# Project presentation

## Course: S3-D-CODE Calculation Code

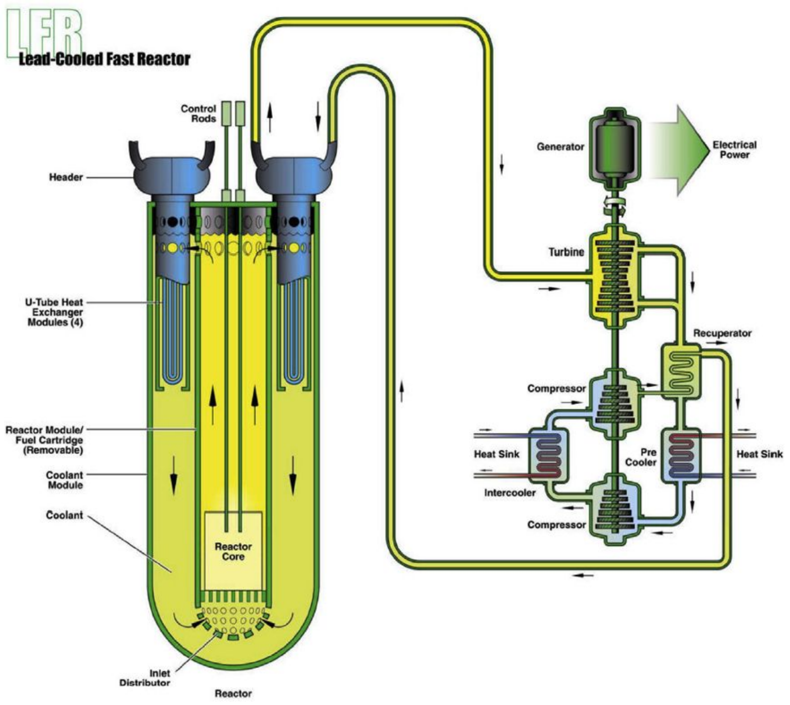
Student: Aurora Jahan

# Project topic

Simulating a Lead cooled fast reactor (LFR) using a lumped parameter coupled neutronics and thermal hydraulics model along with a PID controller

# Reactor description

- **Fast-neutron reactor** cooled by **liquid lead**
- Based on the **European Lead-cooled System (ELSY) DEMO** design
- **24** ductless **fuel assemblies**
- **28×28 square lattice.**
- **Primary circuit lead:** pool > core > hot collector > 8 parallel pipes > 8 pump + SG modules (containing HXs) > pool
- **Secondary circuit water/steam:** pump + SG module > turbine > recuperator (deposits energy) > precooler > compressor 1 > intercooler > compressor 2 > recuperator (gains energy) > pump + SG module




Schematic layout of the LFR power plant (https://www.gen-4.org/)

# Reactor description

Nominal parameters employed in this work for the LFR (Bortot et al., 2011).

| Parameter | Value | Units | Parameter | Value | Units |
|---|---|---|---|---|---|
| Thermal-hydraulics | | | Neutronics (BoC/EoC) | | |
| Core thermal power, $\dot{Q}_0$ | 300 | MW | $\beta$ | 319/323 | pcm |
| Fuel average temperature, $T_f$ | 900 | °C | $\beta_1$ | 6.142/6.224 | pcm |
| Core coolant inlet temperature, $T_{c\ in}$ | 400 | °C | $\beta_2$ | 71.40/72.33 | pcm |
| Core coolant outlet temperature, $T_{c\ out}$ | 480 | °C | $\beta_3$ | 34.86/35.34 | pcm |
| Core coolant mass flow rate, $\Gamma_c$ | 25757 | kg $s^{-1}$ | $\beta_4$ | 114.1/115.5 | pcm |
| Total number of fuel assemblies | 24 | | $\beta_5$ | 69.92/70.75 | pcm |
| Pins per fuel assembly | 744 | | $\beta_6$ | 22.68/22.89 | pcm |
| Total core fuel mass, $m_f$ | 2132 | kg | $\lambda_1$ | 0.0125/0.0125 | $s^{-1}$ |
| Total core coolant mass, $m_c$ | 5429 | kg | $\lambda_2$ | 0.0292/0.0292 | $s^{-1}$ |
| Fuel specific heat, $c_f$ | 376 | J $kg^{-1}$ $K^{-1}$ | $\lambda_3$ | 0.0895/0.0895 | $s^{-1}$ |
| Primary coolant specific heat, $c_c$ | 146 | J $kg^{-1}$ $K^{-1}$ | $\lambda_4$ | 0.2575/0.2573 | $s^{-1}$ |
| SG lead mass, $m_{SG}$ | 25757 | kg | $\lambda_5$ | 0.6037/0.6025 | $s^{-1}$ |
| Hot leg time constant, $\tau_{HL}$ | 5.17 | s | $\lambda_6$ | 2.6688/2.6661 | $s^{-1}$ |
| Cold leg time constant, $\tau_{CL}$ | 67.5 | s | $\Lambda$ | 0.8066/0.8498 | µs |
| SG saturated water temperature, $T_{sat}$ | 357 | °C | $\alpha_D$ | −0.15/-0.17 | pcm $K^{-1}$ |
| | | | $\alpha_C$ | −1.2267/-1.995 | pcm $K^{-1}$ |
| | | | $\alpha_A$ | −0.0429/-0.2374 | pcm $K^{-1}$ |
| | | | $\alpha_R$ | −0.7741/-0.7144 | pcm $K^{-1}$ |

# The 6 group non-linear model

$$\frac{d\psi(t)}{dt} = \frac{(\rho - \beta)}{\Lambda}\psi(t) + \sum_{i=1}^{6} \frac{\beta_i}{\Lambda}\eta_i(t)$$

$$\frac{d\eta_i(t)}{dt} = \lambda_i\psi(t) - \lambda_i\eta_i(t) \ ; \ i = 1, ..., 6$$

$$\rho = \alpha_h h + (\alpha_D + \alpha_A)(T_f - T_f^0) + (\alpha_C + \alpha_R)(T_c - T_c^0)$$

$$\frac{dT_f}{dt} = \frac{P^0}{K\tau_f} - \frac{1}{\tau_f}(T_f - T_c)$$

$$\frac{dT_c}{dt} = \frac{1}{\tau_c}(T_f - T_c) - \frac{2}{\tau_0}(T_c - T_{in})$$

$$T_{out} = 2T_c - T_{in}$$

Where $\psi(t) = \frac{P(t)}{P^0}$, $\eta_i(t) = \frac{C_i(t)}{C_i^0}$, $\tau_f = \frac{M_f C_f}{K}$, $\tau_c = \frac{M_c C_c}{K}$, $\tau_0 = \frac{M_c}{G}$.

# Step 1: Linearization of the model

# The 6 group linear model

$$\frac{d\delta\psi}{dt} = \frac{\beta}{\Lambda}\delta\psi + \sum_{i=1}^{6}\frac{\beta_i}{\Lambda}\delta\eta_i$$

$$\frac{d\delta\eta_i}{dt} = \lambda_i\delta\psi - \lambda_i\delta\eta_i \ ; \ i = 1, ..., 6$$

$$\delta\rho = \alpha_h\delta h + (\alpha_D + \alpha_A)\delta T_f + (\alpha_C + \alpha_R)\delta T_c$$

$$\frac{d\delta T_f}{dt} = \frac{P^0}{K\tau_f} - \frac{\delta T_f}{\tau_f} + \frac{\delta T_c}{\tau_f}$$

$$\frac{d\delta T_c}{dt} = \frac{\delta T_f}{\tau_c} - (\frac{2}{\tau_0} + \frac{1}{\tau_c})\delta T_c + \frac{2}{\tau_0}\delta T_{in}$$

$$\delta T_{out} = 2\delta T_c - \delta T_{in}$$

Where $\psi(t) = \frac{P(t)}{P^0}$, $\eta_i(t) = \frac{C_i(t)}{C_i^0}$, $\tau_f = \frac{M_f C_f}{K}$, $\tau_c = \frac{M_c C_c}{K}$, $\tau_0 = \frac{M_c}{G}$.

# State space representation

$$\underline{\dot{X}} = \underline{\underline{A}}.\underline{X} + \underline{\underline{B}}.\underline{U}$$

$$\underline{Y} = \underline{\underline{C}}.\underline{X} + \underline{\underline{D}}.\underline{U}$$

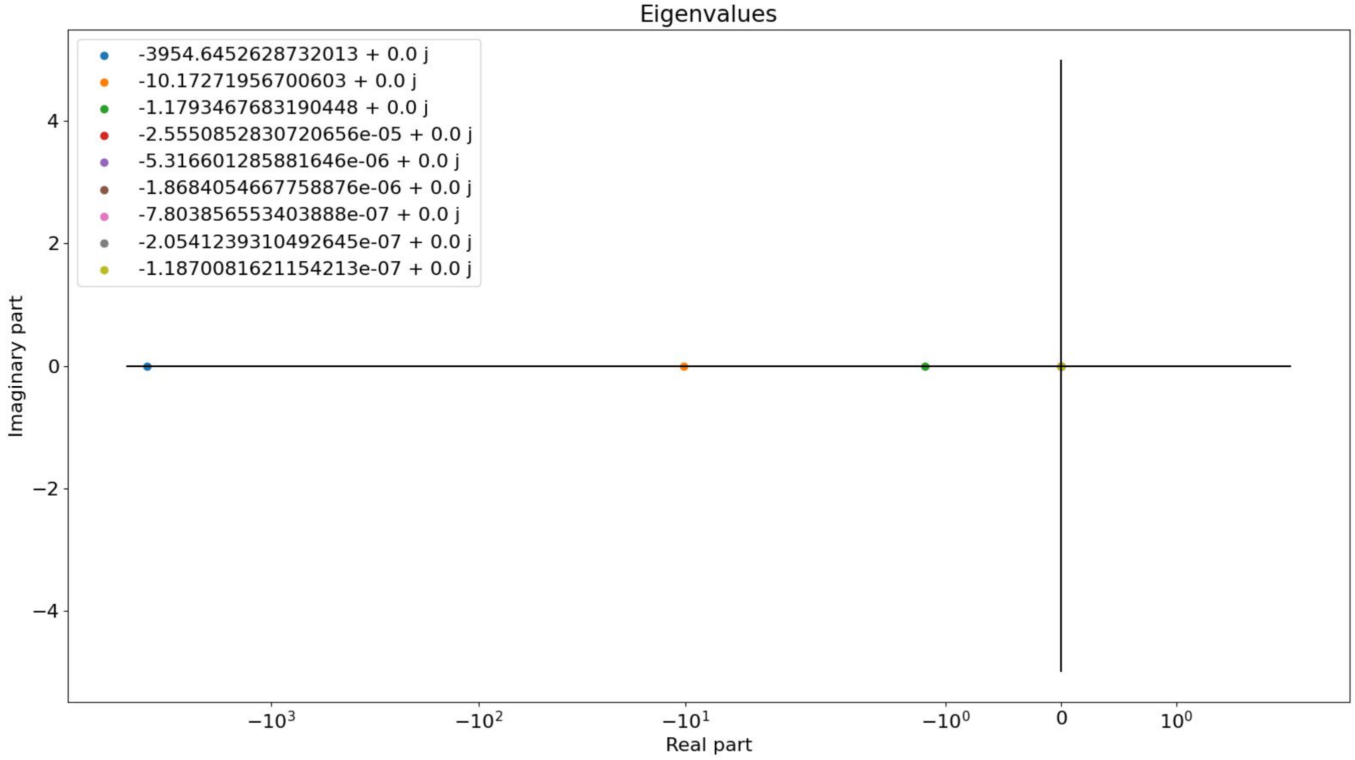
where state variables $X = \begin{pmatrix} \psi \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ T_f \\ T_c \end{pmatrix}$, inputs $U = \begin{pmatrix} h \\ T_{in} \end{pmatrix}$, and outputs $Y = \begin{pmatrix} \psi \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ T_f \\ T_c \\ T_{out} \\ \rho \end{pmatrix}$

