UNIVERSITÄT
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Stable Neo-Hookean Flesh Simulation

Bachelor Thesis

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Institute of Computer Science

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Vorwort

Dies ist ein Vorwort

Abstract

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Chapter 1

Introduction

"Animation offers a medium of story telling and visual entertainment which can bring pleasure and information to people of all ages everywhere in the world."
- Walt Disney

1.1 Motivation

With steadily increasing computational power the demand of better results is constantly increasing. Especially in the field of animation and simulation we are no longer happy with mediocre results. In the entertainment sector the gaming industry and animation studios like Pixar© or Disney© brought us games and movies of highest quality. Both of them have made groundbreaking progress over the years. This is easily seen when we compare today's work with that ten years ago.

As always we have different requirements for each use. In some cases we want to exaggerate a movement or a reaction in a certain way. We can for example create a massive explosion in a movie that would not be half as spectacular in the real world.

In other scenarios we want to come as close as possible to reality. For instance we want an animated character to move and physically interact with its environment as a real human being would. Otherwise our brain would immediately recognize that some things do not add up. The goal here is to bring characters quite literally to life. We can add small details like visible breathing and small wrinkles to have an even more convincing effect. The goal is to create the illusion of a character with personality, thought and emotions. In order to achieve this effect we need the character to move and react nearly physically correct.

In the paper *Stable Neo-Hookean Flesh Simulation* [SGK18] the authors addressed exactly the problem of making an animated movement of a human-like character look as natural as possible. In order to animate a physical movement we first need to understand the physics behind it which lies in the field of continuum mechanics. Unfortunately for most of us it has yet to be learned. The goal of this thesis is for a regular computer science student to give the necessary physical and mathematical background to understand the field of animation and maybe make a contribution to the field.

1.2 Structure

Following up I will give a brief overview of the necessary mathematical background and deliver an introduction in continuum mechanics. Next up I will go through the ideas made in the paper mentioned and include some calculations and visualisations that help for a better understanding.

TODO: Adjust the introduction according to additions in text. Improve quote at beginning. Maybe add some images taken from Incredibles 2 for better visualisation?

Chapter 2

Background

The goal we are striving for is to animate human-like characters. In order to narrow it down even further we concentrate on the behaviour of the flesh of the character. For understanding the thematics of simulating human-like flesh it is necessary to have a basic mathematical background and knowledge of continuum mechanics. The goal of this chapter is to deliver an understanding in the topics mentioned.

2.1 Notation and Convention

At first we will declare the notation used in this thesis to avoid misunderstandings. We will use the common notation used in continuum mechanics taken from the book *Continuum Mechanics* [Spe80]. Additionally we will include some more specific declarations formulated in the paper *Stable Neo-Hookean Flesh Simulation* [SGK18].

2.1.1 General Notation

Scalars are represented by regular, normal-weight variables such as a whereas tensors and matrices are represented by upper-case bold letters as for example $\mathbf{A}$. Vectors will be denoted by bold lower-case variables like $\mathbf{a}$.

2.1.2 Tensor Notation

Furthermore we will use the tensor notation used in the paper *Stable Neo-Hookean Flesh Simulation*. They decided to to define vectorization vec(.) as column-wise flattening of a matrix into a vector ([SGK18], 12:5) similar to Golub and Van Loan (2012) [GV12].

In order to indicate that we are dealing with a vectorized matrix we will use the symbol $\check{\cdot}$ as shown in the following equation:

$$\mathbf{A} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \qquad \mathrm{vec}(\mathbf{A}) = \check{\boldsymbol{a}} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}.$$

Additionally we will have to deal with 4^{th} order tensors in a form of matrix-of-matrices. These matrices are denoted by using blackboard bold:

$$\mathbb{A} = \left[\begin{array}{cc} \begin{bmatrix} a & c \\ b & d \end{bmatrix} & \begin{bmatrix} a & c \\ b & d \end{bmatrix} \\ \begin{bmatrix} a & c \\ b & d \end{bmatrix} & \begin{bmatrix} a & c \\ b & d \end{bmatrix} \end{array} \right] = \left[\begin{array}{cc} [\mathbf{A}_{00}] & [\mathbf{A}_{01}] \\ [\mathbf{A}_{10}] & [\mathbf{A}_{11}] \end{array} \right]$$

If we now vectorize $\mathbb{A}$ we receive the form described in the following:

$$\mathbb{A} = \mathrm{vec}(\mathbb{A}) = [\mathrm{vec}\,(\mathrm{A}_{00})\, |\mathrm{vec}\,(\mathrm{A}_{10})|\, \mathrm{vec}\,(\mathrm{A}_{01})\, |\, \mathrm{vec}\,(\mathrm{A}_{11})]$$

This term above is equivalent to the following notation:

$$\mathbb{A} = \begin{bmatrix} a & e & i & m \\ b & f & j & n \\ c & g & k & o \\ d & h & l & p \end{bmatrix} = \check{\boldsymbol{A}}$$

The advantage of this form is that we can write several expressions as a cross product. We will need this property later to simplify complicated expressions and calculations.

