Université Libre de Bruxelles

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IRIDIA

INFO-H-421 : Heuristic Optimization

Implementation exercise 1 : IIA for PFSP with WT Objective

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Abstract

This work presents an implementation as well as a statistical study of iterative improvement algorithms for the permutation flow-shop scheduling problem (PFSP) with weighted tardiness objective.

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1 Preliminary Remarks

1.1 Randomness

The default pseudo-randomn number generator from the C++11 standard library is used. The seed used for all results shown in this report is `--seed 0`.

1.2 Machine used

mem	3.6 GB
proc	AMD Athlon(tm) II Neo K325 Dual-Core Processor × 2
os	Ubuntu 13.10 64 bits

1.3 Abbreviation used in tables

II	`./run/pfsp-ii`
V	`./run/pfsp-vnd`
S	`--init slack`
R	`--init random`
F	`--pivoting first`
B	`--pivoting best`
E	`--neighborhood exchange`
I	`--neighborhood insert`
T	`--neighborhood transpose`
TEI	`--ordering tei`
TIE	`--ordering tie`

e.g. VSFTIE means `./run/pfsp-vnd --init slack --pivoting first --ordering tie`.

2 Iterative Improvement

2.1 Average relative percentage deviation from the best known solutions

2.1.1 Init

When used with `--neighborhood transpose`, slack heuristic gives always better results than randomn permutation. For other neighborhoods, results are mitigated.

2.1.2 Pivoting

The first improvement approach seems to give globally better results.

2.1.3 Neighborhood

The transpose neighborhood is the least scoring one for all instances and results vary greatly depending on the initial generated solution (combination with slack heuristic giving a better final result than randomn permutation). This can be explained by the fact that the transpose neighborhood is too local.

The insert neighborhood seems to be the best candidate in terms of solution quality.

The exchange neighborhood lies not too far from the insert neighborhood with a reasonable relative percentage deviation.

2.1.4 Remarks

IIRFI and IISFI are the best algorithms for all instances.

2.2 Average computation time

2.2.1 Init

Except for the transpose neighborhood for which runs are too short, for the same combination of other parameters the slack heuristic produces faster runs than a randomn permutation.

2.2.2 Pivoting

The best improvement approach seems to give globally faster runs.

2.2.3 Neighborhood

The transpose neighborhood gives the fastest runs for all instances and all combinations of other parameters.

For the same combination of other parameters the insert neighborhood runs slower than the exchange neighborhood.

2.2.4 Remarks

IIRFE and IIRFI are (in order) the slowest algorithms for all instances.

3 Variable Neighborhood Search

3.1 Average relative percentage deviation from the best known solutions

3.1.1 Init

Cannot conclude without statistical tests.

3.1.2 Ordering

Cannot conclude without statistical tests.

3.2 Average computation time

3.2.1 Init

Slack heuristic produces shorter runs.

3.2.2 Ordering

Cannot conclude without statistical tests.

3.3 Comparison with Iterative Improvement

VND doesn't reach the solution quality obtained with IIRFI or IISFI but gives generally good results compared to the exchange neighborhood used alone.

4 Student t-test and Wilcoxon test

In the tables, both Student t-test and Wilcoxon test p-values are reported. Only the Wilcoxon test p-values will be analyzed. Only the relative percentage deviation has been tested.

Depending on the instances compared, IIRFE, IISBI, IISFE, IISFI are often considered probably similar to the VND approaches.

VND approaches are considered probably relatively close to each other (depending on instance sizes).

5 Running time optimizations

5.1 Delta evaluation

Only the difference between the neighbour and the current solution is evaluated.

5.2 Cache locality

For jobs × machines matrices, a single array is allocated. Row `i` column `j` is accessed through `[i*w+j]` subscripting where `w` is the number of columns. The evaluations functions try to read and write data sequentially.

5.3 Remarks on neighborhood ordering

For the first improvement pivoting approach, the neighborhood ordering seems to have a significant influence. Perturbations close to the end of the schedule have little influence on the solution quality and evaluates rapidly whereas perturbations close to the beginning of the schedule have a high influence on the solution quality and evaluates slowly. For this implementation quality solution has been emphasized and neighborhood walking techniques have been chosen accordingly.

6 Use cases

6.1 Regular usage

Iterative improvement

```
# ./run/pfsp-ii –init slack –pivoting first –neighborhood insert – data/in/100x20_9 -v
```

```
         --init  {slack, random}
     --pivoting  {first, best}
 --neighborhood  {exchange, insert, transpose}
         --seed  long int[] default [high_res_clock::now()]
  -v, --verbose  verbose output
     -h, --help  show help
```

Variable neighborhood descent

```
# ./run/pfsp-vnd –init random –ordering tie – data/in/100x20_9 -v
```

```
        --init  {slack, random}
    --pivoting  {first, best} default first
    --ordering  {tie, tei}
        --seed  long int[] default [high_res_clock::now()]
 -v, --verbose  verbose output
    -h, --help  show help
```

6.2 Generate all data

Output in data/dev, data/test and data/out.

```
# do/all +mytestname
```

A Average relative percentage deviation and computation time

Table 1: avg rel % dev and computation time for 100x20 instances

alg	avg rel % dev	avg time
IIRBE	12.8909	3889 ms
IIRBI	10.7560	8473 ms
IIRBT	143.5449	52 ms
IIRFE	3.5779	20675 ms
IIRFI	1.4422	22761 ms
IIRFT	140.0989	72 ms
IISBE	10.5009	2912 ms
IISBI	5.6719	5691 ms
IISBT	41.6100	40 ms
IISFE	5.1229	6638 ms
IISFI	1.8223	14620 ms
IISFT	38.3003	49 ms
VRFTEI	4.1919	6503 ms
VRFTIE	4.6870	10437 ms
VSFTEI	6.9544	3168 ms
VSFTIE	4.1647	4570 ms

