COMP 540 HW 01

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0. Background Refresher

0.1 Samplers

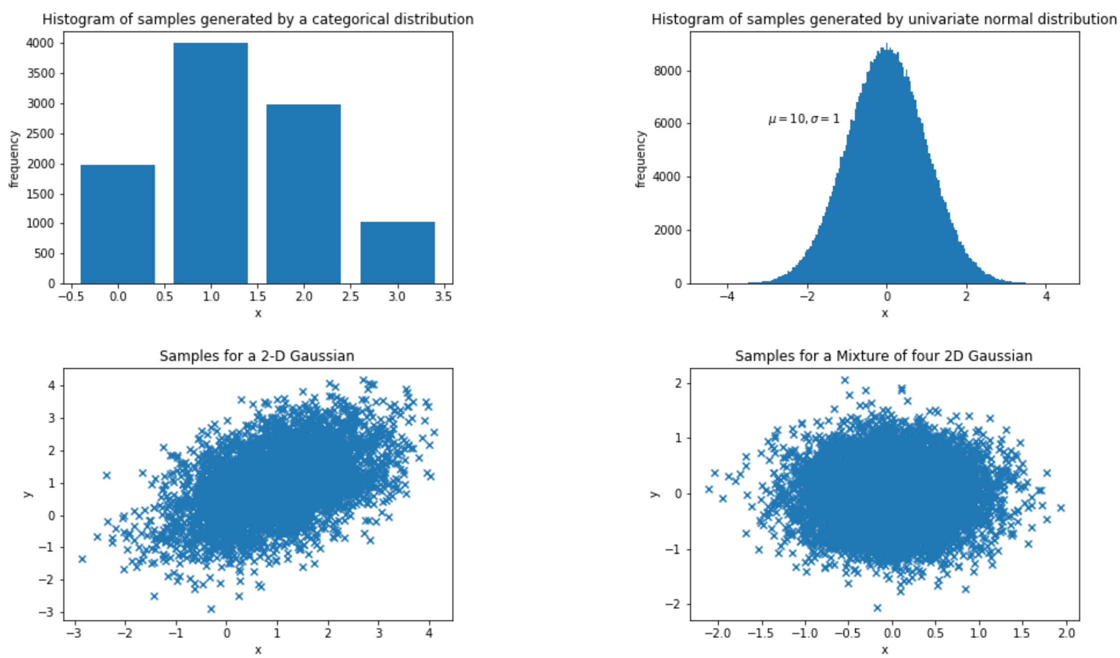


Figure 1: Visualization of four distributions

Figure 1 visualize four distributions. Specifically, for a mixture distribution of four 2D Gaussians, the probability that a sample from this distribution lies within the unit circle centered at $(0.1, 0.2)$ is $p = 0.83736$.

0.2 Prove two independent Poisson random variables are also Poisson variable.

Proof:

Given two independent random variables $X_1 \sim P(\lambda_1)$ and $X \sim P(\lambda_2)$. i.e.,

$$P(X_1 = m) = e^{-\lambda_1}\frac{\lambda_1^m}{m!},\ m = 1, 2, ... \tag{1}$$

and

$$P(X_2 = n) = e^{-\lambda_2}\frac{\lambda_2^n}{n!},\ n = 0, 1, 2, ... \tag{2}$$

Denote sum of them are

$$X = X_1 + X_2 \tag{3}$$

then the probability distribution of X is:

$$\begin{aligned}
P(X = k) &= \sum_{m+n=k} P(X_1 = m, X_2 = n) \\
&\overset{X_1,X_2 indep.R.V.}{=} \sum_{m+n=k} P(X_1 = m)P(X_2 = n) \\
&= \sum_{m+n=k} e^{-\lambda_1}\frac{\lambda_1^m}{m!}e^{-\lambda_2}\frac{\lambda_2^n}{n!} \\
&= \frac{1}{k!}e^{-(\lambda_1+\lambda_2)}\sum_{m+n=k}\frac{k!}{m!n!}\lambda_1^m\lambda_2^n \\
&= \frac{1}{k!}e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^k
\end{aligned} \tag{4}$$

that is, $X = X_1 + X_2 \sim P(\lambda_1 + \lambda_2)$. Q.E.D.

