Digital Logic Theoretical Assignment 3 Solution

1.

State Equation:

$$A(t+1) = J_A A' + K'_A A = (Bx + B'y')A' + (B'xy')'A$$
$$B(t+1) = J_B B' + K'_B B = (A'x)B' + (A + xy')'B$$

Output Equation:

$$z = Ax'y' + Bx'y'$$

State and Output Table

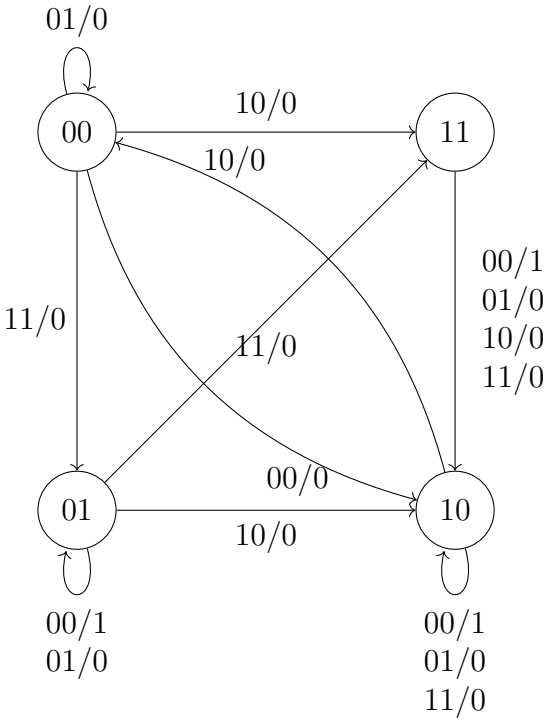
Present State		Input		Next State		Output
A	B	x	y	A	B	z
0	0	0	0	1	0	0
0	0	0	1	0	0	0
0	0	1	0	1	1	0
0	0	1	1	0	1	0
0	1	0	0	0	1	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	1	0
1	0	0	0	1	0	1
1	0	0	1	1	0	0
1	0	1	0	0	0	0
1	0	1	1	1	0	0
1	1	0	0	1	0	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	0	0

Also, you can use FF's inputs to derive state table, instead of using state equations

$$A(t+1) = J_A A' + K'_A A$$
$$B(t+1) = J_B B' + K'_B B$$
$$z = Ax'y' + Bx'y'$$

Current State		Input		Flip-Flop Inputs				Next State		Output
A(t)	B(t)	x	y	J_A	K_A	J_B	K_B	A(t+1)	B(t+1)	z
0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	1	1	1	1	1	0
0	0	1	1	0	0	1	0	0	1	0
0	1	0	0	0	0	0	0	0	1	1
0	1	0	1	0	0	0	0	0	1	0
0	1	1	0	1	0	1	1	1	0	0
0	1	1	1	1	0	1	0	1	1	0
1	0	0	0	1	0	0	1	1	0	1
1	0	0	1	0	0	0	1	1	0	0
1	0	1	0	1	1	0	1	0	0	0
1	0	1	1	0	0	0	1	1	0	0
1	1	0	0	0	0	0	1	1	0	1
1	1	0	1	0	0	0	1	1	0	0
1	1	1	0	1	0	0	1	1	0	0
1	1	1	1	1	0	0	1	1	0	0

State Diagram:



2.

Input Equation:

$$T_A = A + B$$
$$T_B = A' + B$$

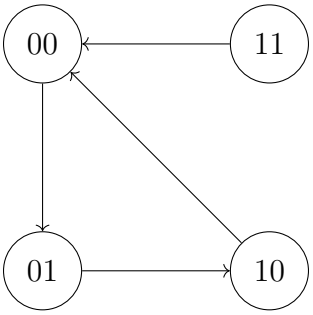
State Equation:

$$A(t+1) = A \oplus T_A = A \oplus (A + B)$$
$$B(t+1) = B \oplus T_B = B \oplus (A' + B)$$

State Table:

Present State		Next State	
A	B	A	B
0	0	0	1
0	1	1	0
1	0	0	0
1	1	0	0

State Diagram:



3.

a)

