristic and constraint propagation using arc consistency checks. Break all ties with the order {I1,I2,I3,I4,I5} and {R1,R2,R3} whenever needed. For each empty cell, write down the values in their remaining domains (i.e. allowable values) both at the start and after you a step (assignment + filtering). An example is done for you for I1. Circle your assignments for each step. Note that if there is a single value left in a domain, this does not mean you have assigned it yet. Do not backtrack but stop if you get an empty domain so the table length should not change. Feel free to use the free-spaces in the cells to work on arc-consistency checks but make sure the your final answer for each cell is clear. (Hint: Drawing the binary constraint graph will help.)

	I1	I2	I3	I4	I5	Variable	Value
Init	R2, R3	R1, R2	R1, R2, R3	R1, R3	R1, R2, R3	I2	R1
Step 1	R2, R3	R1	R2	R3	R1, R2	I4	R3
Step 2	R2, R3	R1	R2	R3	R1, R2	I3	R2
Step 3	R2, R3	R1	R2	R3	R1, R2	I1	R2 (R3)
Step 4	R2 (R3)	R1	R2	R3	R1 (R1, R3)	I5	R1
Final	R2 (R3)	R1	R2	R3	R1		



As it can be seen, after the first assignment and enforcing arc consistency, we pretty much find a solution. Due to the setup of the problem, we need to carry on with the remaining trivial assignments. The values in parentheses denote the differences in solution if LCV was used.

6. (10 points) **Propositional Logic:** Show whether C is entailed by the following knowledge base or not using **backward chaining**. Any other method will not receive any credit. You can use tables, sketches etc. to show your work. You can loosely follow the algorithms and make smart choices. Indicate which rules were used to come up with the solution and the order of inferring the truth-hoods of the symbols.

$$S1: A \vee \neg B \vee C \vee \neg F \rightarrow \neg A \wedge B \wedge F \Rightarrow C$$

$$S2: C \vee H \vee \neg I \vee \neg J \rightarrow I \wedge J \wedge \neg H \Rightarrow C$$

$$S3: \neg D \vee H \rightarrow \neg H \Rightarrow \neg D$$

$$S4: A \vee \neg D \rightarrow \neg A \Rightarrow \neg D$$

$$S5: \neg B \vee D \vee F \rightarrow B \wedge \neg D \Rightarrow F$$

$$S6: J \vee H \rightarrow \neg H \Rightarrow J$$

$$S7: D \vee \neg F \vee I \rightarrow \neg D \wedge F \Rightarrow I$$

$$S8: \neg A \vee D \rightarrow \neg D \Rightarrow \neg A$$

$$S9: B$$

$$S10: \neg D$$

$$S11: E$$

$$S12: J$$

Note that with $\neg A$, $\neg D$ and $\neg H$, we can build the Horn form as above.

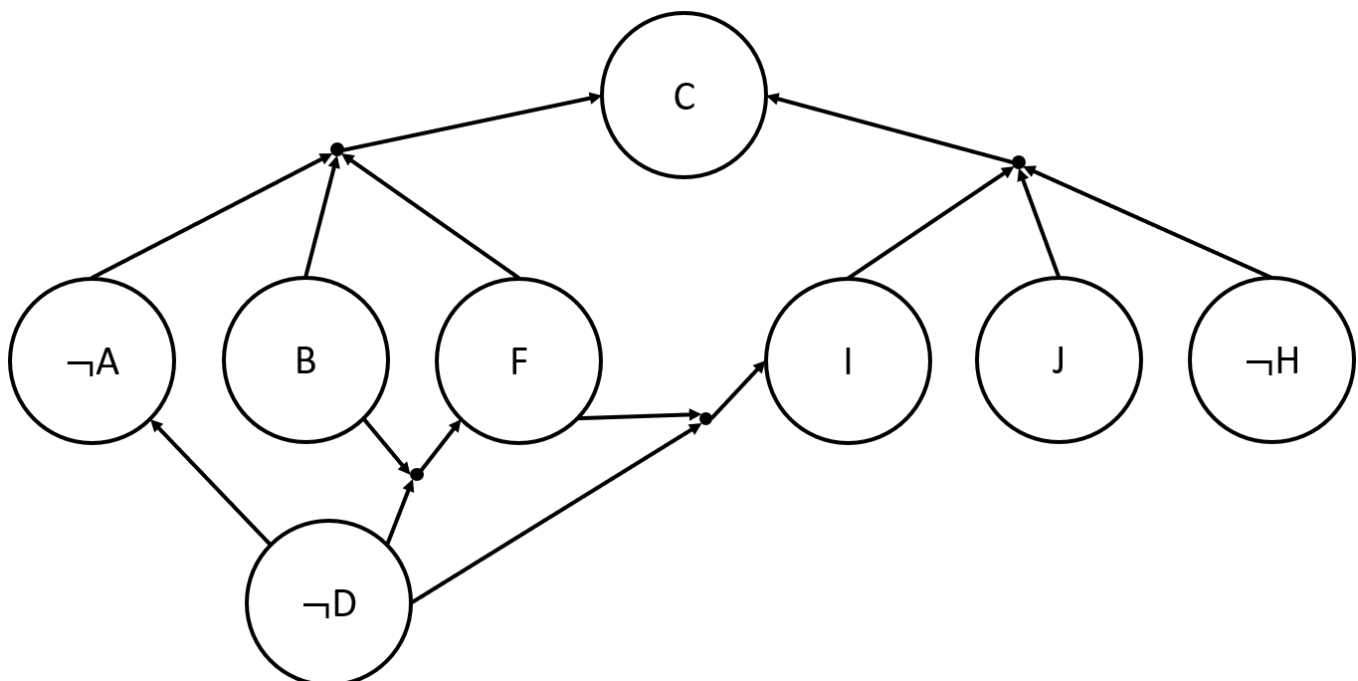
A breadth first approach to adding all the possible premises as subgoals (starting from the query), results in the following order of subgoal additions