Table 2: avg rel % dev and computation time for 100x20 instances (sorted by dev)

alg	avg rel % dev	avg time
IIRFI	1.4422	22761 ms
IISFI	1.8223	14620 ms
IIRFE	3.5779	20675 ms
VSFTIE	4.1647	4570 ms
VRFTEI	4.1919	6503 ms
VRFTIE	4.6870	10437 ms
IISFE	5.1229	6638 ms
IISBI	5.6719	5691 ms
VSFTEI	6.9544	3168 ms
IISBE	10.5009	2912 ms
IIRBI	10.7560	8473 ms
IIRBE	12.8909	3889 ms
IISFT	38.3003	49 ms
IISBT	41.6100	40 ms
IIRFT	140.0989	72 ms
IIRBT	143.5449	52 ms

Table 3: avg rel % dev and computation time for 100x20 instances (sorted by time)

alg	**avg rel % dev**	**avg time**
IISBT	41.6100	40 ms
IISFT	38.3003	49 ms
IIRBT	143.5449	52 ms
IIRFT	140.0989	72 ms
IISBE	10.5009	2912 ms
VSFTEI	6.9544	3168 ms
IIRBE	12.8909	3889 ms
VSFTIE	4.1647	4570 ms
IISBI	5.6719	5691 ms
VRFTEI	4.1919	6503 ms
IISFE	5.1229	6638 ms
IIRBI	10.7560	8473 ms
VRFTIE	4.6870	10437 ms
IISFI	1.8223	14620 ms
IIRFE	3.5779	20675 ms
IIRFI	1.4422	22761 ms

Table 4: avg rel % dev and computation time for 50x20 instances

alg	**avg rel % dev**	**avg time**
IIRBE	59.2969	302 ms
IIRBI	49.3596	600 ms
IIRBT	1033.8080	4 ms
IIRFE	25.7903	1013 ms
IIRFI	15.2501	1293 ms
IIRFT	990.2279	6 ms
IISBE	50.8140	139 ms
IISBI	24.7393	309 ms
IISBT	144.9350	4 ms
IISFE	33.9047	258 ms
IISFI	17.8570	624 ms
IISFT	141.1087	6 ms
VRFTEI	20.9437	380 ms
VRFTIE	20.2999	524 ms
VSFTEI	22.7949	173 ms
VSFTIE	22.1499	208 ms

Table 5: avg rel % dev and computation time for 50x20 instances (sorted by dev)

alg	**avg rel % dev**	**avg time**
IIRFI	15.2501	1293 ms
IISFI	17.8570	624 ms
VRFTIE	20.2999	524 ms
VRFTEI	20.9437	380 ms
VSFTIE	22.1499	208 ms
VSFTEI	22.7949	173 ms
IISBI	24.7393	309 ms
IIRFE	25.7903	1013 ms
IISFE	33.9047	258 ms
IIRBI	49.3596	600 ms
IISBE	50.8140	139 ms
IIRBE	59.2969	302 ms
IISFT	141.1087	6 ms
IISBT	144.9350	4 ms
IIRFT	990.2279	6 ms
IIRBT	1033.8080	4 ms

Table 6: avg rel % dev and computation time for 50x20 instances (sorted by time)

alg	**avg rel % dev**	**avg time**
IISBT	144.9350	4 ms
IIRBT	1033.8080	4 ms
IISFT	141.1087	6 ms
IIRFT	990.2279	6 ms
IISBE	50.8140	139 ms
VSFTEI	22.7949	173 ms
VSFTIE	22.1499	208 ms
IISFE	33.9047	258 ms
IIRBE	59.2969	302 ms
IISBI	24.7393	309 ms
VRFTEI	20.9437	380 ms
VRFTIE	20.2999	524 ms
IIRBI	49.3596	600 ms
IISFI	17.8570	624 ms
IIRFE	25.7903	1013 ms
IIRFI	15.2501	1293 ms

Table 7: avg rel % dev and computation time for 60x20 instances

alg	**avg rel % dev**	**avg time**
IIRBE	28.6631	567 ms
IIRBI	17.3015	1245 ms
IIRBT	346.7143	8 ms
IIRFE	11.7075	2183 ms
IIRFI	5.6391	2687 ms
IIRFT	337.2030	13 ms
IISBE	26.1016	321 ms
IISBI	11.0622	694 ms
IISBT	87.5727	10 ms
IISFE	18.7445	692 ms
IISFI	7.4053	1587 ms
IISFT	86.9753	9 ms
VRFTEI	8.2510	902 ms
VRFTIE	9.4924	1223 ms
VSFTEI	11.0565	510 ms
VSFTIE	11.4802	495 ms

Table 8: avg rel % dev and computation time for 60x20 instances (sorted by dev)

alg	**avg rel % dev**	**avg time**
IIRFI	5.6391	2687 ms
IISFI	7.4053	1587 ms
VRFTEI	8.2510	902 ms
VRFTIE	9.4924	1223 ms
VSFTEI	11.0565	510 ms
IISBI	11.0622	694 ms
VSFTIE	11.4802	495 ms
IIRFE	11.7075	2183 ms
IIRBI	17.3015	1245 ms
IISFE	18.7445	692 ms
IISBE	26.1016	321 ms
IIRBE	28.6631	567 ms
IISFT	86.9753	9 ms
IISBT	87.5727	10 ms
IIRFT	337.2030	13 ms
IIRBT	346.7143	8 ms

Table 9: avg rel % dev and computation time for 60x20 instances (sorted by time)

alg	avg rel % dev	avg time
IIRBT	346.7143	8 ms
IISFT	86.9753	9 ms
IISBT	87.5727	10 ms
IIRFT	337.2030	13 ms
IISBE	26.1016	321 ms
VSFTIE	11.4802	495 ms
VSFTEI	11.0565	510 ms
IIRBE	28.6631	567 ms
IISFE	18.7445	692 ms
IISBI	11.0622	694 ms
VRFTEI	8.2510	902 ms
VRFTIE	9.4924	1223 ms
IIRBI	17.3015	1245 ms
IISFI	7.4053	1587 ms
IIRFE	11.7075	2183 ms
IIRFI	5.6391	2687 ms

