

UNIVERSITY OF FRIBOURG

MASTER THESIS

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**Non-Differentiable Function  
Approximation: Beyond Neural Networks**

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*Author:*  
Corina Masanti

*Supervisor:*  
Dr. Giuseppe Cuccu

*Co-Supervisor:*  
Prof. Dr. Philippe  
Cudré-Mauroux

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eXascale Infolab  
Department of Informatics



# Abstract

Corina Masanti

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Neural networks as generic function approximators can solve many challenging problems. Regardless, they can only be applied successfully for a suited problem structure. A large part of the success of neural networks can be attributed to the efficiency of backpropagation which requires estimating accurate gradients. However, there are many areas where calculating an accurate gradient is non-trivial, including problems in reinforcement learning. In contrast, black-box optimization techniques are less limiting. They presume no constraints on the problem structure, the model, or the solution. With this flexibility, we can study alternative models to neural networks that are yet unexplored in the context of reinforcement learning. This thesis aims to achieve comparable with a function approximator other than neural networks. To this end, I analyzed the performance of polynomial and binary tree models on reinforcement learning benchmark problems. I compared these results to those of neural networks. In the experiments, I used random weight guessing to analyze the models. The results show that the binary tree model can produce results comparable to neural networks and even outperforms them while providing essential advantages. The simple structure of the binary tree model makes hyperparameter tuning straightforward and provides high interpretability.

**Keywords:** black-box optimization, reinforcement learning, binary trees, neural networks, random weight guessing



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## Chapter 1

# Introduction

### 1.1 Reinforcement Learning Control Problems

Control problems appear in the context of both reinforcement learning and control theory. Thus, we often see vocabulary specific to control theory in the context of reinforcement learning. A control problem, generally explained, is a dynamic system described by state variables. Controls or actions determine how the system will behave in the future. The goal of a control problem is to work out a strategy that causes the system to terminate in the desired target state. Reinforcement learning and approaches in control theory offer a framework to solve such control problems (Buşoniu et al., 2018). Even though the two fields embark on the same problem, the research communities remained disjoint primarily, leading to different approaches to the same problem. Reinforcement learning often makes model-free predictions from data, whereas control theory often defines well-specified models (Recht, 2018). This thesis will focus on reinforcement learning frameworks to solve control problems.

Reinforcement learning is an area of machine learning that involves learning the optimal behavior to maximize a reward signal inside a dynamic environment. Trial-and-error interactions with the environment mark the learning process. The learner does not know which actions lead to which rewards until he tries them (Kaelbling, Littman, and Moore, 1996). Reinforcement learning differs from supervised learning, where learning is achieved through a training set with labeled examples. Each sample of the training data represents a description of the situation and a label that marks how the system should react. The learning process aims to generalize the optimal actions given a specific situation so that the system acts correctly in situations outside the training set. In reinforcement learning, there are no labels available beforehand. In addition, we can also not classify reinforcement learning as unsupervised learning, where we typically try to find a hidden structure in a set of unlabeled data. With reinforcement learning, we do not try to find some structure in the data but solely try to maximize the reward signal. Therefore, reinforcement learning is considered a third machine learning paradigm alongside supervised learning, unsupervised learning, and maybe other paradigms. One additional challenge we do not see in the other branches of machine learning is the trade-off between *exploration* and *exploitation*. For a high reward, the reinforcement learning agent should take an action that it has tried in the past and was effective. It should therefore exploit the knowledge gained previously. However, to discover this beneficial action, it has to explore actions it has not taken before (Sutton and Barto, 2018).

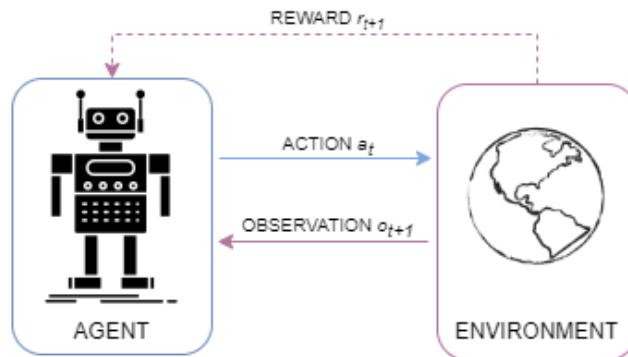
We can represent many problems in machine learning, including reinforcement learning problems, as a function mapping an input space into an output space. The output space or *search space* denotes the space of all feasible solutions. In reinforcement learning, the output space contains any action that could be taken given a

specific situation. The underlying function is the output of the learning process of an algorithm and is often called the *target function*. Optimally, we would derive the formula of the function explicitly. However, even though we suspect such a function exists, we usually do not have enough information to derive a formula directly. Especially in reinforcement learning, the available information is very sparse. Thus, we aim to approximate the target function with *function approximators* by using the available data. Hereafter, I will often talk about models to solve a reinforcement learning task. The models generally represent function approximators to solve a given problem.

To overcome the mentioned difficulties unique to reinforcement learning, we need to experiment with alternative models and strategies than those used for supervised or unsupervised learning.

