~~BEST~~ NECESSARY PRACTICES: CONVERGENCE TESTING

Lecturer: Jared A. Goldberg  (Flatiron Institute/CCA; jgoldberg@flatironinstitute.org)

WHAT ARE WE TRYING TO DO WHEN WE USE MESA?

WE ARE TRYING TO SOLVE THE TIME-DEPENDENT STELLAR STRUCTURE EQUATIONS (IN 1 DIMENSION)

TIME-DEPENDENT 1D STELLAR STRUCTURE EQUATIONS

Conservation of mass: $$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

Conservation of momentum: $$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t}$$

Conservation of energy: $$\frac{\partial l}{\partial m} = \epsilon_{\rm nuc} - \epsilon_\nu - T\frac{\partial s}{\partial t}$$

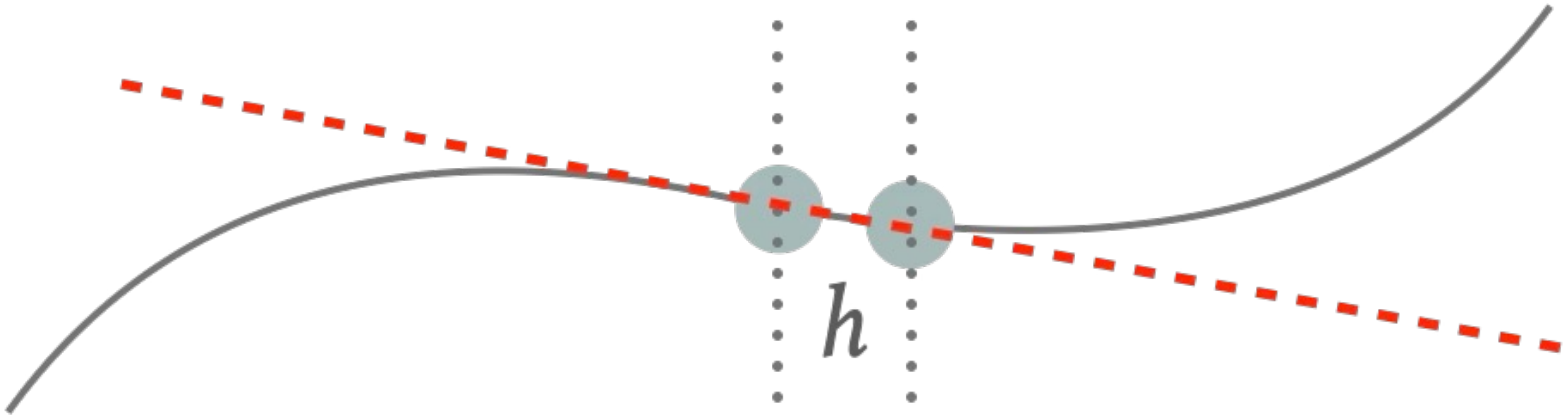
Transport of Heat: $$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4}\frac{T}{P}\nabla \quad \text{where } \nabla = \begin{cases} \nabla_{\rm rad} = \dfrac{3\kappa}{16\pi acG}\dfrac{lP}{mT^4} & \text{if } \nabla_{\rm rad} \le \nabla_{\rm ad} \quad \text{(Radiative)} \\ \nabla_{\rm ad} + \Delta\nabla & \text{if } \nabla_{\rm rad} > \nabla_{\rm ad} \quad \text{(Convective)} \end{cases}$$

Where $\Delta\nabla$ is (often) calculated with Mixing Length Theory

Evolution of composition: $$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\sum_j (1+\delta_{ij}) r_{ij} + \sum_{k,l} r_{kl,i}\right)$$ + mixing, for i = 1 … $N_{\rm species}$

HOW DOES A COMPUTER THINK ABOUT DIFFERENTIAL EQUATIONS?

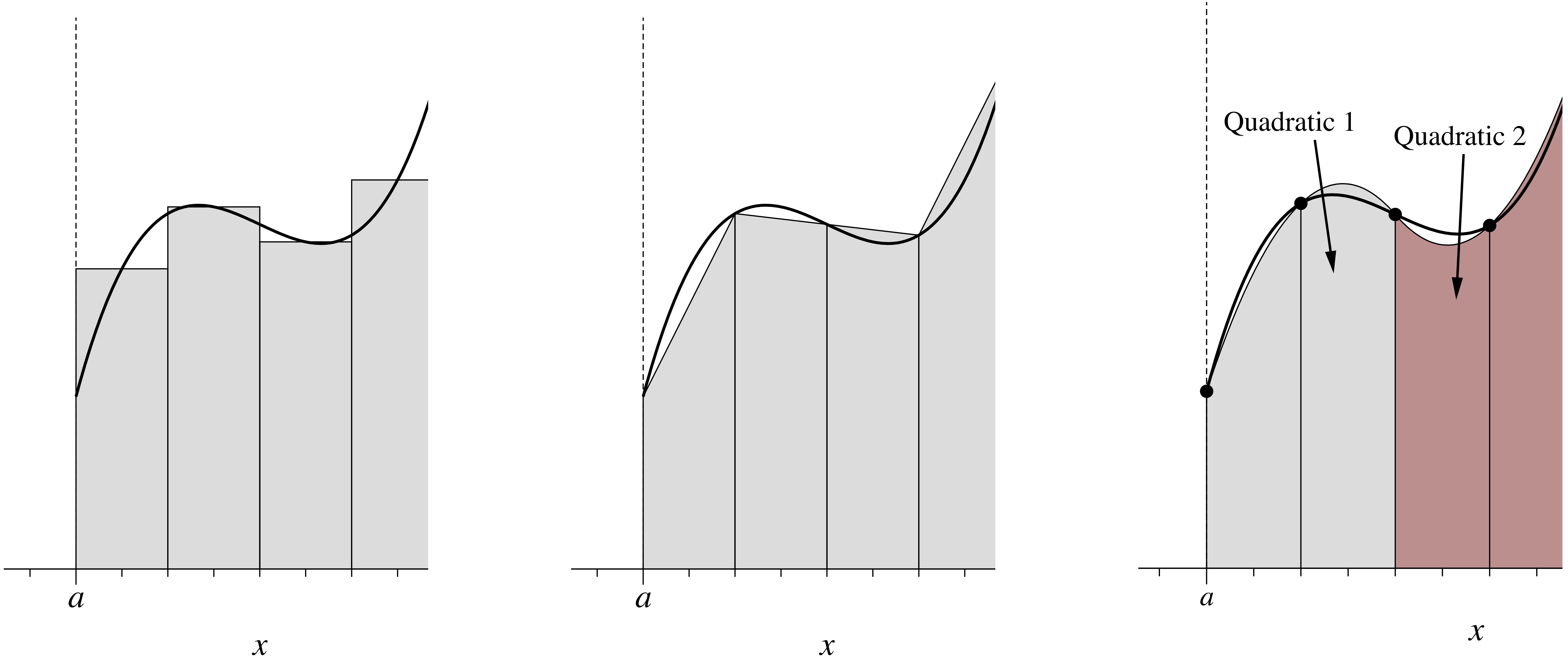
- A derivative is a difference



$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \to \frac{\Delta f}{\Delta x} = \frac{f(x[i+1]) - f(x[i])}{x[i+1]-x[i]}$$