Table 10: avg rel % dev and computation time for 70x20 instances

alg	avg rel % dev	avg time
IIRBE	22.7553	1062 ms
IIRBI	18.4159	2295 ms
IIRBT	282.1761	14 ms
IIRFE	11.5822	4131 ms
IIRFI	3.2664	5667 ms
IIRFT	276.0339	21 ms
IISBE	23.8240	656 ms
IISBI	12.5178	1258 ms
IISBT	79.7734	16 ms
IISFE	13.6888	1567 ms
IISFI	4.9066	3317 ms
IISFT	72.7923	19 ms
VRFTEI	12.2226	1523 ms
VRFTIE	7.7359	2372 ms
VSFTEI	13.0754	872 ms
VSFTIE	7.9641	1120 ms

Table 11: avg rel % dev and computation time for 70x20 instances (sorted by dev)

alg	**avg rel % dev**	**avg time**
IIRFI	3.2664	5667 ms
IISFI	4.9066	3317 ms
VRFTIE	7.7359	2372 ms
VSFTIE	7.9641	1120 ms
IIRFE	11.5822	4131 ms
VRFTEI	12.2226	1523 ms
IISBI	12.5178	1258 ms
VSFTEI	13.0754	872 ms
IISFE	13.6888	1567 ms
IIRBI	18.4159	2295 ms
IIRBE	22.7553	1062 ms
IISBE	23.8240	656 ms
IISFT	72.7923	19 ms
IISBT	79.7734	16 ms
IIRFT	276.0339	21 ms
IIRBT	282.1761	14 ms

Table 12: avg rel % dev and computation time for 70x20 instances (sorted by time)

alg	**avg rel % dev**	**avg time**
IIRBT	282.1761	14 ms
IISBT	79.7734	16 ms
IISFT	72.7923	19 ms
IIRFT	276.0339	21 ms
IISBE	23.8240	656 ms
VSFTEI	13.0754	872 ms
IIRBE	22.7553	1062 ms
VSFTIE	7.9641	1120 ms
IISBI	12.5178	1258 ms
VRFTEI	12.2226	1523 ms
IISFE	13.6888	1567 ms
IIRBI	18.4159	2295 ms
VRFTIE	7.7359	2372 ms
IISFI	4.9066	3317 ms
IIRFE	11.5822	4131 ms
IIRFI	3.2664	5667 ms

Table 13: avg rel % dev and computation time for 80x20 instances

alg	avg rel % dev	avg time
IIRBE	22.7754	1589 ms
IIRBI	14.9170	3667 ms
IIRBT	201.0257	24 ms
IIRFE	8.2952	7078 ms
IIRFI	3.8864	9576 ms
IIRFT	195.8405	34 ms
IISBE	19.6925	1047 ms
IISBI	11.0241	2203 ms
IISBT	58.9942	23 ms
IISFE	11.0078	2669 ms
IISFI	4.3316	5939 ms
IISFT	56.9208	25 ms
VRFTEI	7.4860	2759 ms
VRFTIE	9.4001	3865 ms
VSFTEI	7.5906	1536 ms
VSFTIE	6.6299	1962 ms

Table 14: avg rel % dev and computation time for 80x20 instances (sorted by dev)

alg	avg rel % dev	avg time
IIRFI	3.8864	9576 ms
IISFI	4.3316	5939 ms
VSFTIE	6.6299	1962 ms
VRFTEI	7.4860	2759 ms
VSFTEI	7.5906	1536 ms
IIRFE	8.2952	7078 ms
VRFTIE	9.4001	3865 ms
IISFE	11.0078	2669 ms
IISBI	11.0241	2203 ms
IIRBI	14.9170	3667 ms
IISBE	19.6925	1047 ms
IIRBE	22.7754	1589 ms
IISFT	56.9208	25 ms
IISBT	58.9942	23 ms
IIRFT	195.8405	34 ms
IIRBT	201.0257	24 ms

Table 15: avg rel % dev and computation time for 80x20 instances (sorted by time)

alg	**avg rel % dev**	**avg time**
IISBT	58.9942	23 ms
IIRBT	201.0257	24 ms
IISFT	56.9208	25 ms
IIRFT	195.8405	34 ms
IISBE	19.6925	1047 ms
VSFTEI	7.5906	1536 ms
IIRBE	22.7754	1589 ms
VSFTIE	6.6299	1962 ms
IISBI	11.0241	2203 ms
IISFE	11.0078	2669 ms
VRFTEI	7.4860	2759 ms
IIRBI	14.9170	3667 ms
VRFTIE	9.4001	3865 ms
IISFI	4.3316	5939 ms
IIRFE	8.2952	7078 ms
IIRFI	3.8864	9576 ms

Table 16: avg rel % dev and computation time for 90x20 instances

alg	**avg rel % dev**	**avg time**
IIRBE	15.2368	2607 ms
IIRBI	12.4893	5779 ms
IIRBT	160.2339	38 ms
IIRFE	5.2240	12403 ms
IIRFI	2.5965	14820 ms
IIRFT	152.3347	54 ms
IISBE	14.5337	1675 ms
IISBI	8.6073	3582 ms
IISBT	55.3209	36 ms
IISFE	8.2186	4007 ms
IISFI	2.4010	9674 ms
IISFT	49.4902	38 ms
VRFTEI	5.7313	4584 ms
VRFTIE	5.7316	6610 ms
VSFTEI	8.2814	2200 ms
VSFTIE	3.7770	3155 ms