# State space representation (cont.)

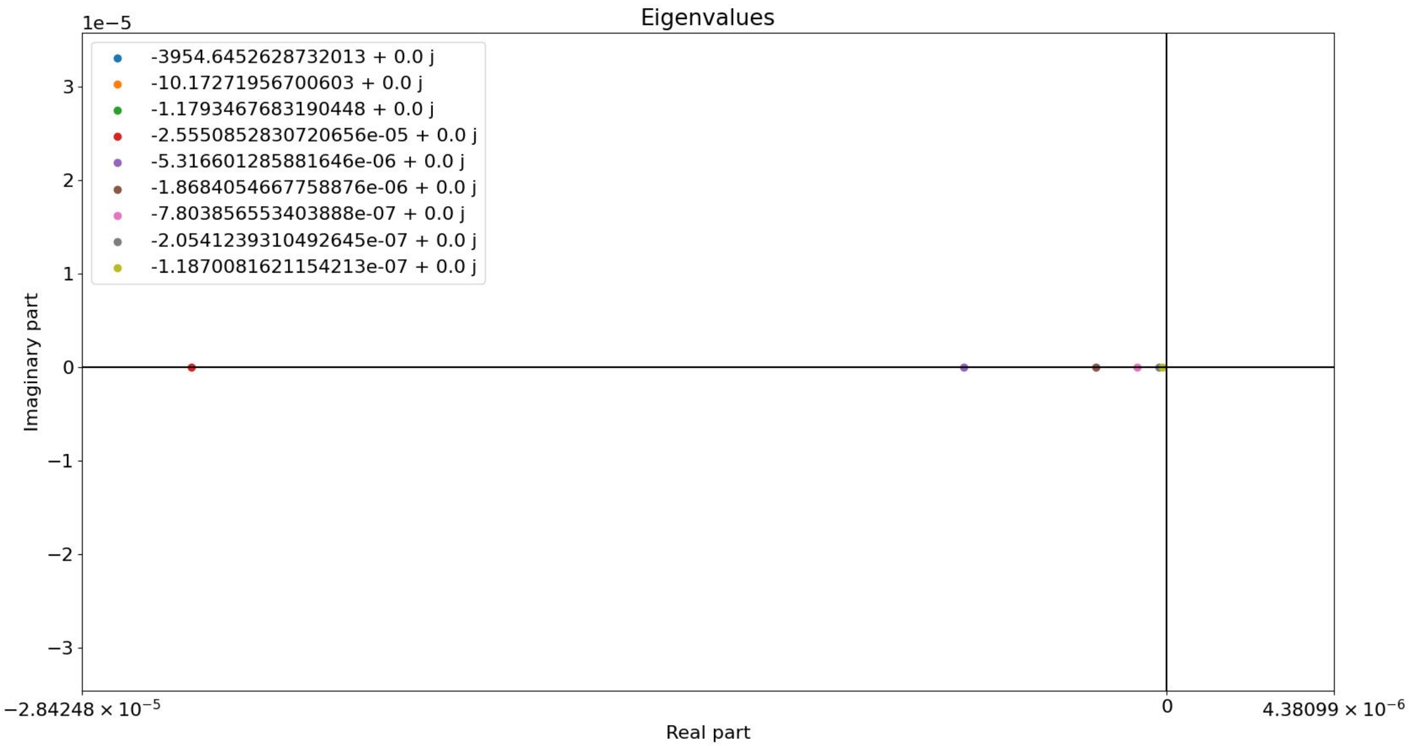
$$A = \begin{pmatrix} -\frac{\beta}{\Lambda} & \frac{\beta_1}{\Lambda} & \frac{\beta_2}{\Lambda} & \frac{\beta_3}{\Lambda} & \frac{\beta_4}{\Lambda} & \frac{\beta_5}{\Lambda} & \frac{\beta_6}{\Lambda} & \frac{\alpha_f}{\Lambda} & \frac{\alpha_c}{\Lambda} \\ \lambda_1 & -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\lambda_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -\lambda_4 & 0 & 0 & 0 & 0 \\ \lambda_5 & 0 & 0 & 0 & 0 & -\lambda_5 & 0 & 0 & 0 \\ \lambda_6 & 0 & 0 & 0 & 0 & 0 & -\lambda_6 & 0 & 0 \\ \frac{P^0}{K\tau_f} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau_f} & \frac{1}{\tau_f} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_c} & -(\frac{2}{\tau_0} + \frac{1}{\tau_c}) \end{pmatrix} \quad B = \begin{pmatrix} \frac{\alpha_h}{\Lambda} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{2}{\tau_0} \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_f & \alpha_c \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \\ \alpha_h & 0 \end{pmatrix}$$

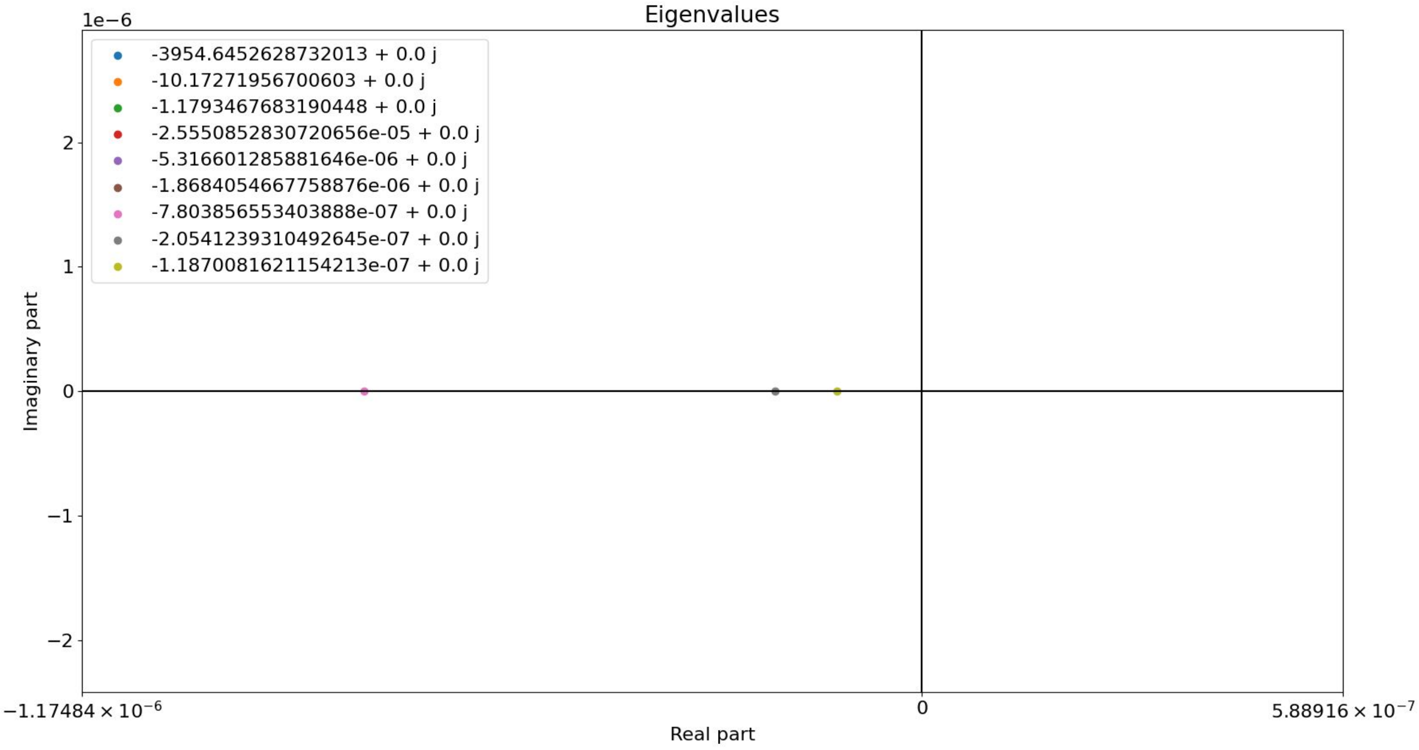
# Eigenvalues



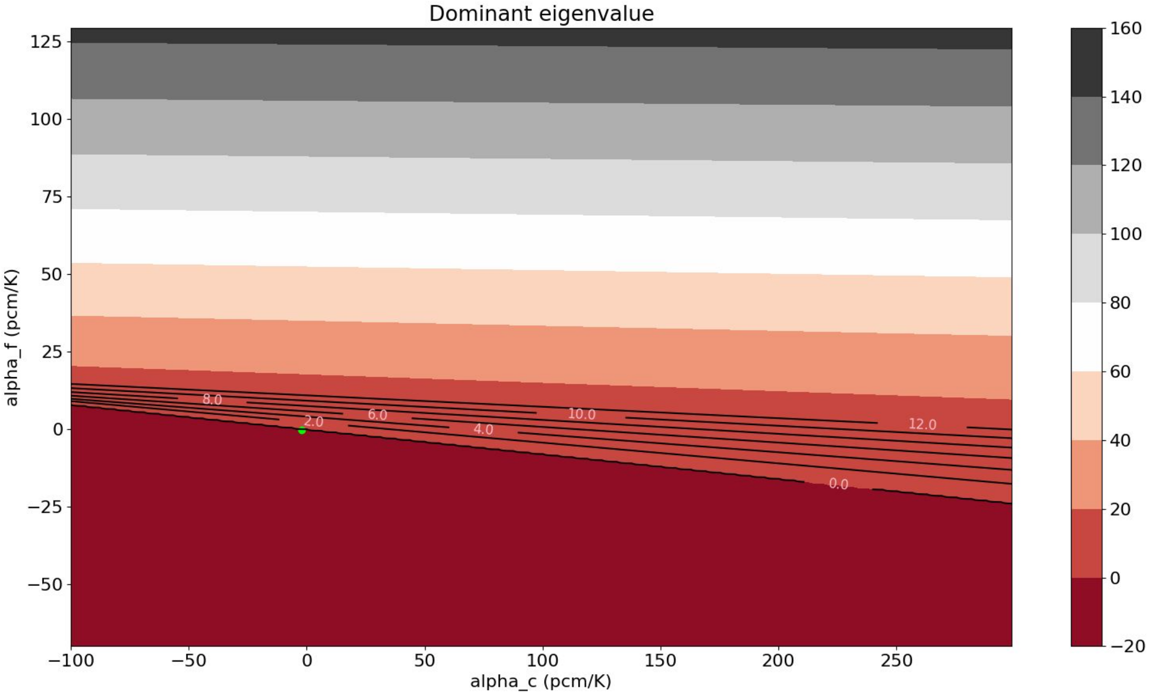
# Eigenvalues (cont.)



