Program Analysis : Assignment Two

Q1.

a)

Procedure: `FindSumAndProduct(A)`

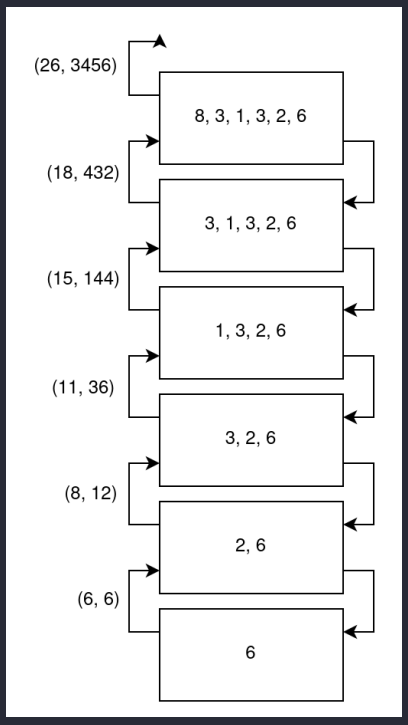
Input: A sequence of real values `A[1...n]`

Output: Where no element in A is equal to 0, a pair $(sum, product)$ containing the sum and product values of the elements in A. Where an element in A is equal to zero, the output should be the pair $(null, null)$

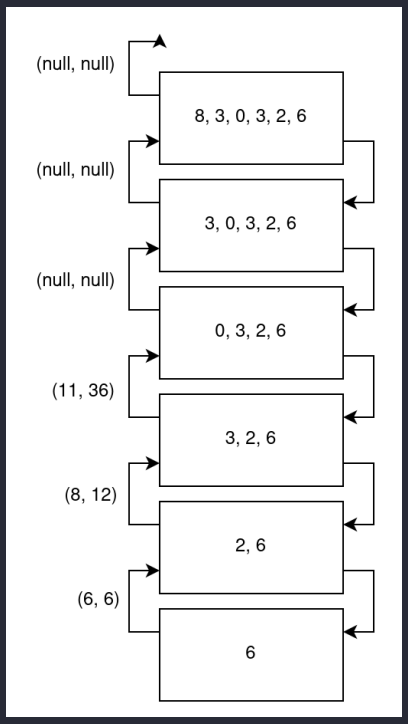
```
current = A[0]

if n == 1 then
    return (current, current)

else
    m = n - 1
    pair = FindSumAndProduct(A[1...m])

    if current == 0 or pair == (null, null) then
        return (null, null)
    end if

    return (pair[0] + current, pair[1] * current)
end if
```

b)

Sequence = (8, 3, 1, 3, 2, 6)



Sequence = (8, 3, 0, 3, 2, 6)



c)

$$T(n) = \begin{cases} T(n-1) + c_1 \; if \; n > 1 \\ c_2 \; otherwise \end{cases}$$

d)

$$T(n) = T(n-1) + c_1$$
$$T(1) = c_2$$

$$T(n-1) = T(n-2) + c_1$$
$$T(n) = T(n-2) + 2c_1$$

$$T(n-2) = T(n-3) + c_1$$
$$T(n) = T(n-3) + 3c_1$$

k iterations

$$T(n) = T(n-k) + kc_1$$

$$k = n - 1$$
$$T(n) = T(n-(n-1)) + c_1(n-1)$$
$$= T(1) + c_1(n-1)$$
$$= c_2 + c_1(n-1)$$

Q2.

$$T(n) = T(n-1) + 2(n-1) + c_1$$

$$T(2) = c_2$$

$$T(n-1) = T(n-2) + 2(n-2) + c_1$$

$$T(n) = T(n-2) + 2(n-2) + 2(n-1) + 2c_1$$

$$= T(n-2) + 2(n-2+n-1) + 2c_1$$

$$T(n-2) = T(n-3) + 2(n-3) + c_1$$

$$T(n) = T(n-3) + 2(n-3) + 2(n-2) + 2(n-1) + 3c_1$$

$$= T(n-3) + 2(n-3+n-2+n-1) + 3c_1$$

k iterations:

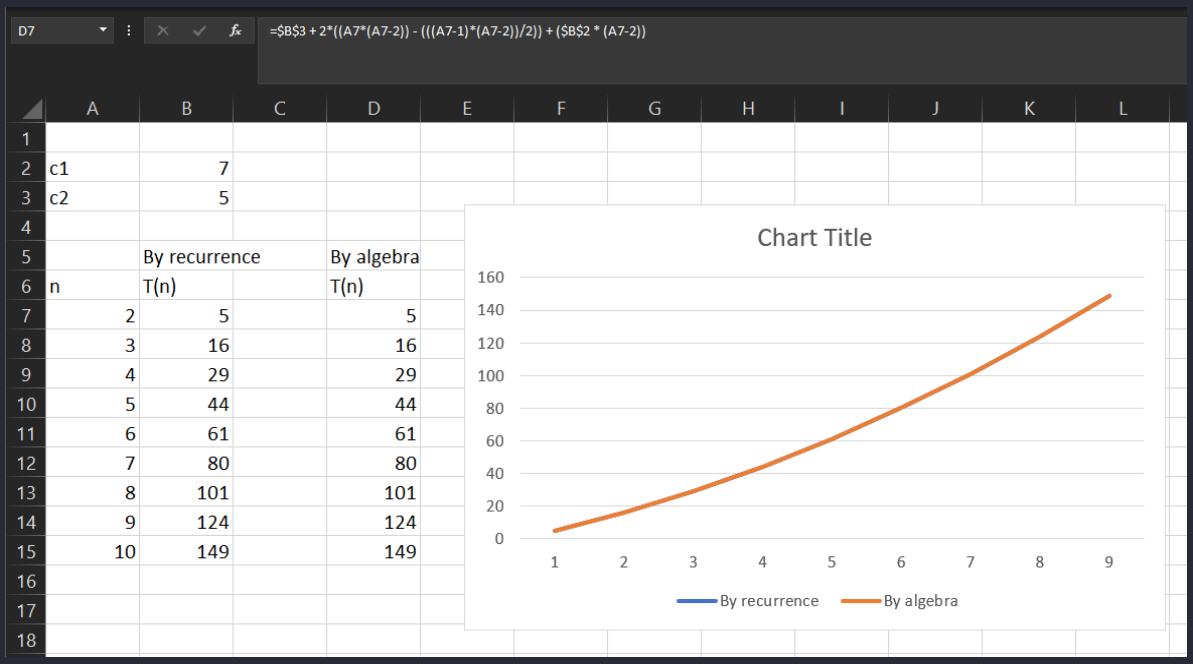
$$T(n) = T(n-k) + 2\left(kn - \sum_{1}^{i=k} i\right) + kc_1$$

$$k = n-2$$

$$T(n) = T(n-(n-2)) + 2\left(n(n-2) - \sum_{1}^{i=n-2} i\right) + c_1(n-2)$$

$$= T(2) + 2\left(n(n-2) - \frac{(n-1)(n-2)}{2}\right) + c_1(n-2)$$

$$= c_2 + 2\left(n(n-2) - \frac{(n-1)(n-2)}{2}\right) + c_1(n-2)$$



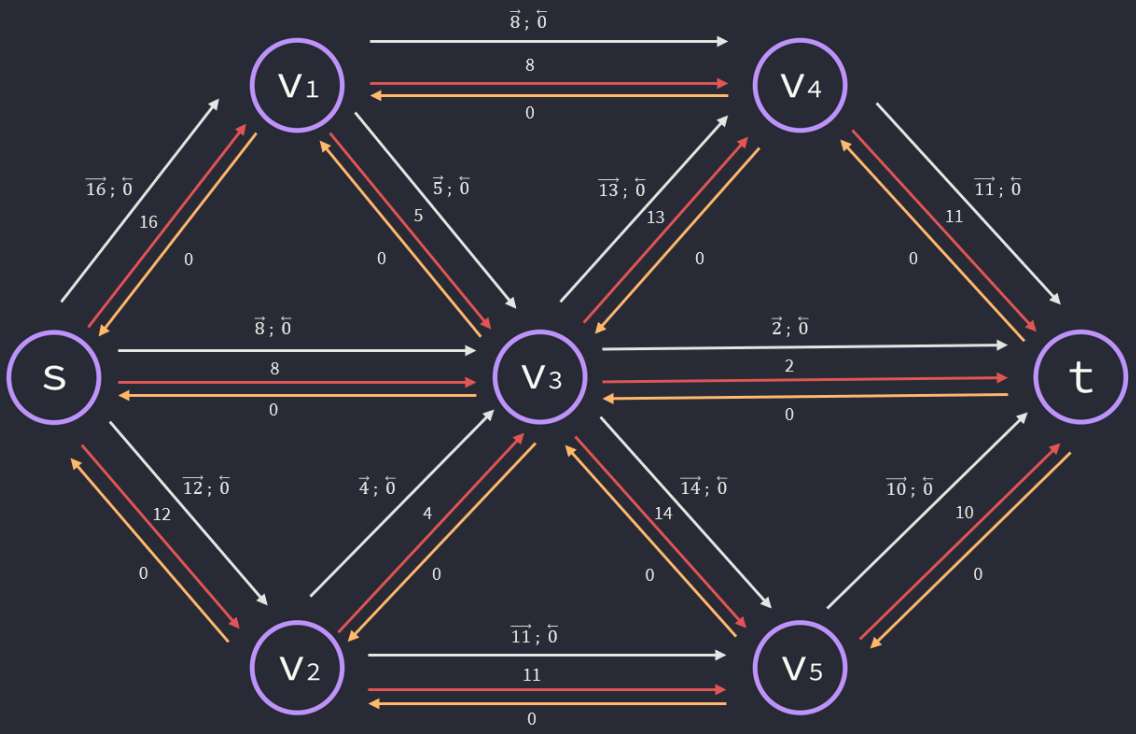
	A	B	C	D
1				
2	c1	7		
3	c2	5		
4				
5		By recurrence		By algebra
6	n	T(n)		T(n)
7	2	5		5
8	3	16		16
9	4	29		29
10	5	44		44
11	6	61		61
12	7	80		80
13	8	101		101
14	9	124		124
15	10	149		149
16				
17				
18				