Table 17: avg rel % dev and computation time for 90x20 instances (sorted by dev)

alg	**avg rel % dev**	**avg time**
IISFI	2.4010	9674 ms
IIRFI	2.5965	14820 ms
VSFTIE	3.7770	3155 ms
IIRFE	5.2240	12403 ms
VRFTEI	5.7313	4584 ms
VRFTIE	5.7316	6610 ms
IISFE	8.2186	4007 ms
VSFTEI	8.2814	2200 ms
IISBI	8.6073	3582 ms
IIRBI	12.4893	5779 ms
IISBE	14.5337	1675 ms
IIRBE	15.2368	2607 ms
IISFT	49.4902	38 ms
IISBT	55.3209	36 ms
IIRFT	152.3347	54 ms
IIRBT	160.2339	38 ms

Table 18: avg rel % dev and computation time for 90x20 instances (sorted by time)

alg	**avg rel % dev**	**avg time**
IISBT	55.3209	36 ms
IIRBT	160.2339	38 ms
IISFT	49.4902	38 ms
IIRFT	152.3347	54 ms
IISBE	14.5337	1675 ms
VSFTEI	8.2814	2200 ms
IIRBE	15.2368	2607 ms
VSFTIE	3.7770	3155 ms
IISBI	8.6073	3582 ms
IISFE	8.2186	4007 ms
VRFTEI	5.7313	4584 ms
IIRBI	12.4893	5779 ms
VRFTIE	5.7316	6610 ms
IISFI	2.4010	9674 ms
IIRFE	5.2240	12403 ms
IIRFI	2.5965	14820 ms

B Statistical tests on average relative percentage deviation

Table 19: Student t-test and Wilcoxon test results for 100x20 instances

	IIRBI	IIRBT	IIRFE	IIRFI	IIRFT	IISBE	IISBI	IISBT	IISFE	IISFI	IISFT	VRFTEI	VRFTIE	VSFTEI	VSFTIE
IIRBE	2.710e-01 3.750e-01	1.524e-08 1.953e-03	7.299e-06 1.953e-03	9.121e-07 1.953e-03	1.016e-08 1.953e-03	2.685e-02 2.734e-02	5.812e-04 1.953e-03	1.345e-07 1.953e-03	7.877e-07 1.953e-03	8.000e-06 1.953e-03	1.382e-07 1.953e-03	7.797e-05 1.953e-03	7.768e-05 1.953e-03	7.835e-03 9.766e-03	1.406e-04 1.953e-03
IIRBI		1.245e-08 1.953e-03	4.187e-04 1.953e-03	3.503e-05 1.953e-03	8.856e-09 1.953e-03	8.318e-01 8.457e-01	6.414e-03 5.859e-03	1.243e-08 1.953e-03	4.079e-03 9.766e-03	6.356e-06 1.953e-03	2.147e-07 1.953e-03	1.780e-04 1.953e-03	4.460e-04 1.953e-03	8.312e-04 1.953e-03	2.719e-06 1.953e-03
IIRBT			6.716e-09 1.953e-03	6.484e-09 1.953e-03	3.669e-02 4.883e-02	9.167e-09 1.953e-03	1.268e-08 1.953e-03	1.684e-07 1.953e-03	1.151e-08 1.953e-03	1.177e-08 1.953e-03	1.311e-07 1.953e-03	1.231e-08 1.953e-03	9.520e-09 1.953e-03	6.346e-09 1.953e-03	9.798e-09 1.953e-03
IIRFE				2.089e-03 5.859e-03	4.719e-09 1.953e-03	5.077e-05 1.953e-03	4.458e-02 6.445e-02	3.521e-09 1.953e-03	1.150e-01 1.602e-01	1.352e-01 8.398e-02	1.574e-08 1.953e-03	4.630e-01 5.566e-01	2.435e-01 4.922e-01	1.293e-02 1.367e-02	5.366e-01 9.219e-01
IIRFI					4.160e-09 1.953e-03	1.089e-06 1.953e-03	2.513e-03 3.906e-03	2.875e-10 1.953e-03	2.484e-03 9.766e-03	7.081e-01 4.316e-01	1.091e-09 1.953e-03	1.489e-03 5.859e-03	5.792e-03 9.766e-03	6.210e-04 1.953e-03	7.741e-03 2.734e-02
IIRFT						5.665e-09 1.953e-03	8.383e-09 1.953e-03	1.025e-07 1.953e-03	8.389e-09 1.953e-03	7.942e-09 1.953e-03	6.886e-08 1.953e-03	8.686e-09 1.953e-03	6.998e-09 1.953e-03	4.501e-09 1.953e-03	6.820e-09 1.953e-03
IISBE							2.246e-03 5.859e-03	8.072e-09 1.953e-03	3.344e-04 1.953e-03	4.712e-06 1.953e-03	2.172e-08 1.953e-03	2.624e-04 1.953e-03	8.200e-04 1.953e-03	9.324e-03 1.367e-02	9.774e-05 1.953e-03
IISBI								7.508e-09 1.953e-03	7.040e-01 5.566e-01	5.068e-03 9.766e-03	2.898e-08 1.953e-03	2.596e-01 1.934e-01	4.658e-01 5.566e-01	3.171e-01 2.754e-01	2.172e-01 2.324e-01
IISBT									1.260e-08 1.953e-03	1.288e-09 1.953e-03	2.215e-02 6.445e-02	1.379e-09 1.953e-03	4.131e-09 1.953e-03	4.390e-09 1.953e-03	2.526e-09 1.953e-03
IISFE										5.068e-03 9.766e-03	2.364e-08 1.953e-03	3.108e-01 4.316e-01	6.394e-01 6.953e-01	2.643e-01 2.754e-01	3.856e-01 6.250e-01
IISFI											2.928e-09 1.953e-03	1.704e-02 1.367e-02	1.274e-02 2.734e-02	2.193e-03 1.953e-03	7.659e-03 1.367e-02
IISFT												6.471e-09 1.953e-03	1.884e-08 1.953e-03	8.078e-08 1.953e-03	1.155e-08 1.953e-03
VRFTEI													5.878e-01 7.695e-01	4.054e-02 3.711e-02	9.657e-01 1.000e+00
VRFTIE														9.561e-02 6.445e-02	5.217e-01 4.922e-01
VSFTEI															1.307e-02 2.734e-02