## 1.2 Classic Reinforcement Learning

In reinforcement learning, we have an agent, also called a controller for control problems, and an environment. The agent can take on actions to interact with the environment. On the other hand, the environment always holds a specific state that the agent's actions can alter. The agent's goal in classic reinforcement learning is to learn a policy  $\pi$  that drives the system into the desired state by taking the right actions. For example, the goal might be to solve skill games like Ball-in-a-Cup or Kendama (Kober, Mohler, and Peters, 2010). As mentioned before, we do not have labeled data beforehand as we do in supervised learning problems. We only get a scalar reward signal that indicates how well the agent performs. The agent's goal is to maximize the cumulative rewards received by the environment. Therefore, the agent needs to learn optimal behavior only through trial-and-error interaction with the environment. This aspect makes the problem structure much more challenging. Figure 1.1 illustrates the loop between the agent and the environment. The agent



**FIGURE 1.1: Interaction loop between an reinforcement learning agent and the environment.** At timestep  $t$ , the agent receives the observation  $o_t$  from the environment. The agent decides to select action  $a_t$  and interacts with the environment. The environment changes and returns the next observation  $o_{t+1}$  and the reward  $r_{t+1}$  to the agent.

has received the observation  $o_t$  from the environment and therefore knows the environment's current state. The agent then performs some action  $a_t$  in the environment. The environment's state changes according to the received action and returns an observation  $o_{t+1}$  indicating the current state of the environment and a reward  $r_{t+1}$ . One interaction between the agent and the environment represents one *timestep*.

The rewards correspond to an underlying reward function specific to the environment. The reward function is a function that maps the current state of the environment, the current action, and the subsequent state into a scalar value. It can be deterministic or can contain a random component making it stochastic. Depending on the problem, the reward can be sparse or dense. Sparse reward means that the reward function returns zero in most of its domain and only returns a meaningful reward in a few rare settings or at the end of the experiment. That introduces an immense challenge since we cannot know which actions led to success or failure. Optimization problems with this setting are generally challenging to solve. Dense reward environments return a meaningful reward at almost every step, making the optimization process much more manageable.

Learning a policy that maximizes cumulative rewards is generally non-trivial. If the learning algorithm is not explorative enough, we might get stuck in a local optimum and never reach the goal. However, if there is too much exploration, we receive less reward. Thus, there needs to be a balance between exploration and exploitation. In addition, there is often a time delay for the reward. The consequence of an action might not be immediately visible to the agent. This problem is known as the credit-assignment problem in the literature (Sutton and Barto, 2018).

We can solve reinforcement learning problems with methods based on value functions or policy search. The value function predicts the future reward of landing in a specific state. It indicates how good or bad being in a particular state is. The agent chooses the best action in the state with methods based on value functions. Methods based on policy search directly search for the optimal policy  $\pi^*$  and do not need to maintain a value function model (Arulkumaran et al., 2017).

As interest in neural networks and deep learning increased, deep reinforcement learning, which uses a neural network to estimate policies or value functions, attracted more attention.

### 1.2.1 Neural Networks as Models

Artificial neural networks are machine learning algorithms inspired by the functionalities of the human brain. They are applied successfully in many areas including in reinforcement learning problems. In 1958, Frank Rosenblatt implemented the perceptron (Rosenblatt, 1958). The perceptron represents a simplified version of a biological neuron. It was an early type of what later should be called artificial neural networks. On this idea, the concept of neural networks was born and further developed throughout the years.

Neural networks in the modern sense consist of connected neurons or nodes that imitate the biological neurons of the brain, sending signals to each other. A neuron receives one or multiple input values and calculates its output value. The information is then passed onto the next neuron. Figure 1.2 illustrates the structure of one artificial neuron. To calculate the output value, the neuron receives two input values  $x_1, x_2$  and one bias term  $b$ . We often use a bias with neural networks, allowing us to make affine transformations to the data. The neuron uses the assigned weights  $\mathbf{w}$  to calculate the weighted sum of input values. In more detail, the calculation is the following:

$$\mathbf{w}^T \mathbf{x} = w_1 x_1 + w_2 x_2 + b$$

For convenience, the following images of neural networks will not show the bias term explicitly.

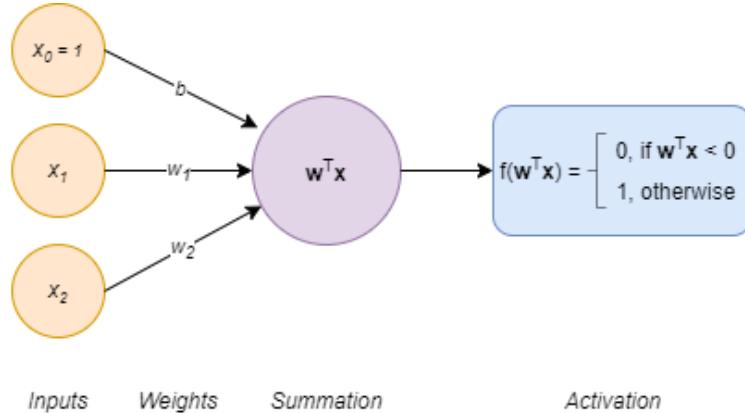


FIGURE 1.2: Illustration of an artificial neuron. The image illustrates how an artificial neuron works. The neuron receives two input values  $x_1, x_2$  and one bias term  $b$ . It calculates the weighted sum of the inputs. The activation function then defines the neuron's output passed to the next neuron.

