



~~BEST~~ NECESSARY PRACTICES: CONVERGENCE TESTING

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**WHAT ARE WE TRYING TO DO
WHEN WE USE MESA?**

WE ARE TRYING TO SOLVE THE
TIME-DEPENDENT STELLAR
STRUCTURE EQUATIONS (IN 1
DIMENSION)

TIME-DEPENDENT 1D STELLAR STRUCTURE EQUATIONS

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Conservation
of mass:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

Conservation
of momentum:

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2} \frac{\partial^2 r}{\partial t}$$

Conservation
of energy:

$$\frac{\partial l}{\partial m} = \epsilon_{\text{nuc}} - \epsilon_\nu - T \frac{\partial s}{\partial t}$$

Transport
of Heat:

$$\frac{\partial T}{\partial m} = -\frac{Gm}{4\pi r^4} \frac{T}{P} \nabla \quad \text{where } \nabla = \begin{cases} \nabla_{\text{rad}} = \frac{3\kappa}{16\pi acG} \frac{lP}{mT^4} & \text{if } \nabla_{\text{rad}} \leq \nabla_{\text{ad}} \text{ (Radiative)} \\ \nabla_{\text{ad}} + \Delta\nabla & \text{if } \nabla_{\text{rad}} > \nabla_{\text{ad}} \text{ (Convective)} \end{cases}$$

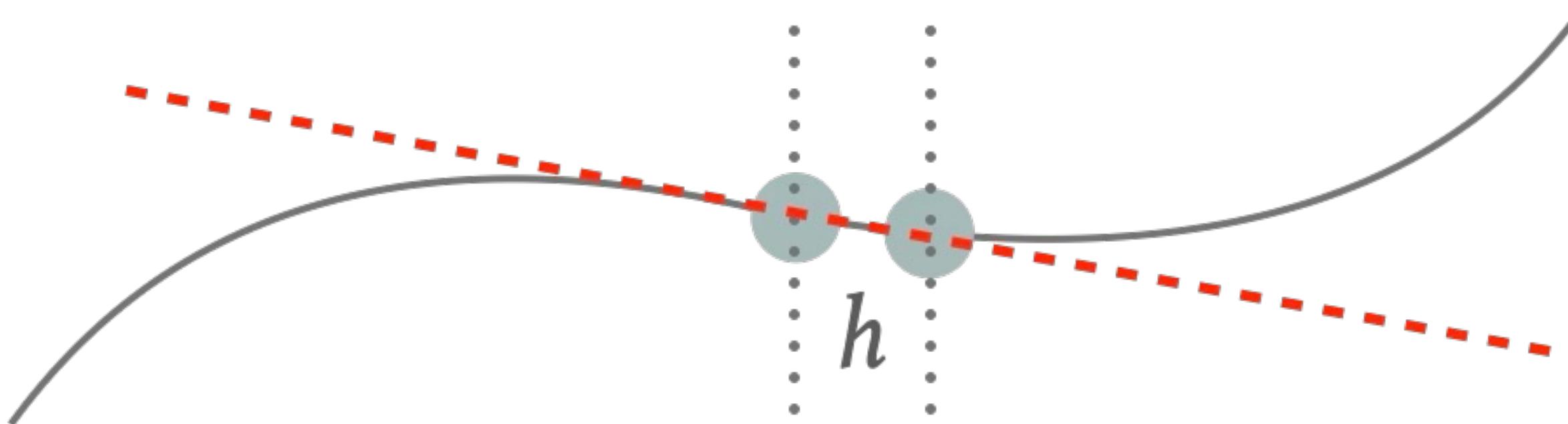
Where $\Delta\nabla$ is (often) calculated with Mixing Length Theory

Evolution
of composition:

$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho} \left(-\sum_j (1 + \delta_{ij}) r_{ij} + \sum_{k,l} r_{kl,i} \right) + \text{mixing, for } i = 1 \dots N_{\text{species}}$$

HOW DOES A COMPUTER THINK ABOUT DIFFERENTIAL EQUATIONS?