Table 20: Student t-test and Wilcoxon test results for 50x20 instances

	IIRBI	IIRBT	IIRFE	IIRFI	IIRFT	IISBE	IISBI	IISBT	IISFE	IISFI	IISFT	VRFTEI	VRFTIE	VSFTEI	VSFTIE
IIRBE	4.237e-01 3.750e-01	2.105e-03 1.953e-03	1.050e-03 1.953e-03	2.888e-04 1.953e-03	2.422e-03 1.953e-03	4.176e-01 1.934e-01	9.331e-03 9.766e-03	1.136e-04 1.953e-03	2.273e-02 1.953e-02	4.383e-04 1.953e-03	1.153e-03 1.953e-03	3.938e-04 1.953e-03	8.102e-04 1.953e-03	5.302e-03 3.906e-03	5.760e-04 1.953e-03
IIRBI		2.042e-03 1.953e-03	3.310e-02 9.766e-03	9.663e-03 1.953e-03	2.376e-03 1.953e-03	9.005e-01 1.309e-01	6.473e-02 1.953e-02	7.896e-05 1.953e-03	1.473e-01 1.309e-01	2.763e-02 5.859e-03	3.821e-04 1.953e-03	2.944e-02 1.953e-03	2.914e-02 1.953e-03	4.773e-03 1.953e-03	2.370e-02 3.906e-03
IIRBT			1.746e-03 1.953e-03	1.649e-03 1.953e-03	1.246e-02 1.953e-03	2.102e-03 1.953e-03	1.773e-03 1.953e-03	3.078e-03 1.953e-03	2.028e-03 1.953e-03	1.726e-03 1.953e-03	2.590e-03 1.953e-03	1.786e-03 1.953e-03	1.851e-03 1.953e-03	1.931e-03 1.953e-03	1.750e-03 1.953e-03
IIRFE				2.851e-04 1.953e-03	2.002e-03 1.953e-03	1.195e-03 3.906e-03	8.242e-01 8.457e-01	1.939e-05 1.953e-03	1.346e-01 4.883e-02	3.453e-02 2.734e-02	7.125e-05 1.953e-03	1.625e-01 3.223e-01	2.223e-01 3.750e-01	6.928e-01 6.250e-01	2.367e-01 3.223e-01
IIRFI					1.887e-03 1.953e-03	3.689e-04 1.953e-03	1.055e-01 6.445e-02	1.658e-05 1.953e-03	4.674e-03 1.367e-02	3.827e-01 2.754e-01	4.462e-05 1.953e-03	9.588e-02 1.602e-01	2.941e-01 2.324e-01	3.396e-01 6.445e-02	1.458e-02 3.711e-02
IIRFT						2.414e-03 1.953e-03	2.026e-03 1.953e-03	3.614e-03 1.953e-03	2.338e-03 1.953e-03	1.973e-03 1.953e-03	3.046e-03 1.953e-03	2.040e-03 1.953e-03	2.115e-03 1.953e-03	2.231e-03 1.953e-03	2.007e-03 1.953e-03
IISBE							7.956e-06 1.953e-03	1.524e-04 1.953e-03	6.041e-02 8.398e-02	4.349e-04 1.953e-03	6.605e-04 1.953e-03	5.566e-04 1.953e-03	2.414e-04 1.953e-03	2.784e-02 4.883e-02	4.025e-03 3.906e-03
IISBI								4.882e-05 1.953e-03	2.372e-01 2.754e-01	2.058e-01 1.934e-01	1.324e-04 1.953e-03	5.017e-01 7.695e-01	3.954e-01 6.250e-01	8.571e-01 9.219e-01	7.035e-01 8.457e-01
IISBT									1.039e-04 1.953e-03	3.371e-05 1.953e-03	7.044e-01 6.953e-01	2.123e-05 1.953e-03	3.938e-05 1.953e-03	5.510e-05 1.953e-03	3.188e-05 1.953e-03
IISFE										7.951e-03 1.953e-02	3.996e-04 1.953e-03	3.635e-02 6.445e-02	3.735e-02 1.953e-02	6.278e-02 6.445e-02	2.921e-02 1.953e-02
IISFI											1.062e-04 1.953e-03	3.966e-01 3.223e-01	5.673e-01 6.250e-01	5.752e-01 3.223e-01	2.274e-01 3.750e-01
IISFT												9.295e-05 1.953e-03	1.760e-04 1.953e-03	2.172e-04 1.953e-03	8.267e-05 1.953e-03
VRFTEI													8.011e-01 8.457e-01	8.144e-01 9.219e-01	7.479e-01 9.219e-01
VRFTIE														7.571e-01 6.953e-01	7.111e-01 1.000e+00
VSFTEI															9.212e-01 6.250e-01