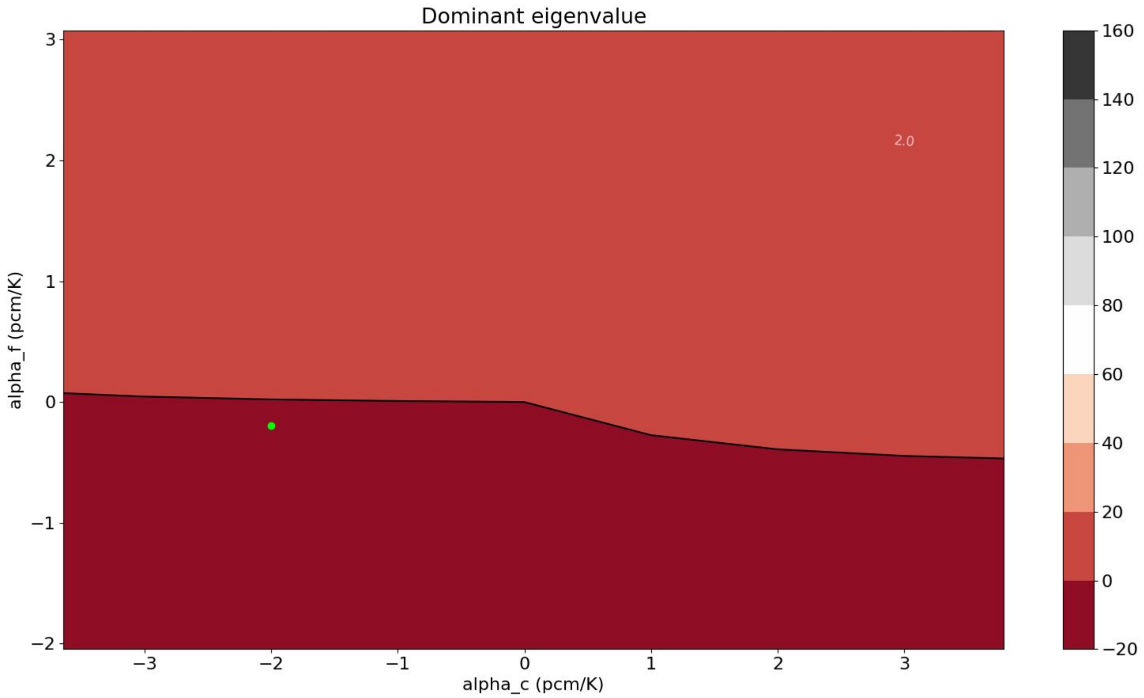
# Eigenvalues (cont.)



# Stability map

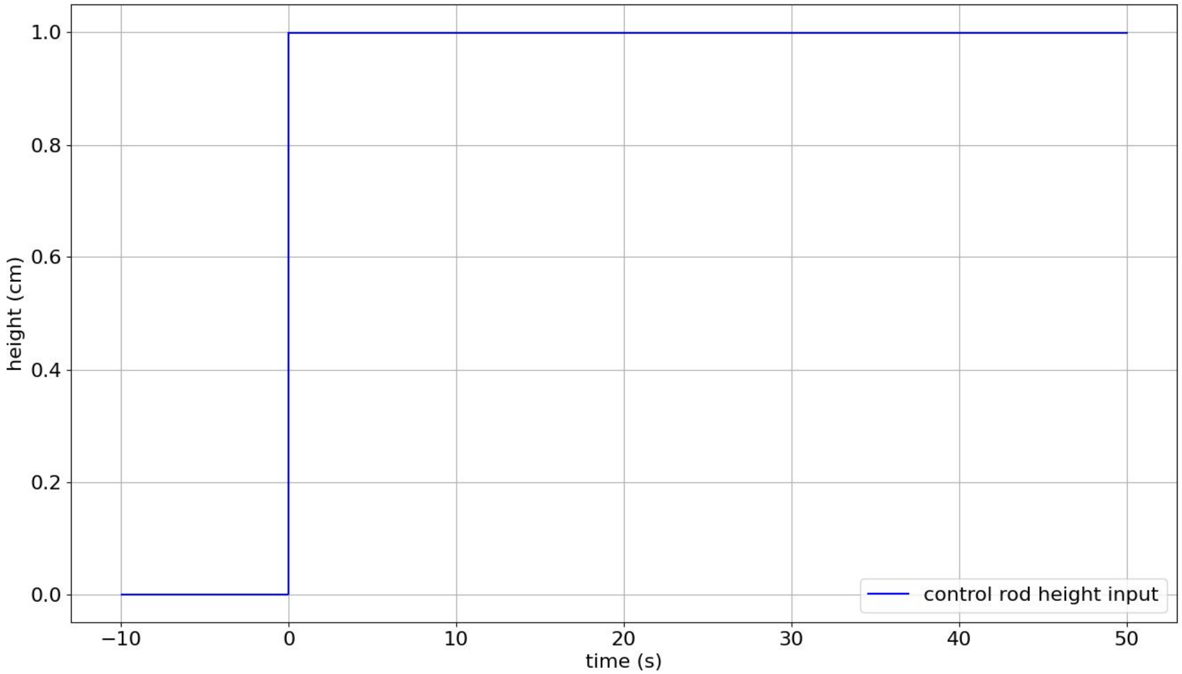


# Stability map



# Step 2: Solving the non-linear model

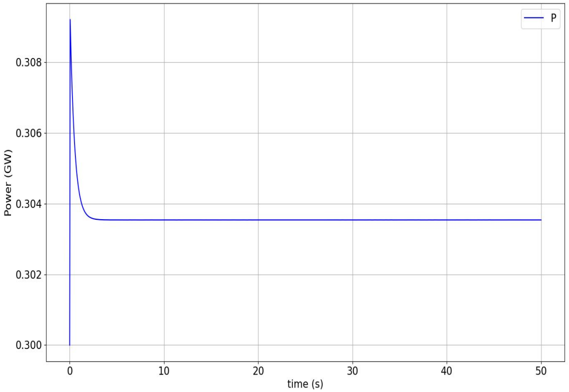
# Step input of control rod height



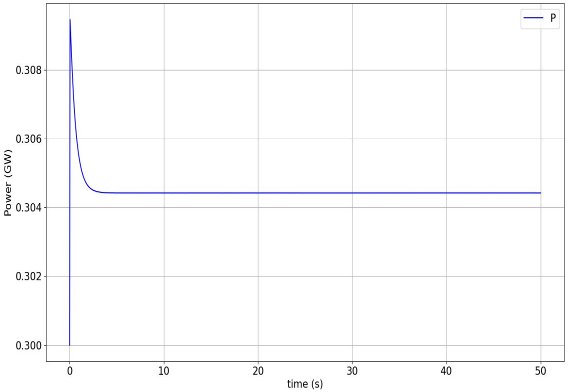
# Part 1: verifying the stability of the non-linear model by running it with various alpha_C and alpha_f values from the stability map