Q3.

		w												
		0	1	2	3	4	5	6	7	8	9	10	11	12	13
i	6	0	1	2	3	4	5	6	7	8	9	10	11	12	13
	5	0	1	2	3	4	5	6	7	8	9	10	11	12	12
	4	0	1	2	3	4	5	6	7	8	9	10	11	11	11
	3	0	1	2	3	4	5	6	6	6	6	6	6	6	6
	2	0	1	2	3	3	3	3	3	3	3	3	3	3	3
	1	0	0	2	2	2	2	2	2	2	2	2	2	2	2
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Q4.

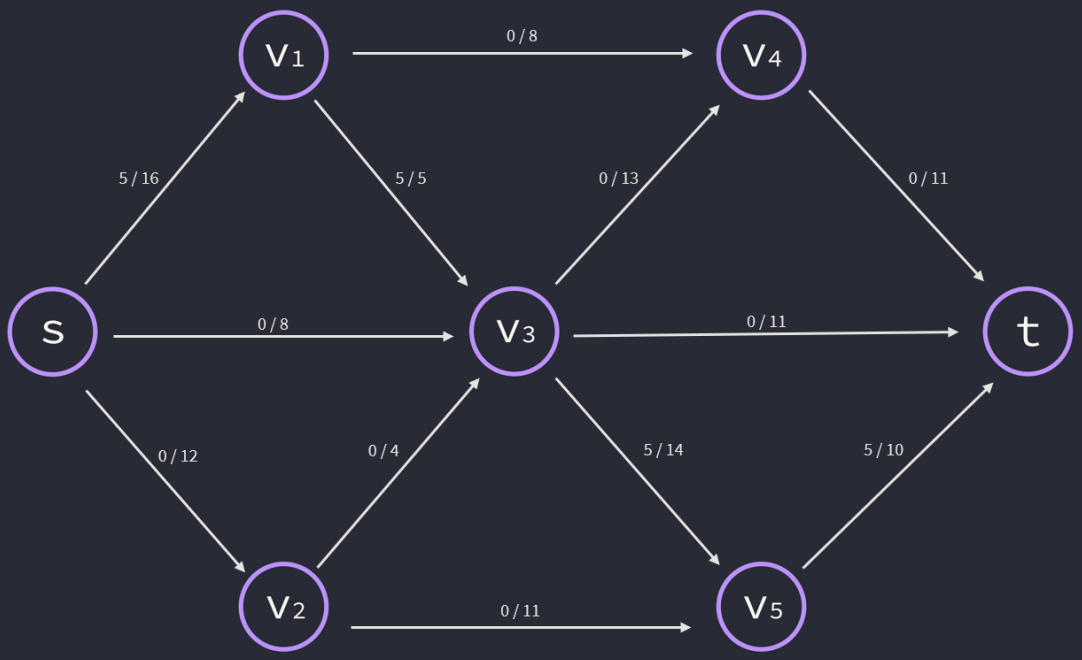
a)