2.1.3 Summary

A quick overview of the notation used so far:

> a: Scalar
> $\mathbf{A}$: Matrix or tensor
> $\mathbf{a}$: Vector
> $\text{vec}(\mathbf{A}) = \check{\boldsymbol{a}}$: Vectorized matrix

2.2 Mathematical Background

Since mathematics play an important role in our field of interest we need to build a solid background before we go further into the physics part. This chapter should cover all the important concepts used later in the calculations. A basic understanding of linear algebra is assumed.

2.2.1 Matrices

At first we will discuss the physical or geometrical meaning of some common matrix properties.

2.2.2 Singular Value Decomposition

The singular value decomposition (SVD) will play an important role in the following. It is important for our application since it represents the best possible approximation of a given matrix by a matrix of low rank. This approximation can be looked at as a compression of the data given ([LM15], S. 295).

TODO: Complete this section. Add lemmas and theorems used afterwards in the calculations. Possible adjustments may come at the end.

2.3 Continuum Mechanics

In this section we will give a broad introduction into the field of Continuum Mechanics. In Continuum Mechanics we are less interested in small particles like atoms or molecules of an object but rather concentrate on pieces of matter which are in comparison very large. We are therefore concerned with the mechanical behavior of solids and fluids on the macroscopic scale ([Spe80], p. 1).

2.4 Deformation

When applying a force over an object naturally the object itself undergoes a deformation. In the following we will be consistent with most previous literature in continuum mechanics and use the term strain as a measure of deformation and stress as the force per unit area.

$$\begin{aligned} &\textit{Strain} = \textit{measure of deformation} \\ &\textit{Stress} = \textit{force per unit area} \end{aligned}$$

TODO: Decide whether to include the above paragraph and how to connect it better to the paper.

Now let us have a look at a deformation in a rather mathematical sense. Graphically we can imagine a deformation with the help of a two dimensional deformation map as shown in Fig. 2.1. Here we have on the left side an ellipse that represents an object or material in its rest state. A function ϕ maps this rest state of the ellipse to a deformed state shown in Fig. 2.1 on the right side. Mathematically spoken this means that we can map each point of a chosen object from its rest state to a deformed one.

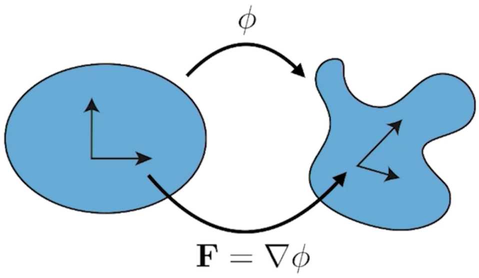

Figure 2.1: Deformation Map [Pix]

In conclusion the function ϕ describes our deformation. If we derive ϕ we can calculate the deformation gradient $\mathbf{F}$ which serves as a measure of the deformation in the following sense:

$$\begin{aligned}
&\text{Deformation gradient:} && \mathbf{F} = \nabla\,\phi \\
&\text{Length changes:} && I_C = \operatorname{tr}\left(\mathbf{F}^T\mathbf{F}\right) \\
&\text{Volume changes:} && J = \det(\mathbf{F})
\end{aligned}$$

TODO: Adjust so the equal sign is at the same place in each column.

2.4.1 Deformation Gradient

The deformation gradient $\mathbf{F}$ offers us as said previously a measurement of the deformation. With its help we can amongst other things calculate the volume and length change an object undergoes during a deformation. For our needs we define the deformation gradient exactely as the authors of the paper [SGK18] did:

$$\mathbf{F} = \left[\, f_0 \,\middle|\, f_1 \,\middle|\, f_2 \,\right] = \begin{bmatrix} f_0 & f_3 & f_6 \\ f_1 & f_4 & f_7 \\ f_2 & f_5 & f_8 \end{bmatrix}$$

TODO: Add some examples. Already include definitions of the paper?
Some more sources:
http://www.continuummechanics.org/deformationgradient.html

2.5 Material Constants

When we look at a deformation of an object we need to consider the material the object consists of. A material can be very stiff like steel or easily deformable ones like robber.