0.3 Proof question 3

$$\begin{aligned}
p(x_1,x_0) &= p(x_1|x_0)p(x_0) \\
&= \alpha e^{-\frac{(x_1-x_0)^2}{2\sigma^2}}\alpha_0 e^{-\frac{(x_0-\mu_0)^2}{2\sigma_0^2}}
\end{aligned} \tag{5}$$

The probability of $X_1 = x_1$ is,

$$\begin{aligned}
p(X_1 = x_1) &= \int p(X_1 = x_1, X_0 = x_0)\,dx_0 \\
&= \alpha_0\alpha\int exp\left(-\frac{1}{2}\left(\frac{(x_0-\mu_0)^2}{\sigma_0^2} + \frac{(x_1-x_0)^2}{\sigma^2}\right)\right)dx_0 \\
&= \frac{\alpha_0\alpha}{A}exp\left(-\frac{1}{2}\frac{(x_1-\mu_0)^2}{\sigma^2+\sigma_0^2}\right)
\end{aligned}$$

Here A is a constant. So

$$p(X_1 = x_1) = \alpha_1 exp\left(-\frac{1}{2}\frac{(x_1-\mu_1)^2}{\sigma_1^2}\right)$$

And

$$\mu_1 = \mu_0$$

$$\sigma_1^2 = \sigma^2 + \sigma_0^2$$

$$\alpha_1 = \frac{\alpha_0\alpha}{A} = \alpha_0\alpha\int exp\left(-\frac{1}{2}\frac{x_0^2 - 2\left(\frac{\sigma^2\mu_0+\sigma_0^2x_1}{\sigma^2+\sigma_0^2}\right)x_0 + \left(\frac{\sigma^2\mu_0+\sigma_0^2x_1}{\sigma^2+\sigma_0^2}\right)^2}{\sigma_0^2\sigma^2/\left(\sigma^2+\sigma_0^2\right)}\right)dx_0$$

0.4 Eigenvalues

$$\boldsymbol{A} = \left[\begin{array}{cc} 13 & 5 \\ 2 & 4 \end{array} \right]$$

$$\begin{array}{rcl} \boldsymbol{AX} & = & \lambda \boldsymbol{X} \\ (\boldsymbol{A} - \lambda \boldsymbol{I})\boldsymbol{X} & = & 0 \\ \left[\begin{array}{cc} 13-\lambda & 5 \\ 2 & 4-\lambda \end{array} \right] \boldsymbol{X} & = & 0 \end{array}$$

$\lambda_1 = 3,\ \boldsymbol{X}_1 = \begin{pmatrix} -1/2 \\ 1 \end{pmatrix}$
$\lambda_2 = 14,\ \boldsymbol{X}_2 = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

0.5 Matrix multiplication

$A = \left[\begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array} \right]$, $B = \left[\begin{array}{cc} 5 & 6 \\ 7 & 8 \end{array} \right]$, we have $(A+B)^2 \neq A^2 + 2AB + B^2$
$A = \left[\begin{array}{cc} 0 & 1 \\ 0 & 1 \end{array} \right]$, $B = \left[\begin{array}{cc} 1 & 1 \\ 0 & 0 \end{array} \right]$, $A, B \neq 0$ and we have $AB = 0$

0.6 Question 6

$$\begin{array}{rcl} A^T A & = & (I - 2uu^T)^T (I - 2uu^T) \\ & = & (I - 2uu^T)(I - 2uu^T) \\ & = & I - 2uu^T - 2uu^T + 4u(u^T u)u^T \\ & = & I \end{array} \tag{6}$$

0.7 Convex function

0.7.1 Triple

for $x \geq 0$

$$f(x) = 3x^3 \tag{7}$$

$$f^{'}(x) = 3x^2 \tag{8}$$

$$f^{''}(x) = 6x \geq 0 \tag{9}$$

so,$f(x) = x^3$ is convex.

0.7.2 Two dimension

For any $\lambda \in [0,1]$,
we have $\lambda \geq 0, 1 - \lambda \geq 0$

For any $(x_1, y_1), (x_2, y_2) on R^2$

$$\begin{aligned} f(\lambda(x_1,y_1)+(1-\lambda)(x_2,y_2)) &= max(\lambda(x_1,y_1)+(1-\lambda)(x_2,y_2)) \\ &\le max(\lambda(x_1,y_1))+max((1-\lambda)(x_2,y_2)) \\ &= \lambda max(x_1,y_1)+(1-\lambda)max(x_2,y_2) \\ &= \lambda f(x_1,y_1)+(1-\lambda)f(x_2,y_2) \end{aligned} \tag{10}$$

So, $f(x)$ is convex.

0.7.3 Plus

Because f is convex on S, for $\lambda \in [0,1]$ and all $x_1, x_2 \in S$, we have

$$f(\lambda x_1 + (1-\lambda)x_2) \le \lambda f(x_1) + (1-\lambda)f(x_2) \tag{11}$$

Also,

$$g(\lambda x_1 + (1-\lambda)x_2) \le \lambda g(x_1) + (1-\lambda)g(x_2) \tag{12}$$

Let $h = f + g$,

$$\begin{aligned} h(\lambda x_1+(1-\lambda)x_2) &= f(\lambda x_1+(1-\lambda)x_2)+g(\lambda x_1+(1-\lambda)x_2) \\ &\le \lambda f(x_1)+(1-\lambda)f(x_2)+\lambda g(x_1)+(1-\lambda)g(x_2) \\ &= \lambda(f(x_1)+g(x_1))+(1-\lambda)(f(x_2)+g(x_2)) \\ &= \lambda h(x_1)+(1-\lambda)h(x_2) \end{aligned} \tag{13}$$

So, h is convex, i.e. $f + g$ is convex.

