KOÇ UNIVERSITY

College of Engineering

COMP 341: INTRODUCTION TO ARTIFICIAL INTELLIGENCE

Midterm 2

Spring 2024, 28/04/2024

Duration: 110 Minutes

Name: Solutions

ID: 00110001 00110000 00110000

- This exam contains 10 pages including this cover page and 6 questions. Check to see if any pages are missing. Put your initials on the top of every page, in case the pages become separated.
- By submitting this exam, you **agree** to fully comply with Koç University Student Code of Conduct, and accept any punishment in case of failure to comply. If you do not submit this exam on time, you will receive 0 credits.
- The exam is **closed book** and **closed notes**. You are **not** allowed to use any additional material including any electronic equipment such as computers and mobile phones.
- You are expected to be able provide clear and concise answers. Gibberish will not receive any credit. Your answers need to be readable by a human, illegible writing is not gradable.
- Read each question carefully and make sure to follow instructions. The written instructions take precedence over an answer that the instructor might give you during the exam, unless the instructor makes a class wide announcement.
- Do not write in the table below.

Question:	1	2	3	4	5	6	Total
Points:	21	15	12	20	17	15	100
Score:							

1. (21 points) True or False :

   False ___ In adversarial search, evaluation functions must approximate the true terminal values.

   True ___ Chance nodes can be used to model stochastic opponents.

   False ___ In probability theory, the following always holds: $P(X|Y, Z) = P(X|Y)P(Y|Z)$.

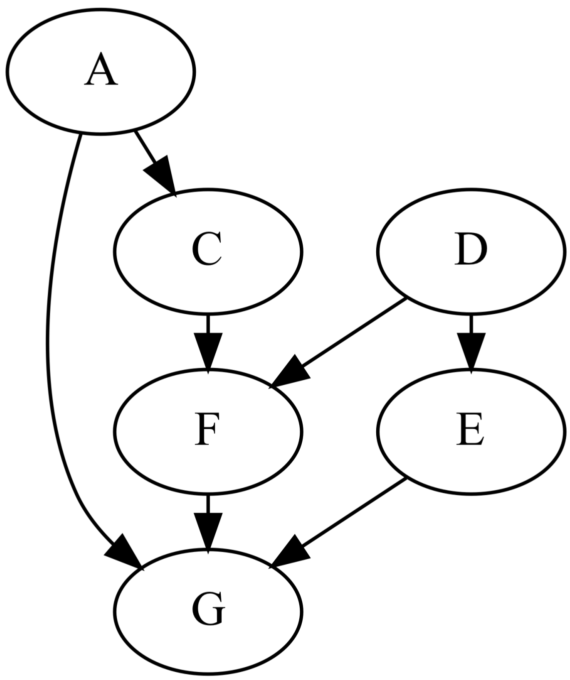
   True ___ If X and Y are conditionally independent given Z, then the following always holds: $P(X|Y, Z) = P(X|Z)$.

   False ___ Bayesian Networks cannot represent arbitrary joint distributions.

   True ___ The computational complexity of variable elimination is determined by the largest factor generated during the process.

   False ___ We need to calculate the weight of each sample for Gibbs sampling.

2. (15 points) Answer the conditional independence questions as True or False based on the given Bayesian Network.



   False ___ A and F are independent. Path A-C-F

   True ___ A and F are independent given C. Observing C blocks the path

   False ___ A and F are independent given C and G. Observing G enables the path A-G-F

   False ___ E and F are independent. Path E-D-F

   True ___ E and F are independent given D. Observing D blocks the path

3. (12 points) You are playing a game where you need to damage your opponents. You have 100 base damage and fixed 80% critical hit bonus. Critical hits happen based on your critical hit chance which is 0% without any gear. Gear can give you extra damage and/or critical hit chance.

   For example, an item that gives you +20 damage will lead to 120 damage. An item with +20 damage and 20% critical chance will either lead to 120 damage without a critical hit or $120 \times 1.8 = 216$ damage with a critical hit (which happens with 20% chance).