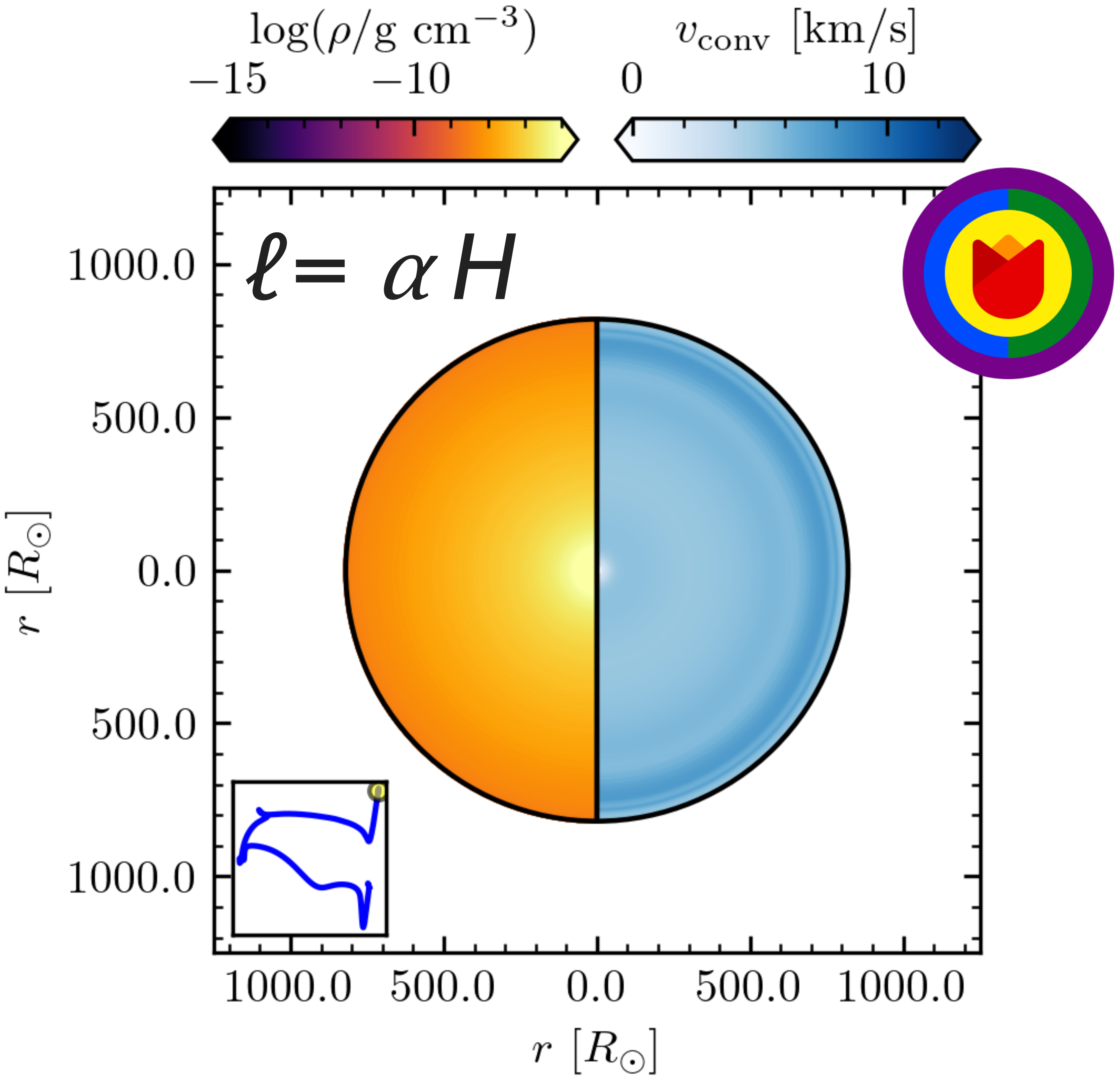
HOW DOES A COMPUTER THINK ABOUT DIFFERENTIAL EQUATIONS?

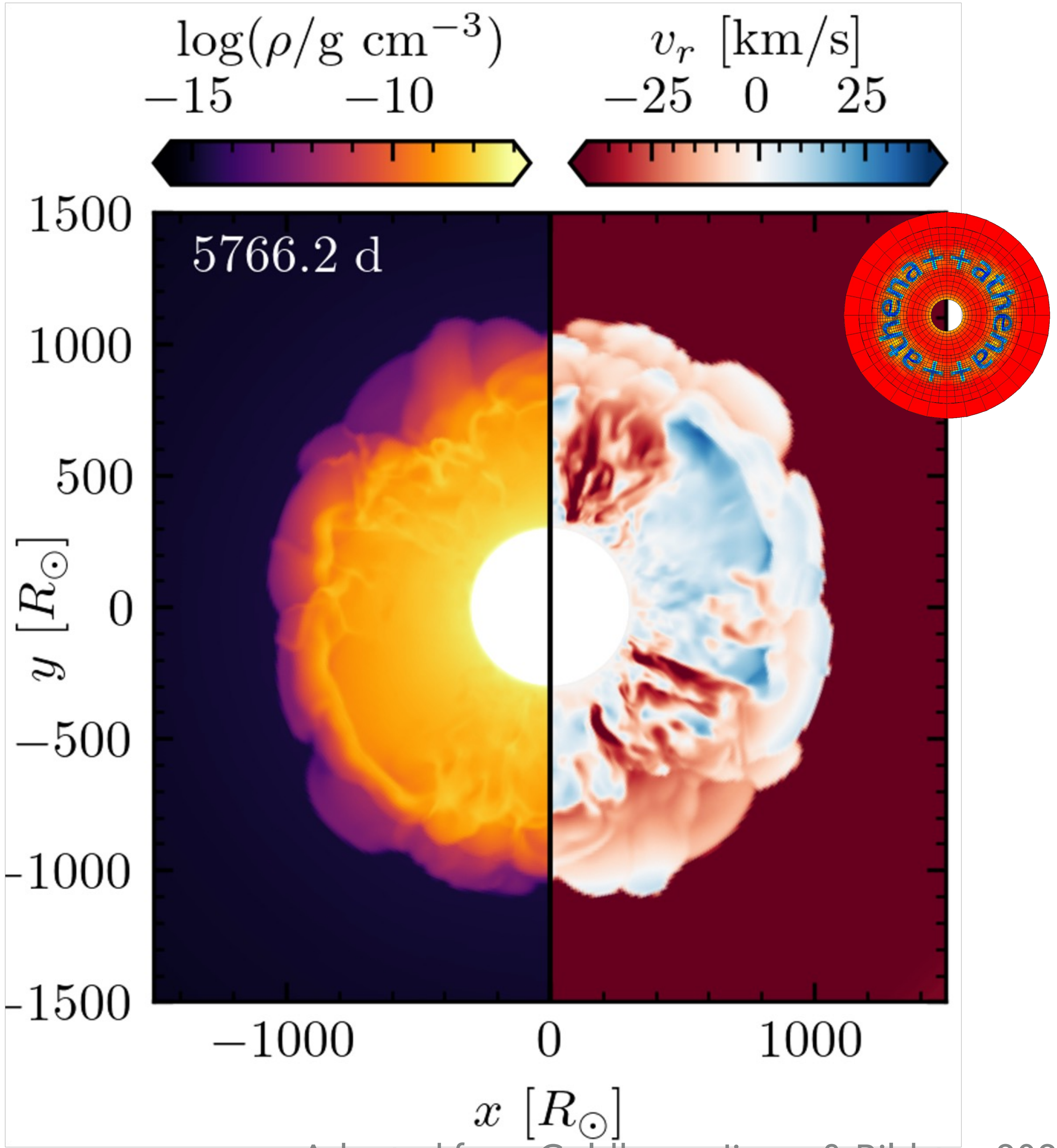
- An integral is a sum $\int_{\text{core}}^{\text{surface}} f(x)\mathrm{d}x \to \sum_{k=nz}^{k=1} f(x[k])\,\Delta x_k$



