Study plan for data adaptive selection of truncation level

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(Initially intended for no extrapolation, just TMLE of truncation induced target parameter).

I want to understand if choosing δ by starting at a say 0.2 and moving to zero until $\frac{d}{d\delta}\hat{\Psi}_n(\delta) + \frac{1}{n}\frac{d}{d\delta}\hat{\sigma}_n(\delta) = 0$ can work.

Define

$$
\begin{aligned}
\delta_n^0 &= \text{argmin}_\delta E[(\hat{\Psi}_n(\delta) - EY_d)^2] = \text{argmin}\text{MSE}_n(\delta) \\
\delta_n^1 &= \text{argmin}_\delta b_0^2(\delta) + \frac{1}{n}\sigma_0^2(\delta) \\
\delta_n^2 &= \text{argmin}_\delta b_0(\delta) + \frac{1}{n}\sigma_0(\delta).
\end{aligned}
$$

Stuff I know:

- $b_0^2(\delta) + \frac{1}{n}\sigma_0^2(\delta) \leq \left(b_0(\delta) + \frac{1}{n}\sigma_0(\delta)\right)^2 \leq 2\left(b_0^2(\delta) + \frac{1}{n}\sigma_0^2(\delta)\right)$

Stuff I need to check:

1. Do I have $\alpha_{i,j}\delta_n^i \leq \delta_n^j \leq \beta_{i,j}\delta_n^i$ for $i,j \in \{0,1,2\}$ and $\alpha_{i,j} > 0, \beta_{i,j} > 0$? In other words, are the three deltas defined above roughly speaking equivalent up to multiplicative constants?
2. Do I have the same for the MSEs wrt EY_d of $\Psi(\delta_n^i)$ and $\Psi(\delta_n^j)$ for pairs (i,j)?
3. Can I estimate δ_n^2 well? Let $\hat{\delta}_n$ its estimate obtained with the procedure described above?
4. Do I have $b_0(\hat{\sigma}_n^2) + \frac{1}{n}\sigma_0(\hat{\sigma}_n^2) \sim b_0(\sigma_n^2) + \frac{1}{n}\sigma_0(\delta_n^2)$?

Notes:

1 and 2: Computing δ_n^0 requires simulations, especially computing MSE_n for a bunch of n

3: to check if $\hat{\delta}_n^2$ estimates δ_n^2 well: plot MSE_n as a function of n, or $n \times \text{MSE}_n$ as a function of n, or $\log \text{MSE}_n$ as a function of $\log n$. The latter will give an idea of the rate.