   You can equip two pieces of gear. In your inventory you have the following:

   - Item 1: +60 damage
   - Item 2: 75% chance to critical hit
   - Item 3: +50 damage and 20% chance to critical hit

   Answer the following with this information. You don't have to calculate the final expressions. If you need to compare them, state what you compare and your decision based on the outcome.

   (a) (3 points) What is your expected damage when you equip items 1 and 2?

   Let's say x is the base damage, y is the additional damage from the items, c is the critical hit bonus and p is the critical chance. You do $(x + y)$ damage with $(1 - p)$ probability and $(x+y)\times(1+c)$ damage with p probability. Then your expected damage is $(x+y)(1-p)+(x+y)(p)(1+k) = (x+y)(1+pk)$. Plug in the numbers to get $(100+60)(1+0.75\cdot0.8) = 160\cdot1.6 = 256$

   Possible versions if the item interactions are misunderstood (2 points): $160 \cdot 0.75 + 100 \cdot 1.8 \cdot 0.25 = 165$ or $60 \cdot 0.75 + (100 \cdot 1.8 + 60)\cdot = 180$

   (b) (4 points) Items with critical chance do not stack additively (e.g. wearing items 2 and 3 does not lead to 95% critical chance) when equiped together. The game calculates whether you will have a critical hit or not independently for each item (e.g. when wearing items 2 and 3, the game checks if item 2 leads to a critical hit with 75%, then if item 3 leads to a critical hit with 20%). In this case, what is your effective critical hit chance when wearing items 2 and 3?

   There are two ways to calculate this.
   First one is calculating the probability of both items not leading to a critical hit and then subtracting this from 1. Not getting a critical from item 2 has 0.6 probability and not getting a critical from item 3 has 0.9 probability. Since they are independent, we can calculate the not getting critical probability as their multiplication which is $(1 - 0.4) \times (1 - 0.1) = 0.54$. Then the overall critical change is 1-0.54=0.46
   The second one is by using the probability of union $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where A and B are the critical events of item 2 and 3 respectively. We know $P(A) = 0.8$ and $P(B) = 0.25$ and need to calculate $P(A \cap B)$, i.e., the case when the game decides to critical hit for both items. We know they are independent events, thus $P(A \cap B) = P(A)P(B)$. For both getting the critical, this is $0.75 \times 0.2 = 0.15$. Then the overall critical chance can be calculated as 0.75 + 0.20 - 0.15 = 0.8

   Not sure about the results if there are any misunderstanding, will need to see the answers.

(c) (3 points) Which item combination would you pick among items 1&2 and items 2&3? Justify your answer with math. (No need to check the items 1&3 combination)

We basically have to compare the expected damage of the combinations with each other:

- Items 1 and 2: We have calculated the expected damage as 256 in part a
- Items 2 and 3: We have the overall critical hit chance from part b. We can calculate the expected damage as $(x + y)(1 + pk) = (100 + 50)(1 + 0.8 \cdot 0.8) = 150 \cdot 1.64 = 246$ (1 point)

We pick the (item 1, item 2) combination since its expected damage is larger than that of (item 2, item 3) combination.

Note that the expected damage of the (item 2, item 3) combination would have been larger if the critical hit chances would stack additively (versus dimishingly).

If there was a misunderstanding, we are looking for comparisons based on expected damage (2 points).

(d) (2 points) As you play, you receive a new item, Item 4, which has a 20% chance to apply poison. When you check the game rules, you see that if you get a critical hit, you cannot poison the enemy for that hit, regardless of the poison chance. The game first checks whether you are getting a critical hit or not. If you are getting a critical hit, it does not perform a poison check. If you are not getting a critical hit then the game checks whether you are poisioning the enemy or not based on the poison chance. What is the effective probability of poisoning the enemy if you are wearing items 3 and 4?