- 1) C
- 2) $\neg A, B, F$ (S1)
- 3) $I, J, \neg H$ (S2)
- 4) $\neg D$ (S10)

and the following order of inference (with the used sentence in parentheses):

- 1) B (S9)
- 2) J (S12)
- 3) $\neg D$ (S10)
- 4) $\neg A$ (S8)
- 5) F (S5)
- 6) C (S1)

The resulting graph can be seen below:



7. (11 points) **First Order Logic:** You are given the following first-order logic knowledge base. Using repeated application of the generalized modus ponens, generate all the facts you can. You are free to select any pick order you want. Write the substitutions, existing facts and the resulting facts clearly in the table below. Note that you can generate negative facts. Extend the table if needed but it can also stay partially empty. Make sure you give the newly generated facts an alias of the form F_i (e.g. the first fact you generate must be F_6)

R1: $\forall x1 \text{ SharpTeeth}(x1) \Rightarrow \text{Carnivore}(x1)$

R2: $\forall x2 \text{ ThreeHorns}(x2) \Rightarrow \text{Triceratop}(x2)$

R3: $\forall x3 \text{ Carnivore}(x3) \wedge \text{BigClaw}(x3) \Rightarrow \text{Deinonychuses}(x3)$

R4: $\forall x4 \text{ Carnivore}(x4) \wedge \text{SmallArms}(x4) \Rightarrow \text{TRex}(x4)$

R5: $\forall x5, y5 \text{ SharpTeeth}(x5) \wedge \neg \text{Deinonychus}(x5) \wedge \text{Triceratop}(y5) \Rightarrow \text{Hunts}(x5, y5)$