- ▶ A derivative is a difference

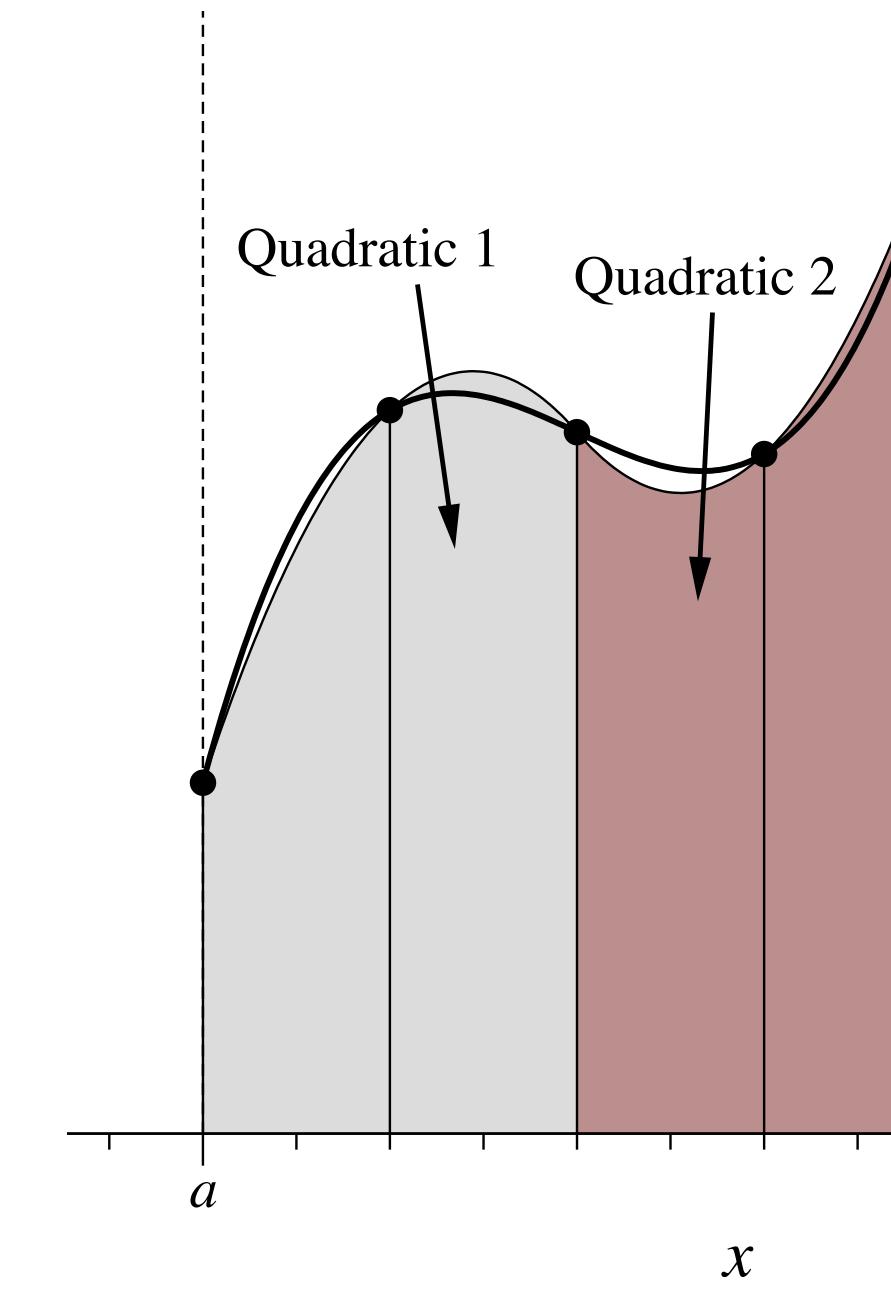
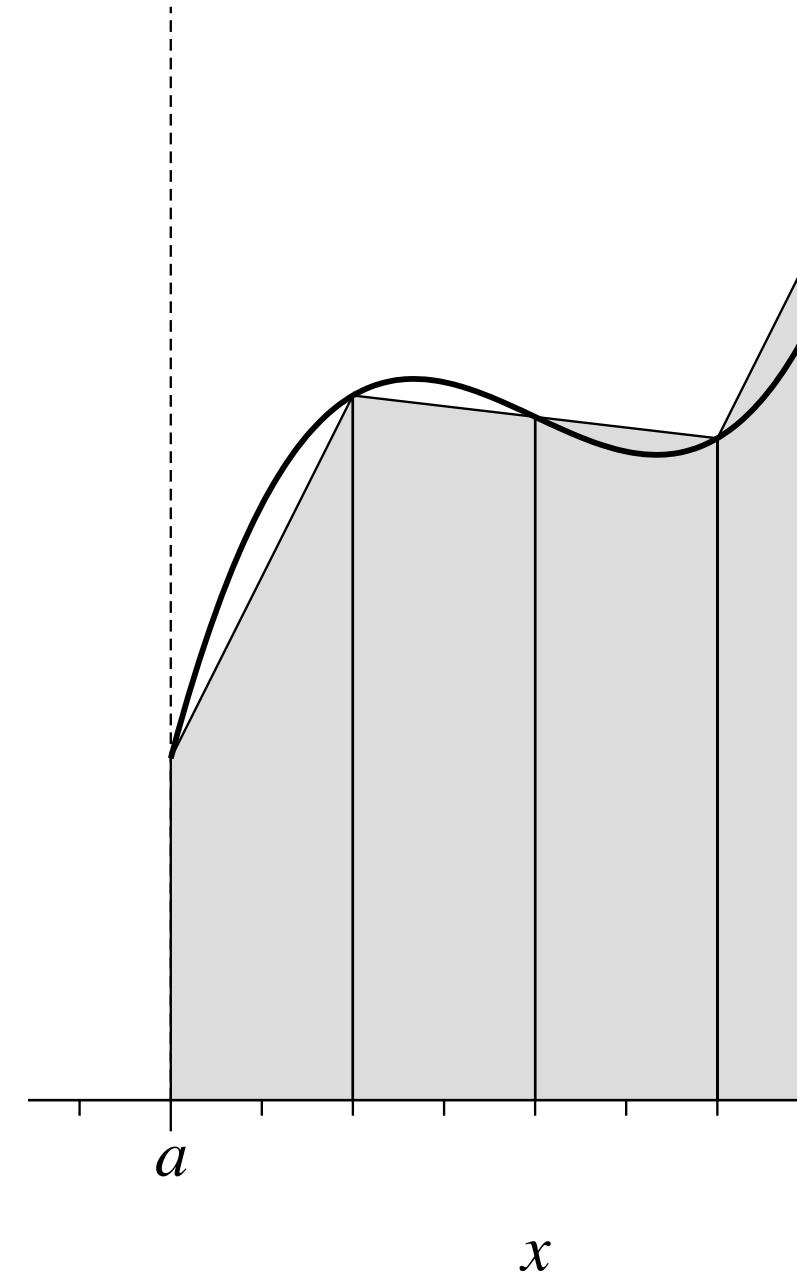
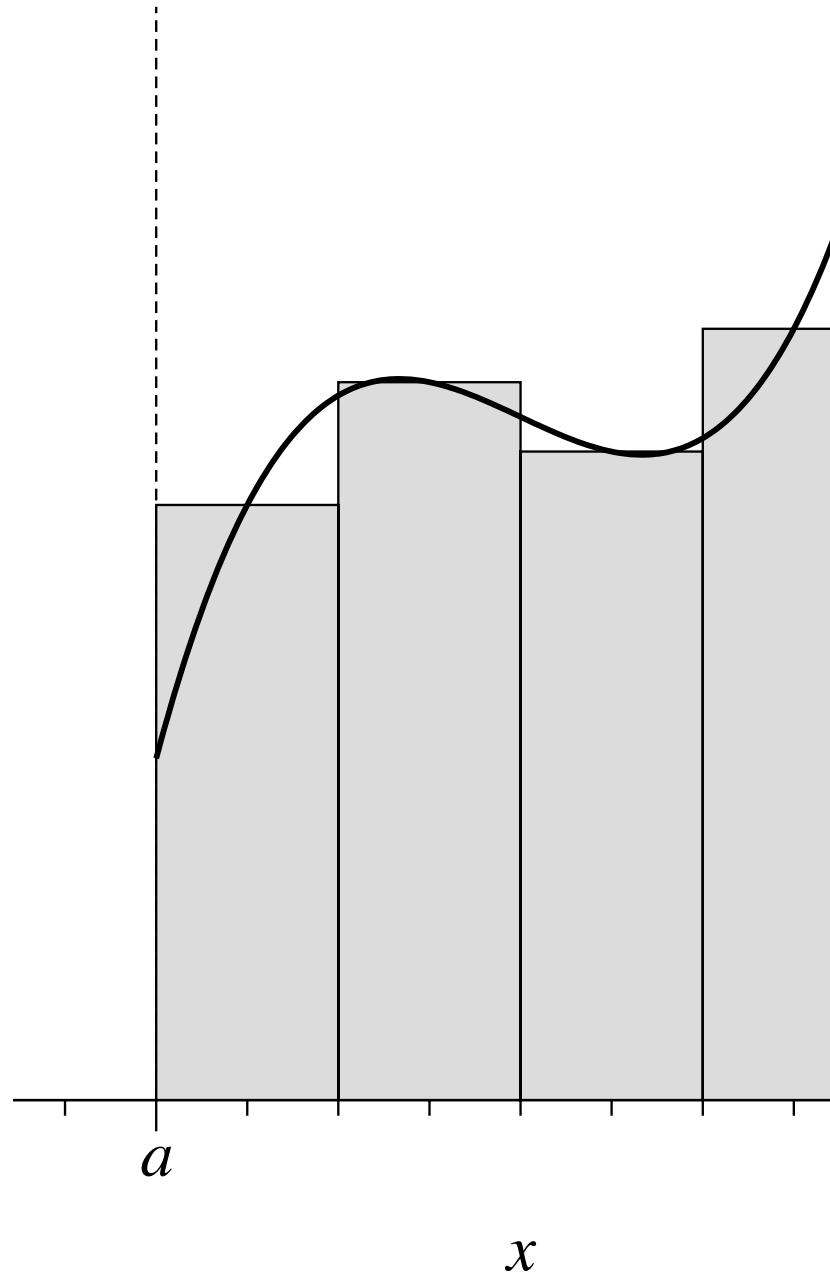


$$\frac{df}{dx} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h} \rightarrow \frac{\Delta f}{\Delta x} = \frac{f(x[i + 1]) - f(x[i])}{x[i + 1] - x[i]}$$

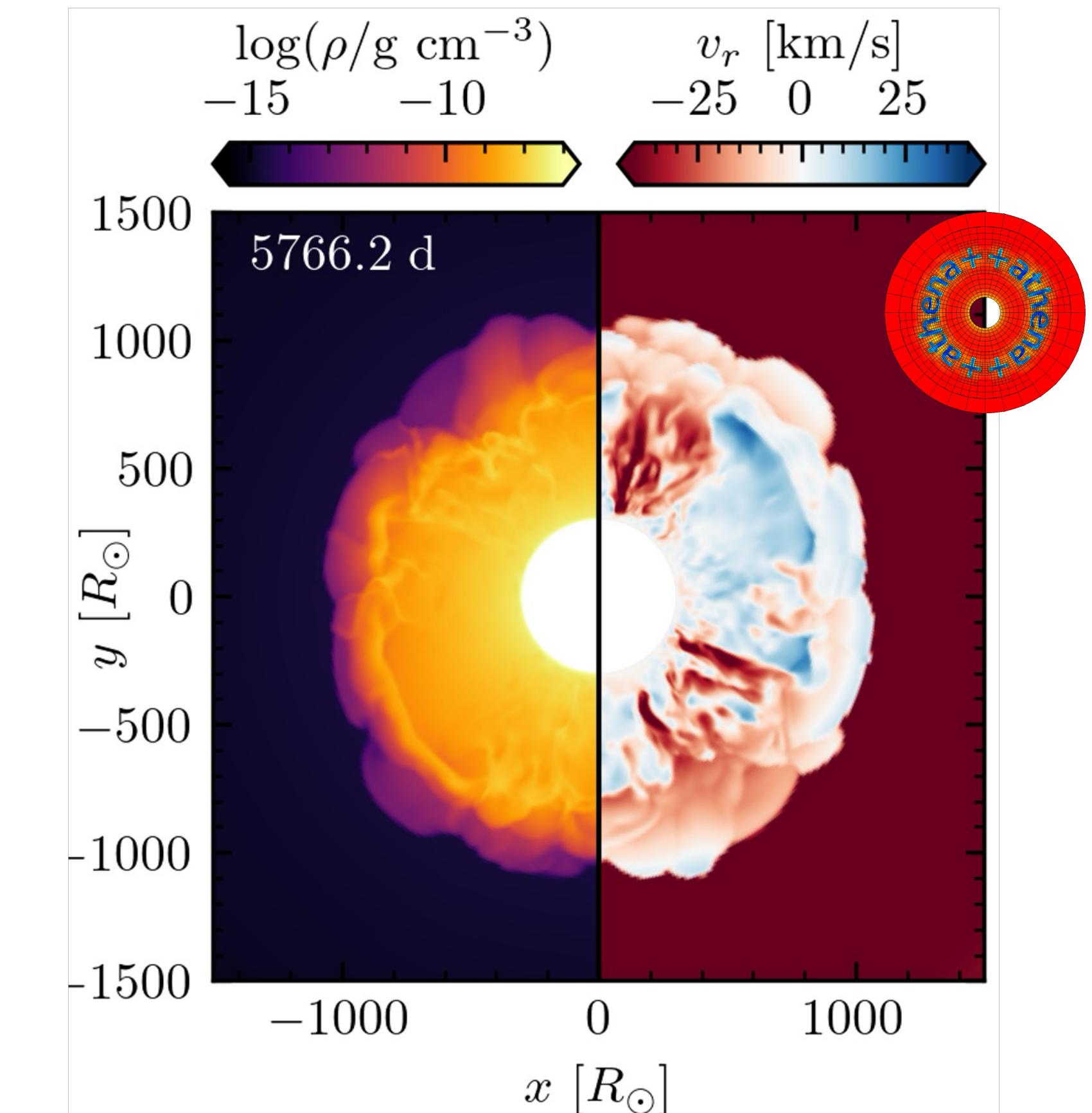
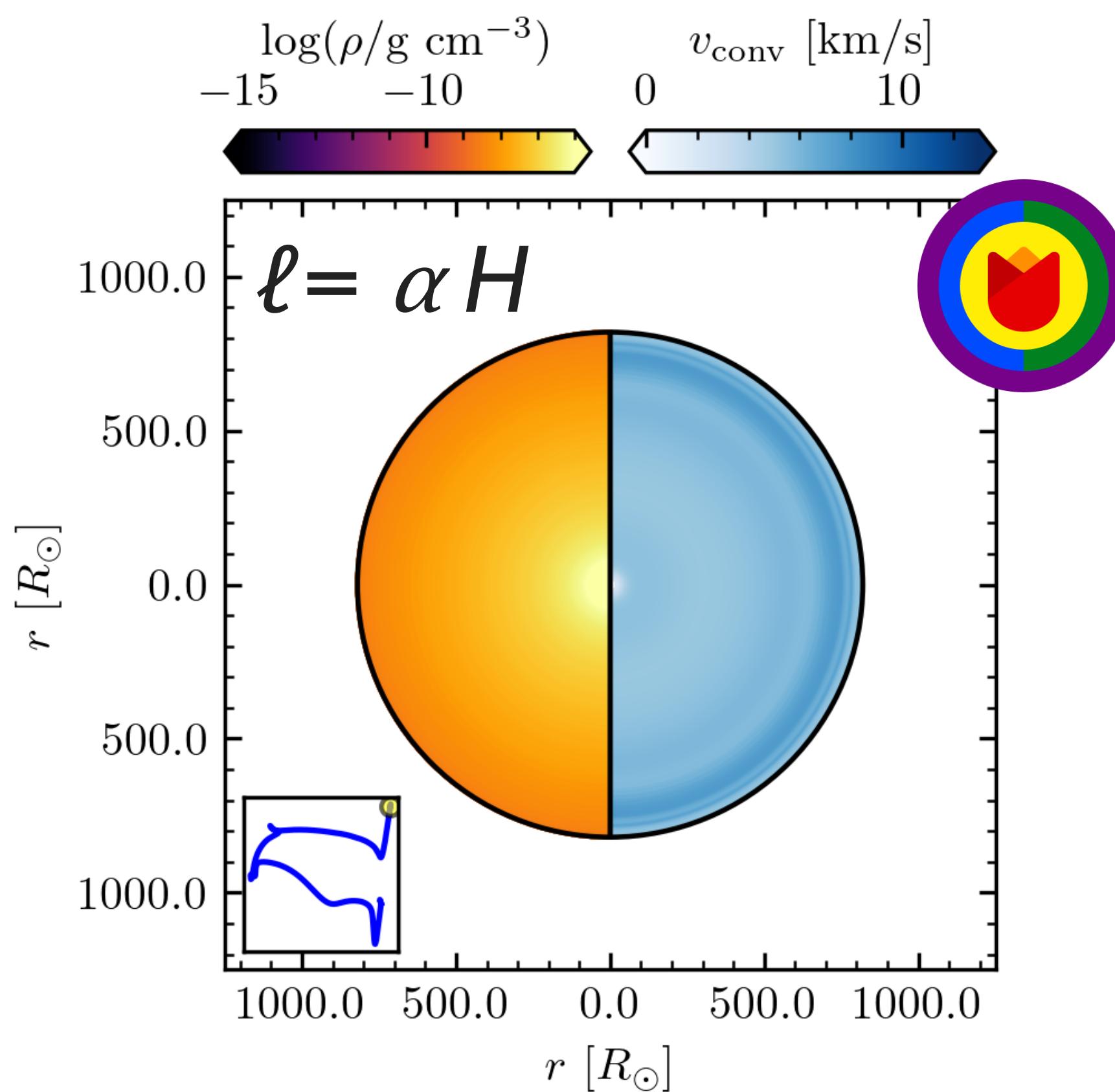
HOW DOES A COMPUTER THINK ABOUT DIFFERENTIAL EQUATIONS?