This is pretty simple, we just multiply the not getting a critical hit probability with poison probability. since you can only poison when there are no critical hits. With item 3, not getting a critical hit has 1-0.2 $= 0.8$ probability. Thus the effective posion probability is $0.8 \cdot 0.2 = 0.16$

4. (20 points) We are still playing the game from the previous question (read its description if you skipped it, you will not need any results from that question) but with different gear. Your current damage is 200, critical hit chance is 25% and critical hit boost is 80%. You have 200 hitpoints. You see an enemy with 500 hitpoints who deals flat 120 damage (it doesn't deal critical hits). You have two actions, hit and retreat. When you retreat, the encounter ends. When you hit, you either have a critical hit or not, as before. Enemy only has the hit action. You take turns hitting each other. Hitpoints are reduced by the same amount of damage dealt. When somebody goes below 0 hitpoints, the encounter ends.

   An example encounter is as follows:

   - You hit the enemy without any critical hit (200 damage). You have 200 hitpoints, and the enemy has 300.
   - The enemy hits you (120 damage). You have 80 hitpoints and the enemy has 300.
   - You hit the enemy with a critical hit (360 damage). You have 80 hitpoints and the enemy has 0. Congratulations, you defeated the enemy!

   Defeating the enemy has +1 utility, losing has -1 utiliy and retreating has 0 utility. You take the first action and want to decide whether to hit the enemy or retreat using expectiminimax search. Answer the following questions based on this information.

   (a) (12 points) Draw the game tree in the next page. Write the hitpoints of you and the enemy next to the non-chance nodes. Write the terminal values below the terminal states.

   Grading (regular):

   - 7 terminal nodes, 3 chance nodes, 3 max nodes (including the root), 3 min nodes. 0.5 point per node for 8 total.
   - If the hitpoints are missing or wrong, 0.25 points per node (instead of 0.5) for 4 points
   - 7 terminal values: Retreats total 1 point, rest 0.5 per node for 3 total.
   - 1 bonus point for overall clarity

   Grading (enemy hits compounded):

   - 6 regular terminal nodes, 1 compound terminal node, 3 chance nodes, 1 root node, 2 non-terminal compound nodes. Regular nodes 0.5, compound nodes 1.
   - Missing or wrong hitpoints get half-points.
   - 7 terminal values: Retreats total 1 point, rest 0.5 per node for 3 total.
   - 1 bonus point for overall clarity

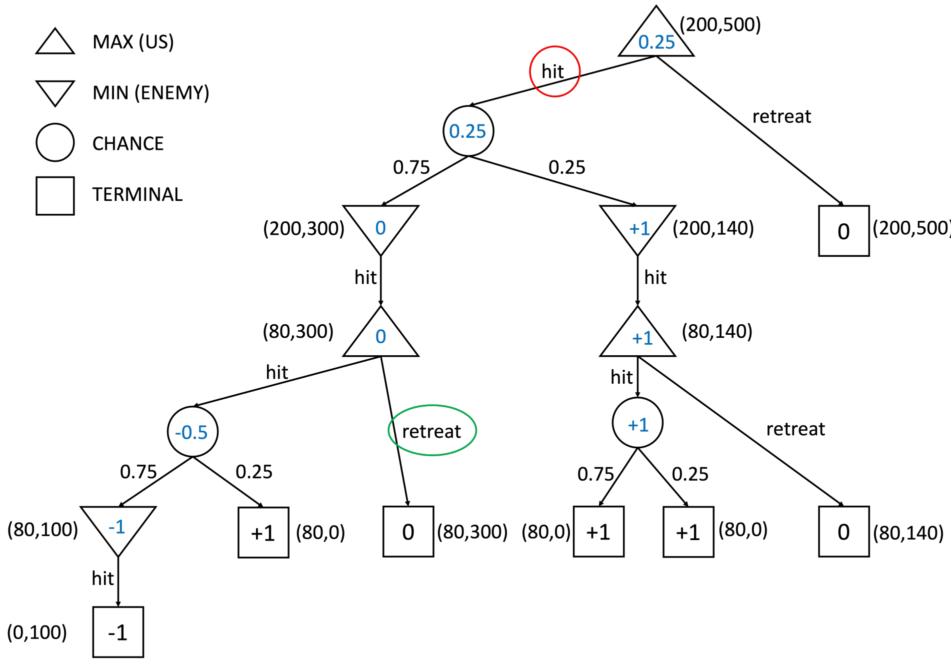
   (b) (6 points) Calculate the values of the non-terminal states and write them on your tree. See the blue numbers in the game tree. 1 per chance node, 1 for the root, 0.5 for the rest for 6 total. If compound states are used, 1 for each node.

