Digital Logic Assignment 1

Solution

September 2023

1 Convert numbers into destination bases

a) $(234.5)_{10}$ in base 3

Solution:

Take integer and repeat the division.

Iterations	Quotient	Remainder	Coefficient
234/3	78	0	$a_0 = 0$
78/3	26	0	$a_1 = 0$
26/3	8	2	$a_2 = 2$
8/3	2	2	$a_3 = 2$
2/3	0	2	$a_4 = 2$

Thus the integer part is $(22200)_3$

Take fraction part and repeat the multiplication.

Iterations	Integer	Fraction	Coefficient
0.5*3	1	.5	$a_{-1} = 1$
0.5*3	1	.5	$a_{-2} = 1$

Thus the fraction part is $(.11)_3$

So the final answer should be $(22200.11)_3$.

b) $(234.5)_{10}$ in base 12

Solution:

Take integer and repeat the division.

Iterations	Quotient	Remainder	Coefficient
234/12	19	6	$a_0 = 6$
19/12	1	7	$a_1 = 7$
1/12	0	1	$a_2 = 1$

Thus the integer part is $(176)_{12}$

Take fraction part and repeat the multiplication.

Iterations	Integer	Fraction	Coefficient
0.5*12	6	0	$a_{-1} = 6$

Thus the fraction part is $(.6)_{12}$

So the final answer should be $(176.6)_{12}$.

c) $(435)_6$ in base 10

Solution:

$4 * 6^2 + 3 * 6^1 + 5 * 6^0 = 167$

Thus the final answer is 167_{10}.

d) $(10110.0101)_2$ in base 8 Solution:

Divide the binary into groups

$$010\,110.010\,100$$

Thus the binary is 26.24 in octal.

2 Correct number systems for arithmetic operations

a) 1234 + 5432 = 6666

Solution:

Since all numbers are smaller than 7 and no carry is needed, the operation is correct in any number system that radix $r \geq 7$.

b) 302/20=12.1

Solution:

Assume the radix of the number system is r, then

$$\begin{aligned} 3r^2 + 2 &= (r + 2 + r^{-1})2r \\ r^2 - 4r &= 0 \\ r &= 4 \end{aligned}$$

Thus the radix is 4.

3 Simplify the Boolean expressions to the indicated number of literals

a) Simplify (a'+c)(a'+c')(a+b+c'd) to 4 literals

Solution:

$$\begin{aligned} (a' + c)(a' + c')(a + b + c'd) &= (a' + cc')(a + b + c'd) \\ &= a'(a + b + c'd) \\ &= a'a + a'(b + c'd) \\ &= a'(b + c'd) \end{aligned}$$

b) Simplify abc'd+a'bd+abcd to 2 literals

Solution:

$$\begin{aligned} abc'd + a'bd + abcd &= bd(ac' + a' + ac) \\ &= bd(a(c' + c) + a') \\ &= bd(a + a') \\ &= bd \end{aligned}$$

4 Simplify the Boolean expressions to minimum number of literals

a) (a+c)(a'+b+c)(a'+b'+c)

Solution:

$$\begin{aligned} (a + c)(a' + b + c)(a' + b' + c) &= (a + c)(a' + (b + c)(b' + c)) \\ &= (a + c)(a' + c + bb') \\ &= (a + c)(a' + c) \\ &= c + aa' \\ &= c \end{aligned}$$

b) F(a,b,c)=∑(0,1,2,3,5)

Solution:

$$
\begin{aligned}
F(a,b,c) = \sum(0,1,2,3,5) &= a'b'c' + a'b'c + a'bc' + a'bc + ab'c \\
&= a'b'(c'+c) + a'b(c'+c) + ab'c \\
&= a'b' + a'b + ab'c \\
&= a'(b'+b) + ab'c \\
&= a' + ab'c \\
&= (a'+a)(a'+b'c) \\
&= a' + b'c
\end{aligned}
$$

5 Convert to sum of minterm and product of maxterm

a) F(a,b,c,d)=bd'+acd'+ab'c+a'c'

Solution:

$$
\begin{aligned}
F(a,b,c,d) &= (a+a')b(c+c')d' + a(b+b')cd' + ab'c(d+d') + a'(b+b')c'(d+d') \\
&= abcd' + abc'd' + a'bcd' + a'bc'd' + ab'cd' + ab'cd + a'bc'd + a'b'c'd + a'b'c'd' \\
&= \sum(1110, 1100, 0110, 0100, 1010, 1011, 0101, 0001, 0000) \\
&= \sum(0, 1, 4, 5, 6, 10, 11, 12, 14) \\
&= \prod(2, 3, 7, 8, 9, 13, 15)
\end{aligned}
$$

b) F(x,y,z)=(x'+z)(y+x')

Solution:

$$
\begin{aligned}
F(x,y,z) &= (x' + yy' + z)(x' + y + zz') \\
&= (x'+y+z)(x'+y'+z)(x'+y+z') \\
&= \prod(4,5,6) \\
&= \sum(0,1,2,3,7)
\end{aligned}
$$

6 Simplify the Boolean functions

a) To indicated literals, algebraic method.

Solution:

$$
\begin{aligned}
F_1 &= A'BC' + A'BC + ABC \\
&= A'BC' + A'BC + A'BC + ABC \\
&= A'B(C+C') + (A'+A)BC \\
&= A'B + BC \\
&= B(A'+C)
\end{aligned}
$$

$$
\begin{aligned}
F_2 &= A'B'C' + A'BC' + AB'C + ABC \\
&= A'(B+B')C' + A(B+B')C \\
&= A'C' + AC \\
&= (A \oplus C)'
\end{aligned}
$$

b) To indicated literals, using k-map.