0.7.4 Multiply

Let $h = fg$

$$\begin{aligned} h^{''} &= (f^{'}g + fg^{'})^{'} \\ &= f^{''} + 2f^{'}g^{'} + fg^{''} \end{aligned} \tag{14}$$

Obviously, $f \ge 0$, $f^{''} \ge 0$, $g \ge 0$, $g^{''} \ge 0$ on S.
Let the minimum of both f and g be x_0, then when $x \le x_0$, $f(x_0) \le 0$ and $g(x_0) \le 0$; when $x \ge x_0$, $f(x_0) \ge 0$ and $g(x_0) \ge 0$, i.e. $f^{'}g^{'} \ge 0$. So

$$h^{''} \ge 0 \tag{15}$$

h is convex, i.e. $f + g$ is convex.

0.8 Entropy of categorical distribution

The entropy of a categorical distribution on K values is

$$H(p) = -\sum_{i=1}^{K} p_i \log(p_i), \tag{16}$$

with constraint that

$$\sum_{i=1}^{K} p_i = 1. \tag{17}$$

Using Lagrange Multiplier, one can combine the above two equations into:

$$L(p, \lambda) = -\sum_{i=1}^{K} p_i \log(p_i) + \lambda(\sum_{i=1}^{K} p_i - 1). \tag{18}$$

Taking derivative of all unknown variables gives:

$$\frac{\partial L}{\partial p_i} = -(\log p_i + 1) + \lambda = 0, \;\; i = 1, 2, ... \tag{19}$$

Substituting p_i with λ yields

$$p_i = \frac{1}{K}, \;\; i = 1, 2, ..., K. \tag{20}$$

Q.E.D.

1. Locally weighted linear regression.

1.1 Expression of $J(\theta)$

Define $\boldsymbol{X}$, $\boldsymbol{W}$ in the following way:

$$\boldsymbol{X} = \begin{bmatrix} ----\left(x^{(1)}\right)^T---- \\ ----\left(x^{(2)}\right)^T---- \\ \vdots \\ ----\left(x^{(m)}\right)^T---- \end{bmatrix} \Longleftrightarrow \boldsymbol{X}_{i,j} = (x^{(i)})_j \tag{21}$$

$$\boldsymbol{W} = \begin{bmatrix} w^{(1)} & & & \\ & w^{(2)} & & \\ & & \ddots & \\ & & & w^{(m)} \end{bmatrix} \Longleftrightarrow \boldsymbol{W}_{i,j} = \delta_{i,j} w^{(i)} \tag{22}$$

Substituting $\boldsymbol{X}$, $\boldsymbol{W}$ into $(\boldsymbol{X}\theta - \boldsymbol{y})^T \boldsymbol{W} (\boldsymbol{X}\theta - \boldsymbol{y})$ yields

$$\begin{aligned}
(\boldsymbol{X}\theta - \boldsymbol{y})^T \boldsymbol{W} (\boldsymbol{X}\theta - \boldsymbol{y}) &= \sum_{i,j} \left[(\boldsymbol{X}\theta - \boldsymbol{y})^T\right]_{1,i} \boldsymbol{W}_{i,j} (\boldsymbol{X}\theta - \boldsymbol{y})_{j,1} \\
&= \sum_{i,j} \delta_{i,j} w^{(i)} \left[(\boldsymbol{X}\theta - \boldsymbol{y})^T\right]_{1,i} (\boldsymbol{X}\theta - \boldsymbol{y})_{j,1} \\
&= \sum_{i} w^{(i)} \left[(\boldsymbol{X}\theta - \boldsymbol{y})_{i,1}\right]^2 \\
&= \sum_{i} w^{(i)} \left[\left(x^{(i)}\right)^T \theta - y^{(i)}\right]^2 \\
&= \sum_{i} w^{(i)} \left[\theta x^{(i)} - y^{(i)}\right]^2.
\end{aligned}$$

So $J(\theta) = \frac{1}{2}(\boldsymbol{X}\theta - \boldsymbol{y})^T \boldsymbol{W}(\boldsymbol{X}\theta - \boldsymbol{y})$. Q.E.D.