The activation function  $f$  then defines the output value that should be passed into the next neuron. This example uses a step function with a threshold of 0. Many other functions can be used as activation functions. Another commonly used function is the sigmoid function. Figure 1.3 shows the formula and a plot of the logistic sigmoid function in 2D. The sigmoid function is a mathematical function that maps

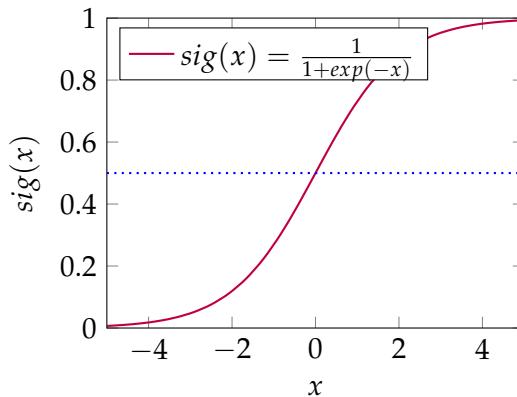


FIGURE 1.3: Sigmoid function. The figure shows a plot of the logistic sigmoid function. The function maps an arbitrary input space into the range between 0 and 1. The output of the function can be interpreted as a probability. The sigmoid function helps scale data into a meaningful value.

an arbitrary input space into an output space with a small range, for example, 0 and 1. The function has a characteristic S-shaped curve. We can interpret the output space of the sigmoid function as a probability.

In a neural network, the neurons are arranged in connected layers. A network has at least an input and an output layer. We can further expand it with one or more hidden layers. Depending on the model, the connectivity between the layers differs. Figure 1.4 shows a sketch of a network with three hidden layers that are fully connected.

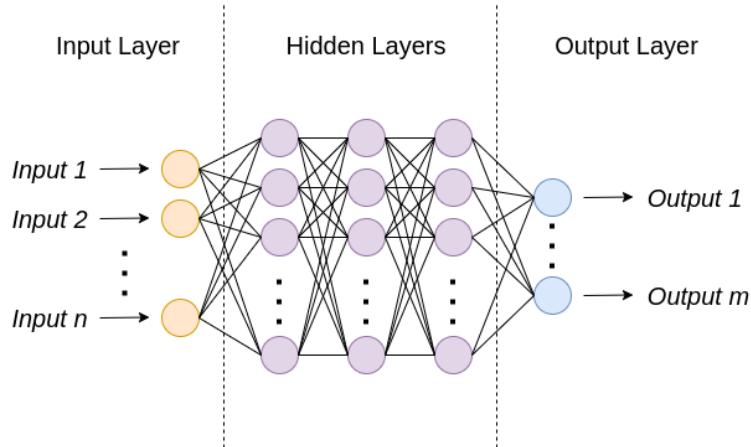


FIGURE 1.4: **Sketch of a neural network.** The image shows a sketch of a neural network with three fully connected hidden layers. The network expects  $n$  inputs and has  $m$  possible outputs.

A neural network with one or more hidden layers defines a non-linear function. It is theoretically able to represent any function. Thus, neural networks are generic function approximators. In addition, neural networks are highly expressive and flexible. In the context of reinforcement learning, they can be used to approximate a value function or a policy function. A large part of the success of neural networks can be traced back to the use of *backpropagation*. Backpropagation uses information from the previous epoch (i.e., iteration) to adjust the network weights using the error gradient. Deep reinforcement learning combines reinforcement learning with the methods of deep learning. Deep neural networks have multiple hidden layers, allowing them to represent data with multiple levels of abstractions (LeCun, Bengio, and Hinton, 2015). Deep reinforcement learning has been used successfully in many applications, including robotics, video games, and computer vision (François-Lavet et al., 2018).

The backpropagation algorithm's efficiency in calculating the gradient of an objective function is a crucial component of deep learning's success. However, there are certain limitations. For the backpropagation algorithm to be efficient, it relies on calculating accurate gradients. In fields like reinforcement learning, where we only have a reward signal that might be time delayed, this is a non-trivial task. In addition, there is a risk of getting stuck in a local optimum (Ha, 2017). These limits raise the question of which alternative models or techniques could be used that overcome these difficulties.

### 1.3 OpenAI Gym

The research community requires high-quality benchmarks to compare algorithms and models applied to problems in reinforcement learning. There were already a few released benchmarks like the Arcade Learning Environment (Bellemare et al., 2013) or the RLLab benchmark for continuous control (Duan et al., 2016). OpenAI Gym<sup>1</sup> combined the elements of these existing benchmarks in one toolkit. It provides a collection of tasks (called *environments*) with varying levels of difficulties and challenges. The interface is the same for all environments, making it very convenient to

<sup>1</sup><https://www.gymlibrary.ml/>

use. The environments represent general reinforcement learning problems, including control problems, widely used in the literature to compare different approaches. To interact with an environment, we first have to initialize it. We can do this with the predefined function `make()`. The following Python code shows the initialization of the environment `CartPole`:

```
1 import gym
2 env = gym.make("CartPole-v0")
```

In the setting of reinforcement learning, we assume that an agent is acting inside the dynamic environment. In each step, the agent can take some action and receive a reward and a new observation from the environment. The environment predefines both the action and observation space. In practice, an agent returns an action  $a_0$  with which we call the function `step()` of the environment. The next code snippet shows an example of this behavior:

```
1 observation, reward, done, info = env.step(a0)
```

The observation and reward returned by the environment apply to the current timestep. OpenAI Gym focuses on episodic reinforcement learning, where the agent acts in a series of episodes with a fixed length. The initial state of the environment in each episode is sampled randomly. An episode ends when specific termination criteria are met. For example, when the task is solved or when we max out on timesteps (Brockman et al., 2016).

## 1.4 Direct Policy Search

Although using value functions to solve reinforcement learning problems has worked well in many applications, there are limitations. If the state space is not entirely observable, convergence problems can arise (El-Fakdi, Carreras, and Palomeras, 2005). Additionally, in traditional value-based methods like Q-learning or temporal difference learning, the value function is computed iteratively with the help of bootstrapping. That often results in a bias in the quality assessment of state-action pairs for continuous state spaces. Another disadvantage of these methods is that the value functions are often discontinuous (Deisenroth, Neumann, Peters, et al., 2013).