   (c) (1 point) What is the best action at the start? From the tree we can see that the best action at the start is hit (red circle). However, we will accept retreat if your tree calculations resulted in this (even if wrong)

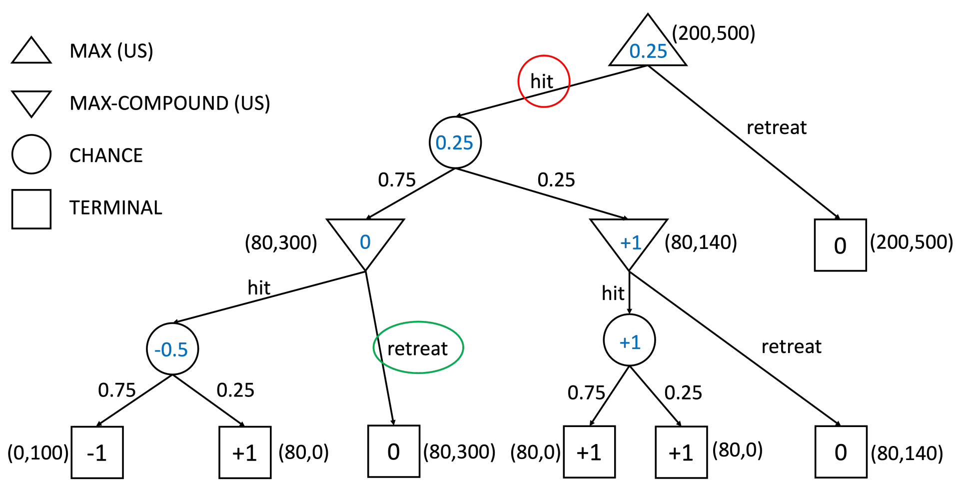
   (d) (1 point) Let's say that you decided to hit at the start. What is your best action (after the enemy's action) if you do not get a critical hit in your first hit? From the tree we can see that the best action at this point is retreat (green ellipse). However, we will accept hit if your tree calculations resulted in this (even if wrong)

(Reserved for Q4)

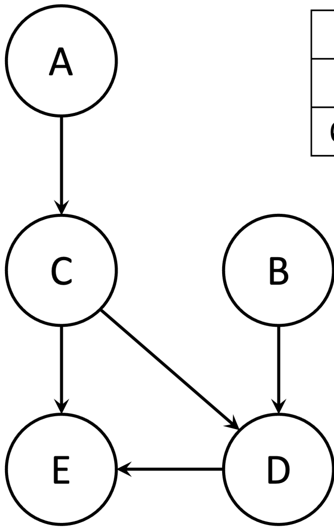
Regular



If the enemy hit is compounded (since it only has one action, we do not need to explicitly put its turn in the tree). Note that left most terminal node is also compound.



5. (17 points) Answer the following questions given the Bayesian Network below.



P(A)	
+a	-a
0.4	0.6

P(B)	
+b	-b
0.6	0.4

	P(C|A)	
A	+c	-c
+a	0.25	0.75
-a	0.4	0.6

		P(D|B,C)	
B	C	+d	-d
+b	+c	0.2	0.8
+b	-c	0.8	0.2
-b	+c	0.6	0.4
-b	-c	0.5	0.5

		P(E|C,D)	
C	D	+e	-e
+c	+d	0.2	0.8
+c	-d	0.7	0.3
-c	+d	0.6	0.4
-c	-d	0.1	0.9

(a) (3 points) What is the joint distribution, $P(A, B, C, D, E)$?
$P(A, B, C, D, E) = P(A)P(B|A)P(C|A, B)P(D|B)P(E|C, D)$