1.2 Closed formed solution of θ

$$
\begin{aligned}
\frac{\partial J(\theta)}{\partial \theta} &= \frac{1}{2}\frac{\partial}{\partial \theta}\left[(\boldsymbol{X}\theta - \boldsymbol{y})^T \boldsymbol{W}(\boldsymbol{X}\theta - \boldsymbol{y})\right] \\
&= \frac{1}{2}\frac{\partial}{\partial \theta} tr\left[\boldsymbol{y}^T\boldsymbol{W}\boldsymbol{X}\boldsymbol{\theta} - \boldsymbol{y}^T\boldsymbol{W}\boldsymbol{y} - \boldsymbol{\theta}^T\boldsymbol{X}^T\boldsymbol{W}\boldsymbol{X}\boldsymbol{\theta} + \boldsymbol{\theta}^T\boldsymbol{X}^T\boldsymbol{W}\boldsymbol{y}\right] \\
&= \boldsymbol{X}^T\boldsymbol{W}\boldsymbol{y} - \boldsymbol{X}^T\boldsymbol{W}\boldsymbol{X}\boldsymbol{\theta}
\end{aligned}
$$

where in the third line we used the following two equations[1]:

$$
\frac{\partial tr(\boldsymbol{A}^T\boldsymbol{B})}{\partial \boldsymbol{A}} = \boldsymbol{B}^T \tag{23}
$$

and

$$
\frac{\partial tr(\boldsymbol{A}^T\boldsymbol{B}\boldsymbol{A})}{\partial \boldsymbol{A}} = \boldsymbol{B}\boldsymbol{A} + \boldsymbol{B}^T\boldsymbol{A} \tag{24}
$$

Therefore the closed form solution for θ is

$$
\theta = (\boldsymbol{X}^T\boldsymbol{W}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{W}\boldsymbol{y}. \tag{25}
$$

1.3 Batch gradient descent for locally weighted linear regression

The derivative of J_θ is:

$$
\frac{\partial}{\partial \theta} J(\theta) = \sum_{i=1}^{m} w^{(i)}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}. \tag{26}
$$

Therefore we have the following algorithm:

Algorithm 1: Batch gradient descent algorithm for locally weighted linear regression

Result: The estimated parameters θ for locally weighted linear regression

while θ *does not converge* **do**
    for *every* j **do**
        $\theta_j = \theta_j - \alpha \sum_{i=1}^m w^{(i)}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}$;
    end
end

---

[1] Take the second equation for example:

$$
\begin{aligned}
\left[\frac{\partial tr(\boldsymbol{A}^T\boldsymbol{B}\boldsymbol{A})}{\partial \boldsymbol{A}}\right]_{i,j} &= \frac{\partial A^T_{l,m}B_{m,n}A_{n,l}}{\partial A_{i,j}} = \frac{\partial A_{m,l}B_{m,n}A_{n,l}}{\partial A_{i,j}} \\
&= \delta_{m,i}\delta_{l,j}B_{m,n}A_{n,l} + A_{m,l}B_{m,n}\delta_{n,i}\delta_{l,j} \\
&= B_{i,n}A_{n,j} + A_{m,j}B_{m,i} \\
&= (\boldsymbol{B}\boldsymbol{A})_{i,j} + (\boldsymbol{B}^T\boldsymbol{A})_{i,j},
\end{aligned}
$$

hence leading to $\frac{\partial tr(\boldsymbol{A}^T\boldsymbol{B}\boldsymbol{A})}{\partial \boldsymbol{A}} = \boldsymbol{B}\boldsymbol{A} + \boldsymbol{B}^T\boldsymbol{A}$

2. Properties of linear regression estimator.

2.1 Prove $E[\theta] = \theta^*$

Proof:

The following facts:

1. $y^{(i)} = \theta^{*T} x^{(i)} + \epsilon^{(i)}$,
2. $\epsilon^{(i)}, i = 1, 2, ..., m$ are *i.i.d.* of $N(0, \sigma^2)$,

indicate that:

given fixed arbitrary $x^{(i)}$ and fixed unknown parameter θ^*, $y^{(i)}$, $1 \leq i \leq m$ are *i.i.d. of* $N(\theta^{*T} x^{(i)}, \sigma^2)$

$$p(y^{(i)}|x^{(i)}, \theta^*) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(y^{(i)} - \theta^{*T} x^{(i)})^2}{2\sigma^2}}. \tag{27}$$

The least-square estimate of θ^* is θ given by

$$\begin{aligned} \theta &= (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{y} & (28) \\ &\overset{denote: (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \equiv \boldsymbol{A}}{=} \boldsymbol{A}\boldsymbol{y} & (29) \end{aligned}$$

The expectation of θ is

$$\begin{aligned} E[\theta] &= E[\boldsymbol{A}\boldsymbol{y}] & (30) \\ &= E \begin{bmatrix} \sum_j A_{1,j} y^{(j)} \\ \sum_j A_{2,j} y^{(j)} \\ \vdots \\ \sum_j A_{d+1,j} y^{(j)} \end{bmatrix}. & (31) \end{aligned}$$