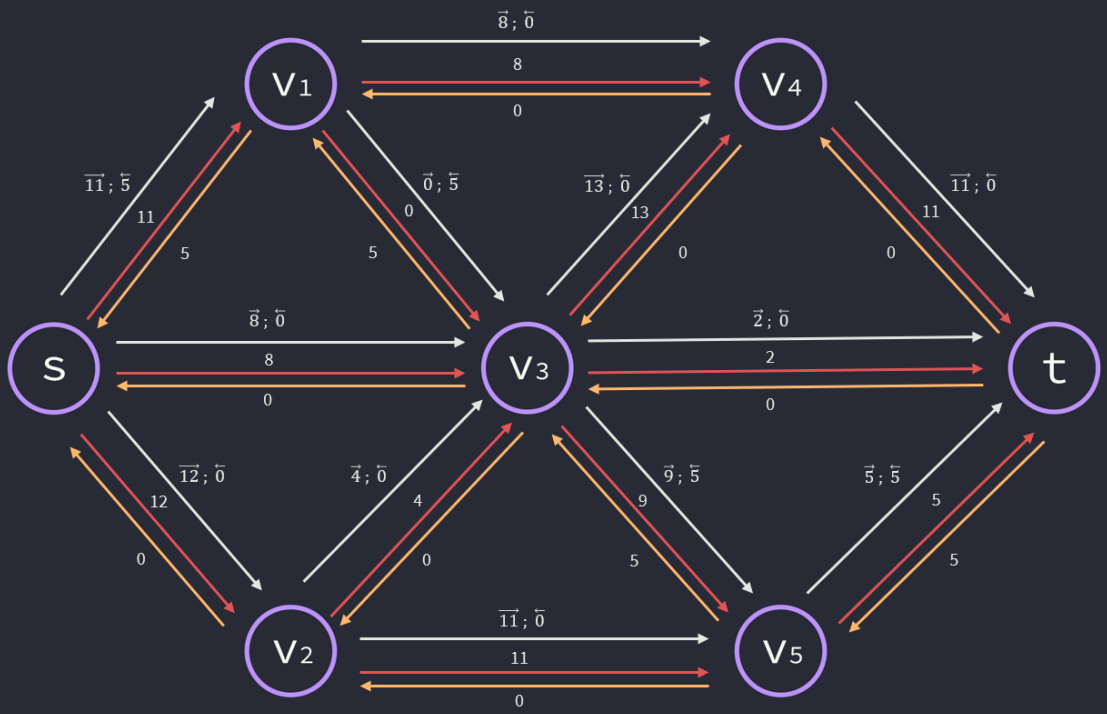
b)

(v_1, v_3)

c)


v1
v4
s
v3
t
v2
v5
0 / 8
5 / 16
5 / 5
0 / 13
0 / 11
0 / 8
0 / 11
0 / 12
0 / 4
5 / 14
5 / 10
0 / 11

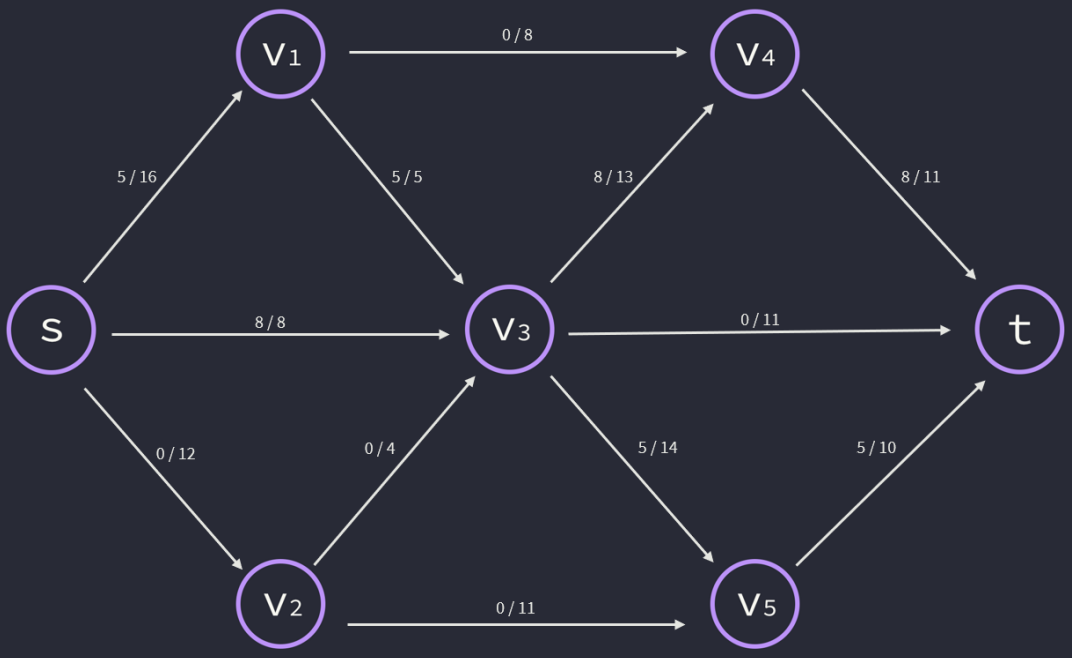
d)