Input Equation:

$$J_1 = X \qquad K_1 = (XQ_2')'$$
$$J_2 = X \qquad K_2 = (XQ_1)'$$

State Equation:

$$Q_1(t+1) = J_1Q_1' + K_1'Q_1 = XQ_1' + XQ_1Q_2'$$
$$Q_2(t+1) = J_2Q_2' + K_2'Q_2 = XQ_2' + XQ_1Q_2$$

Output Equation:

$$F = X \oplus Q_2'$$

State Table:

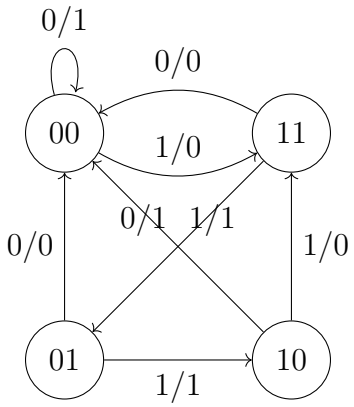
Present State		Input	Next State		Output
Q_1	Q_2	X	Q_1	Q_2	F
0	0	0	0	0	1
0	0	1	1	1	0
0	1	0	0	0	0
0	1	1	1	0	1
1	0	0	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	0	1	1

Also, you can use FF's inputs to derive state table, instead of using state equations

$$
\begin{aligned}
Q_1(t+1) &= J_1 Q_1' + K_1' Q_1 \\
Q_2(t+1) &= J_2 Q_2' + K_2' Q_2 \\
F &= X \oplus Q_2'
\end{aligned}
$$

Current State		Input	Flip-Flop Inputs				Next State		Output
$Q_1(t)$	$Q_2(t)$	X	J_1	K_1	J_2	K_2	$Q_1(t+1)$	$Q_2(t+1)$	F
0	0	0	0	1	0	1	0	0	1
0	0	1	1	0	1	1	1	1	0
0	1	0	0	1	0	1	0	0	0
0	1	1	1	1	1	1	1	0	1
1	0	0	0	1	0	1	0	0	1
1	0	1	1	0	1	0	1	1	0
1	1	0	0	1	0	1	0	0	0
1	1	1	1	1	1	0	0	1	1

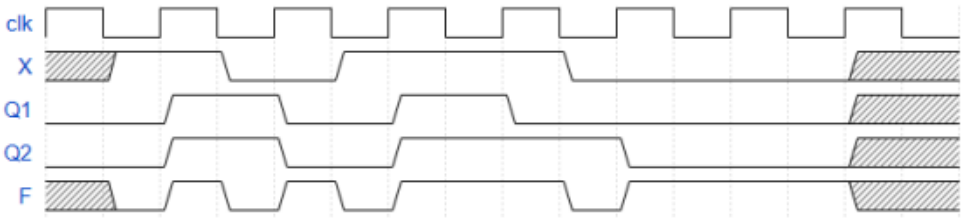
State Diagram:



b)

This is a Mealy machine, because the output is determined by the input and the present state. For example, when the present state is 00, the output is different when the input is 0 or 1.

c)



4. Simplified Version

State Table:

Present State	Input	Next State
Q	X	Q
a	0	b
a	1	a
b	0	d
b	1	c
c	0	d
c	1	c
d	0	a
d	1	d

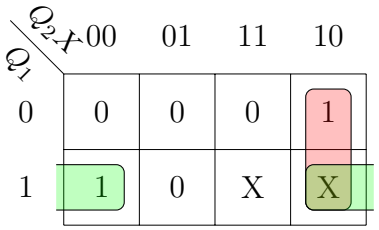
Reduced State Table:

Present State	Input	Next State
Q	X	Q
a	0	b
a	1	a
b	0	d
b	1	b
d	0	a
d	1	d

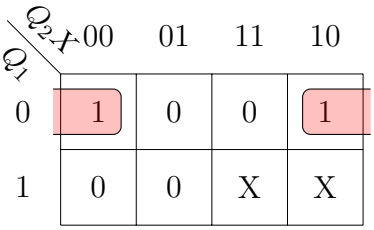
Let $a = 00$, $b = 01$, $d = 10$, then the reduced state table is: **State Table with TFF Input:**

Present State		Input	Next State		TFF Input	
Q_1	Q_2	X	Q_1	Q_2	T_1	T_2
0	0	0	0	1	0	1
0	0	1	0	0	0	0
0	1	0	1	0	1	1
0	1	1	0	1	0	0
1	0	0	0	0	1	0
1	0	1	1	0	0	0
1	1	0	X	X	X	X
1	1	1	X	X	X	X