(b) (14 points) Calculate $P(B|+a, +d)$ using exact inference. You can leave all the numbers as fractions. If you are out of time, write your variable elimination plan for partial points.
We are going to provide the variable elimination answer. Initial factors:

- $f_1(+) = P(+a)$: 1 entry
- $f_2(B) = P(B)$: 2 entries
- $f_3(+a, C) = P(C|+a)$: 2 entries
- $f_4(B, C, +d) = P(+d|B, C)$: 4 entries
- $f_5(C, +d, E) = P(E|C, +d)$: 4 entries

Hidden variables are C and E. We need to join their factors and sum them out. To remove C first, we need to join f_3, f_4 and f_5 which will give us 8 entries. To remove E first, we only need to sum it out from f_5 without any joins! However, there is another shortcut. If you notice, when we sum out E, we would only get 1s as we are just summing out a complete distribution. As a result, we can ignore f_5 and just start with removing C using f_3 and f_4.

Step 1: Join $f_3(+a, C)$ and $f_4(B, C, +d)$ to get $f_6(+a, B, C, +d)$

B	C	A	D	$f_6((+a, B, C, +d)$
$+b$	$+c$	$+a$	$+d$	$0.25 \cdot 0.2 = 0.05$
$+b$	$-c$	$+a$	$+d$	$0.75 \cdot 0.8 = 0.60$
$-b$	$+c$	$+a$	$+d$	$0.25 \cdot 0.6 = 0.15$
$-b$	$-c$	$+a$	$+d$	$0.75 \cdot 0.5 = 0.375$

Step 2: Marginalize out C from f_6 to get $f_7(+a, B, +d)$

B	A	D	$f_7(+a, B, +d)$
$+b$	$+a$	$+d$	$0.05 + 0.600 = 0.650 = 26/40$
$-b$	$+a$	$+d$	$0.15 + 0.375 = 0.525 = 21/40$

Step 3: Join $f_2(B)$ and $f_7(+a, B, +d)$ to get $f_8(+a, B, +d)$

B	A	D	$f_8(+a, B, +d)$
$+b$	$+a$	$+d$	$26/40 \cdot 0.6 = 156/400 = 39/100$
$-b$	$+a$	$+d$	$21/40 \cdot 0.4 = 84/400 = 21/100$

Step 4: Normalize $f_8(+a, B, +d)$ to get the desired probability $P(B| + a, +d)$

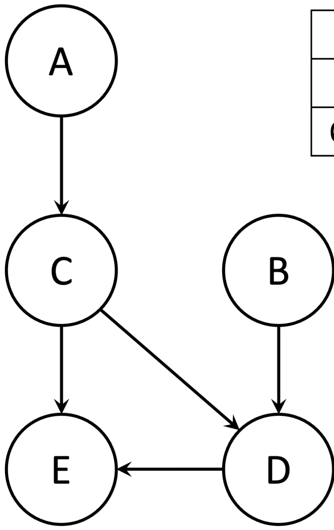
B	A	D	$P(B| + a, +d)$
$+b$	$+a$	$+d$	$39/(39 + 21) = 39/60 = 0.65$
$-b$	$+a$	$+d$	$21/(39 + 21) = 21/60 = 0.35$

Grading: Calculation errors can be ignored as long as we can see individual math steps. If unclear, just 0 points. Breakdown is below.

- Removing E (either trivially or with calculations): 2 points
- Removing C (1 join and 1 sum out): 3 + 3 points
- Joining remaining things for B: 3 points
- Correct normalization: 2 points
- Overall legibility and clarity 1 point

If there are intermediate steps (e.g. first join f_1 and f_2, then join the resulting with f_9), we look at the entire thing (e.g. 3 points for 2 steps)

6. (15 points) Answer the following questions given the same Bayesian Network as before.