Since expectations has the following property (regardless of the independence of Z_k):

$$E[Z_1 + Z_2 + ... + Z_l] = \sum_k Z_k, \tag{32}$$

we have

$$\begin{aligned} E[\theta] &= E \begin{bmatrix} \sum_j A_{1,j} y^{(j)} \\ \sum_j A_{2,j} y^{(j)} \\ \vdots \\ \sum_j A_{d+1,j} y^{(j)} \end{bmatrix} = \begin{bmatrix} \sum_j A_{1,j} E[y^{(j)}] \\ \sum_j A_{2,j} E[y^{(j)}] \\ \vdots \\ \sum_j A_{d+1,j} E[y^{(j)}] \end{bmatrix} & (33) \\ &= \boldsymbol{A} \begin{bmatrix} E[y^{(1)}] \\ E[y^{(2)}] \\ \vdots \\ E[y^{(m)}] \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \theta^{*T} x^{(1)} \\ \theta^{*T} x^{(2)} \\ \vdots \\ \theta^{*T} x^{(m)} \end{bmatrix} & (34) \\ &= \boldsymbol{A}\boldsymbol{X}\theta^* = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{X} \theta^* & (35) \\ &= \theta^* & (36) \end{aligned}$$

where in the second step the following equation is used: $E[y^{(i)}] = \theta^{*T} x^{(i)}$ (trivial to obtain as $y^{(i)}$ observe normal distribution). Therefore $E[\theta] = \theta^*$, implying that the estimation θ is unbiased. Q.E.D

2.2 Prove $Var(\theta) = \left(\boldsymbol{X}^T \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \sigma^2$

Denote $\boldsymbol{A} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T$, therefore $\theta = \boldsymbol{A}\boldsymbol{y}$.

$$
\begin{aligned}
Var(\theta) &= Var(\boldsymbol{A}\boldsymbol{y}) && (37)\\
&= Var(\begin{bmatrix} \sum_j A_{1,j} y^{(j)} \\ \sum_j A_{2,j} y^{(j)} \\ \vdots \\ \sum_j A_{m,j} y^{(j)} \end{bmatrix}) = \begin{bmatrix} \sum_j A_{1,j} Var(y^{(j)}) \\ \sum_j A_{2,j} Var(y^{(j)}) \\ \vdots \\ \sum_j A_{m,j} Var(y^{(j)}) \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} Var(y^{(1)}) \\ Var(y^{(2)}) \\ \vdots \\ Var(y^{(m)}) \end{bmatrix}, && (38)
\end{aligned}
$$

where in the second line we made used of the property of variance:

$$
Var(\sum_k Z_k) = \sum Var(Z_k),\ Z_k \ is\ independent\ from\ each\ other. \quad (39)
$$

Substituting $Var(y^{(i)}) = \sigma^2$ $i = 1, 2, ..., m$, we have $Var(\theta) = \boldsymbol{A}\sigma^2 = \left(\boldsymbol{X}^T \boldsymbol{X}\right)^{-1} \boldsymbol{X}^T \sigma^2$. Q.E.D.

3. Implementing linear regression and regularized linear regression

3.1.A1

No plot for this question.

3.1.A2

Figure 2 plots the linear fit using parameter obtained through training, while Fig. 3 shows the convergence of the loss function against iteration times.

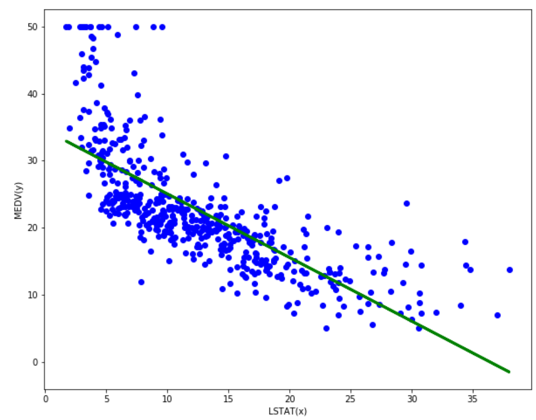


Figure 2: Fitting a linear model to the data

Figure 2 plots the linear fit using parameter obtained through training.

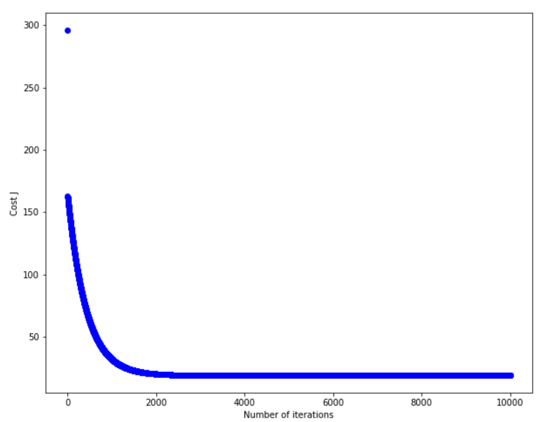


Figure 3: Convergence of Loss function against the number of iterations.

