Stable Neo-Hookean Flesh Simulation

Bachelor Thesis

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Vorwort

Dies ist ein Vorwort

Abstract

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Inhaltsverzeichnis

Kapitel 1

Introduction

1.1 Motivation

The goal of this work is for a regular computer science student to give the necessary physical and mathematical background to understand the field of animation.

1.2 Structure

I will start with the background and then go on to the actual paper.

Kapitel 2

Background

This chapter will provide the necessary background in continuum mechanics and mathematics in order to understand the next chapters.

In this chapter we will examine the topic of the paper *Stable Neo-Hookean Flesh Simulation*. The goal of the paper was to model deformations for virtual characters that have human-like features.

2.1 Notation and Convention

At first we will declare the notation used in this thesis to avoid misunderstandings. We will use the common notation used in continuum mechanics taken from [Spe80]. Additionally we will include the declarations formulated in the paper *Stable Neo-Hookean Flesh Simulation*.

2.1.1 General Notation

Scalars are represented by regular, normal-weight variables such as a whereas tensors and matrices are represented by upper-case bold letters as for example $\mathbf{A}$. Vectors will be denoted by bold lower-case variables like $\mathbf{a}$.

2.1.2 Tensor Notation

Furthermore we will use the tensor notation used in the paper *Stable Neo-Hookean Flesh Simulation*. They decided to to define vectorization vec(.) as column-wise flattening of a matrix into a vector ([SGK18], 12:5) similar to Golub and Van Loan (2012) [**golub2012matrix**].

$$\mathbf{A} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \qquad \mathrm{vec}(\mathbf{A}) = \check{\boldsymbol{a}} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}.$$

In order to indicate that we are dealing with a vectorized matrix we will use the symbol $\check{.}$ as shown above.

Additionally we will have to deal with 4^{th} order tensors in the following form of matrix-of-matrices:

$$\mathbb{A} = \begin{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} & \begin{bmatrix} a & c \\ b & d \end{bmatrix} \\ \begin{bmatrix} a & c \\ b & d \end{bmatrix} & \begin{bmatrix} a & c \\ b & d \end{bmatrix} \end{bmatrix} = \begin{bmatrix} [\mathbf{A}_{00}] & [\mathbf{A}_{01}] \\ [\mathbf{A}_{10}] & [\mathbf{A}_{11}] \end{bmatrix}$$

These matrices are denoted by using blackboard bold.
If we now vectorize $\mathbb{A}$ we receive the following form:

$$\mathbb{A} = \mathrm{vec}(\mathbb{A}) = [\mathrm{vec}\,(\mathrm{A}_{00})\,|\mathrm{vec}\,(\mathrm{A}_{10})|\,\mathrm{vec}\,(\mathrm{A}_{01})\,|\,\mathrm{vec}\,(\mathrm{A}_{11})] = \begin{bmatrix} a & e & i & m \\ b & f & j & n \\ c & g & k & o \\ d & h & l & p \end{bmatrix} = \check{\boldsymbol{A}}$$

The advantage of this form is that we can write several expressions as a cross product which we will need later.

2.1.3 Summary

A quick overview of the used notation:

a: scalar
A: matrix or tensor
a: vector
vec($\mathbf{A}$) = $\check{\boldsymbol{a}}$: vectorized matrix

2.2 Mathematical Background

Since mathematics plays an important role in our field of interest we will build a solid background in this chapter. A basic understanding of linear algebra is assumed.

2.2.1 Matrices

At first we will discuss the physical or geometrical meaning of some common matrix properties.

2.2.2 Singular Value Decomposition

The singular value decomposition (SVD) will play an important role in the following. It is important for our application since it represents the best possible approximation of a given matrix by a matrix of low rank. This approximation can be looked at as a compression of the data given ([LM15], S. 295).

2.3 Continuum Mechanics

In this section we will give a broad introduction the field of Continuum Mechanics. In Continuum Mechanics we are less interested in small particles like atoms or molecules of an object but concentrate on pieces of matter which are in comparison very large. We are therefore concerned with the mechanical behavior of solids and fluids on the macroscopic scale ([Spe80], p. 1).