R6: $\forall x6 \text{ TRex}(x6) \Rightarrow \neg \text{Deinonychus}(x6)$

F1: $\text{ThreeHorns}(\text{Spike})$

F2: $\text{SharpTeeth}(\text{Fido})$

F3: $\text{BigClaw}(\text{Fido})$

F4: $\text{SharpTeeth}(\text{Rover})$

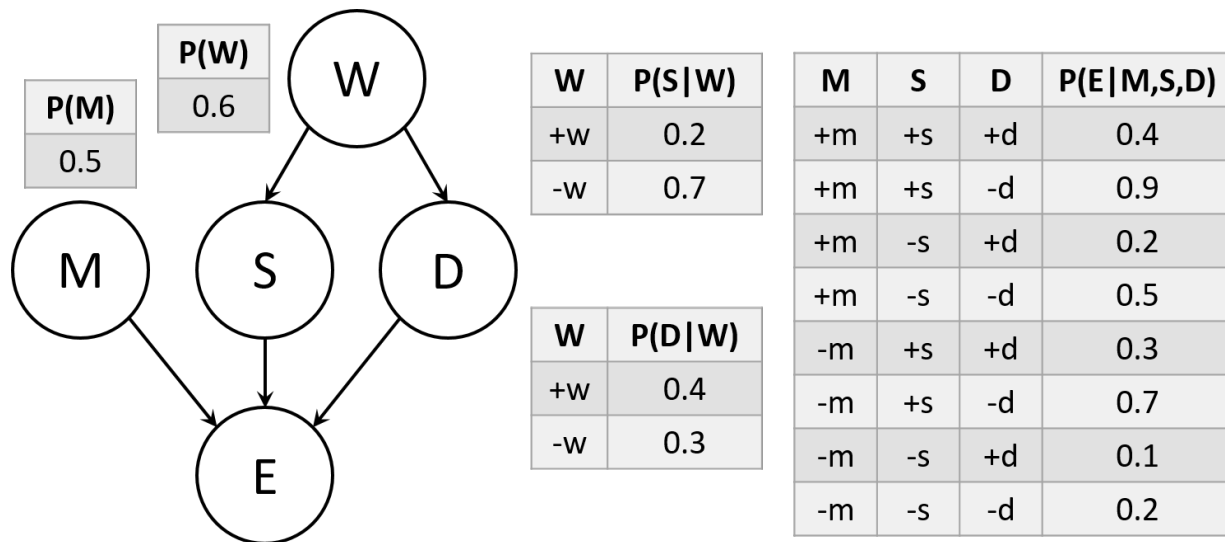
F5: $\text{SmallArms}(\text{Rover})$

R7: $\forall x7 \text{ Deinonychus}(x7) \Rightarrow \neg \text{TRex}(x7)$ (contraposition of R6, done for bonus)

Used Rule	Substitution	Used Facts	New Fact
R1	$x1 \backslash \text{Fido}$	F2	F6: $\text{Carnivore}(\text{Fido})$
R1	$x1 \backslash \text{Rover}$	F4	F7: $\text{Carnivore}(\text{Rover})$
R2	$x2 \backslash \text{Spike}$	F1	F8: $\text{Triceratop}(\text{Spike})$
R3	$x3 \backslash \text{Fido}$	F3, F6	F9: $\text{Deinonychuses}(\text{Fido})$
R4	$x4 \backslash \text{Rover}$	F5, F7	F10: $\text{TRex}(\text{Rover})$
R6	$x6 \backslash \text{Rover}$	F10	F11: $\neg \text{Deinonychuses}(\text{Rover})$
R5	$x5 \backslash \text{Rover}, y5 \backslash \text{Spike}$	F4, F8, F11	F12: $\text{Hunts}(\text{Rover}, \text{Spike})$
R7	$x7 \backslash \text{Fido}$	F9	F13: $\neg \text{TRex}(\text{Fido})$

Grading: First 7 is required (4/3,4/3,4/3,5/3,5/3,4/3,7/3), the last one is bonus (1 point for writing the rule, 1 point for the generated fact)

8. (10 points) **Bayesian Networks:** You like to play real time strategy games. Based on your experience you built a BN which is below. All variables are binary and the probabilities are for the + values:



The variables are:

- **Map size (M):** Whether the map is small (+) or large (-)
 - **Workers (W):** Whether your opponent is concentrating on building workers (+) or not (-)
 - **Soldiers (S):** Whether your opponent is concentrating on building soldiers (+) or not (-)
 - **Defence (D):** Whether your opponent is building defence structures (+) or not (-)
 - **Early rush (E):** Whether your opponent is going to early rush (+) or not (-)
- (a) (2 points) Write down the expression for the joint distribution, $P(M, W, S, D, E)$, of the network.
- $$P(M, W, S, D, E) = P(M)P(W)P(S|W)P(D|W)P(E|M, S, D)$$
- (b) (1 point) Calculate the probability of your opponent building only soldiers and early rushing you in a small map $P(+m, -w, +s, -d, +e)$. You do not need to calculate the final expression (i.e. do not need to perform the additions and multiplications).
- $$P(+m, -w, +s, -d, +e) = P(+m)P(-w)P(+s|-w)P(-d|-w)P(+e|+m, +s, -d)$$
- $$= 0.5 \cdot 0.4 \cdot 0.7 \cdot (1 - 0.3) \cdot 0.9 = 0.0882$$

- (c) (2 points) You also want to check your opponent early rushing in a small map with a large army without other information, i.e. calculate $P(+m, +s, +e)$. You do not need to calculate the final expression (i.e. do not need to perform the additions and multiplications).

