2023 – 2024 Intro to Python with Applications to Finance

Group Project

Objective: Use mean-reversion model to construct currency trading strategy

Mean reversion is a financial term for the assumption that an asset's price will tend to converge to the average price over time. In mathematical terms, mean reversion process (Ornstein-Uhlenbeck process) is described by:

$$dX_t = \theta(\mu - X_t)dt + \sigma dW_t \quad (1)$$

where X_t is the price process, μ is the drift term (long-run mean), θ is the speed of mean reversion, σ is the volatility, and W_t is the standard Brownian motion (i.e. $dW_t \sim N(0, dt)$).

The price X_t have the tendency to converge to the long-run mean μ with the speed θ. However, the noise term dW_t prevents the mean reversion. Thus, higher θ means quicker mean reversion and higher σ means more deviation from the mean.

Econometrically, we can estimate the process by using the following discretization:

$$X_{t+\Delta t} - X_t = \theta(\mu - X_t)\Delta t + \epsilon_t \quad (2)$$

where $X_{t+\Delta t} - X_t$ corresponds to dX_t, ϵ_t corresponds to the noise term σdW_t.

Rearranging the equation (2), we can get:

$$X_{t+\Delta t} - X_t = \theta\mu\Delta t - \theta\Delta t X_t + \epsilon_t \quad (3)$$

Therefore, if we estimate the following time-series regression:

$$\Delta X_t = \alpha + \beta X_{t-1} + \epsilon_t \quad (4)$$

where $\Delta X_t = X_t - X_{t-\Delta t}$, then $\alpha = \theta\mu\Delta t$ and $\beta = -\theta\Delta t X_t$. If the data exhibits the mean reversion feature, then $\alpha > 0$ and $\beta < 0$.

The long run mean μ can be computed as $-\frac{\alpha}{\beta}$ and the speed of mean reversion θ is $-\beta$. As a result, on average it takes $-\frac{1}{\beta}$ days (if we use daily data) for the X_t to converge to the long run mean μ.

Part A:

1) Download the daily EUR/USD exchange rate from 2010 January to 2023 October from Yahoo Finance. Using the full sample, test if EUR/USD

exchange rate exhibits the mean reversion feature. In addition to checking the sign of α and β, extract the t-statistics of the β coefficient and compare it with a critical value (e.g. -1) to see if it is statistically significant. If the full sample does not exhibit the mean reversion feature, is there any sample period during which the EUR/USD exchange rate exhibits mean reversion feature?

2) Construct a dynamic trading strategy (ignoring the transaction costs). One baseline strategy is that, at any given day, estimate the mean reversion model for a past estimation window (e.g. 80 trading days) and extract the estimated value $\hat{\alpha}$, $\hat{\beta}$, and the t-statistics for $\hat{\beta}$ (i.e. $t(\hat{\beta})$). If $\hat{\alpha} > 0$, $\hat{\beta}<0$ and $t(\hat{\beta})$ < Critical value (e.g. -1), then compute the expected change $X_{t+\Delta t} - X_t$ based on today's closing exchange rate X_t and the estimated model parameters $\hat{\alpha}$, $\hat{\beta}$. If $X_{t+\Delta t} - X_t > 0$, then this means EUR is expected to appreciate against USD, we can buy Euro using the closing EUR/USD exchange rate. On the other hand, if $X_{t+\Delta t} - X_t < 0$, then this means EUR is expected to depreciate against USD, we can buy USD using the closing EUR/USD exchange rate. If any of these conditions are not satisfied, we simply do not hold any position in any currency.

   Construct this trading strategy in Python and compute the daily return of this strategy over the entire sample period. Visualize by plotting your strategy's cumulative performance against time. You can assume that you initially hold 1 dollar and then plot how much the 1 dollar will gradually become using this strategy over time.

   Compute some performance metrics of the strategy, such as Sharpe Ratio, Maximum drawdown, etc.

Part B:

3) Try to come up with ways to improve the strategy's performance. For example, specifying a richer version of mean reversion regression model, changing the estimation method and parameters, trading rules, etc.

4) Check if this strategy works for any other currency pairs (e.g. EUR/GBP, GBP/USD, JPY/USD). Is it possible to modify the strategy to jointly trade multiple currency pairs? Please write a Python program to implement your idea.