# Stable regions

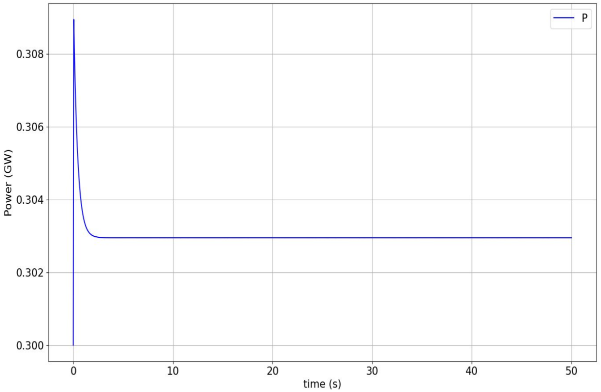
Alpha_c = -1 pcm/K, alpha_f = -1 pcm/K




Alpha_c = -3 pcm/K, alpha_f = -0.5 pcm/K

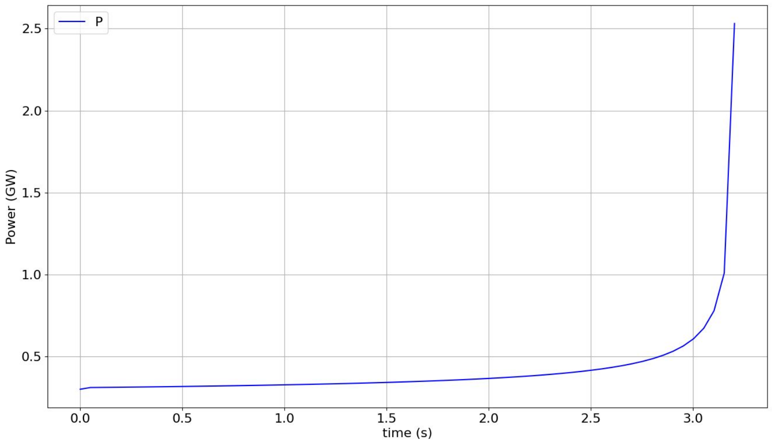



Alpha_c = 1 pcm/K, alpha_f = -1.5 pcm/K

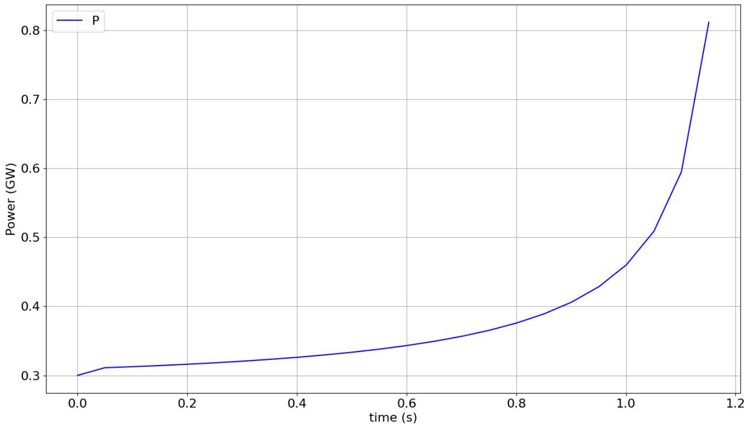


# Unstable regions

Alpha_c = 0 pcm/K, alpha_f = 1 pcm/K

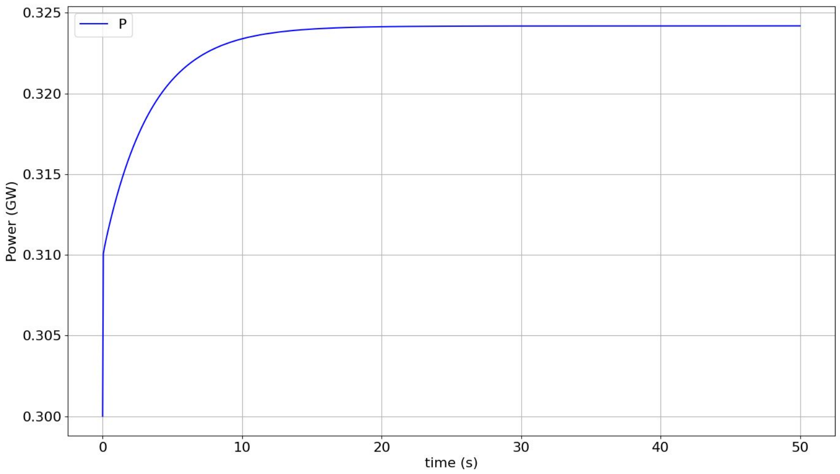



Alpha_c = 2 pcm/K, alpha_f = 2 pcm/K

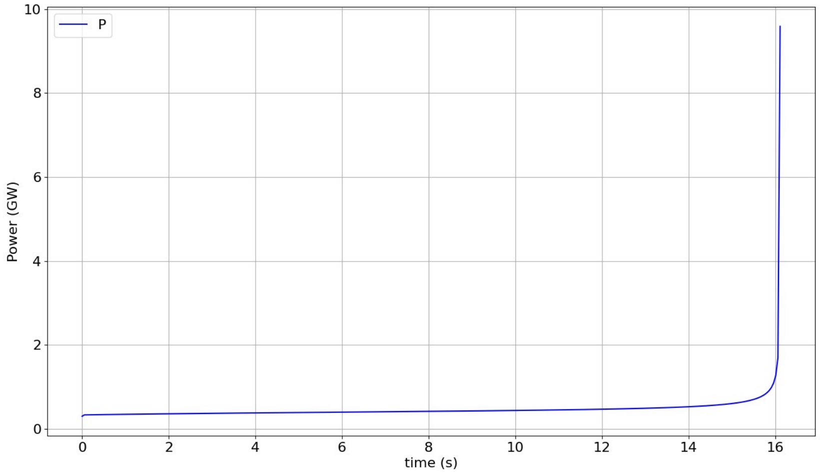


# Unstable region: an interesting phenomenon

Alpha_c = -2 pcm/K, alpha_f = 0.5 pcm/K, dh = 1 cm

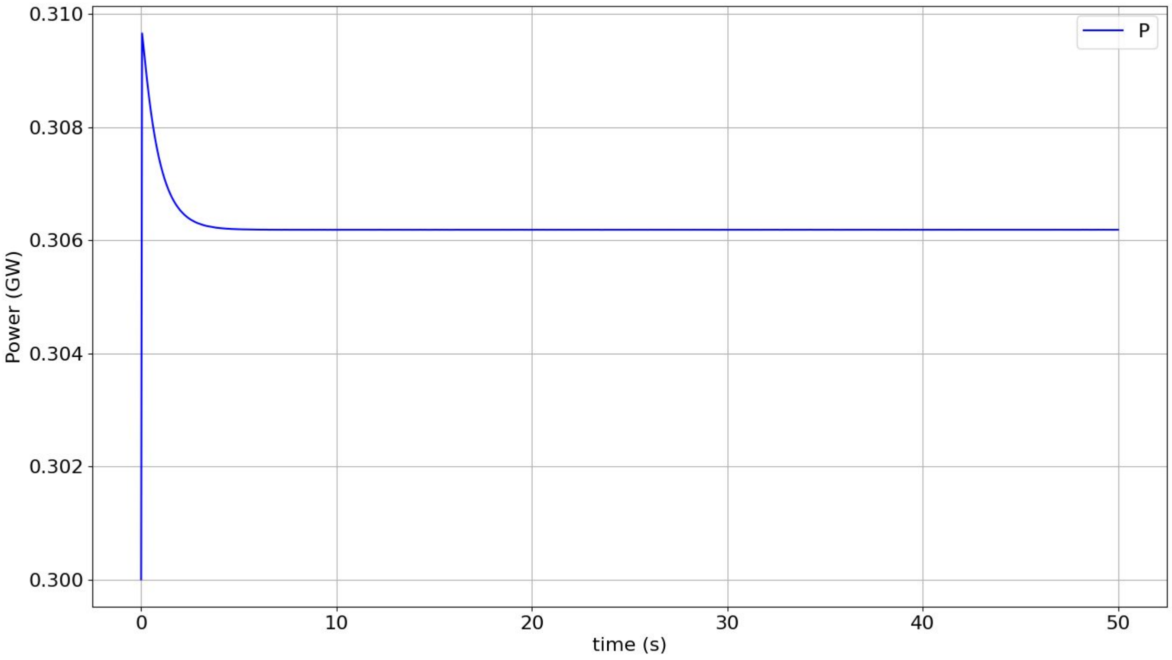



Alpha_c = -2 pcm/K, alpha_f = 0.5 pcm/K, dh = 3 cm

