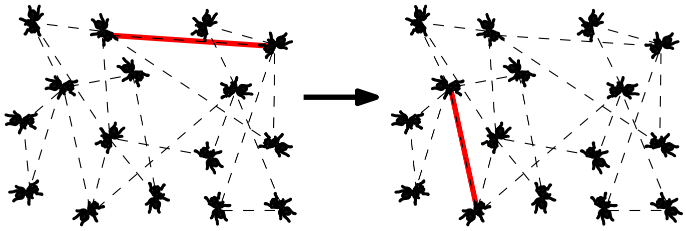
Friend or Foe?
Population Protocols can perform Community Detection

L. Becchetti[1], A. Clementi[2], **Emanuele Natale**[0], F. Pasquale[2], P. Raghavendra[3] and L. Trevisan[3]

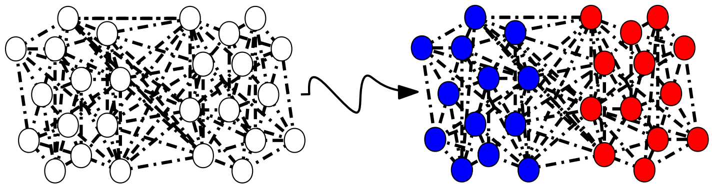
Population protocols

At each round a random edge is chosen and the two corresponding agents interact.



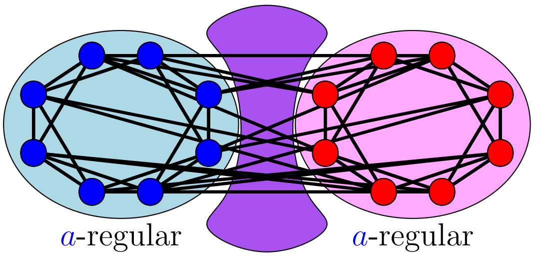
Reconstruction problem

Given graph generated by Regular Stochastic Block Model, find original partition.



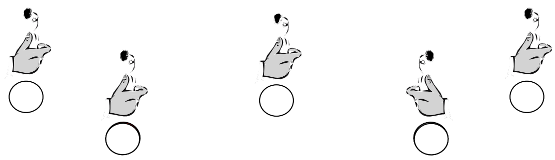
Regular Stochastic Block Model

A graph $G = (V_1 \dot{\bigcup} V_2, E)$ s.t. $|V_1| = |V_2|$, $G|_{V_1}$, $G|_{V_2} \sim$ random a-regular graphs, $G|_{E(V_1,V_2)} \sim$ random b-regular bipartite graph.

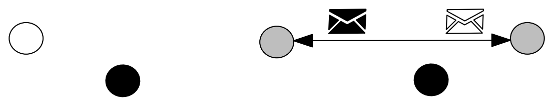


$$\underset{\textit{community}}{\boldsymbol{C}} \quad \overset{\textit{sensitive}}{\boldsymbol{S}} \quad \underset{\textit{labeling}}{\boldsymbol{L}}\ (m,T)$$

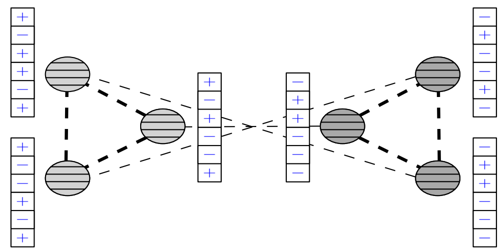
- At the outset $\mathbf{x}_u^{(0)} \sim \mathrm{Unif}(\{-1,+1\}^m)$.



- In each round, the endpoints of the random edge choose a random index $j \in [m]$ and set $\mathbf{x}_u(j) = \mathbf{x}_v(j) = \frac{\mathbf{x}_u(j)+\mathbf{x}_v(j)}{2}$;



- At the T-th update of j-th component, u sets $\mathbf{h}_u(j) = \mathbf{sgn}(\mathbf{x}_u(j))$.



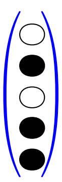
Theorem

$G = (V_1 \bigcup V_2, E)$ Regular Stochastic Block Model s.t. $d\epsilon^4 \gg b \log^2 n$, then w.h.p. $CSL(m,T)$ with $m = \Theta(\epsilon^{-1}\log n)$ and $T = \Theta(\log n)$ labels all nodes but a set U with size $|U| \leq \sqrt{\epsilon} n$, in such a way that

- nodes' labels in the same community agree on at least $5/6$ of entries, and
- nodes' labels in different communities differ in more than $1/6$ of entries.

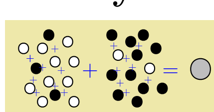
A Taste of Spectral Analysis

CSL is a **linear** dynamics

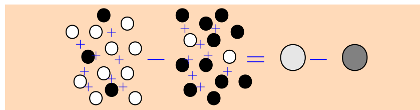
$\mathbf{x}^{(t)} =$ 

$$\boxed{\mathbf{x}^{(t)} = W^{(t)} \cdot \mathbf{x}^{(t-1)} = (W^{(t)} \cdots W^{(1)}) \cdot \mathbf{x}^{(0)}}$$

$\mathbf{E}[W] = P$ transition matrix of r.w.
eigenvec. $\mathbf{v}_1, ..., \mathbf{v}_n$, eigenval. $\lambda_1, ..., \lambda_n$.



$\frac{1}{n}\sum_{u \in V} \mathbf{x}^{(0)}(u)$



$\frac{1}{n}\sum_{u \in V_1} \mathbf{x}^{(0)}(u) - \frac{1}{n}\sum_{u \in V_2} \mathbf{x}^{(0)}(u)$

$$\boxed{\mathbf{x}^{(t)} = \frac{1}{n}(\mathbf{1}^\intercal \mathbf{x}^{(0)})\mathbf{1} + \left(\frac{a-b}{a+b}\right)^t \frac{1}{n}(\chi^\intercal \mathbf{x}^{(0)})\chi + \mathbf{e}^{(t)}}$$

[0] max planck institut informatik
[1] SAPIENZA UNIVERSITÀ DI ROMA
[2] UNIVERSITA' DEGLI STUDI DI ROMA TorVergata
[3] Berkeley UNIVERSITY OF CALIFORNIA