Solution:

Draw the 3-map with the data in truth table for F_1

A \ BC	00	01	11	10
0	0	0	1	1
1	0	0	1	0

Thus F = BC +A'B = B(A'+C)

Draw the 2-map with the data in truth table for F_2 with varible A and C

A \ BC	00	01	11	10
0	1	0	0	1
1	0	1	1	0

Thus F = A'C'+AC = (A$\oplus$ C)'

7 Using K maps to find simpliest sum of peoducts expression

a) F(W,X,Y,Z) = $\sum$(0,2,3,6,7,10,11,12,13,15)

Solution:

WX \ YZ	00	01	11	10
00	1	0	1	1
01	0	0	1	1
11	1	1	1	0
10	0	0	1	1

Thus F(W,X,Y,Z) = X'W'Z'+XY'W+YZ+YW'+X'Y

b) F(A,B,C,D) = $\prod$(1,3,4,5,6,7,9,12,13,14)

Solution:

$$F(A,B,C,D) = \prod(1,3,4,5,6,7,9,12,13,14)$$
$$= \sum(0,2,8,10,11,15)$$

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	1	0
10	1	0	1	1

Thus F(A,B,C,D) = B'D'+ACD

8 Find the simplest sum-of-products expression

Solution:

$$\begin{aligned} f &= abd' + c'd + a'cd' + b'cd' \\ &= ab(c+c')d' + (a+a')(b+b')c'd + a'(b+b')cd' + (a+a')b'cd' \\ &= abcd' + abc'd' + abc'd + ab'c'd + a'bc'd + a'b'c'd + a'bcd' + a'b'cd' + ab'cd' \\ &= \sum(1,2,5,6,9,10,12,13,14) \\ &= \prod(0,3,4,7,8,11,15) \end{aligned}$$

$$\begin{aligned} g &= (a+b+d')(b'+c'+d)(a'+c+d') \\ &= (a+b+c+d')(a+b+c'+d')(a+b'+c'+d)(a'+b'+c'+d)(a'+b+c+d')(a'+b'+c+d') \\ &= \prod(1,3,6,9,13,14) \end{aligned}$$

$$\begin{aligned} F &= fg \\ &= \prod(0,3,4,7,8,11,15)\prod(1,3,6,9,13,14) \\ &= \prod(0,1,3,4,6,7,8,9,11,13,14,15) \\ &= \sum(2,5,10,12) \end{aligned}$$

We can use the map with the sum of minterms function derived from the question.

ab \ cd	00	01	11	10
00	0	0	0	1
01	0	1	0	0
11	1	0	0	0
10	0	0	0	1

Thus F = abc'd'+a'bc'd+b'cd'

9 Simplest sum of products and implement...

a) NAND gates only

Solution:

$$F(A,B,C,D) = \sum(1,2,4,7,8,9,11) + d(0,3,5)$$

With only NAND gates,

AB \ CD	00	01	11	10
00	x	1	x	1
01	1	x	1	0
11	0	0	0	0
10	1	1	1	0

The simplest sum-of-products expression is F=A'B'+A'C'+A'D+B'C'+B'D

From the sum of product function, we can derive the NAND implementation

$$\begin{aligned} F &= A'B' + A'C' + A'D + B'C' + B'D \\ &= [(A'B' + A'C' + A'D + B'C' + B'D)']' \\ &= [(A'B')'(A'C')'(A'D)'(B'C')'(B'D)']' \end{aligned}$$

b) NOR gates implementation Solution:

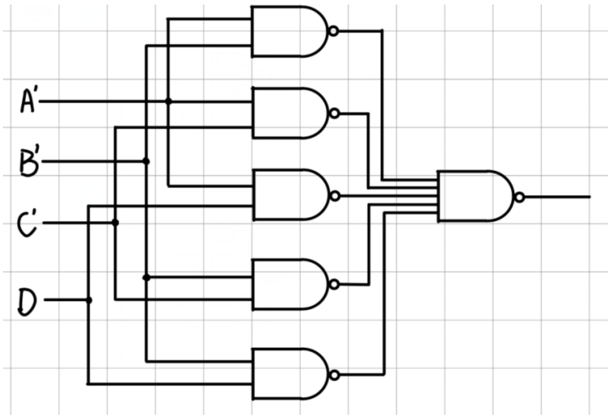
$$\begin{aligned} F &= \sum(1,2,4,7,8,9,11) + d(0,3,5) \\ F' &= \sum(6,10,12,13,14,15) + d(0,3,5) \end{aligned}$$

AB \ CD	00	01	11	10
00	x	0	x	0
01	0	x	0	1
11	1	1	1	1
10	0	0	0	1

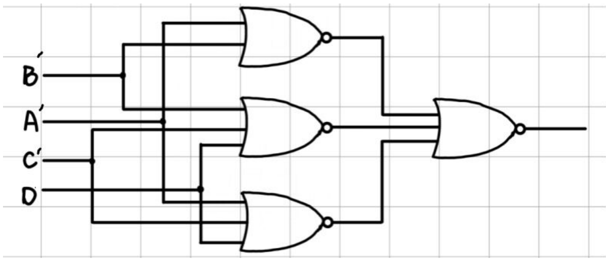
Thus, $F' = AB + BCD' + ACD'$ and $F = (AB + BCD' + ACD')' = (A' + B')(B' + C' + D)(A' + C' + D)$
From the product of sum function, we can derive the NOR implementation

$$\begin{aligned} F &= (A' + B')(B' + C' + D)(A' + C' + D) \\ &= [(A' + B')' + (B' + C' + D)' + (A' + C' + D)']' \end{aligned}$$

c) Draw the two logic diagrams



NAND Implementation



NOR Implementation