INF367: Introduction to Quantum Computing & Quantum Machine Learning

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Mandatory Assignment I

Autumn 2024

Available on: Tuesday, September 17th

Due on: Friday, October 4th at 10:00

Submission instructions: Please read carefully.

i. This assignment has two parts, a coding part and a problem solving part. The coding part is to be submitted digitally on `mitt.uib`, the problem solving part can be submitted either digitally or on paper (at the start of the group session). Please make sure to clearly identify your name(s) on the first page of each document/stack of paper.

ii. Please work and submit in groups of three. If you submit on paper, hand in **one version with all names on it.** For digital submission on `mitt.uib`, **the file name(s) must contain the full names of all students in the group.**

iii. You may use **English** and/or **a Scandinavian language**.

iv. In order to pass this assignment, you have to *make an honest attempt* at solving every one of the tasks. What does this mean? If you can, solve everything. If you cannot solve a task, explain how you tried to solve it and at what point you failed (and if possible: why). If you have a solution for part of the problem, or a simpler version of the problem, then do that (with a short explanation).

v. You need to pass **both mandatory assignments** to qualify for the exam. The assignments do not count in your final grade.

vi. You may be requested to meet with the teacher and/or the TA after submission to discuss your solutions. Each student must therefore be prepared to explain their submitted solutions to us. This is to verify that you haven't simply copied from somebody else.

vii. Can you cheat? Of course you can. Will you get away with it? Probably. Will it do you any good? No! You still have to pass the oral exam in the end. This assignment is part of the preparation.

viii. You should have more than enough time to complete this assignment. But still, it cannot hurt to start early!

1 Coding

Please solve the following tasks using Python and the Qiskit library. You can import other standard libraries (such as Numpy) as needed. Write all of your code in a Jupyter notebook. Use a new cell for every task and mark clearly which cell belongs to which task.

For submission, choose a file name that contains "`Quantum2024_MandatoryI_Coding`" and your full names.

1.1 Playing with Quantum Circuits

1. Implement a quantum circuit in Qiskit that produces the following state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|RRLL\rangle - |LLRR\rangle\big)$$

   (The initial state should be $|0000\rangle$). Use Qiskit's `Statevector()` method to verify that you have in fact produced the desired state. Output a drawing of the circuit. *Hint: written in the standard basis, the state $|\psi\rangle$ is:*

$$|\psi\rangle = \frac{1}{\sqrt{8}}\Big(|0001\rangle + |0010\rangle - |0100\rangle + |0111\rangle - |1000\rangle + |1011\rangle - |1101\rangle - |1110\rangle\Big)$$

2. Add a full standard measurement, run the circuit with 1000 shots and plot a histogram.

3. Construct the inverse circuit, i.e. the circuit that turns the initial state $|\psi\rangle$ into $|0000\rangle$. Comment on how you figured out how to construct it and output a drawing.

1.2 Quantum State Tomography

1. Implement the following circuit in Qiskit:

$$|0\rangle \; -\boxed{X}-\boxed{H}-\boxed{T^\dagger}-\boxed{Z}-\boxed{Y}-\boxed{Rx(1.6)}-\boxed{S}- \qquad |\psi\rangle$$

2. Retrieve the statevector at the end by using the command `Statevector()`.

3. Apply measurements in different bases at the end and run the circuit with each of them in a Qiskit simulation (1000 shots each) to approximately reconstruct the output state $|\psi\rangle$. Output drawings of each circuit with measurement.

4. Write a function `reconstruct_state()` that reconstructs a state from measurement results, as described in section 3.5 of the lecture notes. Use your function to estimate the state $|\psi\rangle$ and compare the result to the retrieved statevector.

2 Manual tasks

Please start the solution to each problem/task on a new page. You can either hand in your solutions on paper, or as a single file. We strongly encourage using LaTeX to typeset your solutions. The filename should include "`Quantum2024_MandatoryI`" and your full names.

Please make sure to also have **your name(s) clearly visible somewhere on the first page.**

2.1 Quantum States and Quantum Gates

1. Express the state

$$|\phi\rangle = \frac{1}{\sqrt{3}}\big(\,|\Phi^+\rangle + |\Phi^-\rangle - i \cdot |\Psi^-\rangle\,\big)$$

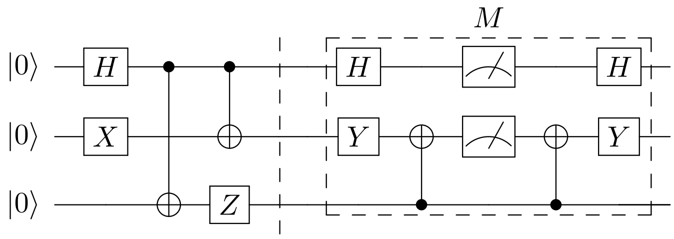
   in the standard basis. Is this state entangled?

2. Apply a Hadamard layer to $|\phi\rangle$ and compute the new state in the standard basis.

3. Now apply a CZ-gate with the bottom qubit as control qubit and compute the new state in the standard basis and the X-basis.

2.2 Measurement Operators

1. Consider the following quantum circuit:



   Express the first part of the circuit (everything left of the barrier) as one unitary operator U. Compute the matrix for U.

2. Compute the quantum state at the barrier. Is it entangled?

3. Define a measurement operator M that is realized by the Block M. You can choose your eigenvalues freely. List all eigenspaces, their dimensionalities and corresponding measurement probabilities.

4. Compute the expectation value and the posterior states.

5. Instead of M, we want to use a different measurement operator:

$$\hat{M} = 1 \cdot |L\Phi^+\rangle \langle L\Phi^+| \; + \; 2 \cdot |L\Phi^-\rangle \langle L\Phi^-| \; + \; 3 \cdot |L\Psi^+\rangle \langle L\Psi^+| \; + \; 4 \cdot |L\Psi^-\rangle \langle L\Psi^-|$$
$$+ 1 \cdot |R\Phi^+\rangle \langle R\Phi^+| \; + \; 2 \cdot |R\Phi^-\rangle \langle R\Phi^-| \; + \; 3 \cdot |R\Psi^+\rangle \langle R\Psi^+| \; + \; 4 \cdot |R\Psi^-\rangle \langle R\Psi^-|$$

   List all eigenspaces, their dimensionalities and corresponding measurement probabilities.

6. Compute the expectation value and the posterior states.

7. How would you realize $\hat{M}$ if your quantum computer could only do (partial) standard measurements? Explain your solution and provide a drawing in circuit notation. (You can draw by hand, or use the Latex package "qcircuit", or use Qiskit's `circuit.draw()` method.)