v1
v4
s
v3
t
v2
v5
8 ; 0
8
0
11 ; 5
11
5
0 ; 5
0
5
13 ; 0
13
0
11 ; 0
11
0
8 ; 0
8
0
2 ; 0
2
0
12 ; 0
12
0
4 ; 0
4
0
9 ; 5
9
5
5 ; 5
5
5
11 ; 0
11
0

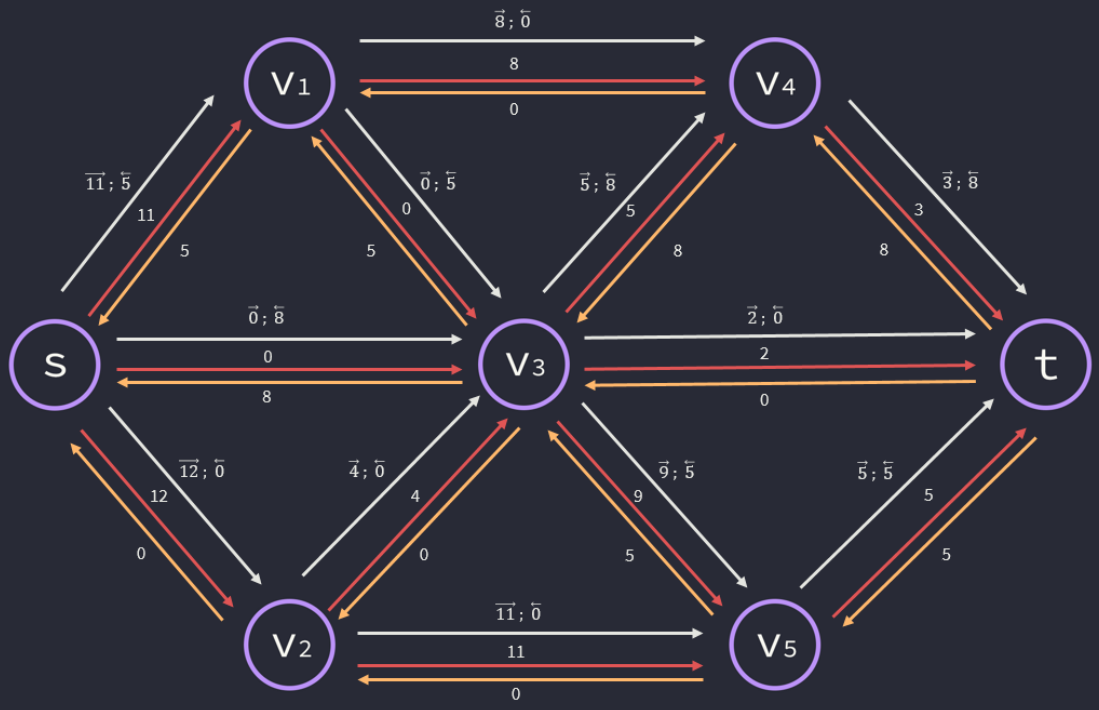
e)

(s, v_3)

f)


v1
v4
s
v3
t
v2
v5
0 / 8
5 / 16
5 / 5
8 / 13
8 / 11
8 / 8
0 / 11
0 / 12
0 / 4
5 / 14
5 / 10
0 / 11