$$\begin{aligned}
 P(+m, +s, +e) &= \sum_{\pm w} \sum_{\pm d} P(+m, w, +s, d, +e) \\
 &= \sum_{\pm w} \sum_{\pm d} P(+m)P(w)P(+s|w)P(d|w)P(+e|+m, +s, d) \\
 &= P(+m)P(+w)P(+s|+w)P(+d|+w)P(+e|+m, +s, +d) \\
 &\quad + P(+m)P(+w)P(+s|+w)P(-d|+w)P(+e|+m, +s, -d) \\
 &\quad + P(+m)P(-w)P(+s|-w)P(+d|-w)P(+e|+m, +s, +d) \\
 &\quad + P(+m)P(-w)P(+s|-w)P(-d|-w)P(+e|+m, +s, -d) \\
 &= 0.5 \cdot 0.6 \cdot 0.2 \cdot 0.4 \cdot 0.4 \\
 &\quad + 0.5 \cdot 0.6 \cdot 0.2 \cdot 0.6 \cdot 0.9 \\
 &\quad + 0.5 \cdot 0.4 \cdot 0.7 \cdot 0.3 \cdot 0.4 \\
 &\quad + 0.5 \cdot 0.4 \cdot 0.7 \cdot 0.7 \cdot 0.9 \\
 &= 0.147
 \end{aligned}$$

- (d) (3 points) You have scouted ahead and saw a contingent of soldiers on the move in a small map. You want to guess if your opponent is likely to do an early rush. Calculate $P(+e|+m, +s)$

$$P(+e|+m, +s) = \frac{P(+m, +s, +e)}{P(+m, +s)} = \frac{P(+m, +s, +e)}{P(+m, +s, +e) + P(+m, +s, -e)}$$

We already have $P(+m, +s, +e)$, let's calculate $P(+m, +s, -e)$.

$$\begin{aligned}
 P(+m, +s, -e) &= \sum_{\pm w} \sum_{\pm d} P(+m)P(w)P(+s|w)P(d|w)P(-e|+m, +s, d) \\
 &= 0.5 \cdot 0.6 \cdot 0.2 \cdot 0.4 \cdot 0.6 + 0.5 \cdot 0.6 \cdot 0.2 \cdot 0.6 \cdot 0.1 + 0.5 \cdot 0.4 \cdot 0.7 \cdot 0.3 \cdot 0.6 + 0.5 \cdot 0.4 \cdot 0.7 \cdot 0.7 \cdot 0.1 \\
 &= 0.053
 \end{aligned}$$

Then we have;

$$P(+e|+m, +s) = \frac{P(+m, +s, +e)}{P(+m, +s, +e) + P(+m, +s, -e)} = \frac{0.147}{0.147 + 0.053} = 0.735$$

- (e) (2 points) You have scouted ahead and saw an opponent army and no defensive structures in a small map. What is the probability of an early rush in this case, i.e. $P(+e|+m, +s, -d)$?

This is a trivial question as the desired probability is already in the CPT of E .

$$P(+e|+m, +s, -d) = 0.9$$

9. (9 points) You are trying to estimate the price of wheat. You talk to a farmer who tells you:

“The crop quality is affected by the amount of sun and the amount of rain. We prefer more sunny days to cloudy days in a given season. Too little or too much rain adversely affect crop quality during the season. Same apply to crop yield (amount of crop that is harvested). Mostly a sunny season and adequate but not too much rain increase crop yield. Crop yield is also increased by using fertilizers. Crop yield, crop quality and demand all affect the price of wheat. ”

Consider all the methods you have learned in the course (and only this course!) so far. Describe how you might approach the problem of estimating the price of wheat. How would you represent the problem? What, if any, assumptions would you make? The more precise you are the more credit you will get. (Hint: It is okay to discretize any continuous variable).

There are two answers (there can be more if you get creative but the midterm wasn't long enough) to this based on the topics we have seen so far: First Order Logic and Bayesian Networks, with the latter being the more obvious choice both because of the language and because of the uncertainty in real life. As a result, I expected you to draw a BN. The grading will be based on the correct variables (nodes), accuracy of the graph topology (connections, casual ones will get more points) and also the domain of the variables (i.e. the values that they can take). I at least expect three values in the domain of rain, the others can be binary. I will also give credit to FOL representations and the grading will be based on the conciseness of the representation and the correctness of the rules. Here is my answer:

