Para una línea recta, χ^2 tiene la siguiente forma

$$\chi^2 = \sum_{i=1}^{N} \frac{(y_i - a_0 - a_1 x_i)^2}{\sigma_i^2}$$

Minimizando

$$\frac{\partial \chi^2}{\partial a_0} = -2 \sum \frac{(y_i - a_0 - a_1 x_i)}{\sigma_i^2} = 0$$

$$\frac{\partial \chi^2}{\partial a_1} = -2 \sum \frac{x_i (y_i - a_0 - a_1 x_i)}{\sigma_i^2} = 0$$

$$\sum \frac{y_i}{\sigma_i^2} = \left(\sum \frac{1}{\sigma_i^2} \right) a_0 + \left(\sum \frac{x_i}{\sigma_i^2} \right) a_1$$

$$\sum \frac{x_i y_i}{\sigma_i^2} = \left(\sum \frac{x_i}{\sigma_i^2} \right) a_0 + \left(\sum \frac{x_i^2}{\sigma_i^2} \right) a_1$$

tenemos un sistema de dos ecuaciones

$$\begin{bmatrix} \sum \frac{y_i}{\sigma_i^2} \\ \sum \frac{x_i y_i}{\sigma_i^2} \end{bmatrix} = \begin{bmatrix} \sum \frac{1}{\sigma_i^2} & \sum \frac{x_i}{\sigma_i^2} \\ \sum \frac{x_i}{\sigma_i^2} & \sum \frac{x_i^2}{\sigma_i^2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Renombrando

$$\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

Obteniendo el inverso

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \sum \frac{x_i^2}{\sigma_i^2} & -\sum \frac{x_i}{\sigma_i^2} \\ -\sum \frac{x_i}{\sigma_i^2} & \sum \frac{1}{\sigma_i^2} \end{bmatrix} \begin{bmatrix} \sum \frac{y_i}{\sigma_i^2} \\ \sum \frac{x_i y_i}{\sigma_i^2} \end{bmatrix}$$

$$\Delta = \left(\sum \frac{1}{\sigma_i^2} \right)\left(\sum \frac{x_i^2}{\sigma_i^2} \right) - \left(\sum \frac{x_i}{\sigma_i^2} \right)^2$$

Tomando $\sigma_i = \sigma$, Encontramos las soluciones para minimos cuadrados

$$\Delta = N \sum x^2 - \left(\sum x \right)^2$$

$$a_0 = \frac{\sum y \sum x^2 - \sum xy \sum x}{\Delta}$$

$$a_1 = \frac{N \sum xy - \sum x \sum y}{\Delta}$$