Derive Input Equation:



T_1



T_2

$$T_1 = Q_1X' + Q_2X'$$
$$T_2 = Q_1'X'$$

4. Non-Simplified Version

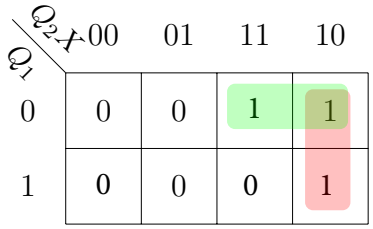
State Table:

Present State	Input	Next State
Q	X	Q(t+1)
a	0	b
a	1	a
b	0	d
b	1	c
c	0	d
c	1	c
d	0	a
d	1	d

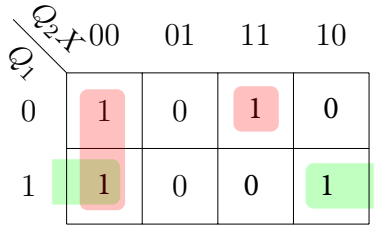
Let $a = 00$, $b = 01$, c = 10, d = 11 then the reduced state table is: **State Table with TFF Input:**

Present State		Input	Next State		TFF Input	
Q_1	Q_2	X	Q_1	Q_2	T_1	T_2
0	0	0	0	1	0	1
0	0	1	0	0	0	0
0	1	0	1	1	1	0
0	1	1	1	0	1	1
1	0	0	1	1	0	1
1	0	1	1	0	0	0
1	1	0	0	0	1	1
1	1	1	1	1	0	0

Derive Input Equation:



T_1



T_2

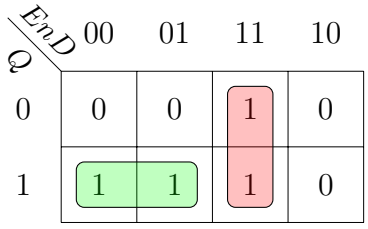
$$T_1 = Q_1'Q_2 + Q_2X'$$
$$T_2 = Q_2'X' + Q_1'Q_2X + Q_1X'$$

5.

State Table:

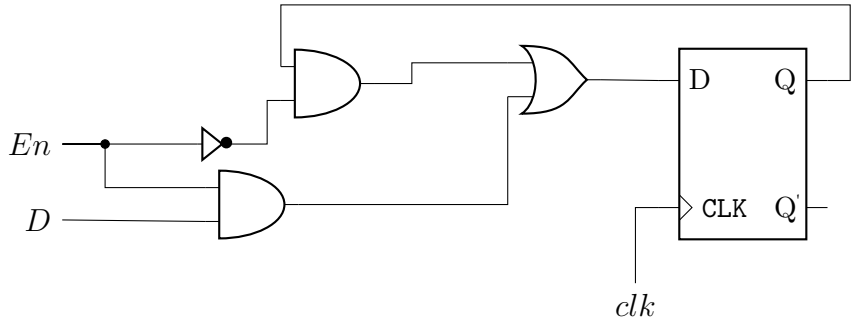
Present State	Input		Next State
Q	En	D	Q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Derive Input Equation:



$$Q(t+1) = EnD + QEn'$$

Block Diagram:



6.

a)

Characteristic Table:

A	B	$Q(t+1)$	Operation
0	0	0	Reset
0	1	$Q(t)$	No Change
1	0	$Q(t)'$	Complement
1	1	1	Set

b)

State Table:

Current State	Input		Next State
Q	A	B	Q
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Simplify Characteristic Equation:

Q \ AB	00	01	11	10
0	0	0	1	1
1	0	1	1	0

$$Q(t+1) = Q'A + QB$$

c)

Excitation Table:

$Q(t)$	$Q(t+1)$	A	B	Operation
0	0	0	X	No Change
0	1	1	X	Set
1	0	X	0	Reset
1	1	X	1	No Change

d)

State Table with JKFF Input:

Current State	JKFF Input		Next State	New FF Input	
Q	J	K	Q	A	B
0	0	0	0	0	X
0	0	1	0	0	X
0	1	0	1	1	X
0	1	1	1	1	X
1	0	0	1	X	1
1	0	1	0	X	0
1	1	0	1	X	1
1	1	1	0	X	0