Problem 3.1.A3: Predicting on unseen data

For lower status percentage = 5, we predict a median home value of 298034.494122

For lower status percentage = 50, we predict a median home value of -129482.128898

3.1.B1: Feature Normalization

No plot for this question.

3.1.B2: Loss function and gradient descent

Figure 4 shows the loss function against the number of iterations.

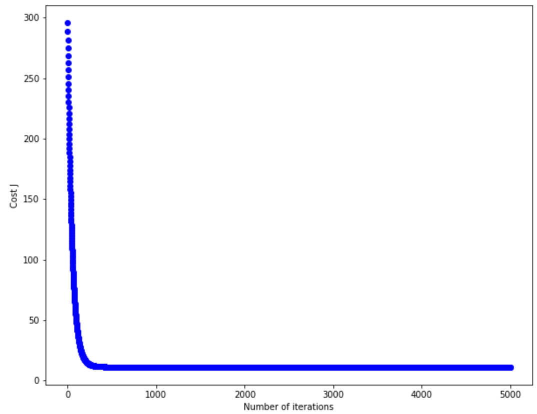


Figure 4: Fitting a linear model to the data

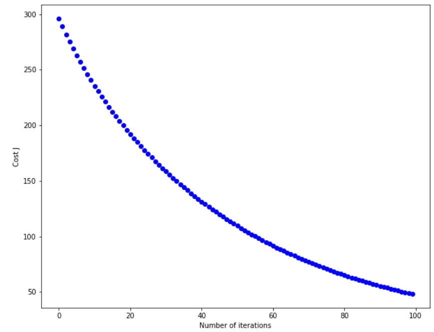
3.1.B3 Making predictions on unseen data

For average home in Boston suburbs, we predict a median home value of 225328.063241

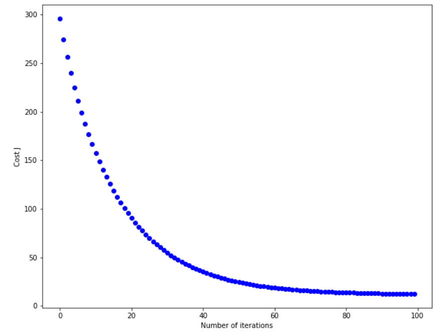
3.1.B4: Normal equations

For average home in Boston suburbs, we predict a median home value of 225328.063241, which is the same as we obtained in subsec. 3.1.B3.

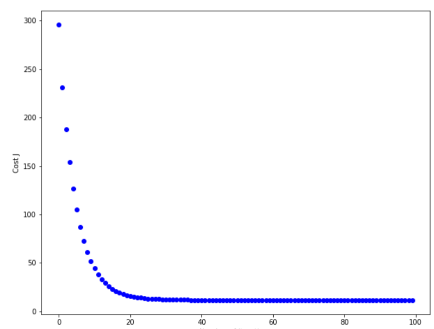
Problem 3.1.B5: Exploring convergence of gradient descent



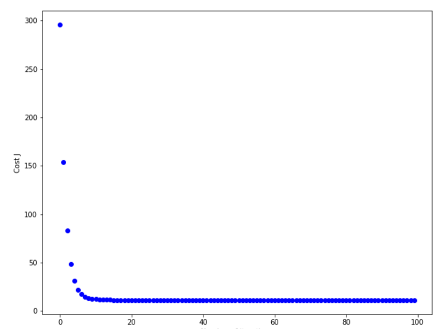
(a) learning rate $\alpha = 0.01$



(b) learning rate $\alpha = 0.03$



(c) learning rate $\alpha = 0.1$



(d) learning rate $\alpha = 0.3$

Figure 5: Convergence of gradient descent for linear regression with multiple variables using different learning rate.

$\alpha = 0.1, 0.3$ and $N_{iteration} = 80$ are good trade off between accuracy and efficiency. By observing Fig. 5d , one can easily find that small α (i.e. $\alpha = 0.01, 0.03$) leads to very slow convergence rate. $\alpha = 0.1, 0.3$ on the other hands, converges swiftly.

3.2.A1: Regularized linear regression cost function

No figures for this question.

3.2.A2: Gradient of the regularized linear regression cost function

Figure 6 shows the fitted curve of the linear model.

3.2.A3: Learning curves

Figure 7 shows the learning curve of the linear model.