Studies suggest that directly approximating a policy can be easier and delivers better results (Sutton et al., 1999; Anderson, 2000). This alternative method to methods based on value functions is called *direct policy search*. With direct policy search, the algorithm directly searches in the search space of policy representations discarding any value function (Wierstra, 2010). Instead of approximating a value function, we approximate the policy directly with an independent function approximator. Thus, we directly approximate the action-state mapping.

## 1.5 Neuroevolution

Neuroevolution is a method that we can use for direct policy search. Like deep reinforcement learning, we employ a neural network as our model. In deep reinforcement learning, we typically use gradient-based learning strategies, whereas, in neuroevolution, we employ evolutionary algorithms to optimize the network parameters. That reduces the risk of being unable to estimate accurate gradients or being stuck in a local optimum. Evolutionary algorithms only require a measure of the

network's performance, making them directly applicable for reinforcement learning problems. Thus, we can use it more widely in various problem classes. Such et al., 2017 showed that genetic algorithms, a subgroup of evolutionary algorithms, work well for the parameter search of neural networks. Even in complex reinforcement learning settings, including Atari and humanoid locomotion, they could achieve results at the scale of traditional deep reinforcement learning frameworks.

## 1.6 Black-Box Optimization

In black-box optimization, the problem structure, as well as the model, remains unknown. Black-box optimization methods optimize a parametrization without any constraints on the problem, model or solution needed. Unlike the gradient-based learning techniques we often see in backpropagation algorithms, there is no constraint on the differentiability. The optimization is solely based on a score or cumulative reward, which makes these methods directly applicable to reinforcement learning problems. Since there are no constraints imposed on the architecture or properties of the model, we can use any function approximator we find suitable for a problem and optimize it with black-box optimization techniques.

In black-box optimization techniques, we optimize a parametrization with the help of the fitness function instead of a reward function. Both are a measure to indicate how well the model performs. Until now, we only talked about reward functions. The reward can be given immediately after combining a state-action pair as in the implementation of OpenAI Gym. The fitness, however, represents the cumulative reward of an episode. In real-life applications, we often only have a cumulative reward instead of getting feedback after each action. Thus, the setting of black-box optimization is more realistic than we see otherwise.

### 1.6.1 Random Weight Guessing

Random Weight Guessing (RWG) is the simplest black-box optimization method. Algorithm 1 shows the procedure with pseudocode. With RWG, we repeatedly ran-

---

**Algorithm 1** RWG as described in Schmidhuber, Hochreiter, and Bengio, 2001

---

```

1: repeat
2:   Randomly sample model weights
3: until model correctly classifies all training sequences
4: return model weights

```

---

domly initialize the weights of a model until the resulting model classifies all training instances correctly. Then we test the model on a separate test set.

Schmidhuber, Hochreiter, and Bengio, 2001 were able to show that simple RWG can even outperform previously proposed more complex algorithms. Naturally, RWG is not a good algorithm for parameter optimization. A problem might require many free parameters or high weight precision in practical applications. In these cases, RWG becomes infeasible for finding a suitable parametrization within a reasonable time. A more appropriate learning technique is suggested for real-life applications. Nevertheless, the results suggest that RWG can be used as an analysis method. Oller, Glasmachers, and Cuccu, 2020 also used an RWG algorithm to analyze the complexity of prevalent OpenAI Gym benchmark problems.

### 1.6.2 Evolution Strategies

Evolution Strategies (ES) belong to the class of evolutionary algorithms and are best suited for continuous optimization. They are directly inspired by the idea of evolution in nature and use the concept of mutation and selection. ES are implemented in a loop where one iteration is called a generation. New individuals (candidate solutions) are generated using the current parents in each generation. We continue with the next generation until a stopping criterion is met. ES are multi-agent algorithms that generate multiple parametrizations that independently explore a different path or area in the optimization space. This exploration prevents us from getting stuck in a local optimum.

**(1 + 1)-ES:** The population model (1 + 1) means that we maintain only one individual. For the next generation, we generate one new offspring as a variation of the parent, drawn from the normal distribution. For this, we adapt the mean of the distribution according to the parent. Then we keep the parent or the child based on which performed had a better fitness score. Algorithm 2 shows this process in pseudocode.

---

#### Algorithm 2 (1 + 1)-ES in $d$ dimensions

---

```

1: Parameter:  $\sigma > 0$ 
2: Initialization:  $x = x_0 \in \mathbb{R}^d$ 
3: while not done do
4:   Sample  $x' \sim \mathcal{N}(x, \sigma^2 I)$ 
5:   if  $f(x') \leq f(x)$  then
6:      $x \leftarrow x'$ 
7: return  $x$ 
```

---

### 1.6.3 Covariance Matrix Adaptation Evolution Strategy

The Covariance Matrix Adaptation Evolution Strategy (CMA-ES) is an advanced  $(\mu, \lambda)$ -ES that adapts not only the mean of the multivariate normal distribution but also updates the covariance matrix. Since we are in the field of black-box optimization techniques, there are no constraints on the problem, model, or solution of the problem. CMA-ES is a method for non-linear, non-convex optimization problems in a continuous domain. It has been successful in many applications with ill-conditioned objective functions and is considered the industry standard. The following technical information has mainly been taken from Hansen, 2016.

