Résumé des algorithmes d'optimisation vus en cours

Nom	Problème	Algorithme	Hypothèses	Convergence
Gradient	$\min_x f(x)$	$x_{k+1} = x_k - \frac{1}{L}\nabla f(x_k)$	f dérivable et ∇f L-lipschitzien $+$ f convexe $+$ f μ-fortement convexe	$f(x_{k+1}) \leq f(x_k)$ $f(x_k) - f(x^*) \leq \dfrac{L\\|x_0 - x^*\\|^2}{2k}$ $f(x_k) - f(x^*) \leq \left(1 - \frac{\mu}{L}\right)^k L\\|x_0 - x^*\\|^2$
Gradient proximal	$\min_x f(x) + g(x)$ $F = f + g$	$x_{k+1} = \mathrm{prox}_{\frac{1}{L}g}\left(x_k - \frac{1}{L}\nabla f(x_k)\right)$	∇f L-lipschitzien $+$ f et g convexes $+$ F μ-fortement convexe	$F(x_{k+1}) \leq F(x_k)$ $F(x_k) - F(x^*) \leq \dfrac{L\\|x_0 - x^*\\|^2}{2k}$ $F(x_k) - F(x^*) \leq \left(1 - \frac{\mu}{L}\right)^k L\\|x_0 - x^*\\|^2$
Uzawa (ou méthode des multiplicateurs)	$\min_x f(x)$ sc: $Ax = b$	$x_{k+1} = \arg\min_x f(x) + \langle \lambda_k, Ax - b\rangle$ $\lambda_{k+1} = \lambda_k + \frac{\mu}{\\|A\\|^2}(Ax_{k+1} - b)$	f μ-fortement convexe	$(x_k, \lambda_k) \to (x^*, \lambda^*)$ (x^*, λ^*) point selle du lagrangien
Décomposition/ coordination	$\min_x f(x) + g(Mx)$	$x_{k+1} = \arg\min_x f(x) + \langle \lambda_k, Mx\rangle$ $z_{k+1} = \arg\min_z g(z) - \langle \lambda_k, z\rangle$ $\lambda_{k+1} = \lambda_k + \frac{\mu}{\\|MM^T + I\\|}(Mx_{k+1} - z_{k+1})$	f et g μ-fortement convexes	$(x_k, z_k, \lambda_k) \to (x^*, Mx^*, \lambda^*)$
ADMM	$\min_x f(x) + g(Mx)$	$x_{k+1} = \arg\min_x f(x) + \langle \lambda_k, Mx\rangle + \dfrac{\gamma}{2}\\|Mx - z_k\\|^2$ $z_{k+1} = \arg\min_z g(z) - \langle \lambda_k, z\rangle + \dfrac{\gamma}{2}\\|Mx_{k+1} - z\\|^2$ $\lambda_{k+1} = \lambda_k + \gamma(Mx_{k+1} - z_{k+1})$	f et g convexes	$\lambda_k \to \lambda^* \in \arg\max_\lambda \mathcal{D}(\lambda)$ $Mx_k - z_k \to 0$ $f(x_k) + g(z_k) \to f(x^*) + g(Mx^*)$