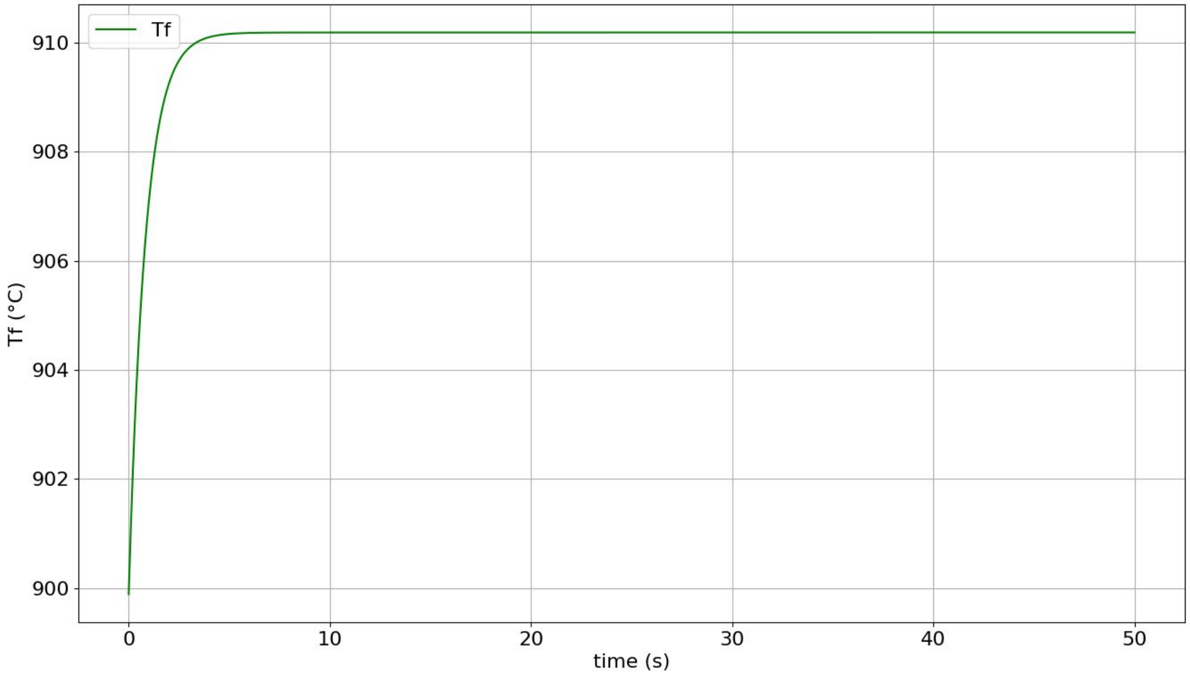


# Part 2: plotting the variables in the output vector

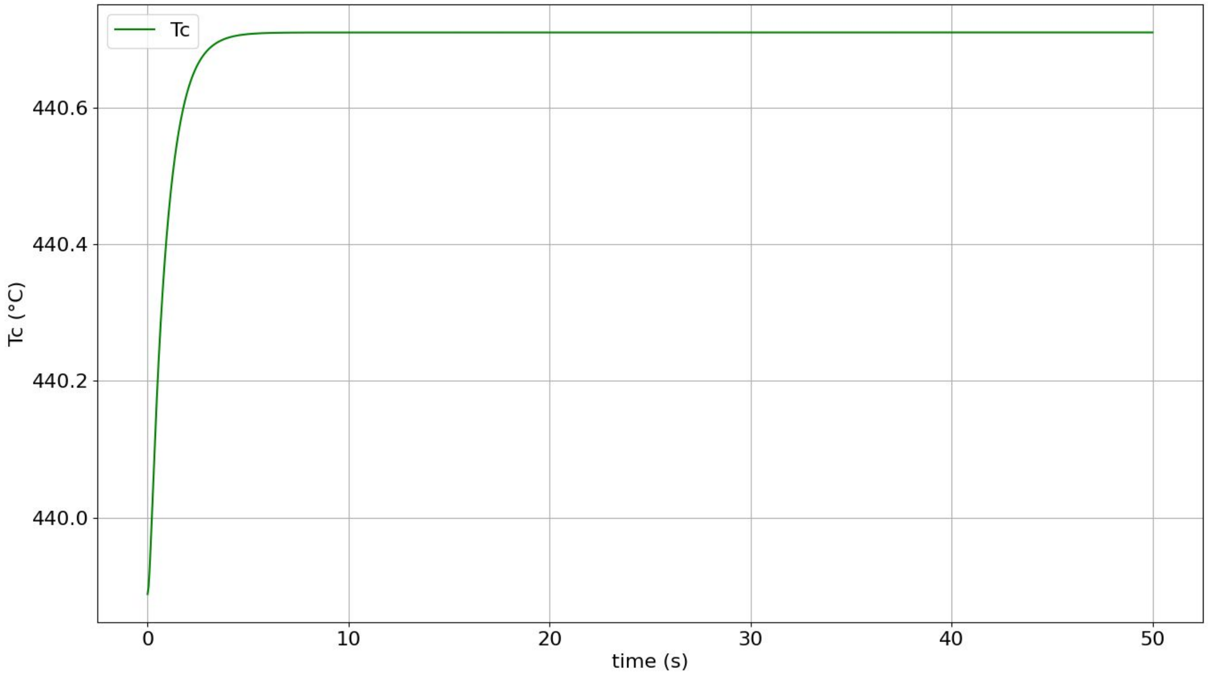
# Response of the reactor



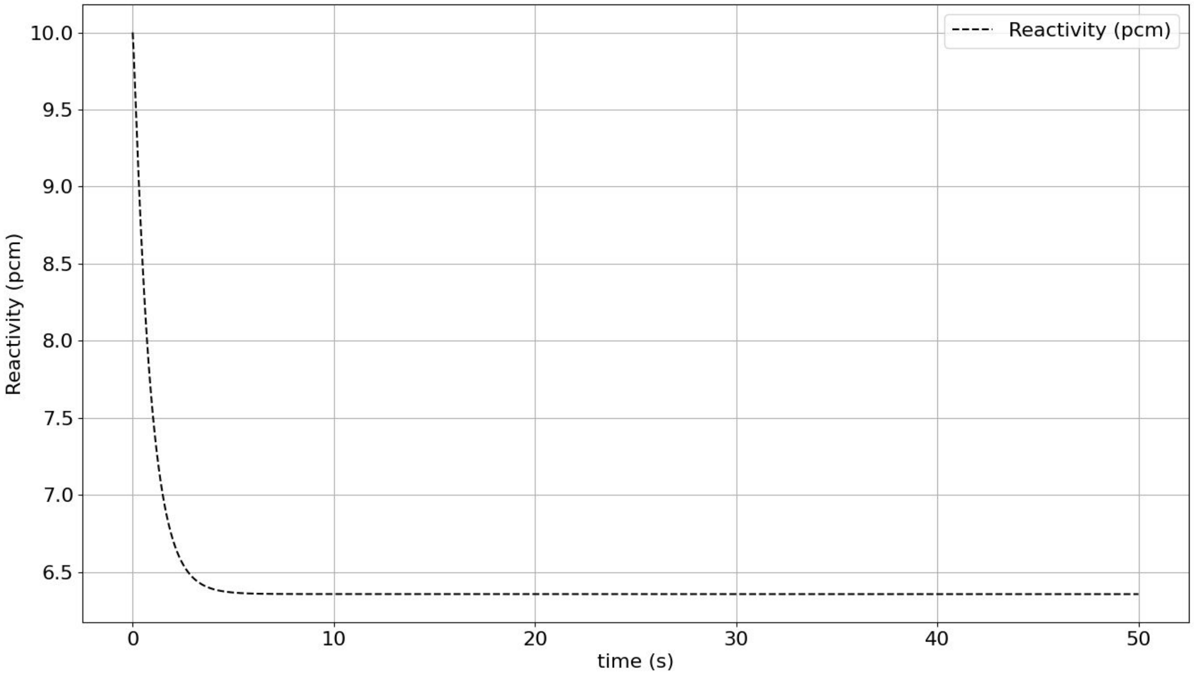
# Response of the reactor (cont.)



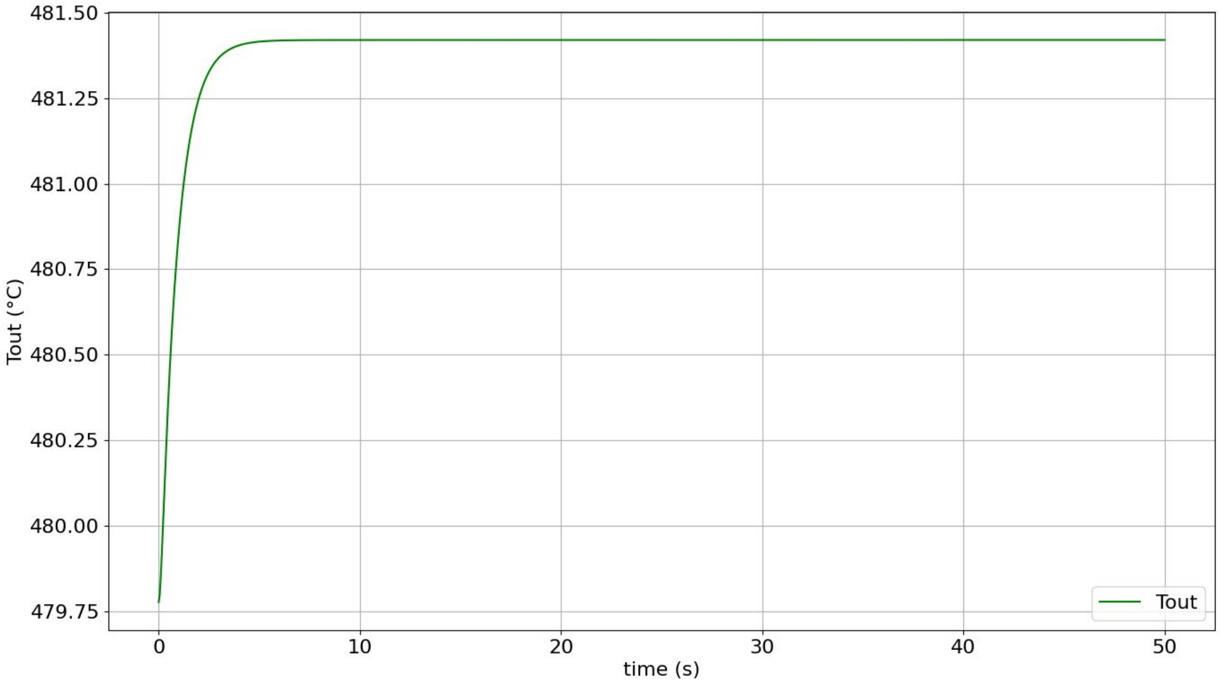
# Response of the reactor (cont.)



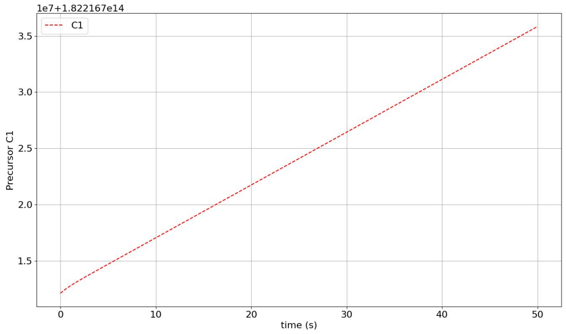
# Response of the reactor (cont.)

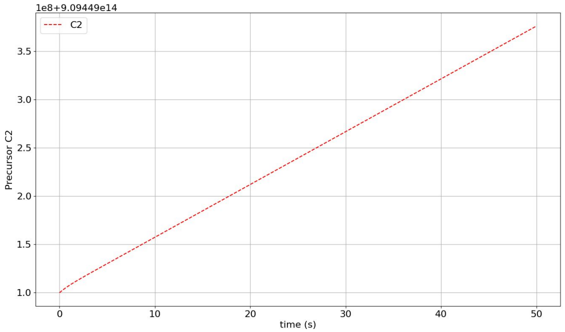


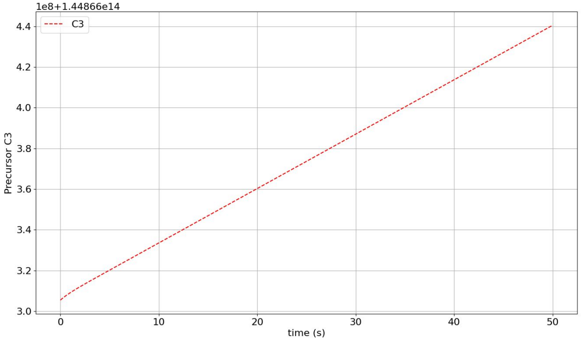
# Response of the reactor (cont.)

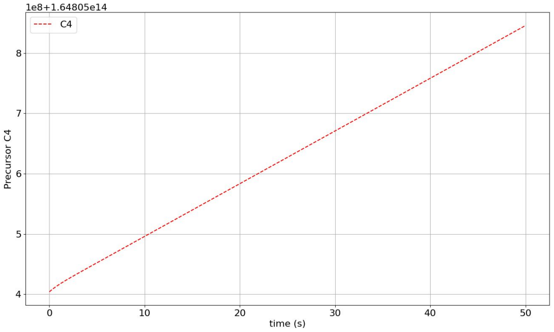


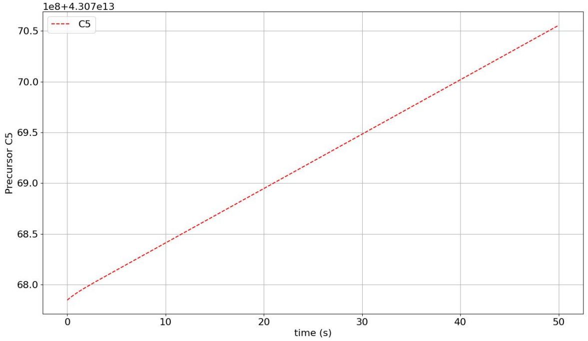
# Response of the reactor (cont.)