**Sampling of the population:** For each iteration of the CMA-ES algorithm, a population of new individuals is generated by sampling a multivariate normal distribution. Formally, for generation  $g + 1$ , the population sampling can be written as

$$\mathbf{x}_k^{(g+1)} \sim \mathbf{m}^{(g)} + \sigma^{(g)} \mathcal{N}(\mathbf{0}, \mathbf{C}^{(g)}) \quad \text{for } k = 1, \dots, \lambda,$$

where  $\mathbf{x}_k^{(g+1)}$  symbolizes the  $k$ -th individual from generation  $g + 1$ ,  $\mathbf{m}^{(g)}$  denotes the mean value of the distribution at generation  $g$ ,  $\sigma^{(g)}$  is the standard deviation at generation  $g$ ,  $\mathbf{C}^{(g)}$  is the covariance matrix at generation  $g$ , and  $\lambda$  is the size of the population. After the sampling of generation  $g + 1$ , we need to adapt the mean and the covariance matrix for the sampling of the next generation. Thus, we need to calculate  $\mathbf{m}^{(g+1)}$  and  $\mathbf{C}^{(g+1)}$ .

**Moving the mean:** At the end of each iteration, we need to update the mean of the distribution. The new mean  $\mathbf{m}^{(g+1)}$  is a weighted average of  $\mu$  selected individuals from the generation  $g + 1$ . Thus, we select  $\mu$  individuals from  $\mathbf{x}_1^{(g+1)}, \dots, \mathbf{x}_{\lambda}^{(g+1)}$ . The formula to update the mean for the next generation is the following:

$$\mathbf{m}^{(g+1)} = \mathbf{m}^{(g)} + c_m \sum_{i=1}^{\mu} w_i (\mathbf{x}_{i:\lambda}^{(g+1)} - \mathbf{m}^{(g)})$$

$$\sum_{i=1}^{\mu} w_i = 1, \quad w_1 \geq w_2 \geq \dots \geq w_{\mu} > 0$$

$c_m \leq 1$  is the learning rate of the algorithm and is usually chosen as  $c_m = 1$ .  $\mu$  is the population size with  $\lambda$  being the population size of the parent population with  $\mu \leq \lambda$ . The weights  $w_{i=1, \dots, \mu} \in \mathbb{R}_{>0}$  are positive coefficients used for the recombination process. The variable  $\mathbf{x}_{i:\lambda}^{(g+1)}$  symbolizes the  $i$ -th best individual from the  $(g + 1)$ -th population in relation to its corresponding fitness score.

**Adaption of the covariance matrix:** In addition to updating the mean of the search distribution, we also need to adapt the covariance matrix. Calculating the adapted covariance matrix is a bit more complex than updating the mean. Thus, I will not go into detail in this case. However, notable is that the covariance matrix adaptation takes the most computational effort.

## 1.7 Research Questions

The main goal of this thesis is to compare alternative models to neural networks. To analyze and compare the different models and their architectures, I formulated the following research questions:

1. How do function approximators other than neural networks compare with the latter?
2. What advantages and disadvantages can we see with other function approximators?
3. How does the bias influence the performance of the model?



## Chapter 2

# Method

### 2.1 OpenAI Gym Environments

OpenAI Gym offers five classic control environments ready to use: `CartPole`, `Acrobot`, `MountainCar`, `MountainCarContinuous`, and `Pendulum`. The initial state of each environment is stochastic. Out of all environments OpenAI Gym provides, the classic control environments are considered the easier ones to solve. Each environment holds an observation and an action space. An agent performs some action from the action space and receives an observation describing how the environment's state changed.

#### 2.1.1 CartPole

The `CartPole` environment corresponds to the pole-balancing problem formulated in *Neuronlike Adaptive Elements That Can Solve Difficult Learning Control Problems* (Barto, Sutton, and Anderson, 1983). A pole is attached to a cart that the agent can move along a one-dimensional track. Fig. 2.1 shows two possible states of this problem. In the image on the left, the pole is perfectly balanced and upright. The right image shows the pole tilted to the left. Thus, the cart should go to the left to correct the angle of the pole.



**FIGURE 2.1: Illustration of the environment CartPole.** The image shows two possible states of the `CartPole` environment. The left image shows a perfectly balanced pole. In the right image, the pole tilts to the left.

**Action Space:** The `CartPole` environment accepts one action per time step and has a discrete action space of  $\{0, 1\}$ . The action indicates the direction in which we push the cart by a fixed force. The actions are described in Table 2.1.

| Action | Description            |
|--------|------------------------|
| 0      | Push cart to the left  |
| 1      | Push cart to the right |

TABLE 2.1: **Action space of the environment CartPole.** Description of all possible actions for the CartPole environment.

**Observation Space:** For CartPole, we receive four observations informing us about the position and velocity of the cart and the pole. Table 2.2 shows all observations with the range of the corresponding values.

| Observation           | Min               | Max             |
|-----------------------|-------------------|-----------------|
| Cart Position         | -4.8              | 4.8             |
| Cart Velocity         | -Inf              | Inf             |
| Pole Angle            | -0.418 rad (-24°) | 0.418 rad (24°) |
| Pole Angular Velocity | -Inf              | Inf             |

TABLE 2.2: **Observation space of the environment CartPole.** Description of all observations for the CartPole environment.

**Reward:** The goal of this task is to keep the pole upright as long as possible. Thus, we get a positive reward of +1 for each step that does not meet the termination criteria. An episode terminates when the pole angle is  $\pm 12^\circ$ , the cart position is  $\pm 2.4$ , or the episode length is higher than 200 (500 for v1).