Adapted from Newman: Computational Physics and slides by Ari Maller

1D "STARS" ARE ENGINEERING AS MUCH AS THEY ARE PHYSICS





Adapted from Goldberg, Jiang, & Bildsten 2022a

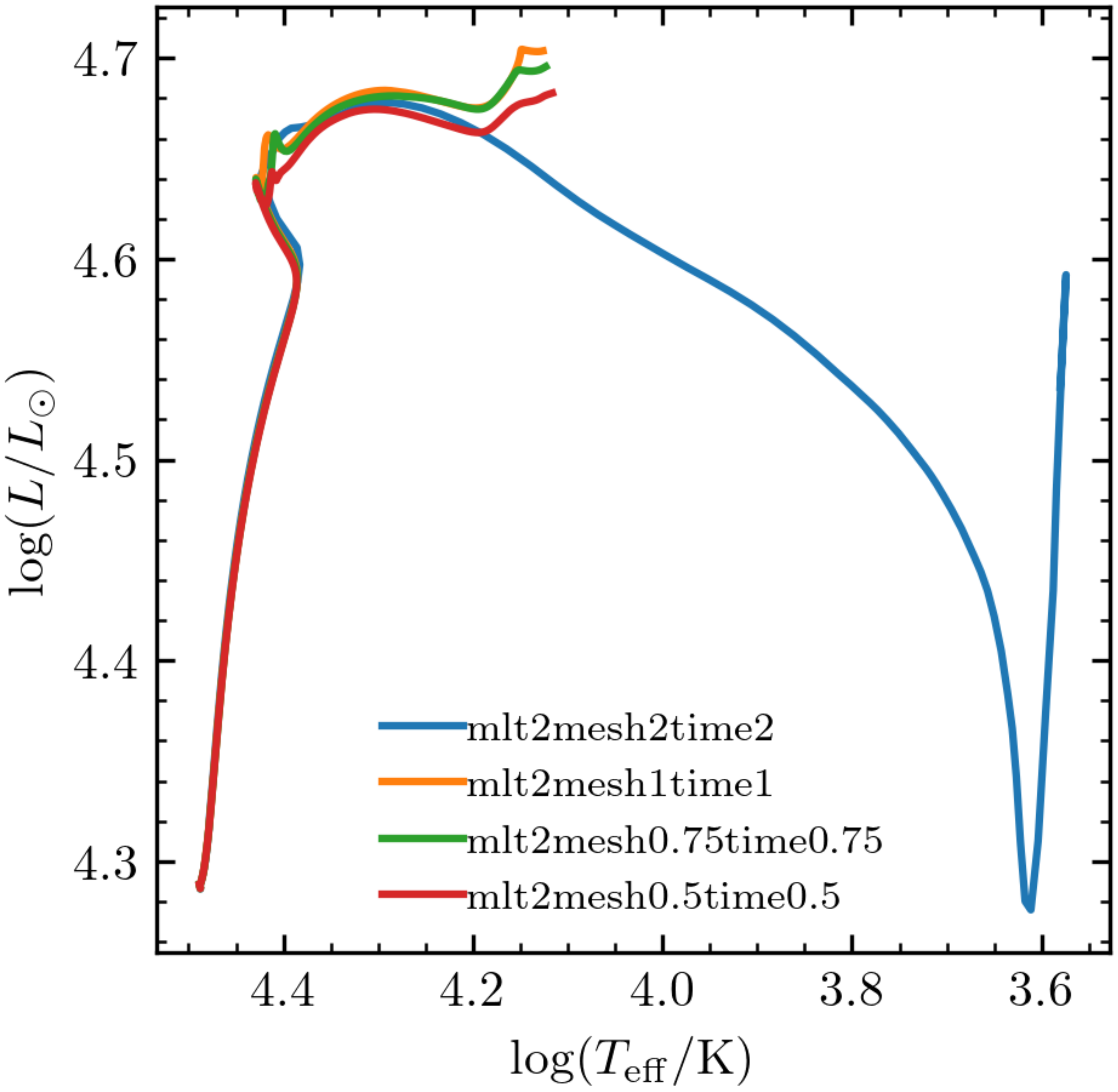
SO: HOW GOOD OF AN APPROXIMATION IS THIS?

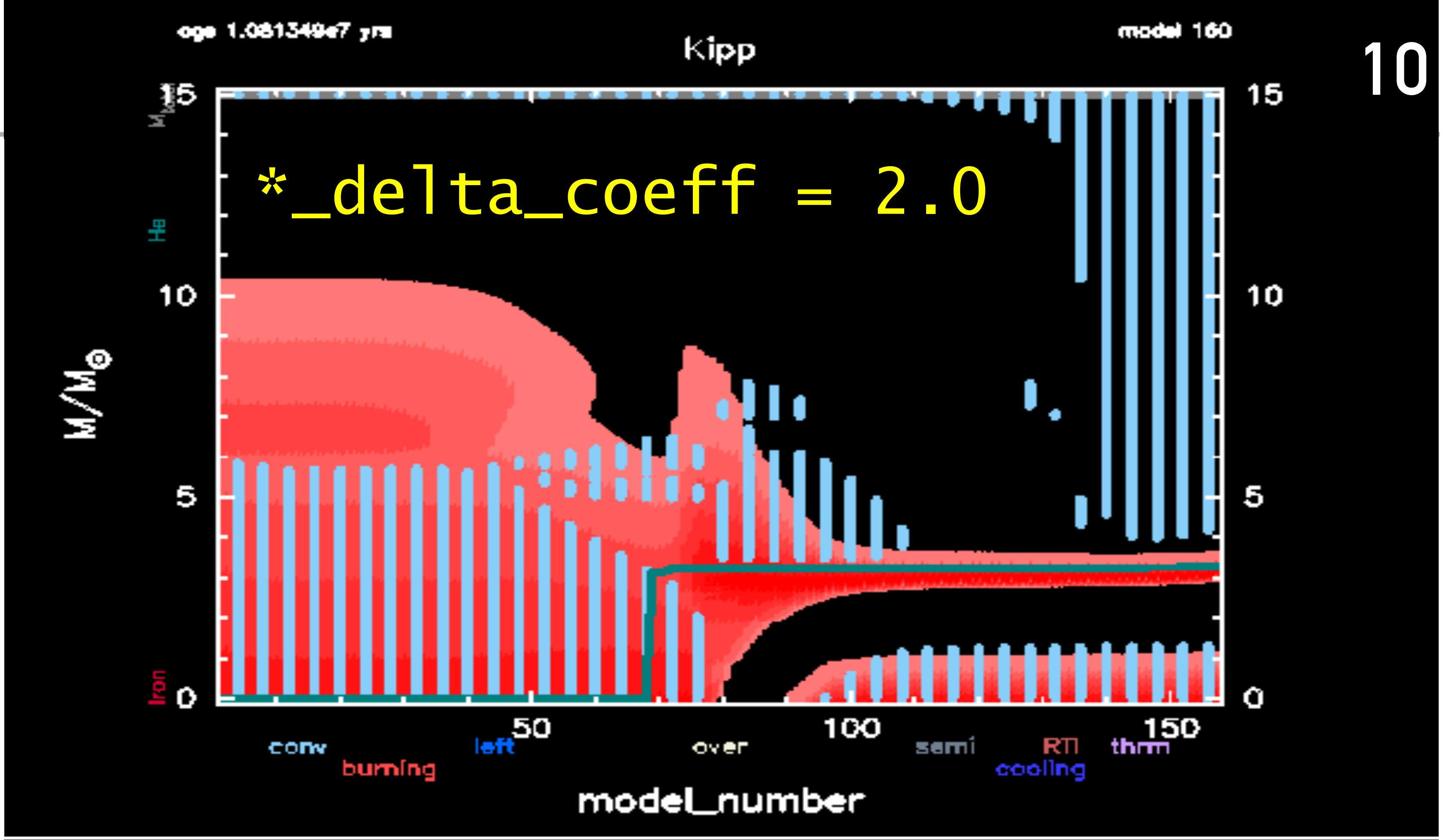
OVERVIEW OF OUR MORNING LAB STRUCTURE:

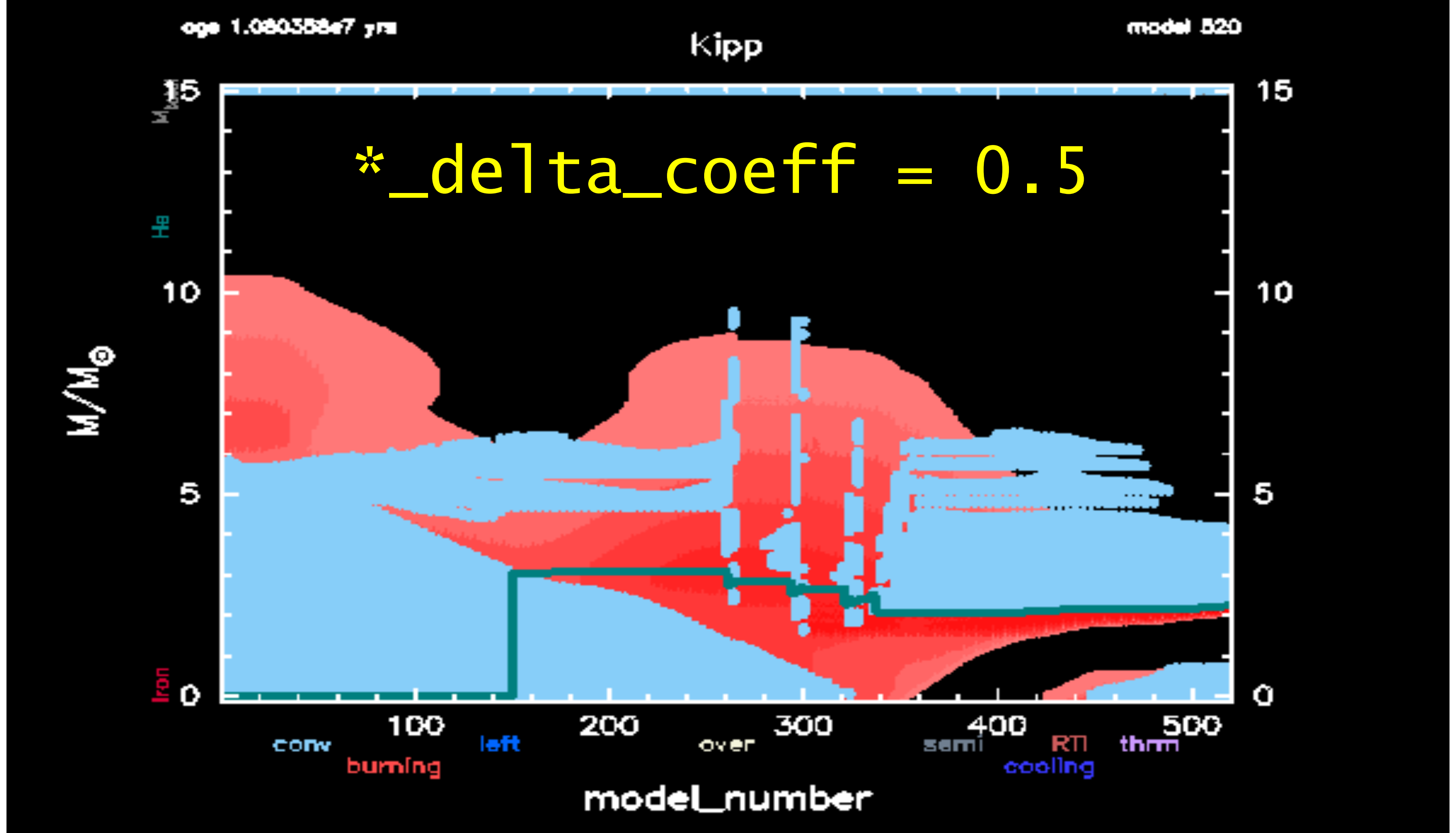
- Mini-mini lab 1: A failed resolution test
- Mini-mini lab 2: A successful resolution test
- Mini-mini lab 3: Testing our 1D “physics” (engineering) assumptions

WHAT HAPPENED?