P(A)	
+a	-a
0.4	0.6

P(B)	
+b	-b
0.6	0.4

	P(C|A)	
A	+c	-c
+a	0.25	0.75
-a	0.4	0.6

		P(D|B,C)	
B	C	+d	-d
+b	+c	0.2	0.8
+b	-c	0.8	0.2
-b	+c	0.6	0.4
-b	-c	0.5	0.5

		P(E|C,D)	
C	D	+e	-e
+c	+d	0.2	0.8
+c	-d	0.7	0.3
-c	+d	0.6	0.4
-c	-d	0.1	0.9

(a) (3 points) Suppose you want to calculate $P(B|+a,+d)$ using rejection sampling. Cross out the samples that would get rejected:

- ~~$-a, -b, +c, +d, -e$~~
- $+a, -b, +c, +d, -e$
- ~~$+a, +b, +c, -d, +e$~~
- $+a, -b, -c, +d, -e$
- ~~$-a, -b, -c, +d, -e$~~
- ~~$-a, +b, +c, -d, +e$~~
- $+a, +b, -c, +d, -e$
- ~~$-a, -b, +c, +d, +e$~~
- ~~$-a, -b, -c, +d, +e$~~
- $+a, +b, +c, +d, -e$

0.25 per correct cross, -0.375 for wrong cross, (round to 1 digit)

(b) (3 points) You have run rejection sampling 1000 times (total samples, before rejection) given the evidence $(+a, +d)$. There are 225 samples left with a total of 150 samples that include $+b$. Is this information enough to calculate the probability $P(+b|+a,+d)$? If so, what is the value? If not, what more information do we need?

It is enough since and we can calculate it as $150/225 = 2/3$.

(c) (4 points) Suppose you want to calculate $P(B|+a, +d)$ using likelihood weighting. Write down all the possible combinations of samples and corresponding weights.

For this problem, all the non-evidence variables (B, C, D) affect the probabilities calculated in weights. Since they are binary, we have $2^3 = 8$ unique cases as follows (0.5 points per item). However, I will also accept the case when E is ignored (with justification!) since it doesn't affect anything (1 point per case if this happens):

- Sample $(+a, +b, +c, +d, +e)$ with weight $P(+a)P(+d|+b, +c) = 0.4 \cdot 0.2 = 0.08$
- Sample $(+a, +b, +c, +d, -e)$ with weight $P(+a)P(+d|+b, +c) = 0.4 \cdot 0.2 = 0.08$
- Sample $(+a, +b, -c, +d, +e)$ with weight $P(+a)P(+d|+b, -c) = 0.4 \cdot 0.8 = 0.32$
- Sample $(+a, +b, -c, +d, -e)$ with weight $P(+a)P(+d|+b, -c) = 0.4 \cdot 0.8 = 0.32$
- Sample $(+a, -b, +c, +d, +e)$ with weight $P(+a)P(+d|-b, +c) = 0.4 \cdot 0.6 = 0.24$
- Sample $(+a, -b, +c, +d, -e)$ with weight $P(+a)P(+d|-b, +c) = 0.4 \cdot 0.6 = 0.24$
- Sample $(+a, -b, -c, +d, +e)$ with weight $P(+a)P(+d|-b, -c) = 0.4 \cdot 0.5 = 0.20$
- Sample $(+a, -b, -c, +d, -e)$ with weight $P(+a)P(+d|-b, -c) = 0.4 \cdot 0.5 = 0.20$

Wrong samples (e.g. involving -a,+d) get -0.25 points. Multiplying together all possible probabilities (i.e. joint likelihood) gets 0 points

(d) (3 points) Now you want to do Gibbs sampling. Your current state is $+a, -b, -c, +d, +e$. You are going to sample the variable **B** next. What is the inference query (i.e. the distribution you will use to sample from)? Simplify this as much as you can using conditional independence.

The query is $P(B|+a, -c, +d, +e)$ (1 points). From the BN, we can see that B is independent of A and E given C and D. Thus, it simplifies to $P(B|-c, +d)$ (2 points). Having $-b$ or $+b$ in the right hand side will get 0 points. Other wrong answers will also get 0.

(e) (2 points) Which conditional probability tables given in the Bayesian Network, would you use to calculate the distribution to perform the Gibbs sampling step from the previous part?

$P(B), P(+d|B, -c)$, will accept the non-evidence versions as well