Table 21: Student t-test and Wilcoxon test results for 60x20 instances

	IIRBI	IIRBT	IIRFE	IIRFI	IIRFT	IISBE	IISBI	IISBT	IISFE	IISFI	IISFT	VRFTEI	VRFTIE	VSFTEI	VSFTIE
IIRBE	1.088e-02 9.766e-03	1.531e-05 1.953e-03	7.785e-04 1.953e-03	1.762e-04 1.953e-03	8.857e-06 1.953e-03	5.297e-01 6.250e-01	6.365e-04 1.953e-03	1.351e-05 1.953e-03	4.248e-03 5.859e-03	8.829e-05 1.953e-03	8.691e-05 1.953e-03	1.454e-04 1.953e-03	9.319e-04 5.859e-03	2.005e-03 3.906e-03	6.926e-04 1.953e-03
IIRBI		1.702e-05 1.953e-03	6.335e-02 6.445e-02	1.193e-03 1.953e-03	9.910e-06 1.953e-03	7.851e-03 1.953e-02	1.163e-01 8.398e-02	7.531e-06 1.953e-03	5.632e-01 9.219e-01	4.096e-03 5.859e-03	5.040e-05 1.953e-03	2.290e-03 1.953e-03	8.653e-03 9.766e-03	8.239e-02 6.445e-02	1.858e-02 4.883e-02
IIRBT			1.818e-05 1.953e-03	1.514e-05 1.953e-03	2.360e-01 3.223e-01	2.146e-05 1.953e-03	1.628e-05 1.953e-03	1.372e-04 1.953e-03	1.445e-05 1.953e-03	1.341e-05 1.953e-03	1.823e-04 1.953e-03	1.540e-05 1.953e-03	1.544e-05 1.953e-03	1.764e-05 1.953e-03	1.483e-05 1.953e-03
IIRFE				1.866e-03 3.906e-03	1.105e-05 1.953e-03	9.102e-05 1.953e-03	7.374e-01 4.922e-01	2.813e-06 1.953e-03	4.483e-02 1.055e-01	5.706e-02 4.883e-02	1.190e-05 1.953e-03	6.177e-02 6.445e-02	3.345e-01 2.754e-01	6.772e-01 5.566e-01	9.351e-01 7.695e-01
IIRFI					9.408e-06 1.953e-03	3.031e-07 1.953e-03	1.928e-02 1.953e-02	2.350e-06 1.953e-03	1.192e-03 1.953e-03	2.318e-01 3.750e-01	1.029e-05 1.953e-03	1.060e-01 1.055e-01	2.052e-02 2.734e-02	2.385e-02 1.953e-02	2.151e-02 9.766e-03
IIRFT						1.328e-05 1.953e-03	1.024e-05 1.953e-03	8.720e-05 1.953e-03	8.501e-06 1.953e-03	8.282e-06 1.953e-03	1.188e-04 1.953e-03	9.320e-06 1.953e-03	9.510e-06 1.953e-03	1.079e-05 1.953e-03	9.128e-06 1.953e-03
IISBE							1.239e-04 1.953e-03	3.067e-05 1.953e-03	3.208e-02 6.445e-02	2.052e-06 1.953e-03	1.306e-04 1.953e-03	6.884e-06 1.953e-03	1.021e-06 1.953e-03	1.437e-05 1.953e-03	5.026e-04 1.953e-03
IISBI								3.926e-06 1.953e-03	2.432e-02 9.766e-03	2.464e-02 6.445e-02	1.628e-05 1.953e-03	2.502e-01 1.934e-01	4.903e-01 4.316e-01	9.978e-01 8.457e-01	8.890e-01 6.953e-01
IISBT									1.604e-05 1.953e-03	3.031e-06 1.953e-03	8.624e-01 6.953e-01	1.266e-06 1.953e-03	4.423e-06 1.953e-03	3.843e-06 1.953e-03	5.824e-06 1.953e-03
IISFE										2.584e-04 1.953e-03	8.005e-05 1.953e-03	2.694e-03 1.953e-03	6.378e-03 9.766e-03	6.310e-02 4.883e-02	5.017e-03 5.859e-03
IISFI											1.589e-05 1.953e-03	6.365e-01 6.953e-01	1.852e-01 1.934e-01	1.504e-01 1.602e-01	4.614e-02 4.883e-02
IISFT												7.053e-06 1.953e-03	2.286e-05 1.953e-03	1.598e-05 1.953e-03	3.296e-05 1.953e-03
VRFTEI													5.764e-01 5.566e-01	2.669e-01 2.754e-01	1.248e-01 1.602e-01
VRFTIE														4.790e-01 3.223e-01	4.337e-01 7.695e-01
VSFTEI															9.075e-01 1.000e+00