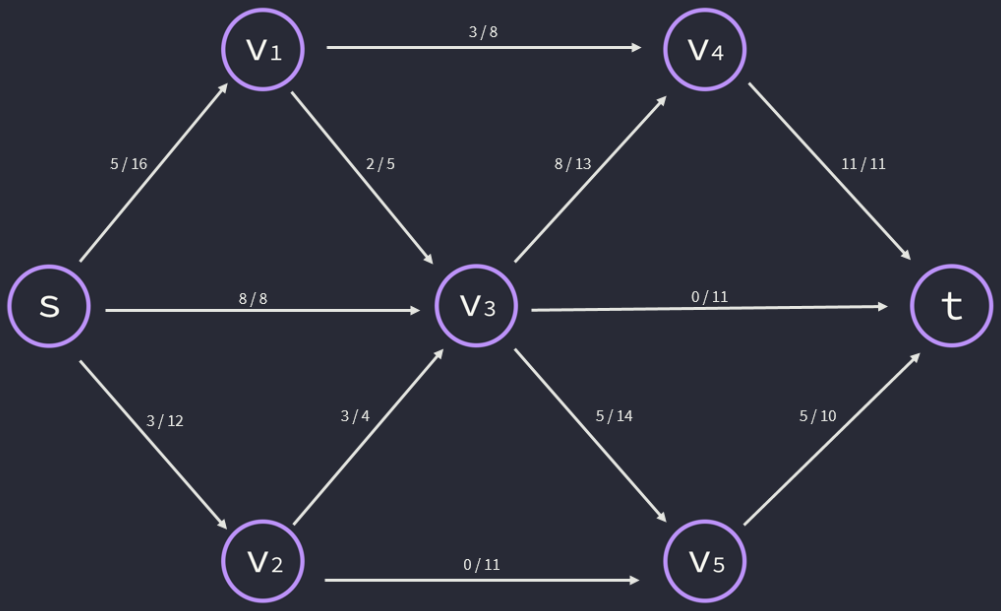
g)


v1
v4
s
v3
t
v2
v5
8 ; 0
8
0
11 ; 5
11
5
0 ; 5
0
5
5 ; 8
5
8
3 ; 8
3
8
0 ; 8
0
8
2 ; 0
2
0
12 ; 0
12
0
4 ; 0
4
0
9 ; 5
9
5
5 ; 5
5
5
11 ; 0
11
0

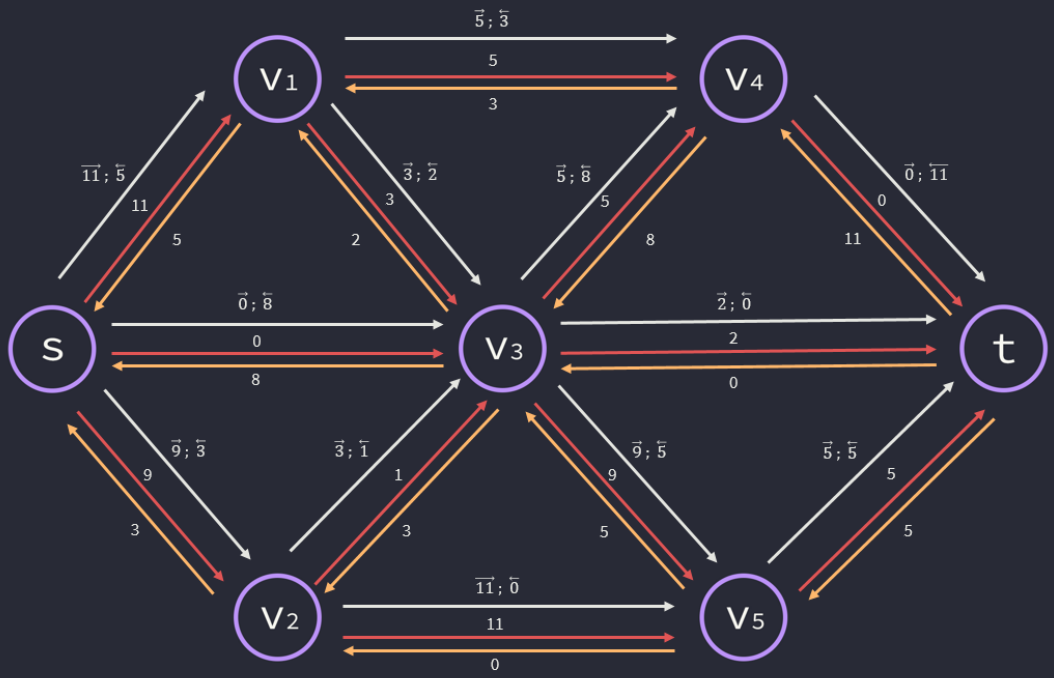
h)

(v_4, t)