- An integral is a sum

$$\int_{\text{core}}^{\text{surface}} f(x) dx \rightarrow \sum_{k=nz}^{k=1} f(x[k]) \Delta x_k$$



1D "STARS" ARE ENGINEERING AS MUCH AS THEY ARE PHYSICS 6



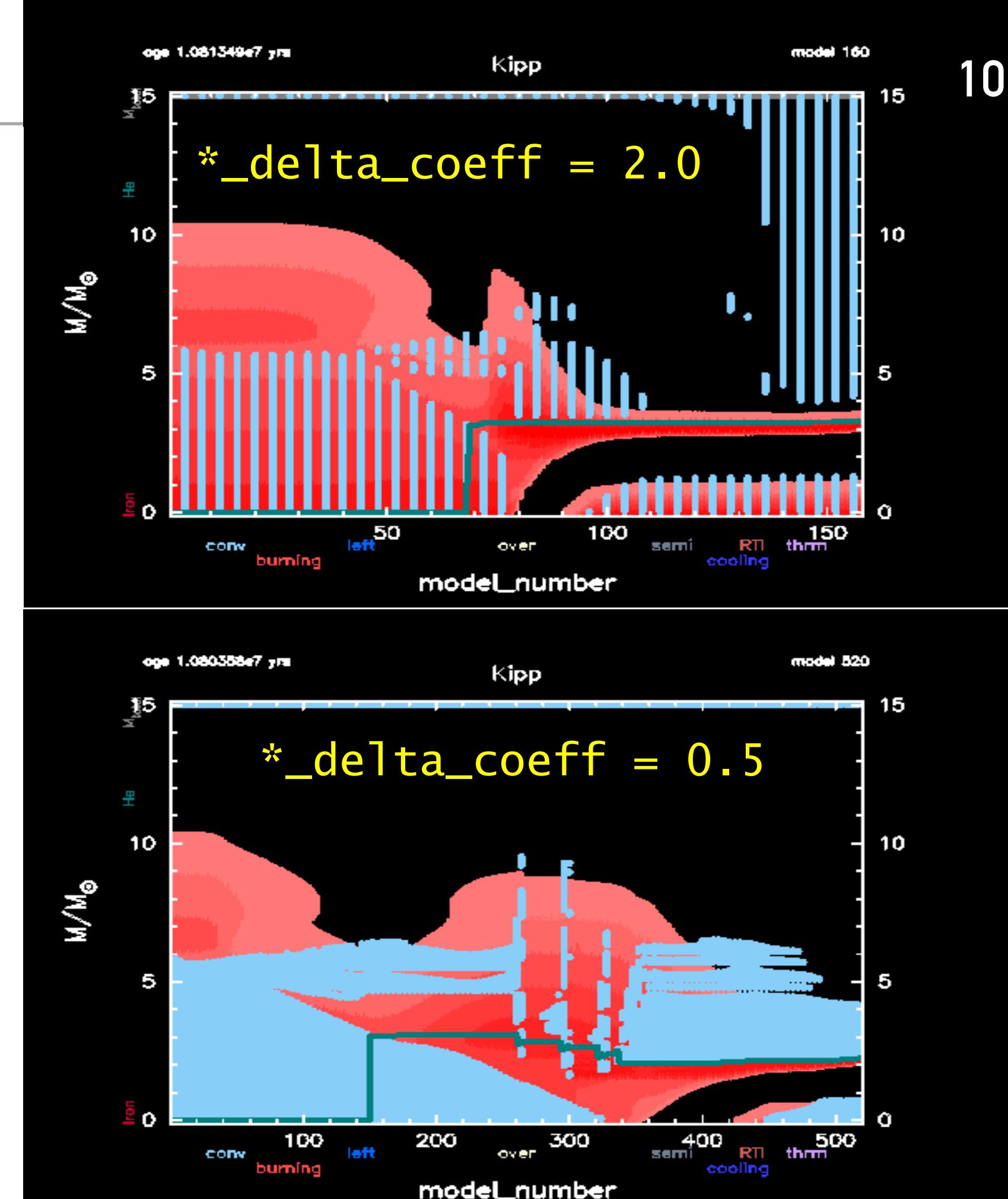
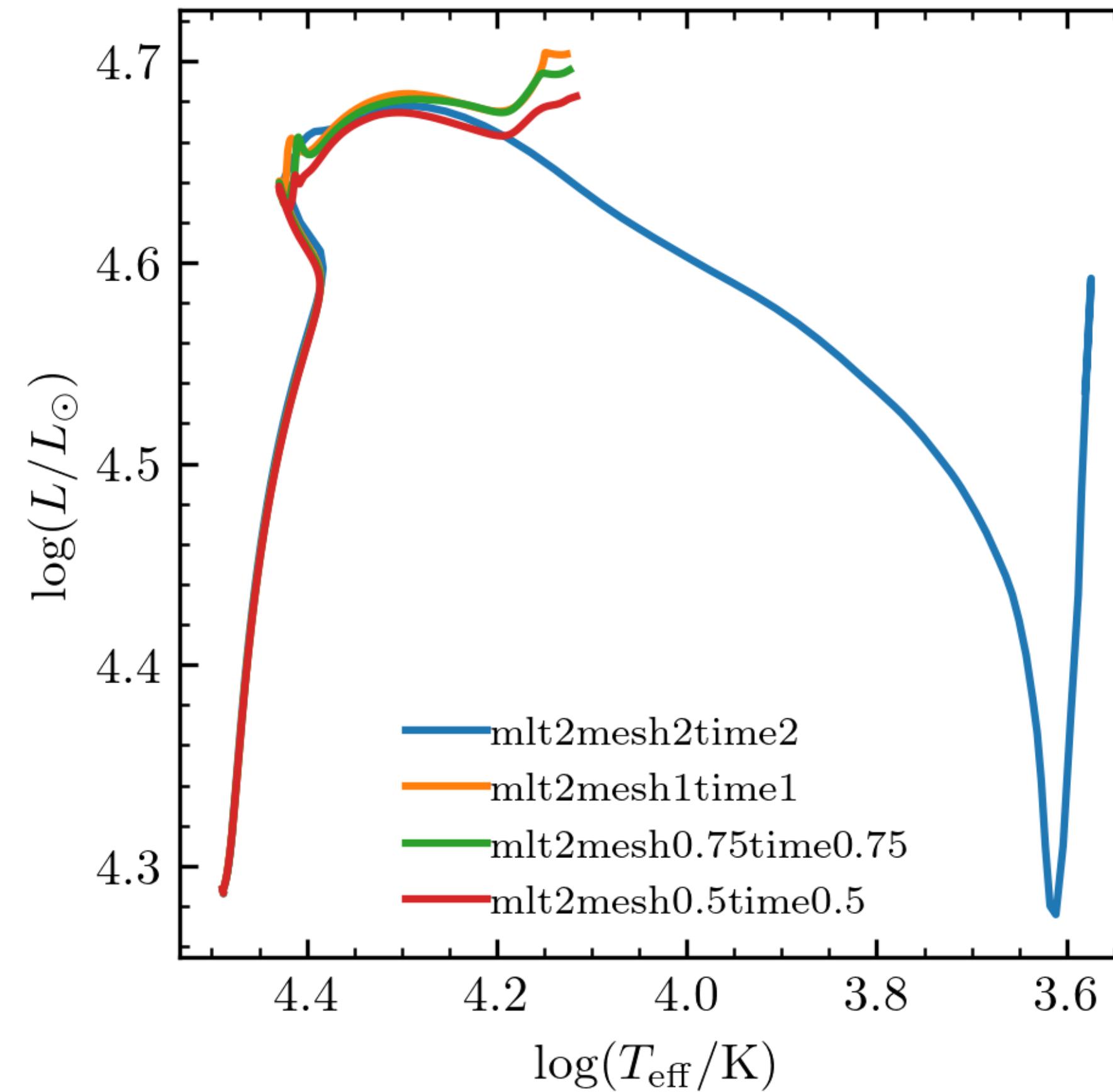
SO: HOW GOOD OF AN APPROXIMATION IS THIS?

OVERVIEW OF OUR MORNING LAB STRUCTURE:

- ▶ Mini-mini lab 1: A failed resolution test
- ▶ Mini-mini lab 2: A successful resolution test
- ▶ Mini-mini lab 3: Testing our 1D “physics” (engineering) assumptions

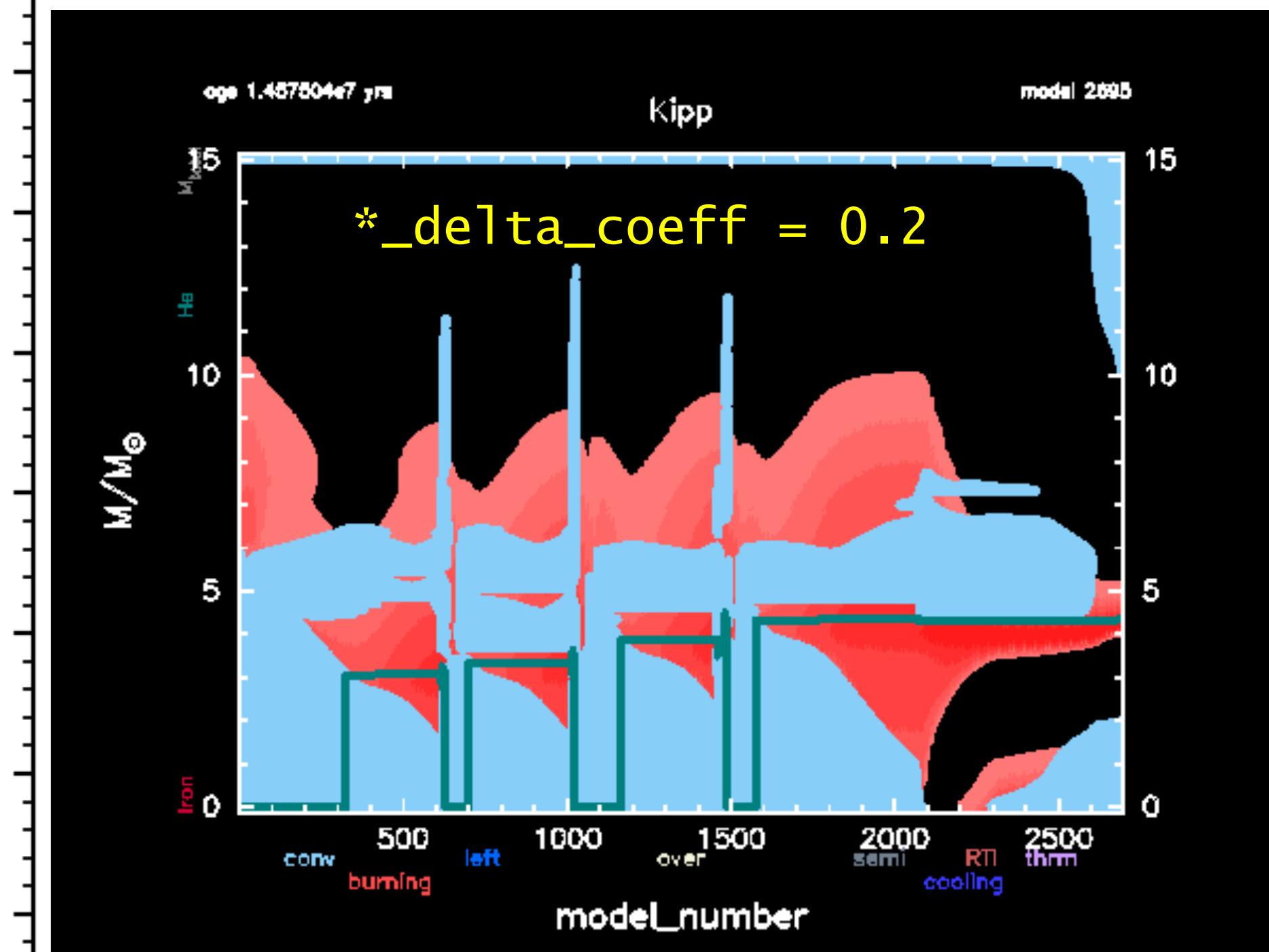
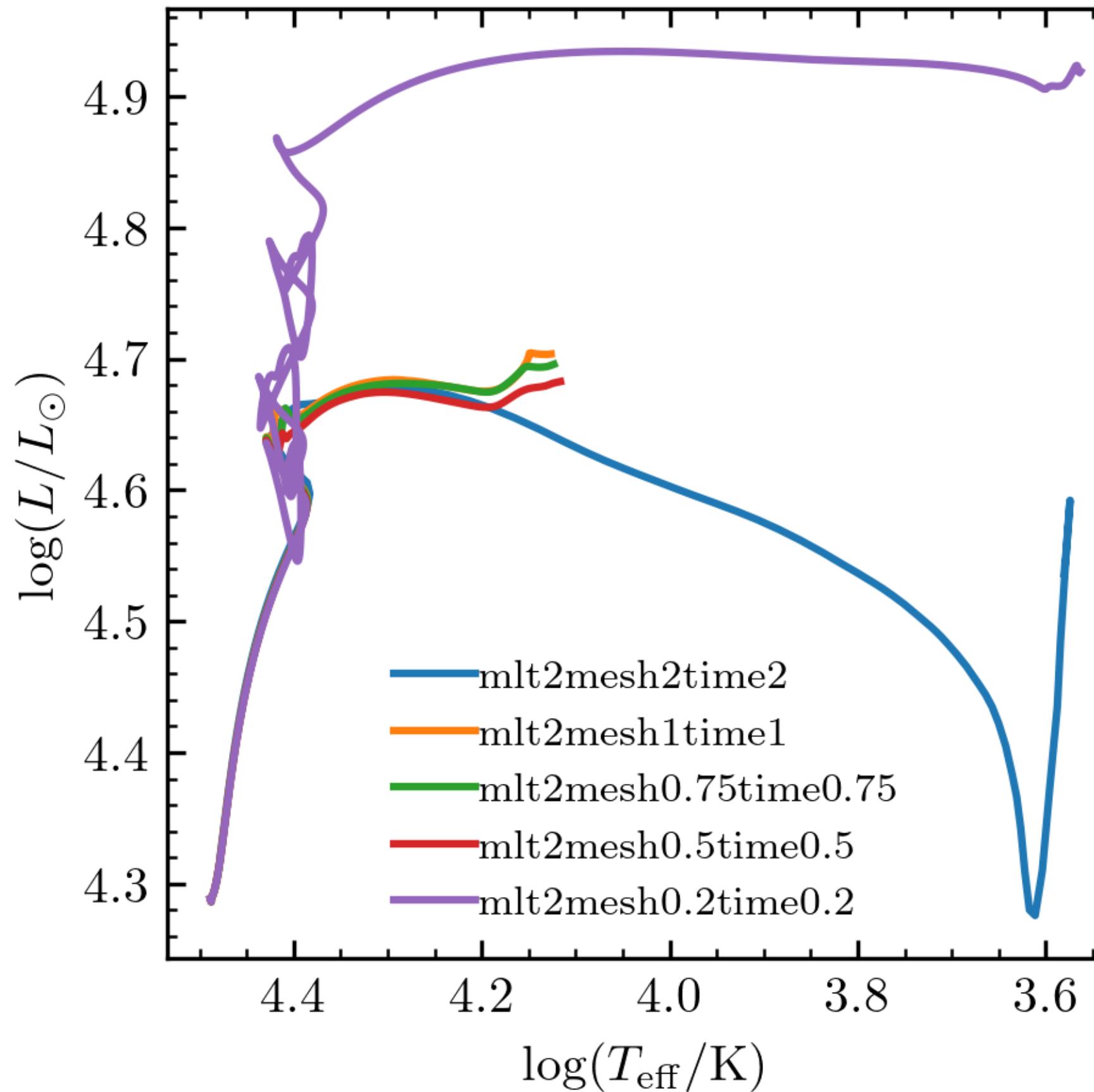
WHAT HAPPENED?

MINI-MINILAB1: RECAP



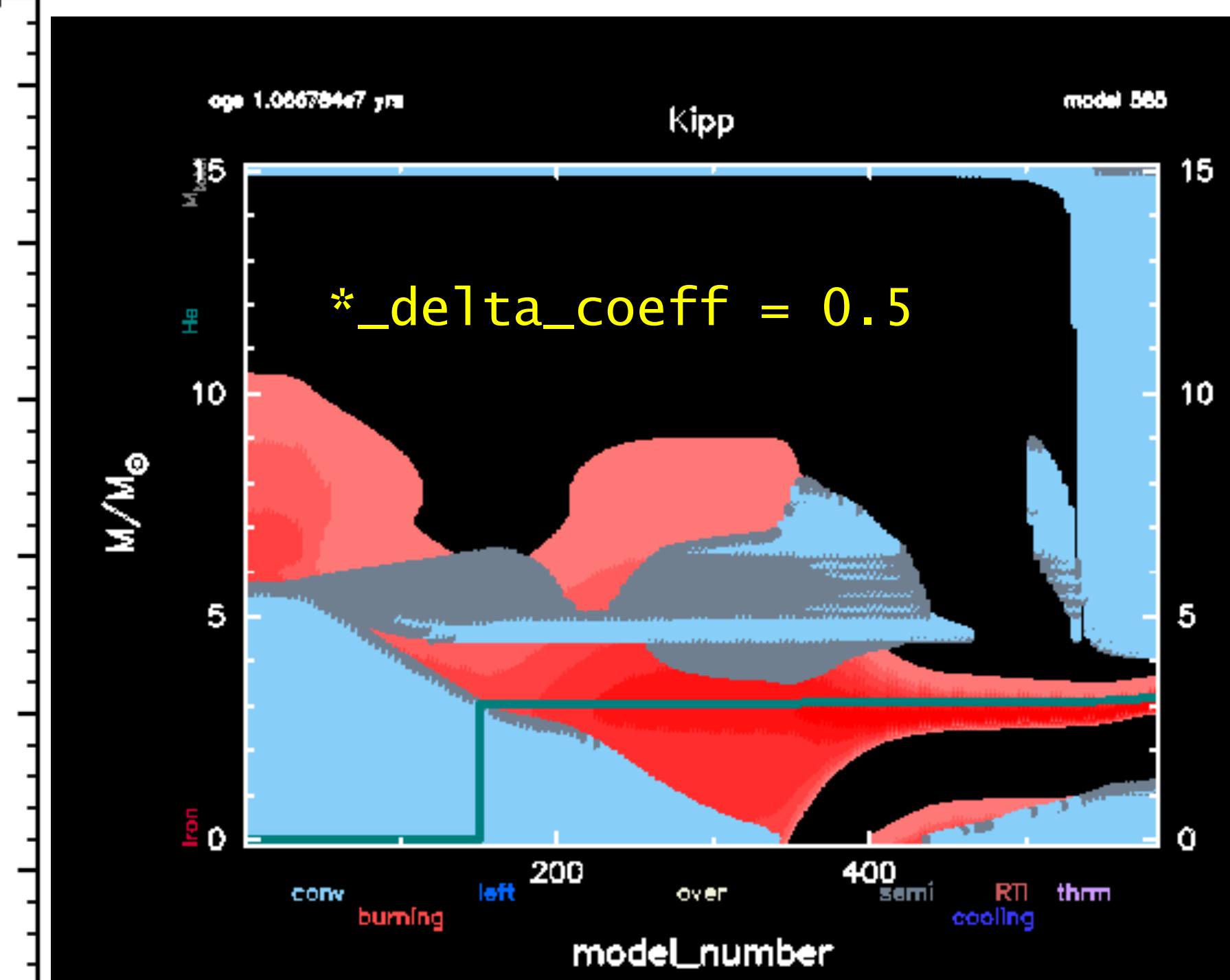
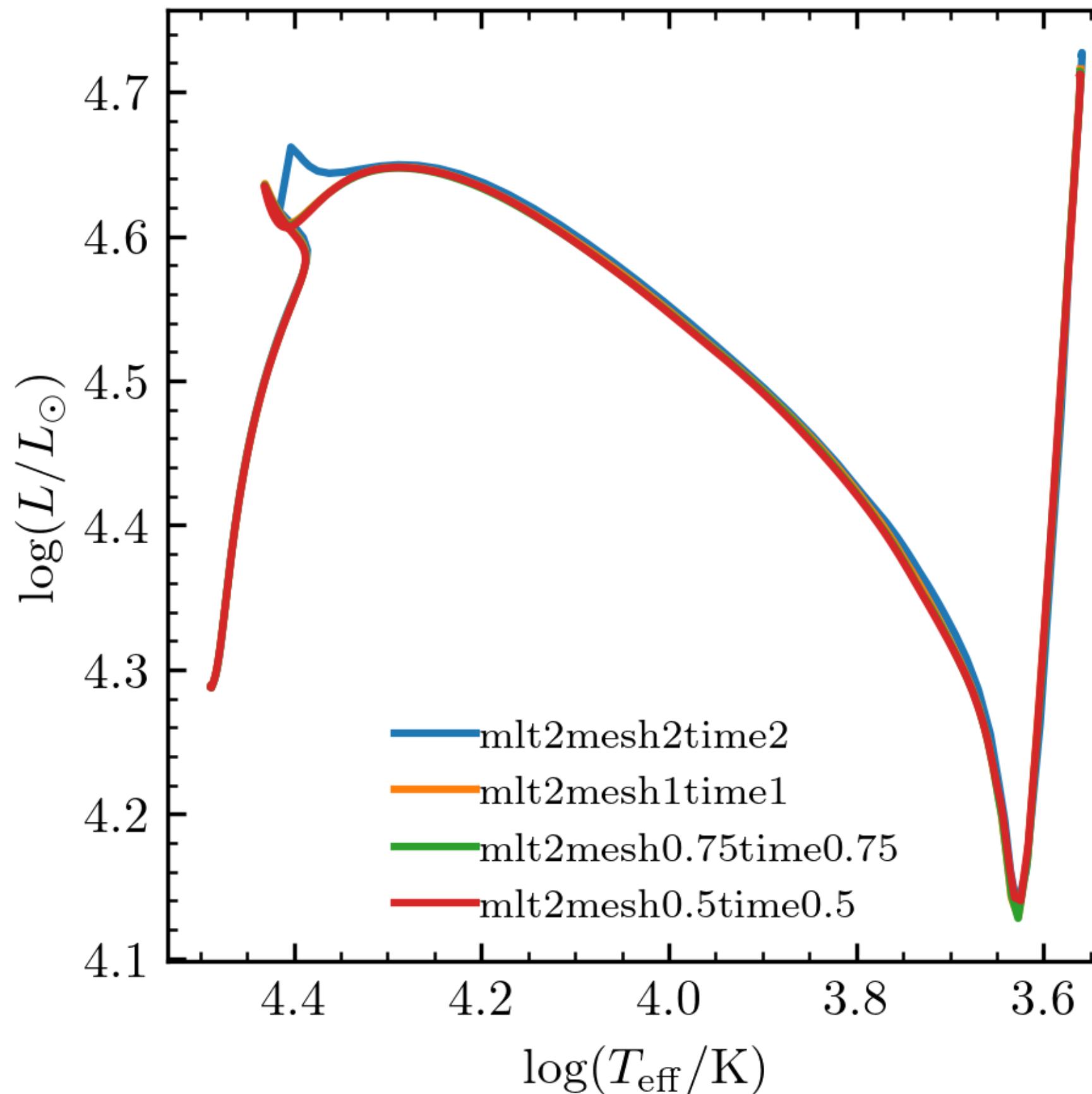
HOME EXERCISE: GO TO EVEN HIGHER RESOLUTION

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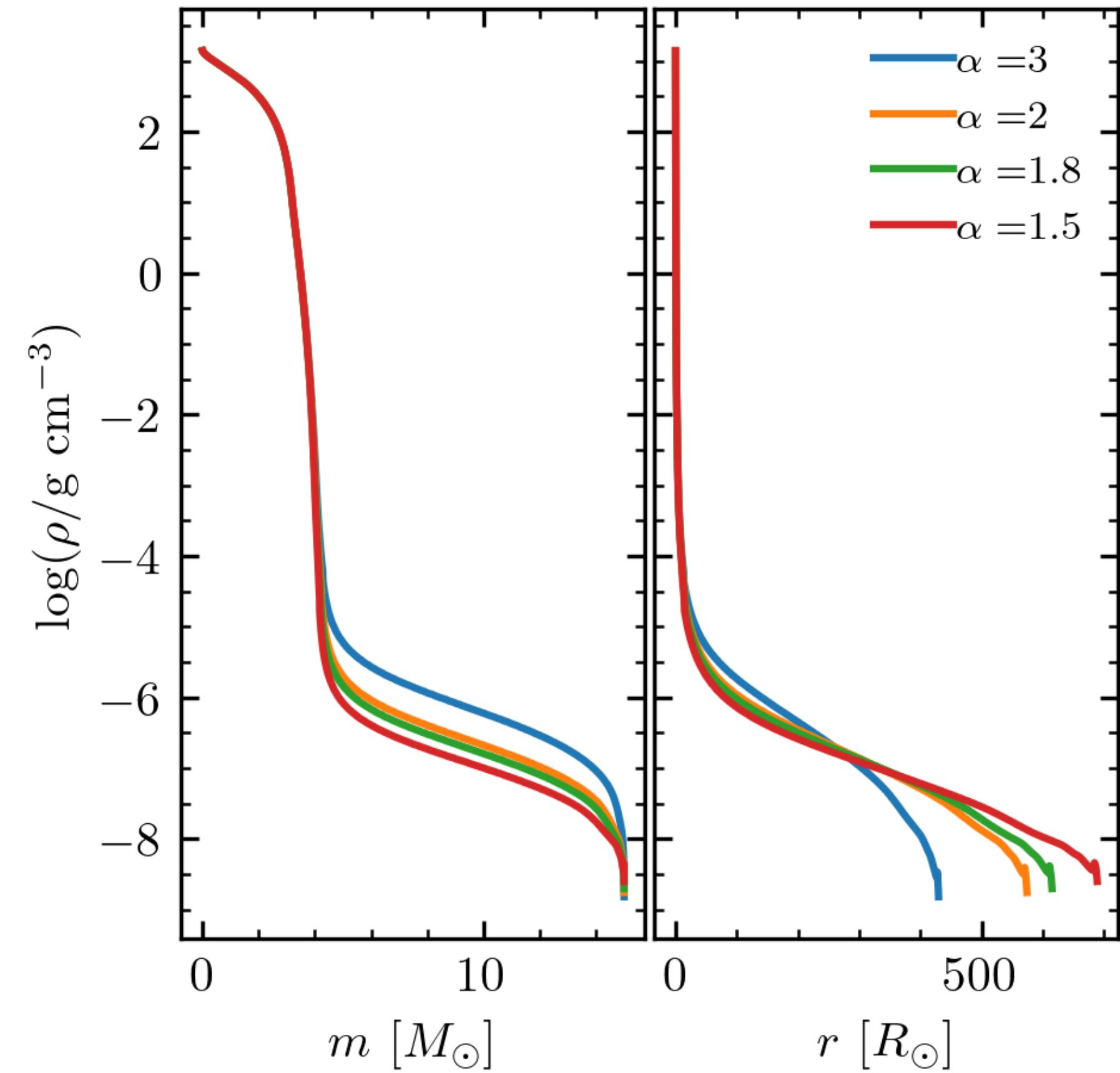
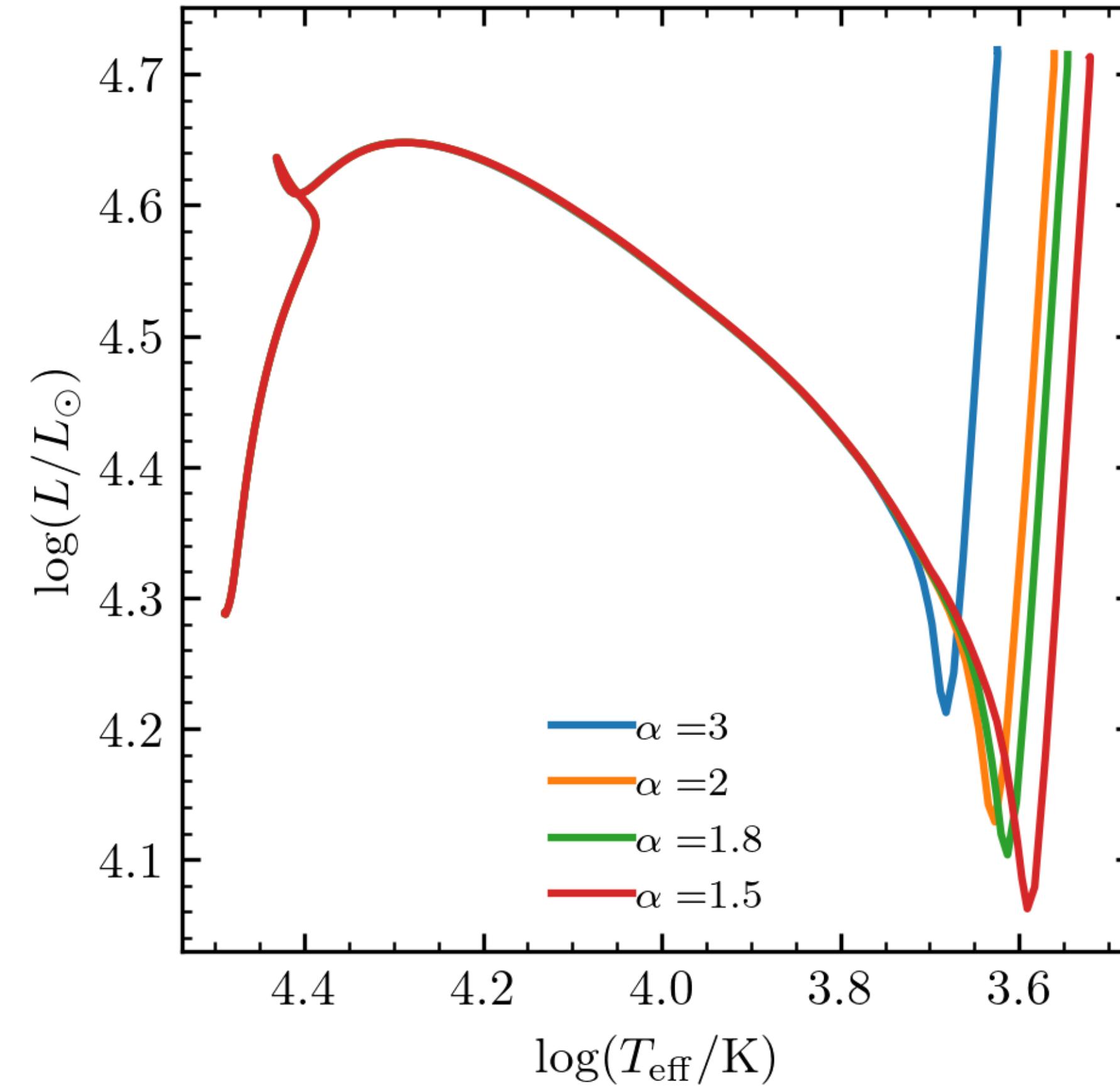


MINI-MINILAB2: RECAP

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MINI-MINILAB3: RECAP



WHY IS THIS SO IMPORTANT?

- ▶ **We want to know that we are solving the equations we think we are solving!**
- ▶ Making your setup as robust as possible isn't just a good practice, it's necessary to make sure you're doing science!
- ▶ Be wary of "magic resolution" - A marginally stable setup for one model likely will not give you consistent results for another model
- ▶ If you adapt your MESA setup from a very old revision, *do not* assume the resolution testing from the old revision will be valid today!
- ▶ **Understanding the physics, understanding the "engineering," and understanding the numerics all go hand-in-hand**
- ▶ **Test, think, explore, and test again!**