Simplify New FF Input:

Q \ JK	00	01	11	10
0	0	0	1	1
1	X	X	X	X

A

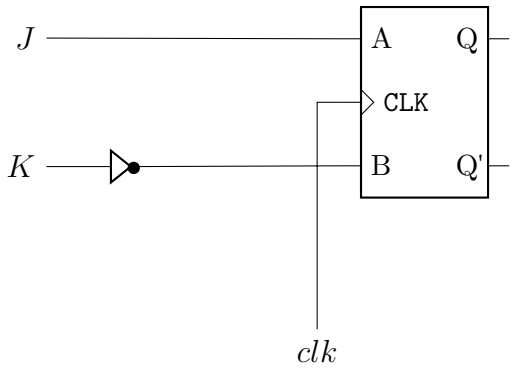
Q \ JK	00	01	11	10
0	X	X	X	X
1	1	0	0	1

B

The simplified JKFF input is:

$$A = J$$
$$B = K'$$

Block Diagram:

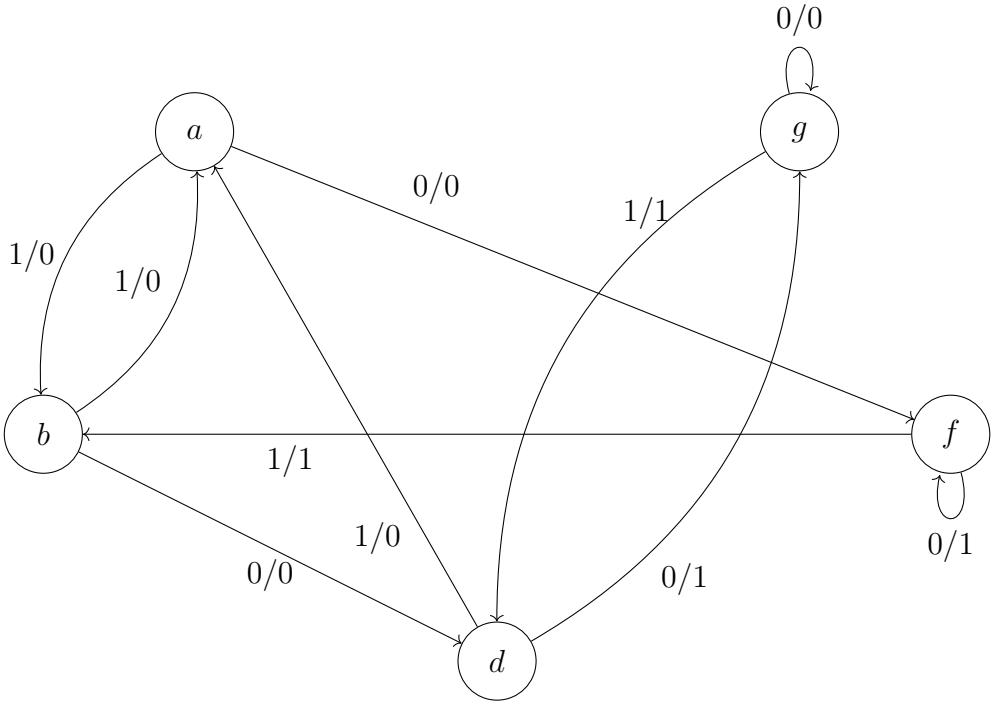


7.

Reduced State Table:

Present State	Next State		Output	
	$X = 0$	$X = 1$	$X = 0$	$X = 1$
a	f	b	0	0
b	d	a	0	0
d	g	a	1	0
f	f	b	1	1
g	g	d	0	1

State Diagram:



Let $a = 000$, $b = 001$, $d = 010$, $f = 011$, $g = 100$.

State Table with JKFF Input:

Present State			Input	Next State			JKFF Input						Output
Q_1	Q_2	Q_3	X	Q_1	Q_2	Q_3	J_1	K_1	J_2	K_2	J_3	K_3	F
0	0	0	0	0	1	1	0	X	1	X	1	X	0
0	0	0	1	0	0	1	0	X	0	X	1	X	0
0	0	1	0	0	1	0	0	X	1	X	X	1	0
0	0	1	1	0	0	0	0	X	0	X	X	1	0
0	1	0	0	1	0	0	1	X	X	1	0	X	1
0	1	0	1	0	0	0	0	X	X	1	0	X	0
0	1	1	0	0	1	1	0	X	X	0	X	0	1
0	1	1	1	0	0	1	0	X	X	1	X	0	1
1	0	0	0	1	0	0	X	0	0	X	0	X	0
1	0	0	1	0	1	0	X	1	1	X	0	X	1
1	0	1	0	X	X	X	X	X	X	X	X	X	X
1	0	1	1	X	X	X	X	X	X	X	X	X	X
1	1	0	0	X	X	X	X	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X	X	X	X	X

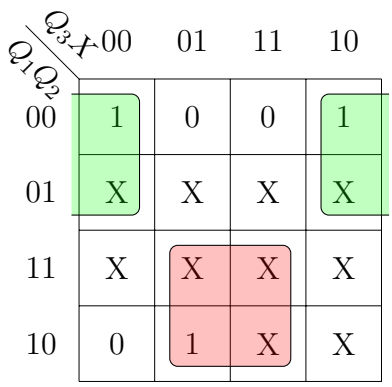
Simplify JKFF Input:

Q_1Q_2 \ Q_3X	00	01	11	10
00	0	0	0	0
01	1	0	0	0
11	X	X	X	X
10	X	X	X	X

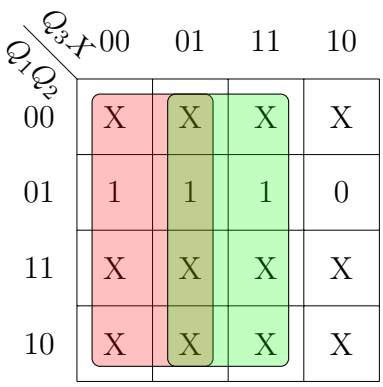
J_1

Q_1Q_2 \ Q_3X	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	X	X	X	X
10	0	1	X	X

