1 Bayes estimator and Bayes risk

1.1 Question 1

Example 1.1 (Temperature-based Energy Consumption Prediction)**.** Let us consider predicting daily energy consumption based on ambient temperature:

- **Input space:** $\mathcal{X} = [-20, 40]$ (temperature in °C)

- **Output space:** $\mathcal{Y} = \mathbb{R}_+$ (energy consumption in kWh)

- **Joint distribution:** $(X, Y) \sim \mathbb{P}$ where $X \sim \text{Uniform}(0, 30)$ and $Y|X = x \sim \mathcal{N}(50 - x, 4)$

- **Loss function:** Squared loss $\ell(y, \hat{y}) = (y - \hat{y})^2$

Corollary 1.2 (Squared Loss Case)**.** *As seen in the lecture notes, for squared loss $\ell(y, \hat{y}) = (y - \hat{y})^2$, the Bayes optimal predictor is:*

$$f^*(x) = \mathbb{E}[Y|X = x]$$

Bayes Optimal Predictor:

$$f^*(x) = \mathbb{E}[Y|X = x] = 50 - x$$

Bayes Risk (using lecture notes formula):

$$R^* = \mathbb{E}_{X,Y}[(Y - f^*(X))^2] = \mathbb{E}_{X,Y}[(Y - (50 - X))^2] = \mathbb{E}[\sigma^2] = 4$$