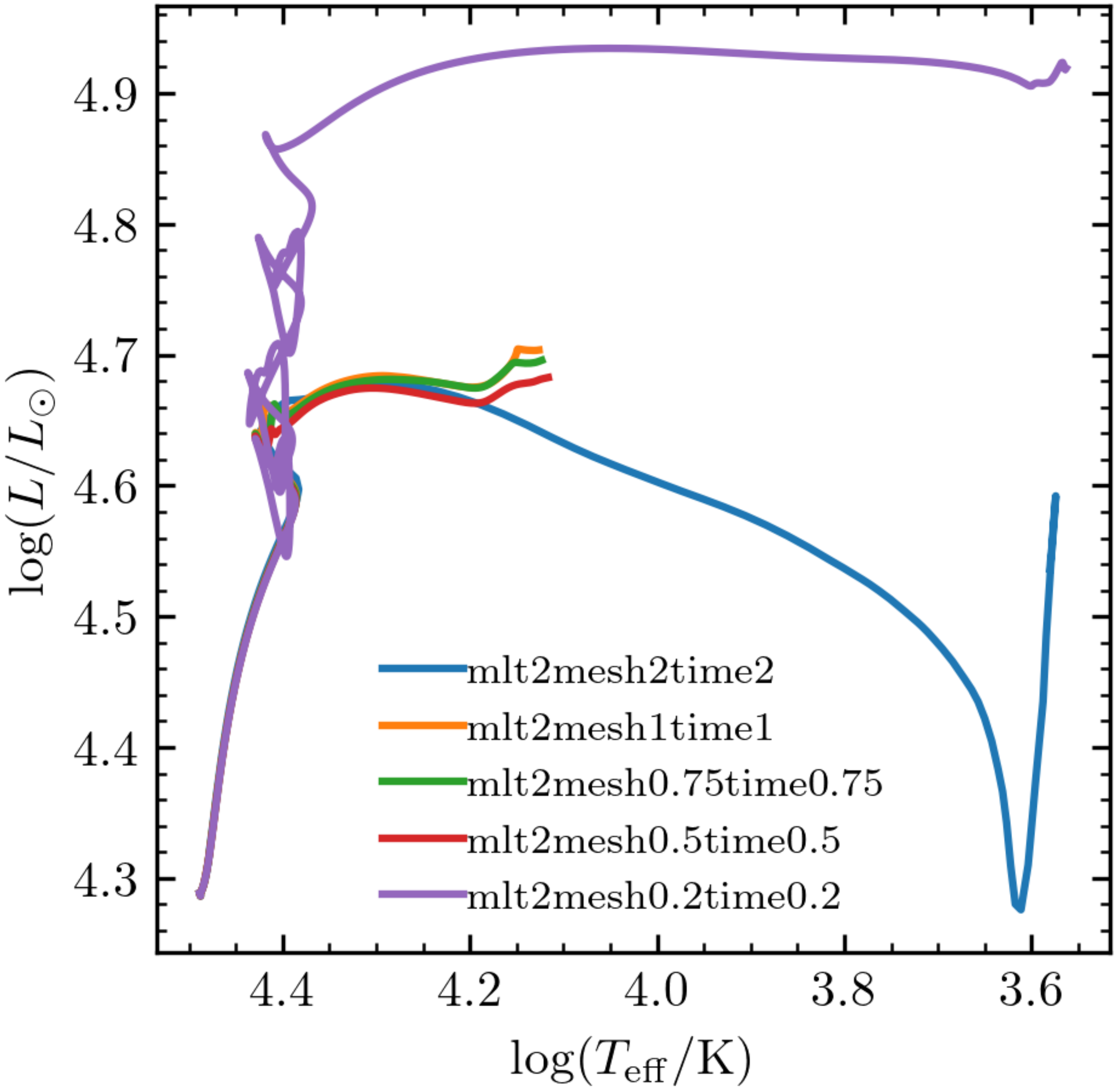
MINI-MINILAB1: RECAP

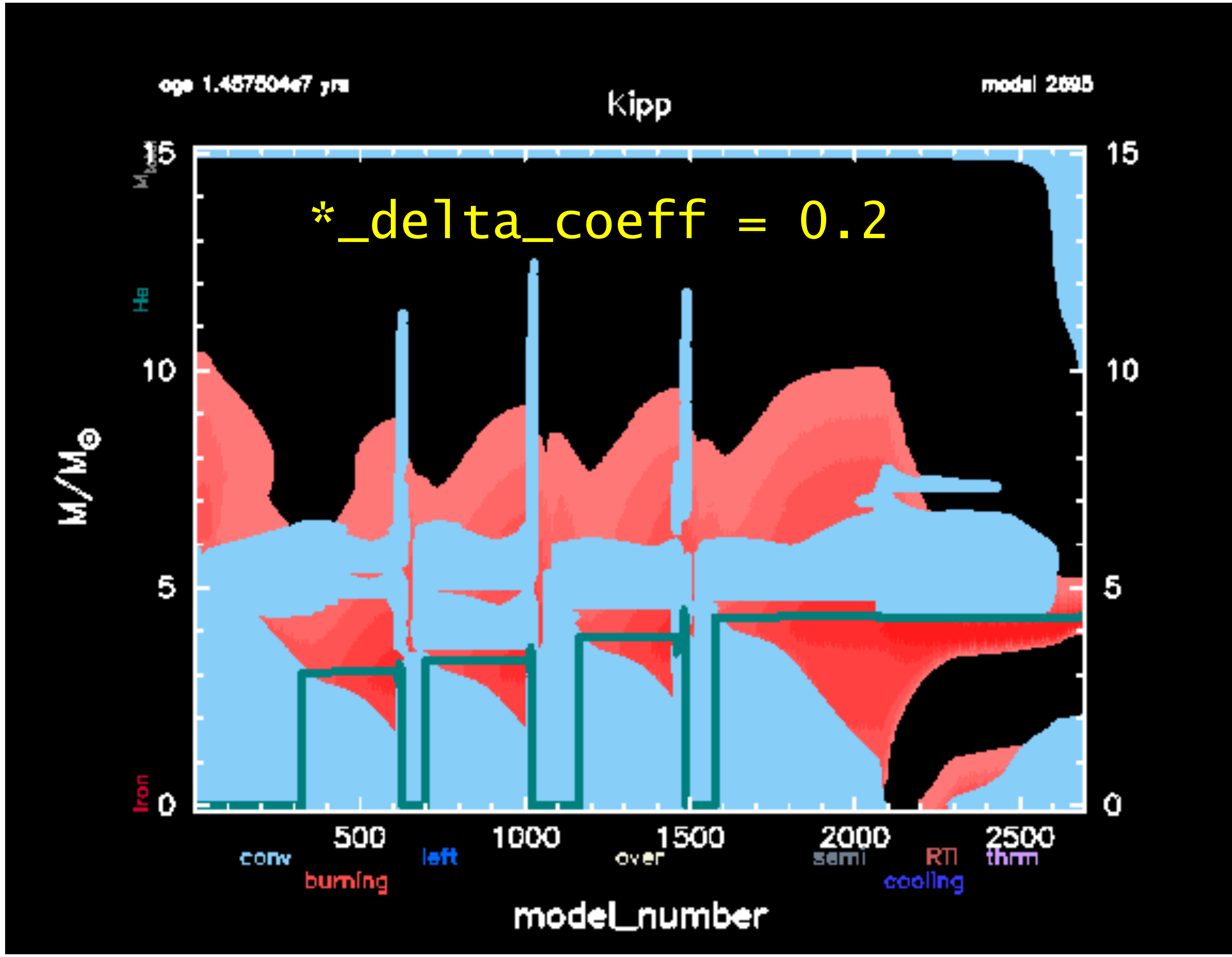




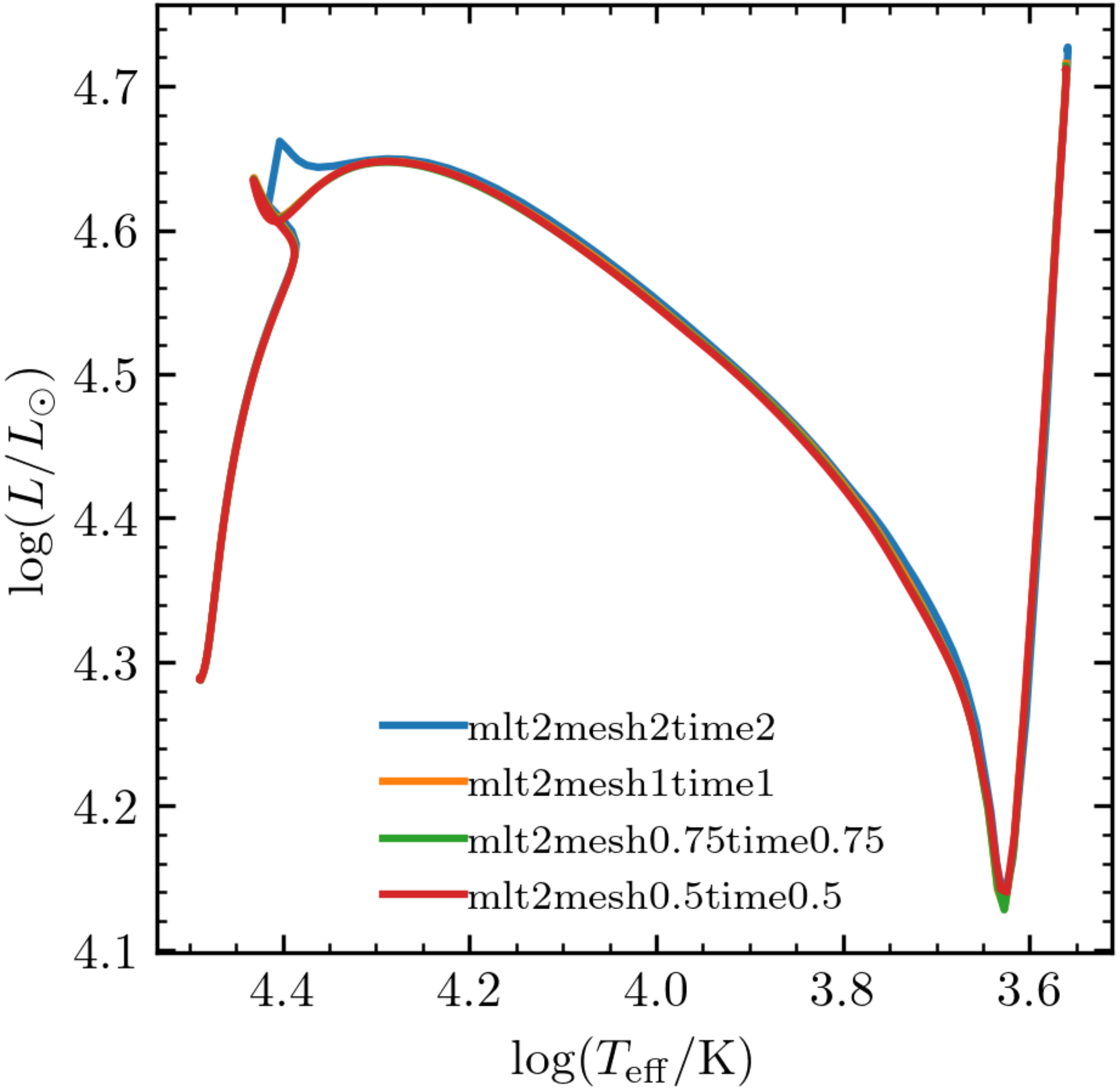


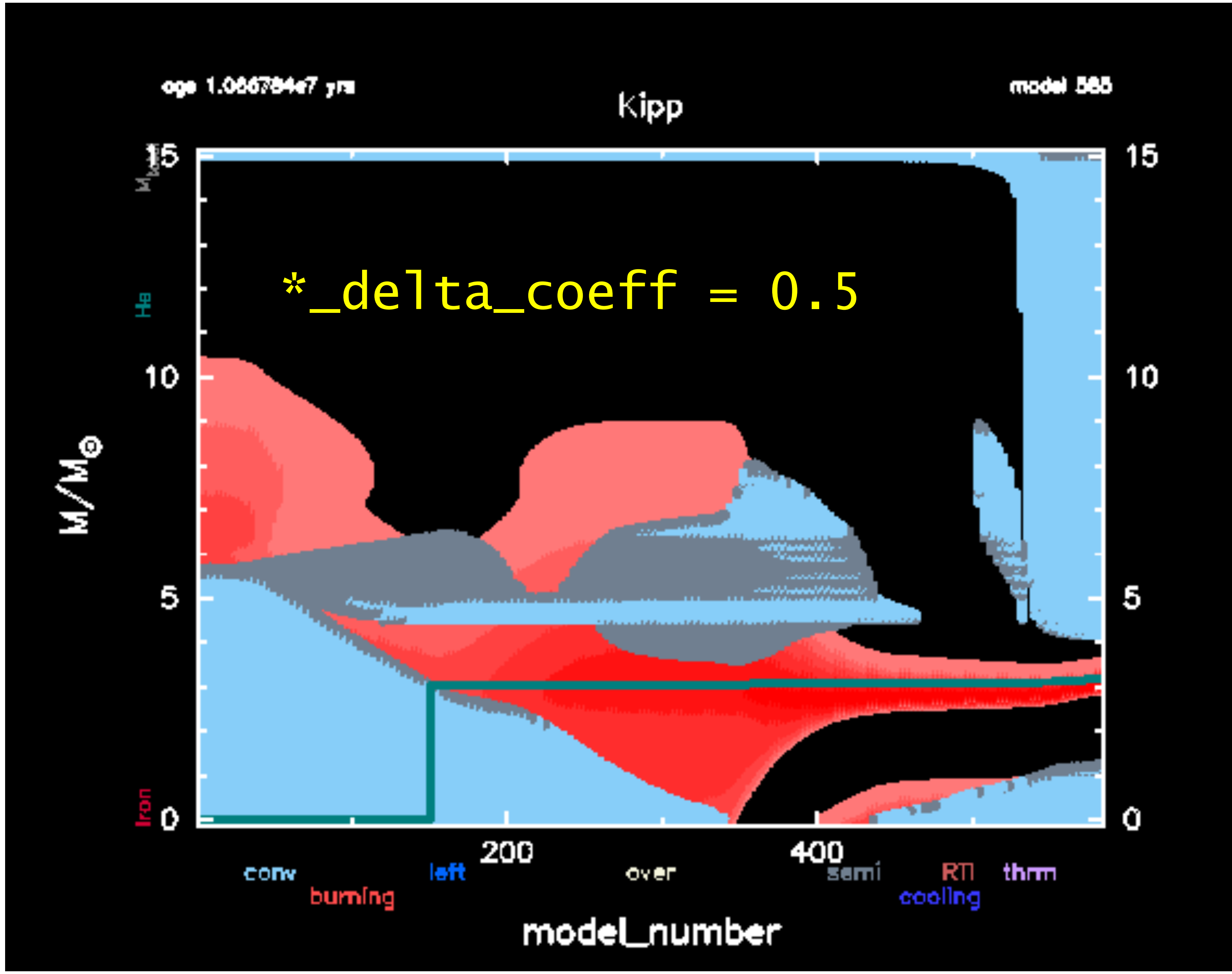
HOME EXERCISE: GO TO EVEN HIGHER RESOLUTION



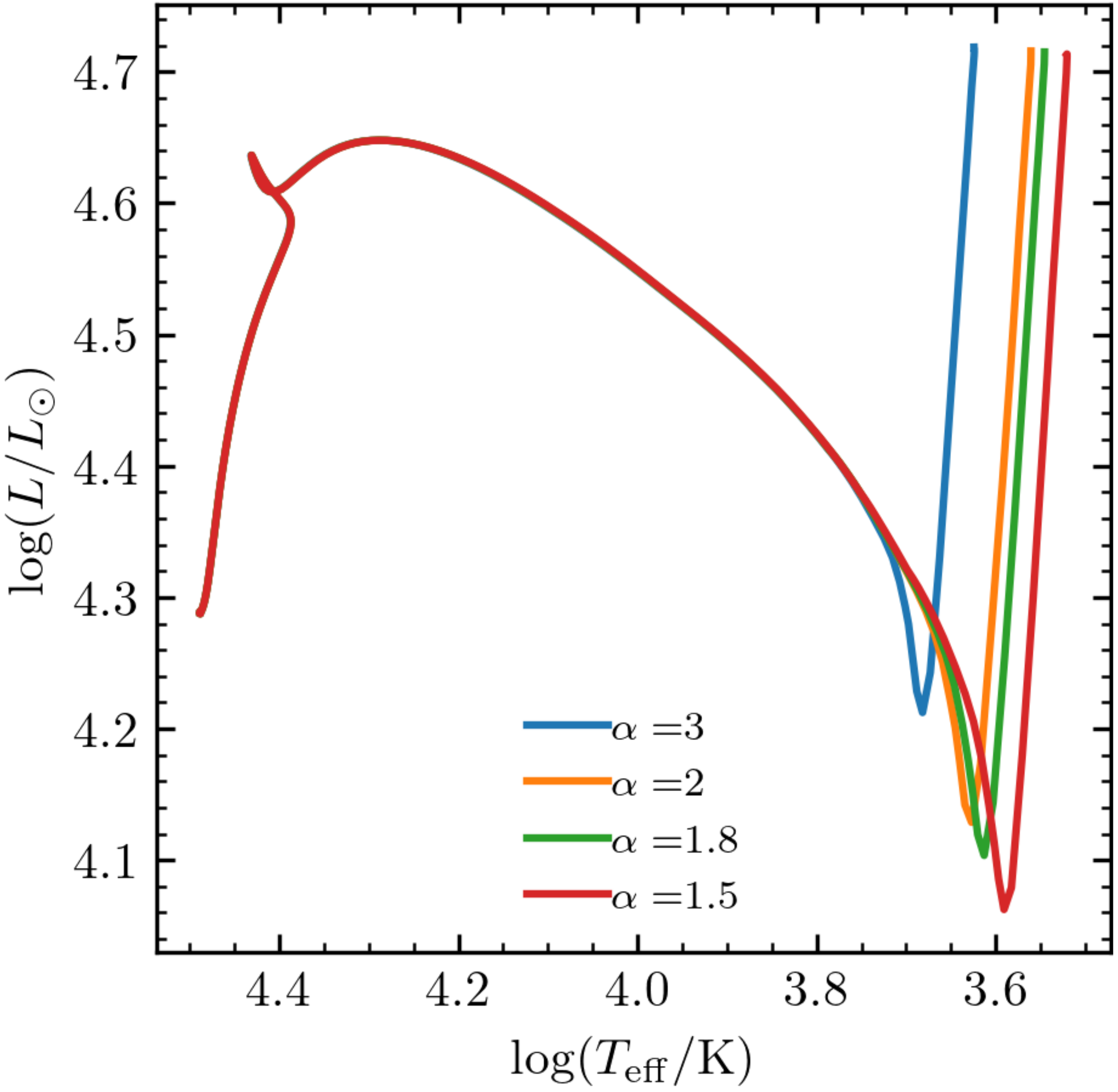