### 2.1.2 Acrobot

The environment Acrobot is based on the paper *Generalization in Reinforcement Learning: Successful Examples Using Sparse Coarse Coding* (Sutton, 1995) and the book *Reinforcement Learning: An Introduction* (Montague, 1999). The system contains a chain with two links, with one end of the chain fixed. The goal is to swing the free end of the chain upwards until reaching a certain height. Fig 2.2 shows in the left image one possible state for this problem with the fixed height indicated by a horizontal line. The right image of the figure illustrates one possible state that reached the target height.

**Action Space:** For Acrobot, the action space is discrete and deterministic, representing the torque applied to the joint between the two links, including the loose end. Table 2.3 shows a description of all possible actions.

| Action | Description                           | Unit         |
|--------|---------------------------------------|--------------|
| 0      | apply -1 torque to the actuated joint | torque (N m) |
| 1      | apply 0 torque to the actuated joint  | torque (N m) |
| 2      | apply 1 torque to the actuated joint  | torque (N m) |

TABLE 2.3: **Action space of the environment Acrobot.** Description of all possible actions for the Acrobot environment.



FIGURE 2.2: **Illustration of the environment Acrobot.** The image on the left shows one possible state of the environment Acrobot. The image on the right shows one possibility of reaching the target height.

**Observation Space:** Acrobot returns six observations that inform us about the two rotational joint angles and their angular velocities. Table 2.4 contains a description for each observation. theta1 is the angle of the first joint, where an angle of 0 indicates that the first link is pointing directly downwards. theta2 is relative to the angle of the first link. An angle of 0 corresponds to having the same angle between the two links.

| Observation                | Min                   | Max                 |
|----------------------------|-----------------------|---------------------|
| Cosine of theta1           | -1                    | 1                   |
| Sine of theta1             | -1                    | 1                   |
| Cosine of theta2           | -1                    | 1                   |
| Sine of theta2             | -1                    | 1                   |
| Angular velocity of theta1 | -12.567 (-4 * $\pi$ ) | 12.567 (4 * $\pi$ ) |
| Angular velocity of theta2 | -28.274 (-9 * $\pi$ ) | 28.274 (9 * $\pi$ ) |

TABLE 2.4: **Observation space of the environment Acrobot.** Description of all observations for the Acrobot environment.

**Reward:** The goal of this task is to reach the target height with as few steps as possible. Thus, each step that does not attain the goal receives a negative reward of -1. Reaching the target height results in a reward of 0. An episode ends after 500 time steps. Acrobot is considered an unsolved environment, meaning there is no reward threshold at which we would consider it solved.

### 2.1.3 MountainCar and MountainCarContinuous

The environments MountainCar and MountainCarContinuous are two different versions of the same problem. The difference between the two is that MountainCar has a discrete action space, and MountainCarContinuous has a continuous action space. The goal of the task is to maneuver a car up a mountain to its target. However, the car's engine is not strong enough to drive up the mountain in one pass. To achieve the goal, we need to accelerate the car strategically. An agent should learn to swing from left to right to reach the target. This task is rather difficult to learn since we counterintuitively need to move in the opposite direction of the target to swing up.

The problem first appeared in Andrew Moore's PhD thesis *Efficient memory-based learning for robot control* (Moore, 1990). Fig 2.3 shows one possible state in the left image and the target state of the two environments in the right image.

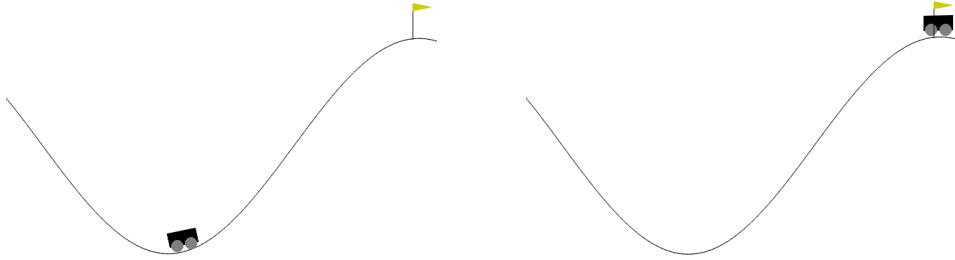


FIGURE 2.3: **Illustration of the environments MountainCar and MountainCarContinuous.** The image shows one possible state of the environments MountainCar and MountainCarContinuous on the left side. The image on the right shows the target state of the two environments.

**Action Space:** The action space for the environment MountainCar consists of three discrete actions. We can accelerate the car to the left or the right, or we do not interact with it. The actions are further described in Table 2.5. On the other hand, the

| Action | Description             | Value | Unit        |
|--------|-------------------------|-------|-------------|
| 0      | Accelerate to the left  | Inf   | position(m) |
| 1      | Don't accelerate        | Inf   | position(m) |
| 2      | Accelerate to the right | Inf   | position(m) |

TABLE 2.5: **Action space of the environment MountainCar.** Description of all possible actions for the MountainCar environment.

environment MountainCarContinuous accepts one continuous action per step. The action represents the directional force on the car and is clipped in the range of [-1, 1] and multiplied by a power of 0.0015.

**Observation Space:** The environments MountainCar and MountainCarContinuous share the same observation space. They return two observations with information about the car's position and velocity. The observations are further described in Table 2.6.

| Observation                          | Min  | Max | Unit        |
|--------------------------------------|------|-----|-------------|
| Position of the car along the x-axis | -Inf | Inf | position(m) |
| Velocity of the car                  | -Inf | Inf | position(m) |

TABLE 2.6: **Observation space of the environment MountainCar.** Description of all observations for the MountainCar and MountainCarContinuous environments.

**Reward:** To solve the task, the car has to reach the flag on the right hill as fast as possible. Each time step in which the agent cannot get to the target, it receives a negative reward of -1 for the environment MountainCar. The task is considered

solved when getting an average reward of -110.0 over 100 consecutive trials. For the environment `MountainCarContinuous`, we receive a negative reward of  $-0.1 * \text{action}^2$  for each step in which the car does not reach the target to penalize actions of large magnitude. When reaching the goal, we receive a positive reward of +100. `MountainCarContinuous` is considered solved when getting an average reward of 90.0 over 100 consecutive trials.

#### 2.1.4 Pendulum

For the environment `Pendulum`, the system consists of a pendulum attached at one end to a fixed point and the other end being free. The goal is to apply torque on the free end to swing it to an upright position. This task is known as the inverted pendulum swing-up problem and is based on the classic problem in control theory. Figure 2.4 shows one possible state of the environment in the left image and the target state in the right one.



FIGURE 2.4: **Illustration of the environment Pendulum.** The left image shows one possible state of the `Pendulum` environment. In the right image, the pendulum is in the upright position, which denotes the target state.

**Action Space:** The environment accepts one action per step. The action represents the torque applied to the free end of the pendulum.

| Action | Description | Min  | Max |
|--------|-------------|------|-----|
| 0      | Torque      | -2.0 | 2.0 |

TABLE 2.7: **Action space of the environment Pendulum.** Description of all possible actions for the `Pendulum` environment.

**Observation Space:** The environment returns three observations. The first two observations represent the x- and y-coordinate of the pendulum's free end in a cartesian coordinate system. The variable theta defines the angle in radians. The third observation is the angular velocity of the free end of the pendulum.

**Reward:** The reward function is defined as  $r = -(theta^2 + 0.1 * theta_dt^2 + 0.001 * torque^2)$ . Regardless of the received reward, an episode ends after 200 time steps.

| Observation        | Min  | Max |
|--------------------|------|-----|
| $x = \cos(\theta)$ | -1.0 | 1.0 |
| $y = \sin(\theta)$ | -1.0 | 1.0 |
| Angular Velocity   | -8.0 | 8.0 |

TABLE 2.8: **Observation space of the environment Pendulum.** Description of all observations for the Pendulum environment.

Pendulum is an unsolved environment meaning that it has no reward threshold at which we consider it solved.

## 2.2 Analysis and Visualization

This thesis aims to deliver a proper analysis of different kinds of models in reinforcement learning. For this, we need appropriate methods and visualizations guaranteeing objectivity in evaluation.

### 2.2.1 RWG-Paper

Oller, Glasmachers, and Cuccu, 2020 deliver an excellent foundation for analysis in their paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing*. Although their focus was not on the model's performance but on the complexity of reinforcement learning benchmarks, their framework can be easily adjusted to fit our purpose. In the paper, the authors analyze and visualize the complexity of standard reinforcement learning benchmarks based on score distribution. They tested their approach on the five classic control benchmark problems from the OpenAI Gym interface discussed in Section 2.1. The authors conducted a fixed series of experiments with a reinforcement learning environment. For these experiments, they used three neural network architectures ( $N_{\text{architectures}} = 3$ ): a network without any hidden layers (0 HL, 0 HU), a network with a single hidden layer of 4 units (1 HL, 4 HU), and a network with two hidden layers of 4 units each (2 HL, 4 HU). With these, they cover a variety of network models that are suited to solve the given tasks.

The evaluation of the benchmark problems should be as objective as possible and should not include bias in the data. To achieve this, the authors did not include learning opportunities for the network models. Instead, they chose the network weights i.i.d. from the standard normal distribution  $\mathcal{N}(0, 1)$  with RWG. This approach assures randomness and no directed learning. They initialized  $10^4$  samples ( $N_{\text{samples}} = 10^4$ ) with different random weights. The number of samples would be too large for a reasonable learning strategy. However, the large number of samples serves a different purpose than optimizing the results. Instead, the aim is to draw statistical conclusions. Each of these neural network samples represents a controller that maps observations to actions in the environment. Chapter 3 aims to explore function approximators other than neural networks representing the controller. In the paper, the authors tested the controllers for each environment during 20 independent episodes ( $N_{\text{episodes}} = 20$ ). They saved the score in the score tensor  $S$  for each episode. Algorithm 3 illustrates the procedure with pseudocode.

After obtaining the scores, the authors calculated the mean performance over all episodes from a sample and its variance. These statistics are significant insights. They can reveal how stable the network models are in completing a task. A low mean value suggests that, in general, the network cannot complete the task. The