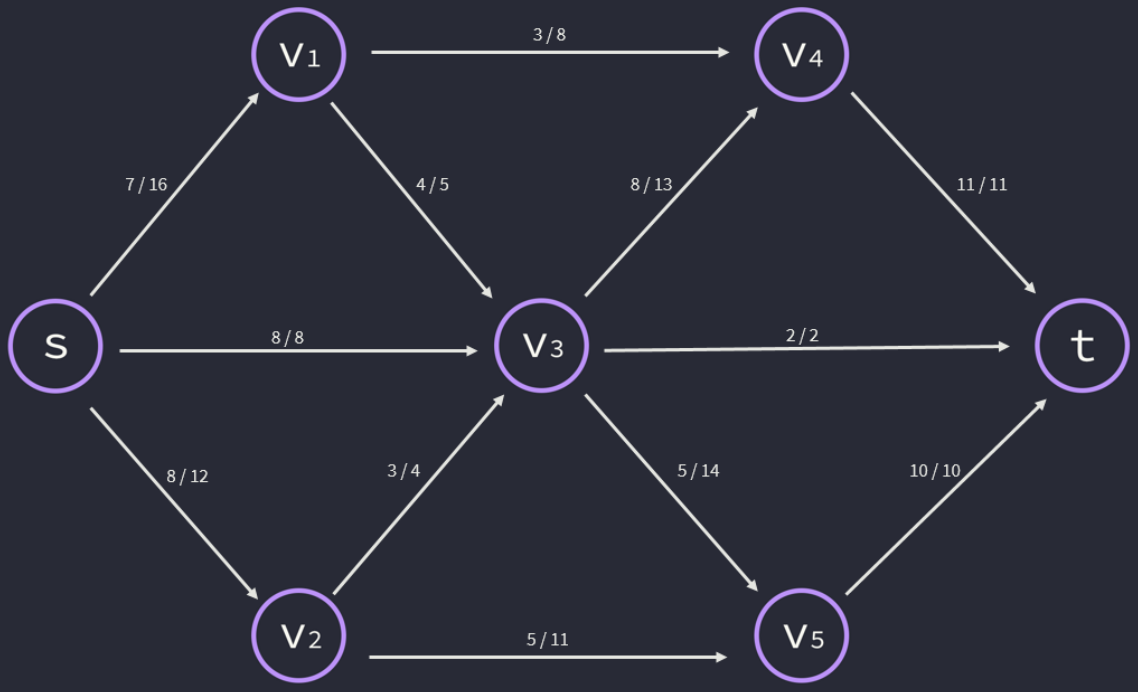
i)