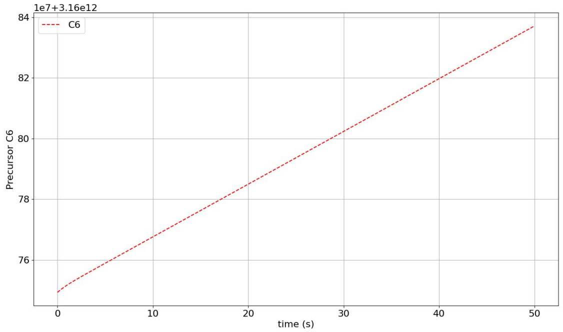


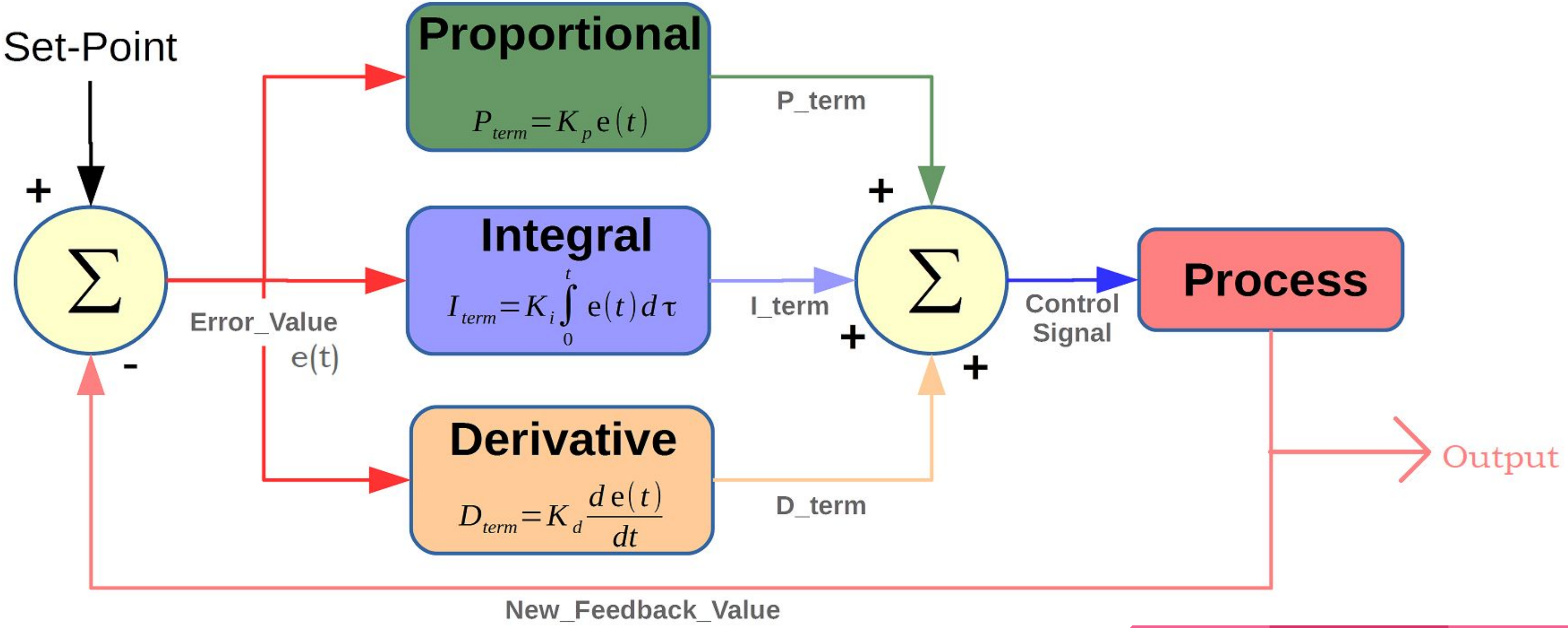



# Step 3: Developing a PID controller for the reactor system

# PID controller



# 6 group controlled non-linear model

$$\frac{d\psi(t)}{dt} = \frac{(\rho - \beta)}{\Lambda}\psi(t) + \sum_{i=1}^{6} \frac{\beta_i}{\Lambda}\eta_i(t)$$

$$\frac{d\eta_i(t)}{dt} = \lambda_i \psi(t) - \lambda_i \eta_i(t) \ ; \ i = 1, ..., 6$$

$$\rho = \alpha_h(K_p e + K_I E) + (\alpha_D + \alpha_A)(T_f - T_f^0) + (\alpha_C + \alpha_R)(T_c - T_c^0)$$

$$\frac{dT_f}{dt} = \frac{P^0}{K\tau_f} - \frac{1}{\tau_f}(T_f - T_c)$$

$$\frac{dT_c}{dt} = \frac{1}{\tau_c}(T_f - T_c) - \frac{2}{\tau_0}(T_c - T_{in})$$

$$T_{out} = 2T_c - T_{in}$$

$$\frac{de}{dt} = -\frac{d\psi}{dt}$$

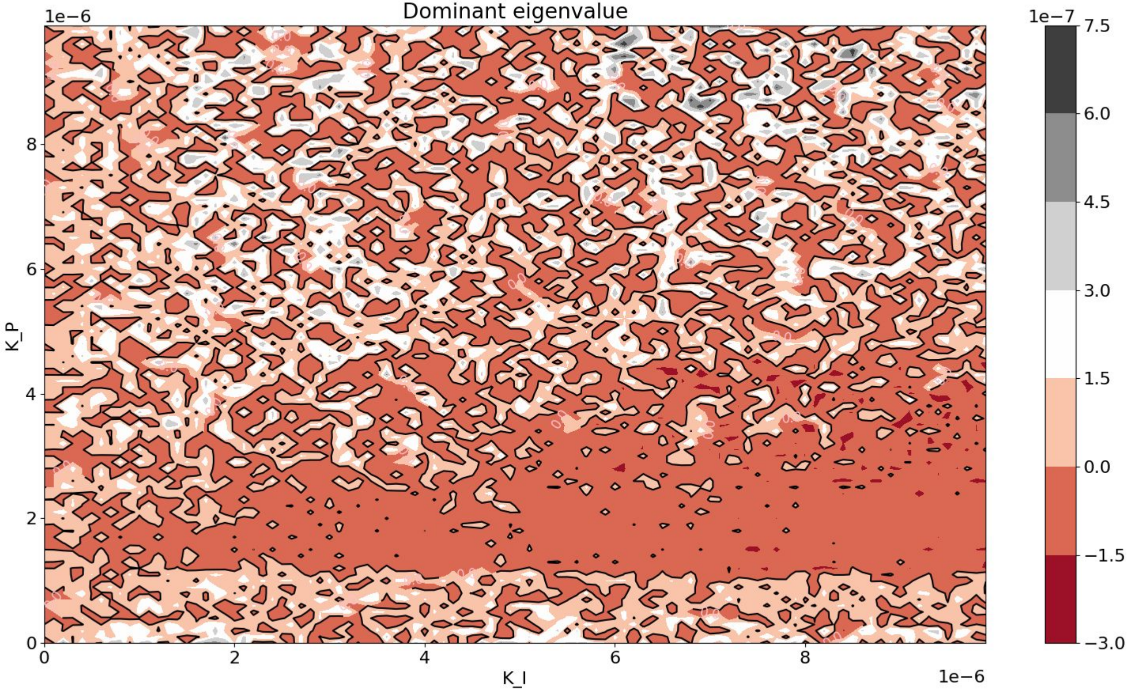
$$\frac{dE}{dt} = e$$

Where $\psi(t) = \frac{P(t)}{P^0}$, $\eta_i(t) = \frac{C_i(t)}{C_i^0}$, $\tau_f = \frac{M_f C_f}{K}$, $\tau_c = \frac{M_c C_c}{K}$, $\tau_0 = \frac{M_c}{G}$, $e(t) = P_{ref} - P(t)$, and $E = \int_0^t e(\tau)d\tau$.