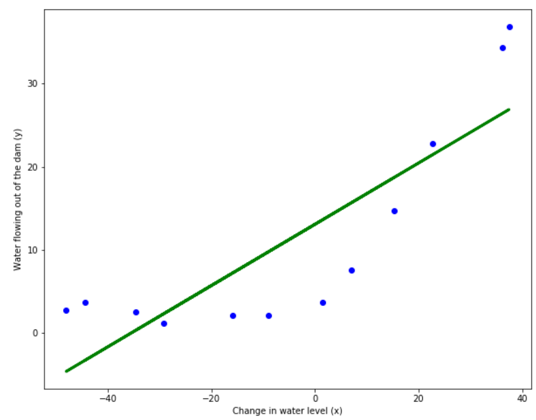


Figure 6: The fitted curve of the linear model.

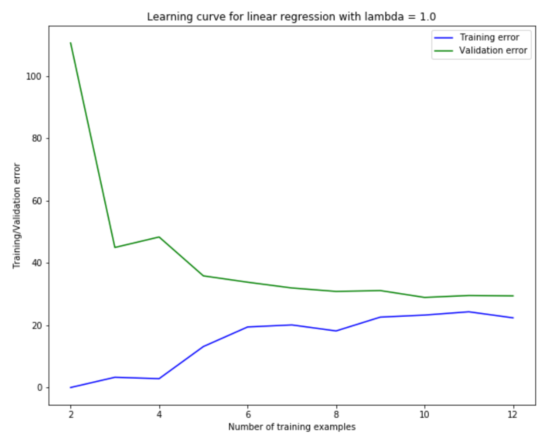
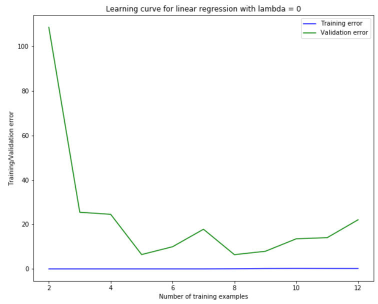


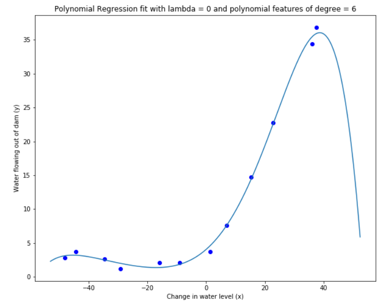
Figure 7: Learning curve of the linear model.

3.2.A4: Adjusting the regularization parameter

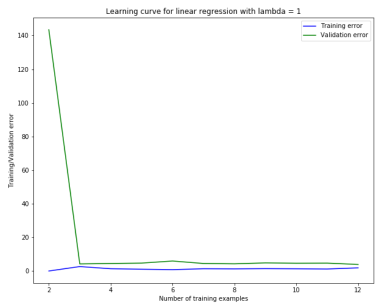
Figure 8 plots the polynomial fit and learning curves for each value of λ, from which we draw the following conclusions:



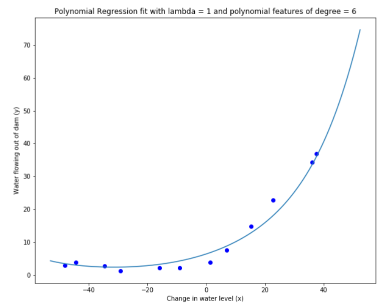
(a) learning rate $\lambda = 0$



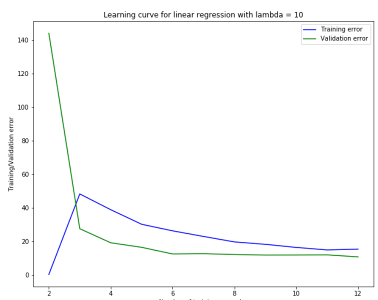
(b) Polynomial fit $\lambda = 0$



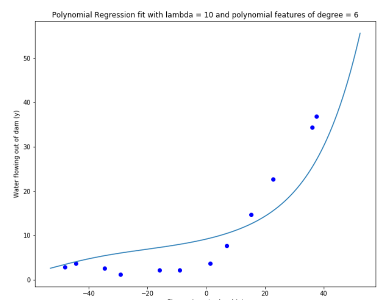
(c) learning rate $\lambda = 1$



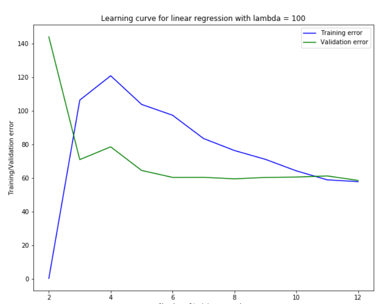
(d) Polynomial fit $\lambda = 1$



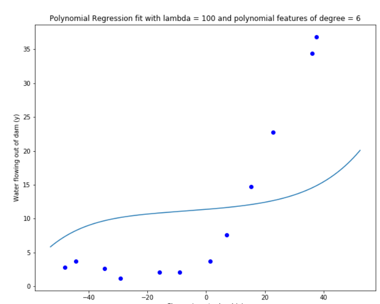
(e) learning rate $\lambda = 10$



(f) Polynomial fit $\lambda = 10$



(g) learning rate $\lambda = 100$



(h) Polynomial fit $\lambda = 100$

Figure 8: Convergence of gradient descent for linear regression with multiple variables using different learning rate.