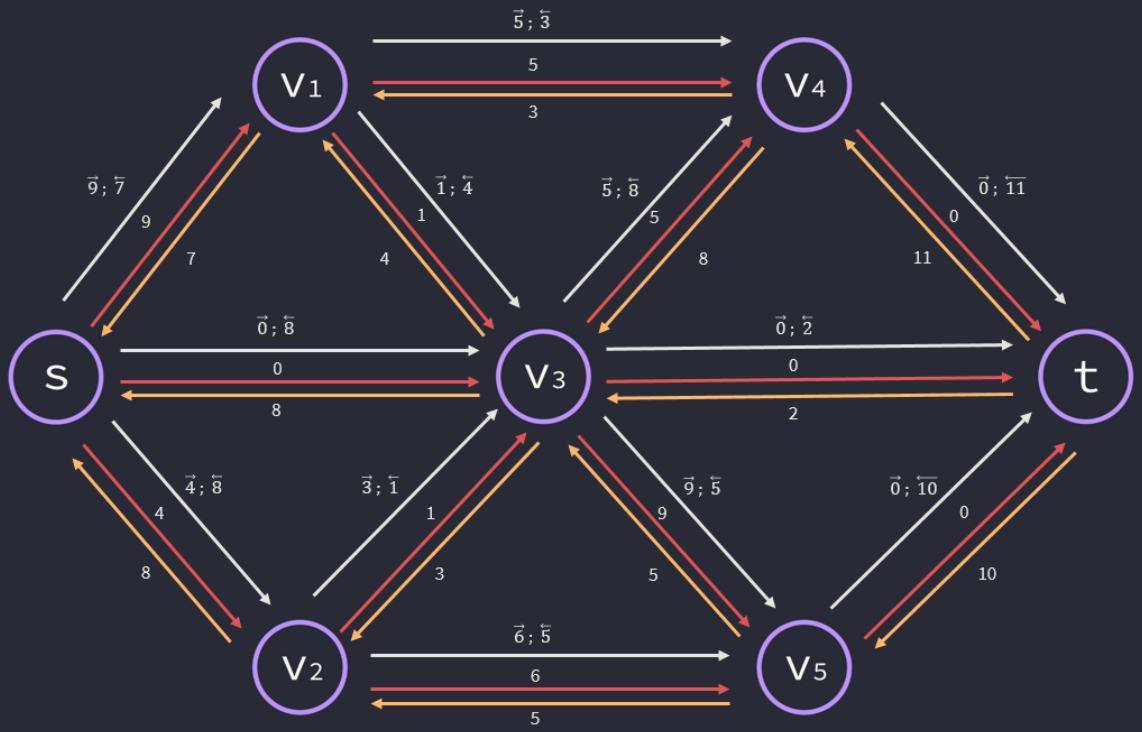
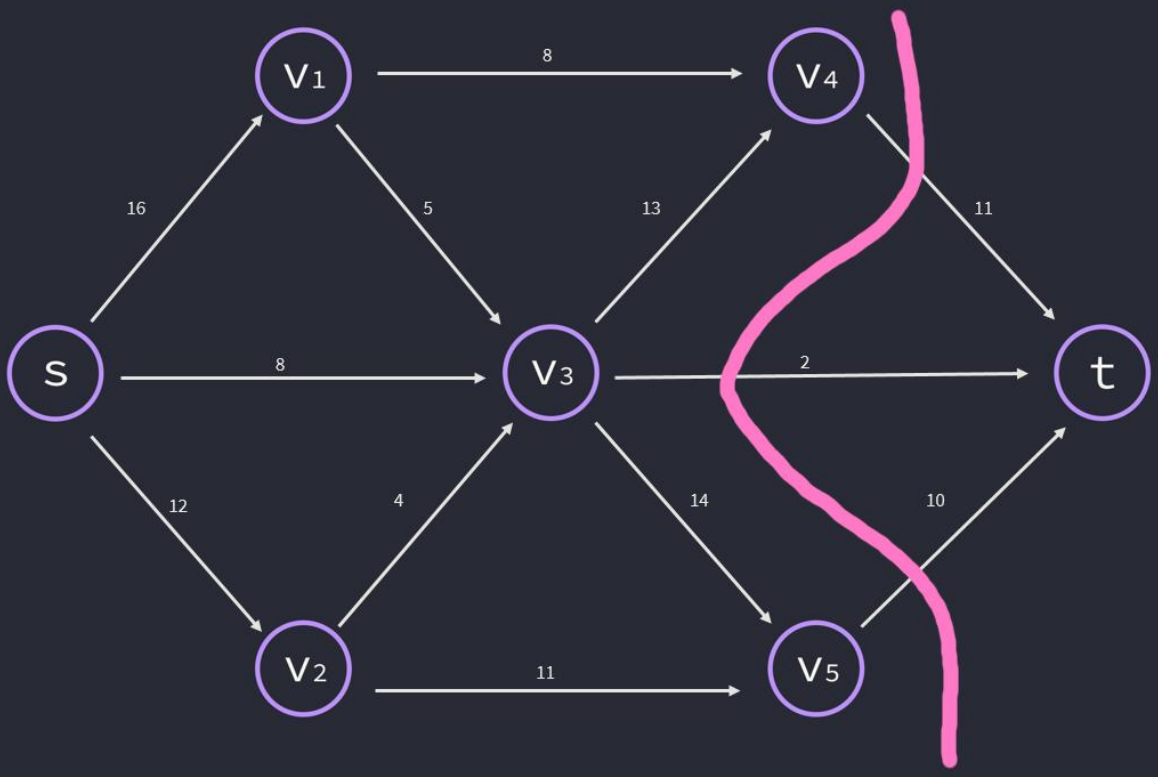

v1
v4
s
v3
t
v2
v5
3 / 8
5 / 16
2 / 5
8 / 13
11 / 11
8 / 8
0 / 11
3 / 12
3 / 4
5 / 14
5 / 10
0 / 11

j)


v1
v4
s
v3
t
v2
v5
5 ; 3
5
3
11 ; 5
11
5
3 ; 2
3
2
5 ; 8
5
8
0 ; 11
0
11
0 ; 8
0
8
2 ; 0
2
0
9 ; 3
9
3
3 ; 1
1
3
9 ; 5
9
5
5 ; 5
5
5
11 ; 0
11
0

k)


v1
v4
s
v3
t
v2
v5
3 / 8
7 / 16
4 / 5
8 / 13
11 / 11
8 / 8
2 / 2
8 / 12
3 / 4
5 / 14
10 / 10
5 / 11


v1
v4
s
v3
t
v2
v5
5 ; 3
5
3
9 ; 7
9
7
1 ; 4
1
4
5 ; 8
5
8
0 ; 11
0
11
0 ; 8
0
8
0 ; 2
0
2
4 ; 8
4
8
3 ; 1
1
3
9 ; 5
9
5
0 ; 10
0
10
6 ; 5
6
5

i)