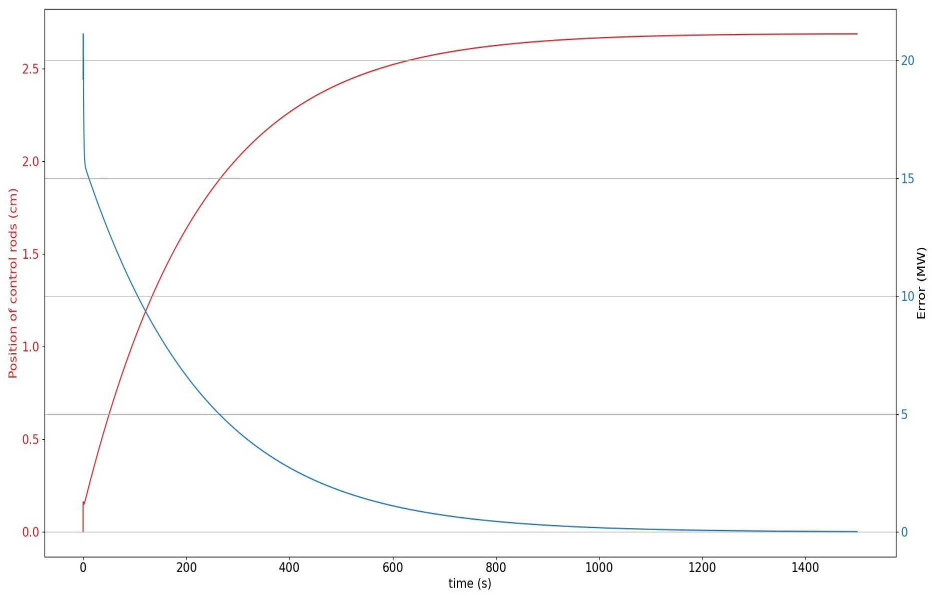
# Studying the stability

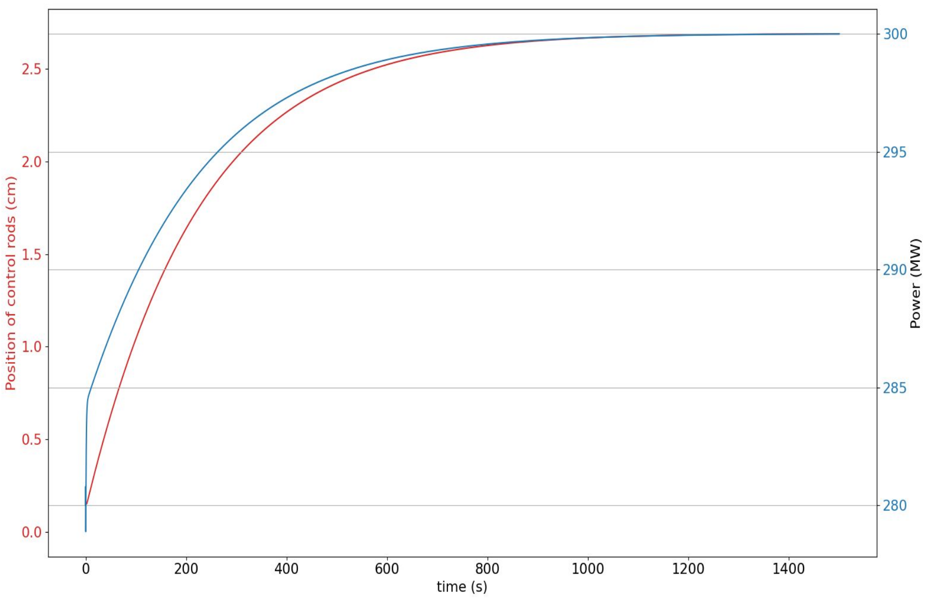
$$A = \begin{pmatrix}
-\frac{\beta}{\Lambda} & \frac{\beta_1}{\Lambda} & \frac{\beta_2}{\Lambda} & \frac{\beta_3}{\Lambda} & \frac{\beta_4}{\Lambda} & \frac{\beta_5}{\Lambda} & \frac{\beta_6}{\Lambda} & \frac{\alpha_f}{\Lambda} & \frac{\alpha_c}{\Lambda} & \frac{\alpha_h K_P}{\Lambda} & \frac{\alpha_h K_I}{\Lambda} \\
\lambda_1 & -\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_2 & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & 0 & 0 & -\lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_4 & 0 & 0 & 0 & -\lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_5 & 0 & 0 & 0 & 0 & -\lambda_5 & 0 & 0 & 0 & 0 & 0 \\
\lambda_6 & 0 & 0 & 0 & 0 & 0 & -\lambda_6 & 0 & 0 & 0 & 0 \\
\frac{P^0}{K\tau_f} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau_f} & \frac{1}{\tau_f} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau_c} & -(\frac{2}{\tau_0} + \frac{1}{\tau_c}) & 0 & 0 \\
\frac{\beta}{\Lambda} & -\frac{\beta_1}{\Lambda} & -\frac{\beta_2}{\Lambda} & -\frac{\beta_3}{\Lambda} & -\frac{\beta_4}{\Lambda} & -\frac{\beta_5}{\Lambda} & -\frac{\beta_6}{\Lambda} & -\frac{\alpha_f}{\Lambda} & -\frac{\alpha_c}{\Lambda} & -\frac{\alpha_h K_P}{\Lambda} & -\frac{\alpha_h K_I}{\Lambda} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}$$