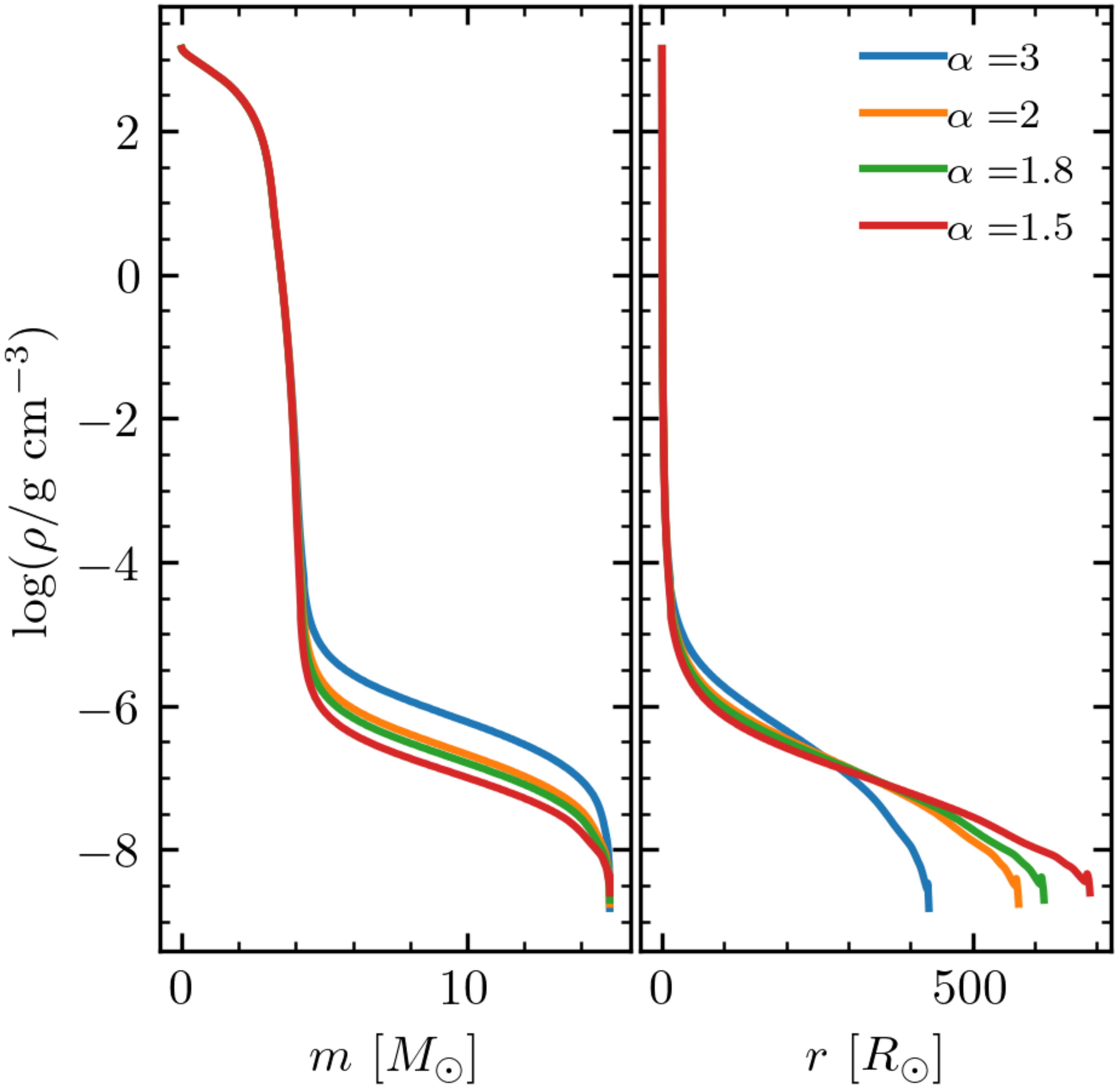
MINI-MINILAB2: RECAP





MINI-MINILAB3: RECAP

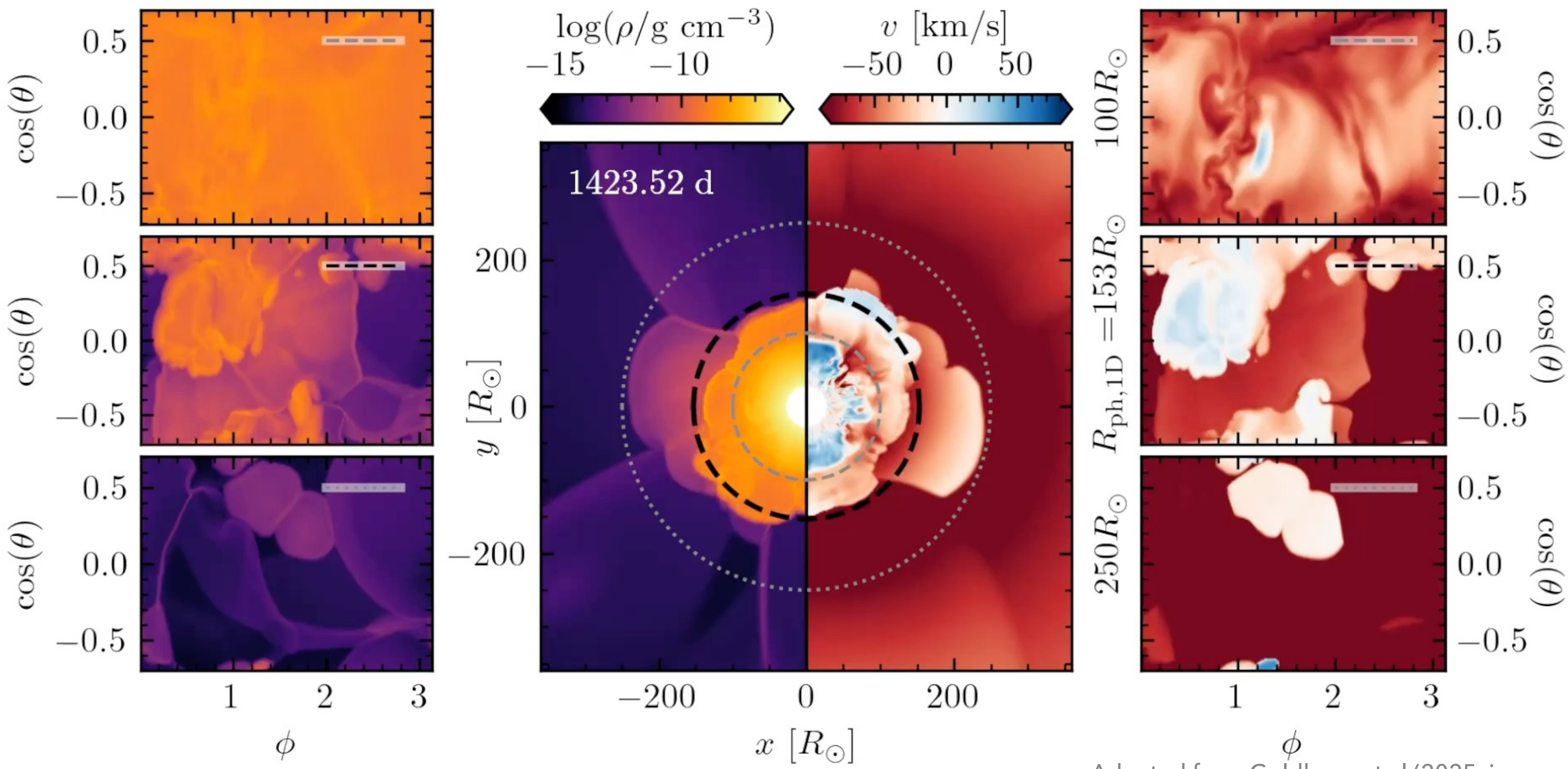




WHY IS THIS SO IMPORTANT?

- **We want to know that we are solving the equations we think we are solving!**
- Making your setup as robust as possible isn't just a good practice, it's necessary to make sure you're doing science!
- Be wary of "magic resolution" – A marginally stable setup for one model likely will not give you consistent results for another model
- If you adapt your MESA setup from a very old revision, ***do not*** assume the resolution testing from the old revision will be valid today!
- **Understanding the physics, understanding the "engineering," and understanding the numerics all go hand-in-hand**
- **Test, think, explore, and test again!**

THANK YOU!!! QUESTIONS? COMMENTS? THOUGHTS? CONCERNS? VIBES?



Adapted from Goldberg et al (2025, in prep.)