The two constants μ and λ that are crucial for us are called *Lamé Parameters*. With the help of these two constants we can calculate the *Poisson's Ratio*:

$$\sigma = \frac{\lambda}{2(\lambda + \mu)} \in [-1, 0.5]$$

The poisson's ratio is of importance for us because it characterizes the materials resistance to volume change. Usually the poisson's ratio of a material is positive. A negative value would mean that the material becomes wider in the cross section when it is stretched. This behaviour is very uncommon in nature. Here are some examples of the poisson's ratio of common materials:

Material	Poisson's ratio
rubber	0.4999
titanium	0.265-0.34
glass	0.18–0.3
cork	0.0

Table 2.1: Different materials with their poisson's ratio

For the simulation of human-like flesh we have to choose a poisson's ratio that is almost 0.5 in order to get realistic results.

TODO: Add some examples and images. Explain poisson's ratio a bit better. Why must poisson's ratio be 0.5? Add some references. Make table more beautiful. Add reference to table, now it is wikipedia (https://en.wikipedia.org/wiki/Poisson
Further reading: http://silver.neep.wisc.edu/ lakes/PoissonIntro.html

2.6 Deformation Energy

In order to get a convincing simulation of high quality we must choose an appropriate energy. In the case of modelling deformations on human-like characters we have to choose an elastic energy. The key property that makes an energy elastic is that if all the forces that are applied over an object add up to zero the object must come back into its rest shape.
The energy then has to be minimized in order to get the results we want.

Definition 1. *This is a definition.*

TODO: Add some exaples and visualisations. What exactly is it and what role does it play in a deformation process?
To include: Piola-Kirchhoff Stress, Cauchy Green invariant, polar decomposition, cauchy green tensor

Chapter 3

Paper

In this chapter we will examine the topic of the paper *Stable Neo-Hookean Flesh Simulation*. The goal of the paper was to model deformations for virtual characters that have human-like features. They concentrated on the deformation energy.

3.1 Energy Formulation

For our needs we need a hyperelastic energy that is stable in the following four important ways:

- Inversion stability
- Reflection stability
- Rest stability
- Meta-stability under degeneracy

TODO: explain each step

3.1.1 Previous Work

Here comes previous work in neo-hookean energy formulation. What is neo-hookean and why do we need it here? And what is wrong with each one.

3.1.2 Stable Neo-Hookean Energy

Conclude to the energy proposed in the paper.

3.2 Energy Analysis

Calculations and Herleitungen

3.2.1 First Piola-Kirchhoff Stress (PK1)

Explain.

3.2.2 The Energy Hessian Terms

Calculations

3.2.3 The Tikhonov, Mu, and Gradient Terms

Calculations

3.2.4 The Volume Hessian

Calculations

3.2.5 The Complete Eigensystem

Calculations

3.3 Experiments with the Code

The authors of the paper *Stable Neo-Hookean Flesh Simulation* [SGK18] kindly provided the implementation for an application of their formulated energy. In this code they implemented the stretch test on a cube. The output were 26 static images with show the deformation in 25 steps.

TODO: Explain how the code is implemented in simple words and how the energy is taken in account with the poisson's ratio.

The following images show the stretch test with $\mu = 1.0$, $\lambda = 10.0$ and a resolution of 10.0 on a tetrahedral and a hexahedral mesh:

(a) Stretch test on a hexahedral mesh

(b) Stretch test on a tetrahedral mesh

Figure 3.1: Stretch test performed on a cube with (a) a hexahedral mesh and (b) a tetrahedral mesh

3.4 Discussion

Stuff, Taylor approx.

List of Figures

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Bibliography

[GV12] H Golub Gene and F Van Loan Charles. *Matrix computations. Vol. 3*. 2012.

[LM15] Joerg Liesen and Volker Mehrmann. *Linear algebra*. 1st ed. 2015. Springer International Publishing, Switzerland 2015: Springer, Cham, 2015. ISBN: 978-3-319-24344-3.

[SGK18] Breannan Smith, Fernando De Goes, and Theodore Kim. "Stable Neo-Hookean Flesh Simulation". In: *ACM Trans. Graph.* 37.2 (Mar. 2018), 12:1–12:15. ISSN: 0730-0301. DOI: `10.1145/3180491`. URL: `http://doi.acm.org/10.1145/3180491`.

[Spe80] A. J. M. Spencer. *Continuum Mechanics*. 2004th ed. 31 East 2nd Street, Mineola, N.Y. 11501: Dover Publications, Inc., 1980. ISBN: 0-486-43594-6 (pbk.)

Online Sources

[Pix] Pixar. *Deformation Map*. URL: https://dl.acm.org/ft_gateway.cfm?id=3180491&ftid=2009597.

Figure Sources

[Pix] Pixar. *Deformation Map.* URL: https://dl.acm.org/ft_gateway.cfm?id=3180491&ftid=2009597.

Erklärung

gemäss Art. 28 Abs. 2 RSL 05

Name/Vorname: ....

Matrikelnummer: ....

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