If λ is too small, the effect of penalty term can be neglected so that the regulation will not be implemented effectively – the training model is still troubled by high-variation/over-fitting issue (as can be seen in Fig....). When λ is too large, the loss function is in fact dominated by the penalty term, which is a slightly similar to the biased issue that we have too strong assumptions on the model. Therefore, the training model turns to be under-fit. Only when λ takes appropriate value that the regulation can works effectively.

3.2.A5 Selecting λ using a validation set

Figure 9 shows the variation in trainng/validation error with respect of regulation parameter λ. For the current model, $\lambda = 1$ is approximately a good choice for the training. We list several reasons to justify our choice:

- The difference between training/validation error is too large when λ is significantly smaller than 1, implying an over-fitting issue – which is true because the penalty term characterized by λ is too small to regulate the effect of excessive features.
- The difference between training/validation error becomes relatively small, but the absolute training/validation error becomes too large when λ gets larger than 1. This indicates that the penalty term dominates the loss function and we actually are making a strong assumption of the target function (of similar form as that of penalty term), i.e., too much bias / not enough variation.
- $\lambda = 1$ makes a good balance to avoid either overfitting or underfitting, i.e. the validation/training error is small (the prediction performance is acceptable); the difference between validation/training error is small (the prediction model is reliable).

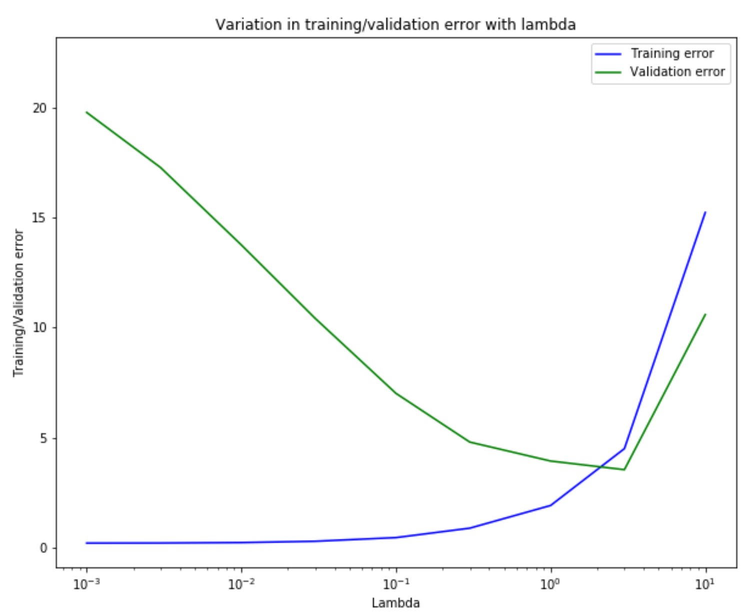


Figure 9: Variation in training/validation error with λ

3.2.A6 Computing test set error

Error when choosing best lamdba 1.0 is 30987.4826556 (USD).

3.2.A7 Plotting learning curves with randomly selected examples

Figure 10 plot the averaged learning curve for $\lambda = 1$.

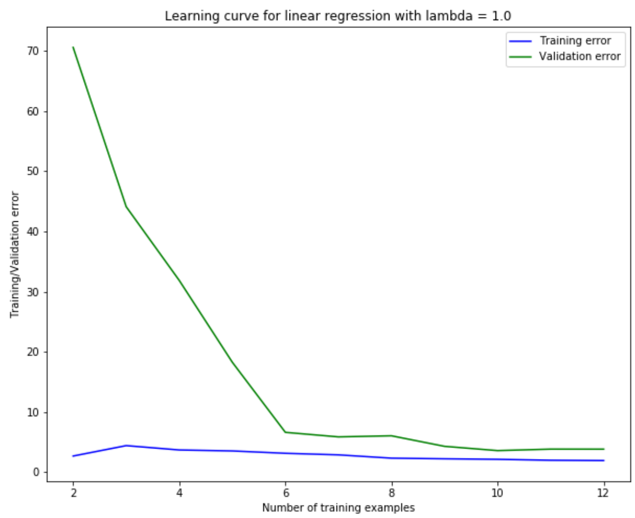


Figure 10: Averaged learning curve for $\lambda = 1$

Extra credit

Please see **bostonexp.pdf** and **bostonexp.ipynb** for our solution.