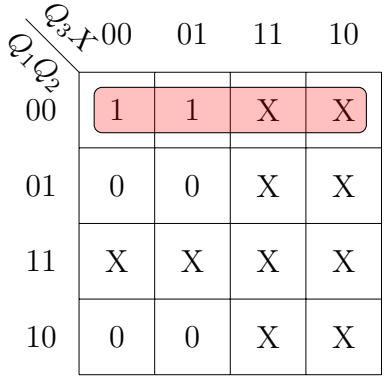
K_1



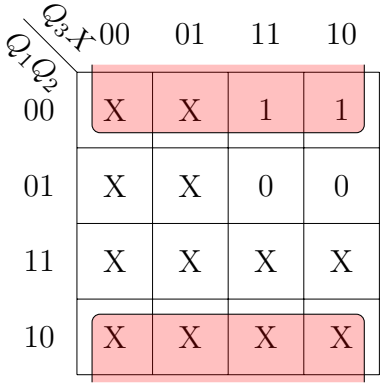
J_2



K_2



J_3



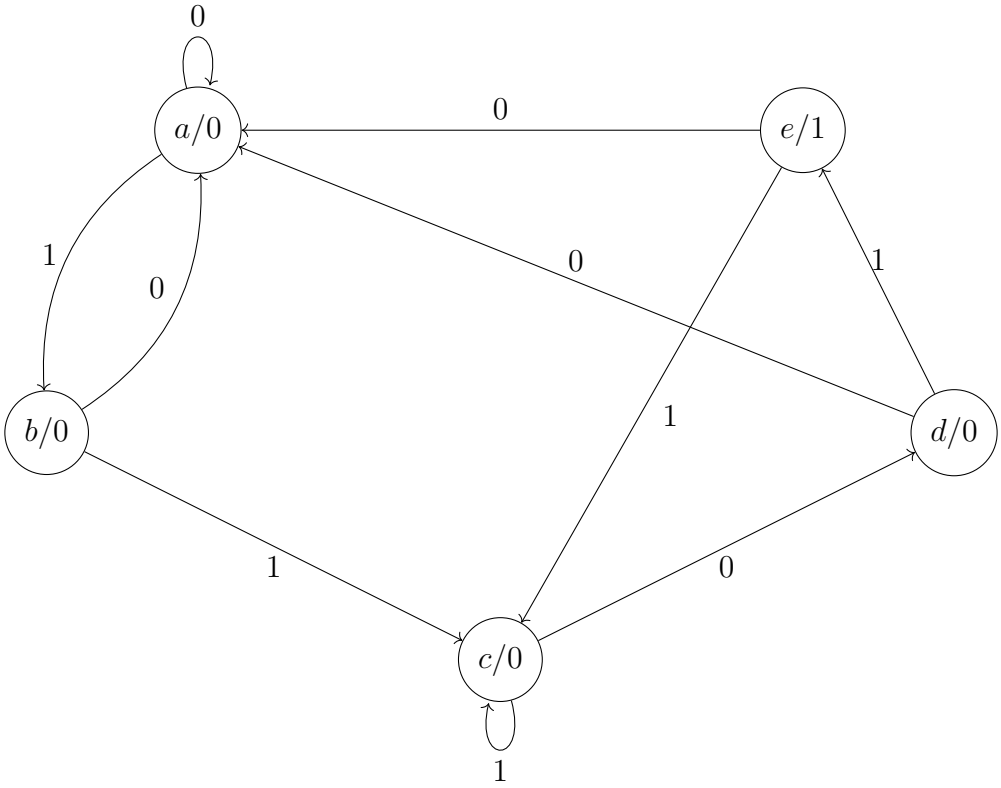
K_3

The simplified JKFF input is:

$$J_1 = Q_2Q_3'X' \qquad K_1 = X$$
$$J_2 = Q_1'X' + Q_1X \qquad K_2 = Q_3' + X$$
$$J_3 = Q_1'Q_2' \qquad K_3 = Q_2'$$

8.

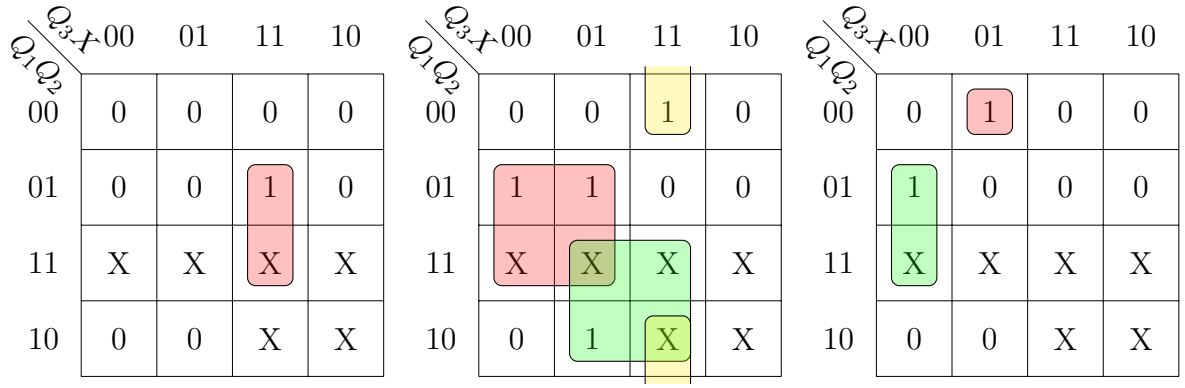
State Diagram:



Let $a = 000$, $b = 001$, $c = 010$, $d = 011$, $e = 100$.
State Table with DFF Input:

Present State			Input	Next State			DFF Input		
Q_1	Q_2	Q_3	X	Q_1	Q_2	Q_3	D_1	D_2	D_3
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	0	0	1
0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	1	0	0	1	0
0	1	0	0	0	1	1	0	1	1
0	1	0	1	0	1	0	0	1	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	1	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	1	0	0	1	0
1	0	1	0	X	X	X	X	X	X
1	0	1	1	X	X	X	X	X	X
1	1	0	0	X	X	X	X	X	X
1	1	0	1	X	X	X	X	X	X
1	1	1	0	X	X	X	X	X	X
1	1	1	1	X	X	X	X	X	X

Simplify DFF Input:



D_1 D_2 D_3

The simplified DFF input is:

$$
\begin{aligned}
D_1 =& Q_2Q_3X \\
D_2 =& Q_2Q_3' + Q_1X + Q_2'Q_3X \\
D_3 =& Q_2Q_3'X' + Q_1'Q_2'Q_3'X
\end{aligned}
$$