Table 22: Student t-test and Wilcoxon test results for 70x20 instances

	IIRBI	IIRBT	IIRFE	IIRFI	IIRFT	IISBE	IISBI	IISBT	IISFE	IISFI	IISFT	VRFTEI	VRFTIE	VSFTEI	VSFTIE
IIRBE	4.802e-02 1.367e-02	1.162e-06 1.953e-03	2.569e-04 3.906e-03	1.826e-06 1.953e-03	1.234e-06 1.953e-03	6.053e-01 4.922e-01	2.531e-03 9.766e-03	3.828e-07 1.953e-03	4.157e-03 1.953e-03	4.726e-05 1.953e-03	1.669e-06 1.953e-03	9.504e-04 1.953e-03	2.198e-05 1.953e-03	4.465e-03 5.859e-03	2.234e-05 1.953e-03
IIRBI		9.731e-07 1.953e-03	4.090e-03 1.367e-02	1.976e-05 1.953e-03	1.069e-06 1.953e-03	9.915e-02 1.602e-01	4.887e-02 6.445e-02	3.671e-07 1.953e-03	3.688e-02 2.734e-02	1.774e-04 1.953e-03	1.842e-06 1.953e-03	1.412e-02 2.734e-02	2.819e-04 1.953e-03	1.727e-02 2.734e-02	1.841e-04 1.953e-03
IIRBT			6.002e-07 1.953e-03	4.380e-07 1.953e-03	3.957e-02 6.445e-02	1.093e-06 1.953e-03	4.323e-07 1.953e-03	7.573e-06 1.953e-03	6.870e-07 1.953e-03	5.121e-07 1.953e-03	4.412e-06 1.953e-03	5.514e-07 1.953e-03	5.577e-07 1.953e-03	6.608e-07 1.953e-03	5.694e-07 1.953e-03
IIRFE				3.350e-06 1.953e-03	6.597e-07 1.953e-03	1.990e-04 1.953e-03	5.459e-01 6.250e-01	1.616e-08 1.953e-03	2.052e-01 2.754e-01	2.225e-03 1.953e-03	9.732e-08 1.953e-03	6.513e-01 8.457e-01	1.639e-02 1.953e-02	4.280e-01 6.250e-01	2.224e-03 1.953e-03
IIRFI					4.652e-07 1.953e-03	1.008e-06 1.953e-03	1.366e-04 1.953e-03	1.072e-08 1.953e-03	5.150e-06 1.953e-03	2.959e-01 3.750e-01	5.354e-08 1.953e-03	1.025e-05 1.953e-03	1.904e-04 1.953e-03	5.106e-04 1.953e-03	4.183e-04 1.953e-03
IIRFT						1.161e-06 1.953e-03	4.645e-07 1.953e-03	8.887e-06 1.953e-03	7.294e-07 1.953e-03	5.449e-07 1.953e-03	5.280e-06 1.953e-03	5.804e-07 1.953e-03	5.856e-07 1.953e-03	7.100e-07 1.953e-03	6.172e-07 1.953e-03
IISBE							2.730e-03 9.766e-03	1.613e-07 1.953e-03	9.739e-04 1.953e-03	2.425e-05 1.953e-03	1.127e-06 1.953e-03	2.153e-04 1.953e-03	1.016e-05 1.953e-03	4.487e-03 9.766e-03	1.397e-05 1.953e-03
IISBI								9.832e-09 1.953e-03	6.036e-01 6.250e-01	2.375e-03 5.859e-03	4.699e-08 1.953e-03	8.837e-01 7.695e-01	4.317e-02 4.883e-02	7.857e-01 9.219e-01	3.997e-02 6.445e-02
IISBT									7.166e-08 1.953e-03	1.429e-08 1.953e-03	1.540e-02 1.953e-02	4.389e-08 1.953e-03	3.659e-08 1.953e-03	1.388e-08 1.953e-03	3.203e-08 1.953e-03
IISFE										2.626e-04 1.953e-03	7.709e-07 1.953e-03	1.302e-01 1.602e-01	5.140e-05 1.953e-03	7.157e-01 4.316e-01	8.351e-04 1.953e-03
IISFI											1.051e-07 1.953e-03	2.180e-03 3.906e-03	1.627e-01 3.223e-01	4.541e-04 1.953e-03	5.706e-02 2.734e-02
IISFT												4.563e-07 1.953e-03	2.649e-07 1.953e-03	2.582e-07 1.953e-03	2.153e-07 1.953e-03
VRFTEI													3.083e-04 1.953e-03	6.186e-01 8.457e-01	3.844e-03 3.906e-03
VRFTIE														2.467e-02 5.859e-03	8.377e-01 9.219e-01
VSFTEI															2.173e-02 9.766e-03

Table 23: Student t-test and Wilcoxon test results for 80x20 instances

	IIRBI	IIRBT	IIRFE	IIRFI	IIRFT	IISBE	IISBI	IISBT	IISFE	IISFI	IISFT	VRFTEI	VRFTIE	VSFTEI	VSFTIE
IIRBE	1.023e-03 1.953e-03	5.018e-07 1.953e-03	4.254e-05 1.953e-03	1.666e-05 1.953e-03	3.326e-07 1.953e-03	2.318e-01 3.750e-01	2.611e-04 3.906e-03	1.621e-06 1.953e-03	3.470e-04 1.953e-03	2.035e-06 1.953e-03	2.324e-06 1.953e-03	2.979e-05 1.953e-03	1.277e-04 1.953e-03	1.631e-05 1.953e-03	1.863e-05 1.953e-03
IIRBI		3.666e-07 1.953e-03	2.487e-03 1.953e-03	1.303e-05 1.953e-03	2.546e-07 1.953e-03	6.311e-02 4.883e-02	2.445e-02 2.734e-02	1.644e-06 1.953e-03	1.355e-02 5.859e-03	5.893e-06 1.953e-03	2.503e-06 1.953e-03	3.741e-04 1.953e-03	1.143e-02 1.953e-02	6.523e-04 3.906e-03	1.610e-04 1.953e-03
IIRBT			3.341e-07 1.953e-03	3.220e-07 1.953e-03	3.368e-02 3.711e-02	4.591e-07 1.953e-03	4.841e-07 1.953e-03	5.014e-06 1.953e-03	3.318e-07 1.953e-03	2.751e-07 1.953e-03	3.713e-06 1.953e-03	3.150e-07 1.953e-03	2.157e-07 1.953e-03	3.408e-07 1.953e-03	4.089e-07 1.953e-03
IIRFE				1.696e-03 3.906e-03	2.404e-07 1.953e-03	1.292e-04 1.953e-03	4.879e-02 4.883e-02	2.527e-07 1.953e-03	3.822e-02 2.734e-02	5.247e-04 1.953e-03	4.154e-07 1.953e-03	4.448e-01 3.750e-01	3.823e-01 3.750e-01	3.807e-01 4.316e-01	3.044e-01 3.750e-01
IIRFI					2.289e-07 1.953e-03	2.427e-05 1.953e-03	3.928e-04 1.953e-03	2.361e-07 1.953e-03	1.850e-05 1.953e-03	6.612e-01 7.695e-01	4.006e-07 1.953e-03	2.020e-02 3.711e-02	6.699e-03 1.367e-02	1.274e-02 1.367e-02	5.711e-02 4.883e-02
IIRFT						3.458e-07 1.953e-03	3.615e-07 1.953e-03	3.641e-06 1.953e-03	2.403e-07 1.953e-03	1.983e-07 1.953e-03	2.607e-06 1.953e-03	2.364e-07 1.953e-03	1.485e-07 1.953e-03	2.487e-07 1.953e-03	2.935e-07 1.953e-03
IISBE							3.857e-03 9.766e-03	1.811e-06 1.953e-03	2.081e-04 1.953e-03	1.393e-05 1.953e-03	4.590e-06 1.953e-03	2.874e-05 1.953e-03	1.319e-03 3.906e-03	6.896e-05 1.953e-03	1.348e-04 1.953e-03
IISBI								8.458e-07 1.953e-03	9.916e-01 8.457e-01	4.160e-06 1.953e-03	1.514e-06 1.953e-03	3.420e-03 5.859e-03	4.318e-01 3.223e-01	2.997e-02 4.883e-02	1.218e-02 1.953e-02
IISBT									6.442e-07 1.953e-03	1.599e-07 1.953e-03	2.247e-02 1.953e-02	3.126e-07 1.953e-03	1.981e-07 1.953e-03	3.382e-07 1.953e-03	8.264e-08 1.953e-03
IISFE										6.772e-05 1.953e-03	1.192e-06 1.953e-03	7.061e-03 5.859e-03	2.914e-01 8.398e-02	1.120e-02 9.766e-03	6.836e-03 1.953e-02
IISFI											2.600e-07 1.953e-03	4.975e-04 1.953e-03	5.602e-03 1.953e-03	4.937e-03 3.906e-03	8.980e-02 1.934e-01
IISFT												5.905e-07 1.953e-03	2.237e-07 1.953e-03	5.321e-07 1.953e-03	1.886e-07 1.953e-03
VRFTEI													2.813e-01 4.316e-01	9.159e-01 1.000e+00	5.094e-01 5.566e-01
VRFTIE														2.486e-01 4.922e-01	1.812e-01 1.934e-01
VSFTEI															5.195e-01 4.922e-01