2.4 Deformation

Graphically we can imagine a deformation with the help of a deformation map. In Fig. 2.1 we have on the left side an ellipse that signifies an object in its rest state. On the right side in the same image we can see the ellipse in a deformed state. We can map each point from its rest state to the deformed one with the help of the function ϕ.

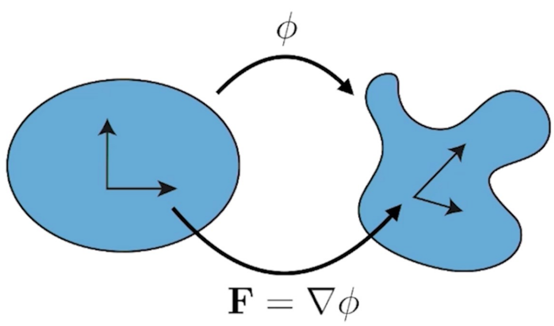


Abbildung 2.1: Deformation Map [Pix]

When applying a force over an object naturally the object itself undergoes a deformation. In the following we will be consistent with most previous literature in continuum mechanics and use the term strain as a measure of deformation and stress as the force per unit area.

$$\begin{aligned} Strain &= measure\ of\ deformation \\ Stress &= force\ per\ unit\ area \end{aligned}$$

2.5 Deformation Gradient

The deformation gradient F is also shown in Fig. 2.1. It offers us a measurement of the deformation. With its help we can amongst other things calculate the volume and length change an object undergoes during a deformation. For our needs we define the deformation gradient as followed:

$$\mathbf{F} = \left[f_0 \middle| f_1 \middle| f_2 \right] = \begin{bmatrix} f_0 & f_3 & f_6 \\ f_1 & f_4 & f_7 \\ f_2 & f_5 & f_8 \end{bmatrix}$$

Measure for the deformation, length and volume change etc. Nonlinear deformations http://www.continuummechanics.org/deformationgradient.html also add some examples

2.6 Material Constants

Naturally the properties of the material the object consists of play an important rule in the deformation process. The two constants μ and λ that are crucial for us are called *Lamé Parameters*. The formula in which they appear is called *Poisso's Ratio* and is of the following form:

$$\sigma = \frac{\lambda}{2(\lambda + \mu)} \in [-1, 0.5]$$

The poisson's ratio is of importance for us since it characterizes the materials resistance to volume change. Usually the poisson's ratio of a material is positive.
For the simulation of human-like flesh we have to choose a poisson's ratio that is almost 0.5 to get realistic results.
further reading: http://silver.neep.wisc.edu/ lakes/PoissonIntro.html

2.7 Deformation Energy

In order to get a convincing simulation of high quality we must choose an appropriate energy. In the case of modelling deformations on human-like characters we have to choose an elastic energy. The key property that makes an energy elastic is that if all the forces that are applied over an object add up to zero the object must come back to its rest shape.
The energy then has to be minimized to get the results we want.

Definition 1. *This is a definition.*

To include: Piola-Kirchhoff Stress, Cauchy Green invariant, polar decomposition, cauchy green tensor

Kapitel 3

Paper

In this chapter we will examine the topic of the paper *Stable Neo-Hookean Flesh Simulation*. The goal of the paper was to model deformations for virtual characters that have human-like features. They concentrated on the deformation energy.

3.1 Energy Formulation

For our needs we need a hyperelastic energy that is stable in the following four important ways:

- Inversion stability
- Reflection stability
- Rest stability
- Meta-stability under degeneracy

TODO: explain each step

3.1.1 Previous Work

Here comes previous work in neo-hookean energy formulation. What is neo-hookean and why do we need it here? And what is wrong with each one.

3.1.2 Stable Neo-Hookean Energy

Conclude to the energy proposed in the paper.

3.2 Energy Analysis

Calculations and Herleitungen

3.2.1 First Piola-Kirchhoff Stress (PK1)

Explain.