# Studying the stability (cont.)

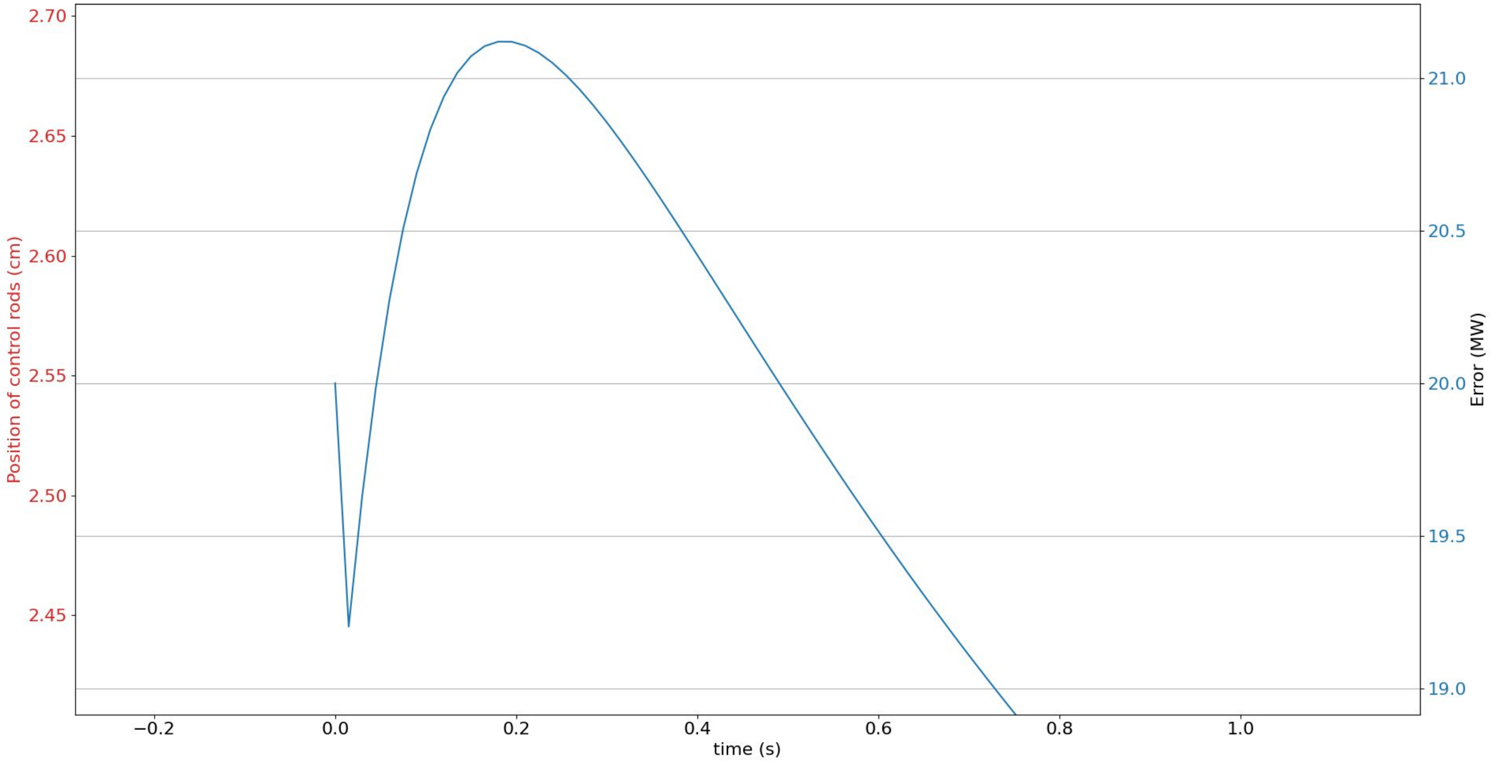


# K_p = 7.5e-6, K_I = 7.5e-7; 20 MW increase in 20 minutes

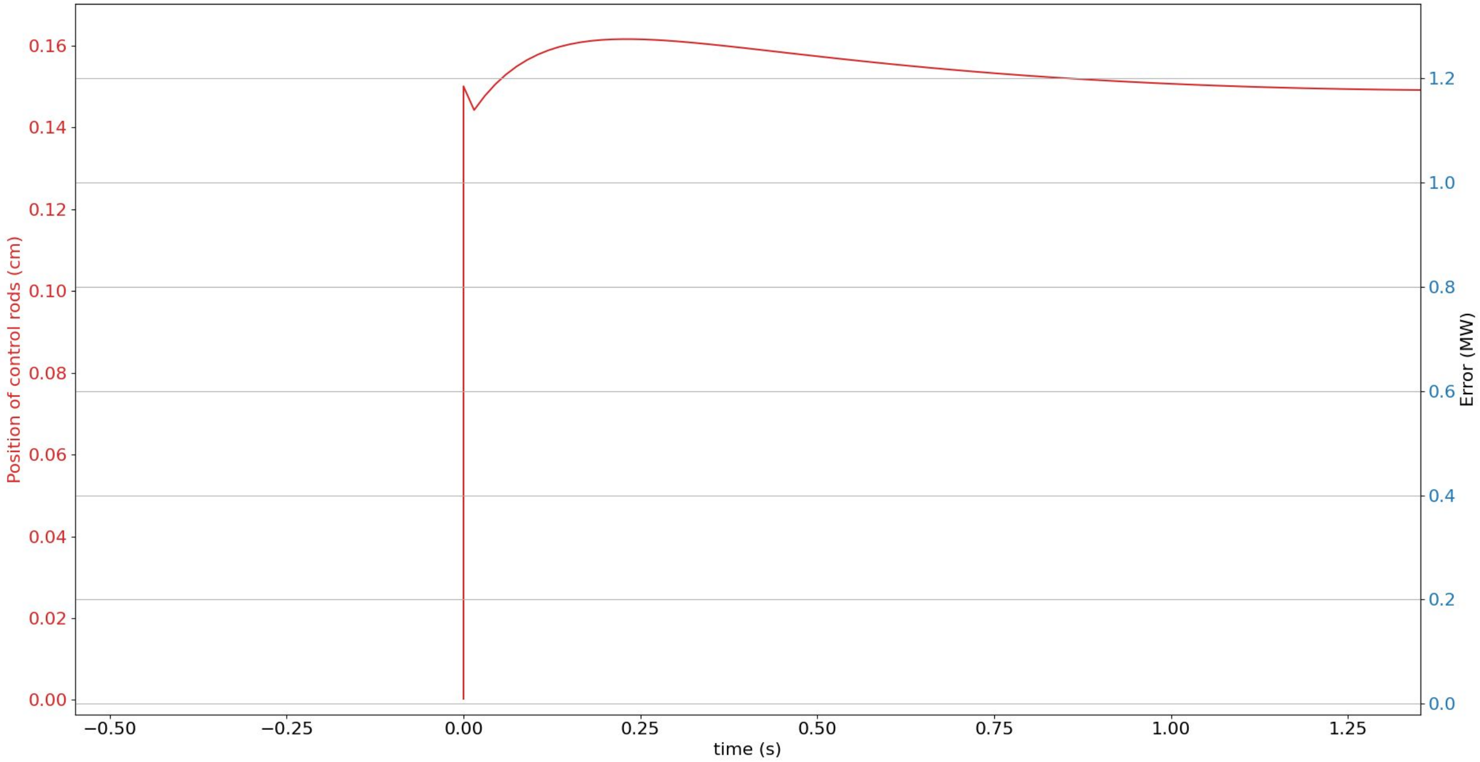


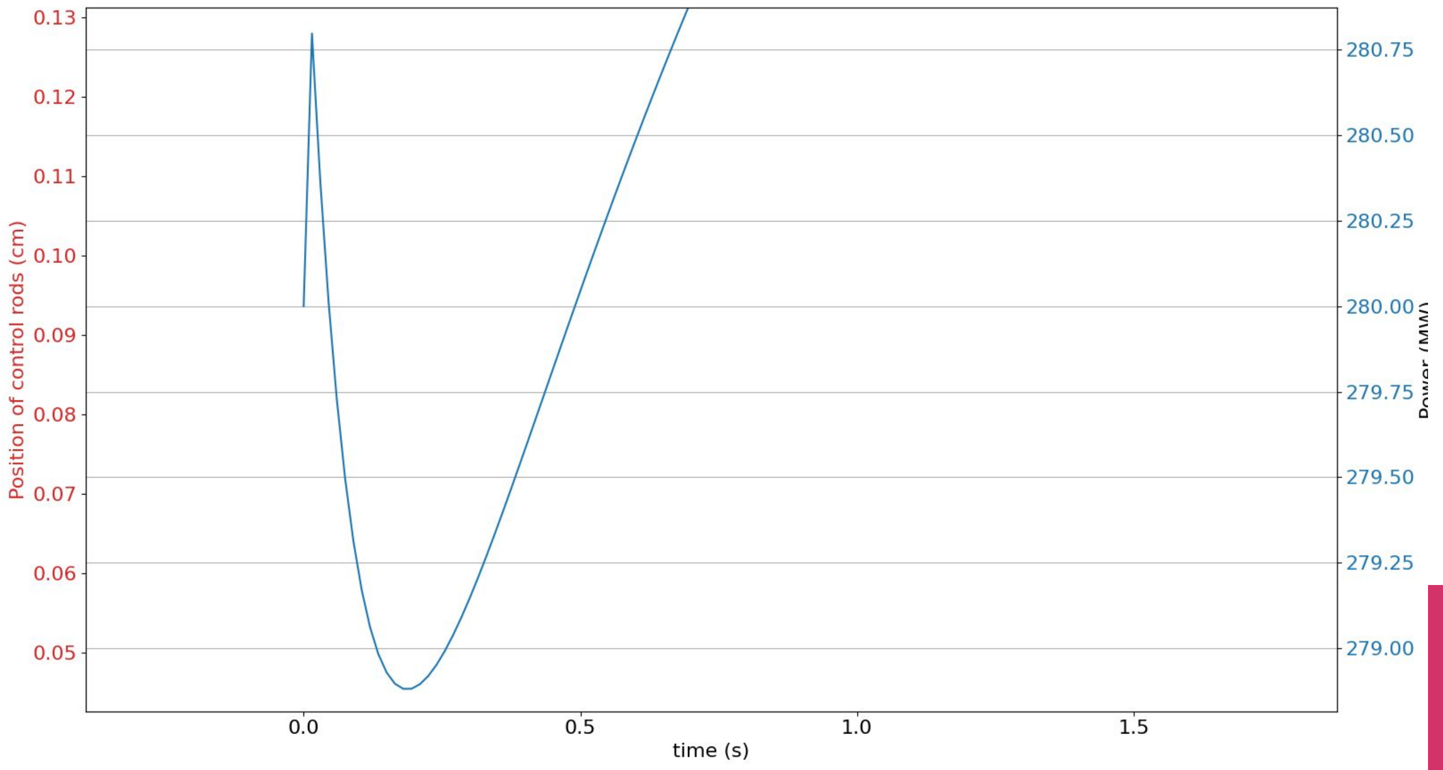
# K_p = 7.5e-6, K_I = 7.5e-7; a closer look



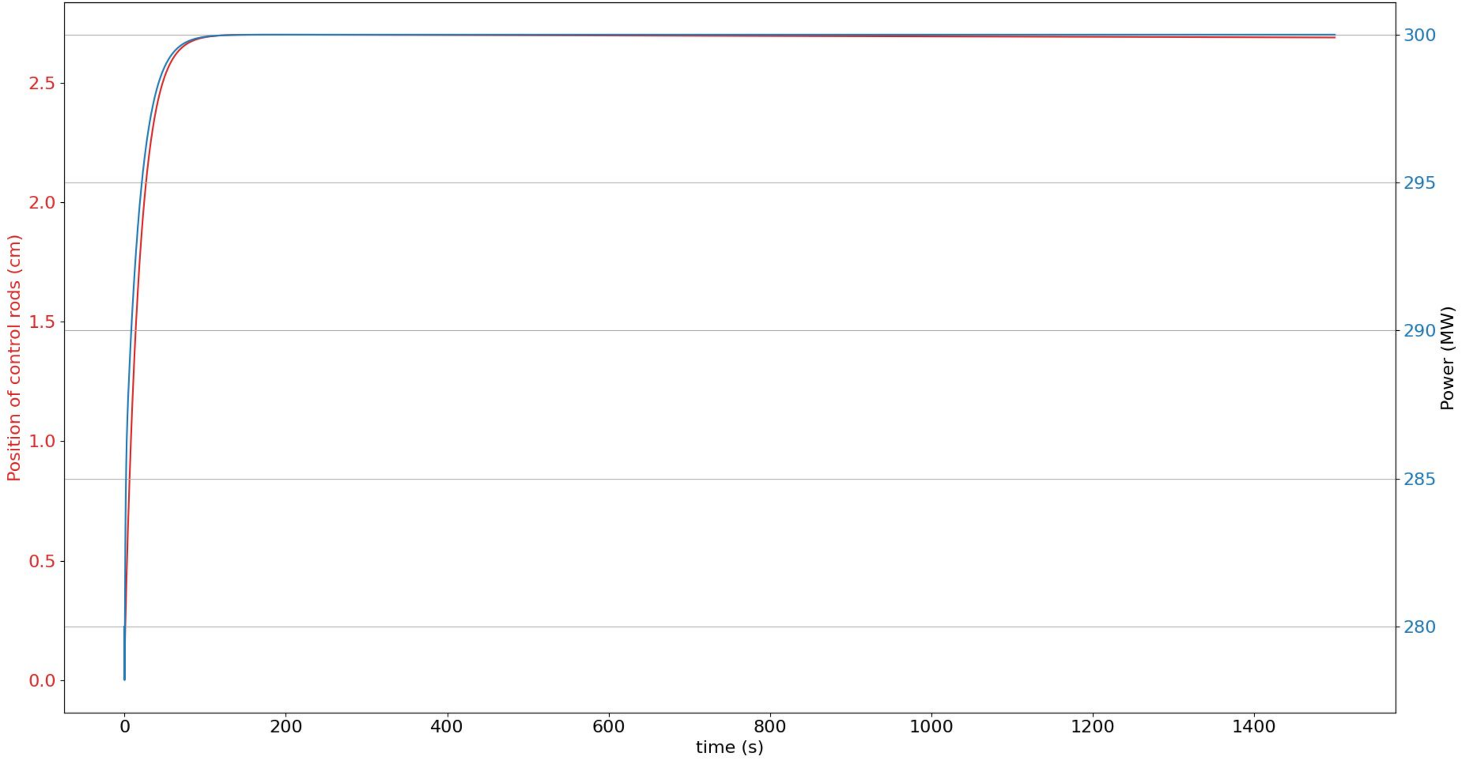
# K_p = 7.5e-6, K_I = 7.5e-7; a closer look (cont.)



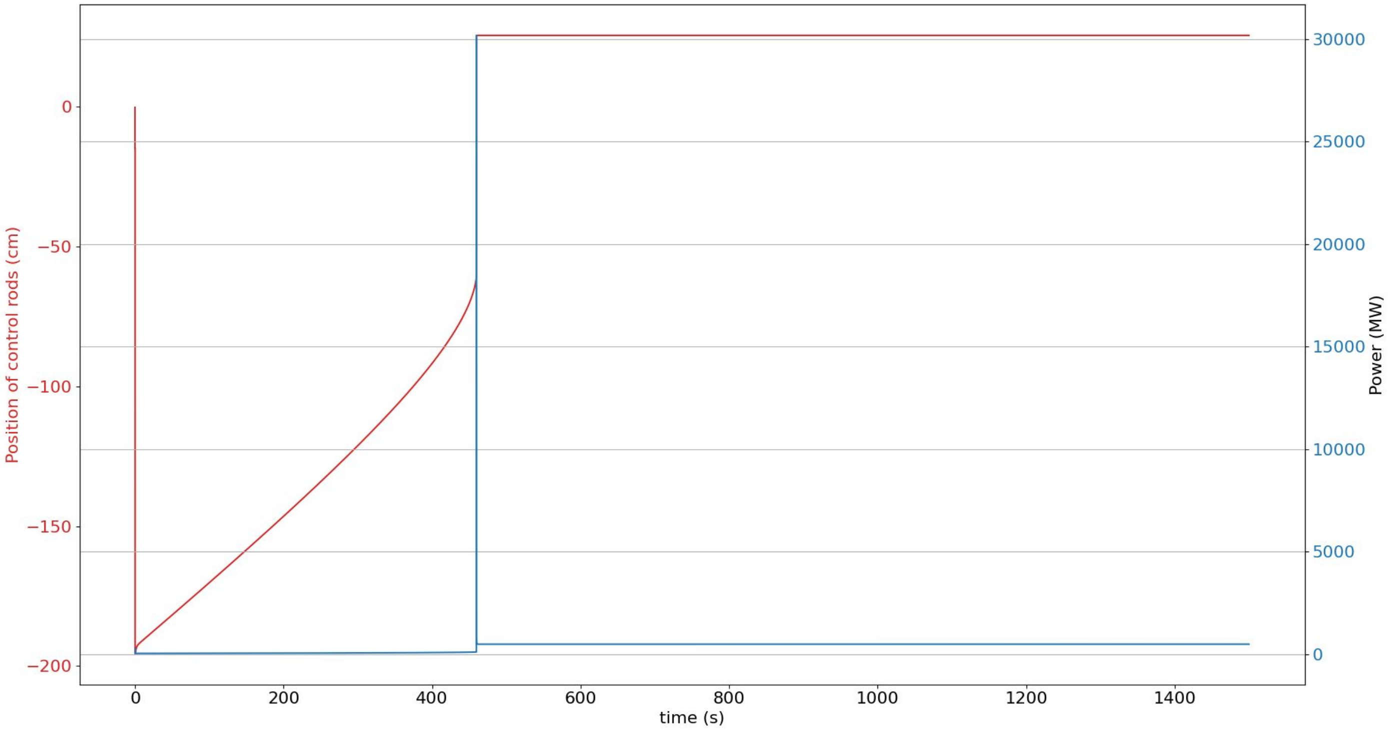
# K_p = 7.5e-6, K_I = 7.5e-7; a closer look (cont.)



# K_p = 2e-6, K_I = 9e-6; another stable pair



# K_p = -7.5e-4, K_I = 7.5e-7; a little nuclear bomb



# Summary

1. We started with a 6 group non-linear coupled neutronics and thermal hydraulics model for an LFR
2. We linearized the model to study the eigenvalues and stability
3. We checked if the non-linear model agreed with the stability study
4. We solved the non-linear model to study the reactor's response to a step input in the control rod position
5. We developed a PID controller for the control rod height
6. We studied the stability in terms of K_P and K_I values.
7. We adjusted the K_p and K_I values to obtain a 1 MW/min increase in power
8. We made sure that the controlled did not produce a power overshoot
9. We looked at some interesting phenomena to gain insight into the dynamics of the reactor model

Thank you! ♡