Table 24: Student t-test and Wilcoxon test results for 90x20 instances

	IIRBI	IIRBT	IIRFE	IIRFI	IIRFT	IISBE	IISBI	IISBT	IISFE	IISFI	IISFT	VRFTEI	VRFTIE	VSFTEI	VSFTIE
IIRBE	1.307e-01 1.602e-01	4.570e-08 1.953e-03	2.243e-06 1.953e-03	2.742e-07 1.953e-03	3.137e-08 1.953e-03	6.574e-01 8.457e-01	1.919e-03 1.953e-03	2.810e-07 1.953e-03	4.460e-06 1.953e-03	1.110e-07 1.953e-03	1.641e-06 1.953e-03	6.462e-07 1.953e-03	1.189e-05 1.953e-03	4.742e-05 1.953e-03	5.857e-07 1.953e-03
IIRBI		4.421e-08 1.953e-03	2.679e-04 1.953e-03	4.974e-04 1.953e-03	3.081e-08 1.953e-03	2.059e-01 2.754e-01	8.023e-02 4.883e-02	2.127e-07 1.953e-03	2.533e-02 3.711e-02	1.945e-04 1.953e-03	6.627e-07 1.953e-03	8.784e-04 1.953e-03	6.640e-04 1.953e-03	6.566e-02 4.883e-02	7.527e-04 1.953e-03
IIRBT			3.184e-08 1.953e-03	3.177e-08 1.953e-03	2.748e-04 1.953e-03	3.485e-08 1.953e-03	5.426e-08 1.953e-03	1.436e-07 1.953e-03	4.314e-08 1.953e-03	1.906e-08 1.953e-03	3.939e-08 1.953e-03	2.290e-08 1.953e-03	2.082e-08 1.953e-03	4.896e-08 1.953e-03	3.372e-08 1.953e-03
IIRFE				1.651e-02 2.734e-02	2.183e-08 1.953e-03	5.428e-05 1.953e-03	3.027e-02 3.711e-02	2.730e-08 1.953e-03	4.424e-03 3.906e-03	4.007e-03 9.766e-03	1.845e-07 1.953e-03	4.366e-01 4.316e-01	5.193e-01 6.953e-01	2.463e-02 2.734e-02	5.856e-02 8.398e-02
IIRFI					2.227e-08 1.953e-03	8.971e-06 1.953e-03	3.371e-04 3.906e-03	4.348e-08 1.953e-03	2.602e-06 1.953e-03	7.992e-01 1.000e+00	2.874e-07 1.953e-03	1.033e-02 3.711e-02	1.726e-02 1.953e-02	4.834e-05 1.953e-03	9.990e-02 6.445e-02
IIRFT						2.435e-08 1.953e-03	3.908e-08 1.953e-03	8.587e-08 1.953e-03	3.051e-08 1.953e-03	1.266e-08 1.953e-03	2.142e-08 1.953e-03	1.562e-08 1.953e-03	1.322e-08 1.953e-03	3.324e-08 1.953e-03	2.280e-08 1.953e-03
IISBE							4.514e-03 9.766e-03	5.309e-08 1.953e-03	9.671e-04 1.953e-03	1.108e-05 1.953e-03	5.393e-07 1.953e-03	8.759e-05 3.906e-03	6.046e-05 1.953e-03	3.381e-03 9.766e-03	5.464e-05 1.953e-03
IISBI								1.195e-07 1.953e-03	7.905e-01 5.566e-01	4.065e-04 3.906e-03	7.558e-07 1.953e-03	8.741e-02 1.309e-01	5.297e-02 6.445e-02	8.348e-01 5.566e-01	6.182e-03 9.766e-03
IISBT									1.346e-07 1.953e-03	1.299e-08 1.953e-03	1.740e-03 3.906e-03	2.618e-08 1.953e-03	1.494e-08 1.953e-03	9.236e-08 1.953e-03	3.017e-08 1.953e-03
IISFE										1.559e-04 1.953e-03	8.415e-07 1.953e-03	2.209e-02 1.953e-02	5.379e-02 4.883e-02	9.380e-01 1.000e+00	1.153e-04 1.953e-03
IISFI											6.427e-08 1.953e-03	5.193e-04 1.953e-03	7.566e-04 1.953e-03	6.247e-04 1.953e-03	1.053e-01 8.398e-02
IISFT												1.342e-07 1.953e-03	8.606e-08 1.953e-03	8.799e-07 1.953e-03	2.412e-07 1.953e-03
VRFTEI													9.997e-01 8.457e-01	5.736e-02 3.711e-02	4.898e-02 6.445e-02
VRFTIE														1.043e-01 1.055e-01	9.911e-02 1.309e-01
VSFTEI															4.441e-05 1.953e-03