3.2.2 The Energy Hessian Terms

Calculations

3.2.3 The Tikhonov, Mu, and Gradient Terms

Calculations

3.2.4 The Volume Hessian

Calculations

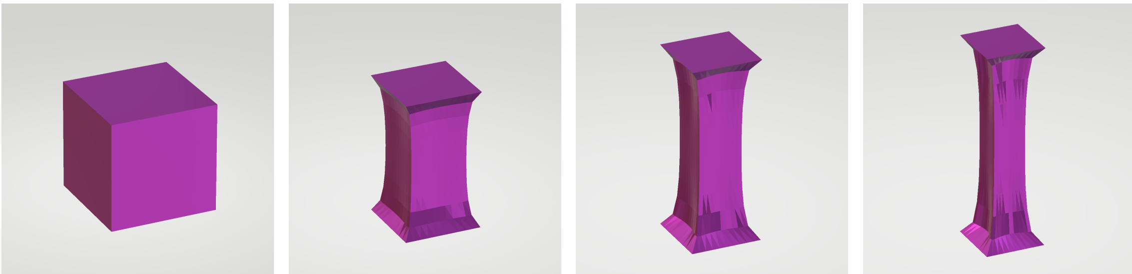
3.2.5 The Complete Eigensystem

Calculations

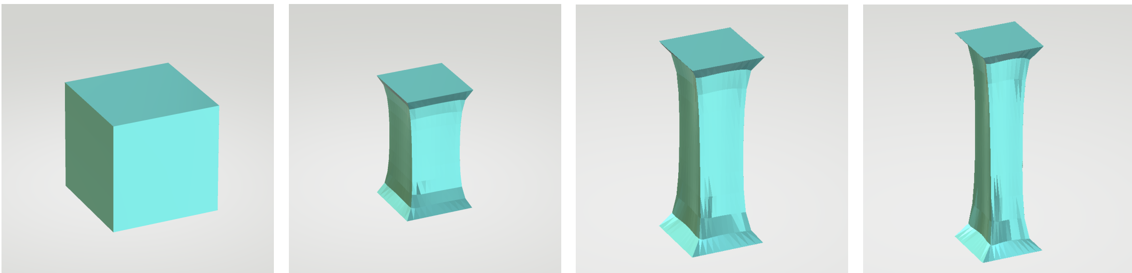
3.3 Experiments with the Code

The authors of the paper *Stable Neo-Hookean Flesh Simulation* [SGK18] kindly provided the implementation for an application of their formulated energy. In this code they implemented the stretch test on a cube. The output were 26 static images with show the deformation in 25 steps. TODO: EXPLAIN HOW THEY DID IT IN SIMPLE WORDS

The following images show the stretch test with $\mu = 1.0$, $\lambda = 10.0$ and a resolution of 10.0 on a tetrahedral and a hexahedral mesh:



(a) Stretch test on a hexahedral mesh



(b) Stretch test on a tetrahedral mesh

Abbildung 3.1: Stretch test performed on a cube with (a) a hexahedral mesh and (b) a tetrahedral mesh

3.4 Discussion

Stuff, Taylor approx.

Abbildungsverzeichnis

Tabellenverzeichnis

Quelltextverzeichnis

Literature

[LM15] Joerg Liesen und Volker Mehrmann. *Linear algebra*. 1st ed. 2015. Springer International Publishing, Switzerland 2015: Springer, Cham, 2015. ISBN: 978-3-319-24344-3.

[Spe80] A. J. M. Spencer. *Continuum Mechanics*. 2004. Aufl. 31 East 2nd Street, Mineola, N.Y. 11501: Dover Publications, Inc., 1980. ISBN: 0-486-43594-6 (pbk.)

Images Resources

[Pix] Pixar. *Deformation Map*. URL: https://dl.acm.org/ft_gateway.cfm?id=3180491&ftid=2009597.

Anhang A

A.1 Diagramm

A.2 Tabelle

A.3 Screenshot

A.4 Graph

Eigenständigkeitserklärung

Hiermit versichere ich, dass ich die vorliegende Masterarbeit selbstständig und nur unter Verwendung der angegebenen Quellen und Hilfsmittel verfasst habe. Die Arbeit wurde bisher in gleicher oder ähnlicher Form keiner anderen Prüfungsbehörde vorgelegt.

Stadt, den xx.xx.xxxx

Max Mustermann