**Algorithm 3** Evaluation process taken from Oller, Glasmachers, and Cuccu, 2020

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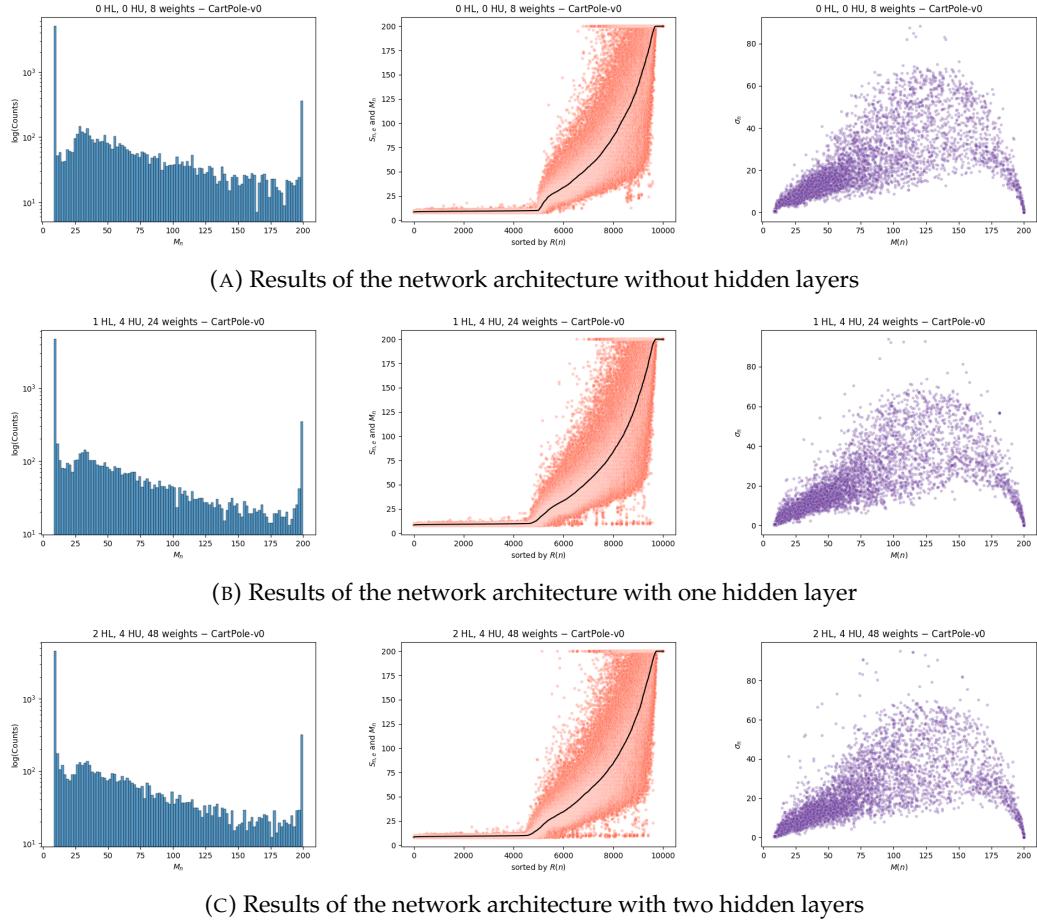
1: Initialize environment
2: Create array  $S$  of size  $N_{\text{architectures}} \times N_{\text{samples}} \times N_{\text{episodes}}$ 
3: for  $n = 1, 2, \dots, N_{\text{samples}}$  do
4:   Sample NN weights randomly from  $\mathcal{N}(0, 1)$ 
5:   for  $e = 1, 2, \dots, N_{\text{episodes}}$  do
6:     Reset the environment
7:     Run episode with NN
8:     Store accrued episode reward in  $S_{a,n,e}$ 
```

---

variance gives us further insight into the score distribution. It illustrates how spread out the scores are from their respective mean score. A high value means that we have high variability. A controller is valuable if it can solve a specific task reliably and stable. Therefore, we strive for a high value for the mean and a low value for the variance. However, training a network with random weight guessing should generally not result in a stable controller. If this is the case, we can assume that the task to solve was too trivial and is not valuable for evaluation measurements. In the illustrations of the paper, the authors ranked the samples according to their mean scores. They then visualized their results with three plots: a log-scale histogram of the mean scores, a scatter plot of the sample scores over their rank, and a scatter plot of score variance over the mean score.

**Reproduction of the results:** I reproduced the results of the authors following the mentioned methodology. My findings for the environment CartPole are displayed in Figure 2.5. The plots illustrate the results for each of the three network architectures. Each row shows the histogram of the mean score values in the left image, the scatter plot of all scores over their rank in the image in the middle, and the scatter plot of the score variance over the mean score in the right image for the corresponding network architecture. There are few differences, but overall all network architectures deliver similar insights.

The histogram plots show that most networks receive a low score. Since the weights of the networks were chosen with RWG, this is relatively unsurprising. Nevertheless, there is still a significant amount of networks that were able to achieve a high mean value or even the maximum value of 200. With a score of 200, the network solved the task in each episode. Therefore, the network could reliably solve the task without any learning technique involved. This behavior should not be the case for a complex task. Furthermore, in the scatter plot in the middle, we can see that the line plot of the mean scores is a continuous increasing line without any jumps. Thus, a suited learning algorithm should generally be able to learn the task incrementally without converging into a local optimum. At the top of the scatter plot, we can see quite a few data points with a score of 200 that have a relatively low mean score. This behavior indicates that a network that generally performs poorly can still solve the task with the right initialization conditions. Lastly, in the scatter plot on the right, we can see the distribution of the variance according to the mean value. On the left side, we have low scores of variance corresponding with a low mean value. These networks were consistently unable to achieve a high score. We can expect most networks to be in this area without any training. However, the data points are spread out in the middle of the plot. For a high variance, the scores of a network differ highly from the mean value. Thus, we might get lucky and receive a



**FIGURE 2.5: Results of the benchmark evaluation.** The plots show a log-scale histogram of the mean scores (left images), a scatter plot of the sample scores over their rank (middle images), and a scatter plot of score variance over the mean score (right image). As expected with RWG, most networks could not solve the given task. However, a significant number of samples still achieve a mean score of 200. That suggests that the environment is trivial to solve.

high score depending on initialization conditions, but we might also get a low score. These networks are inconsistent and unstable. On the right side of the scatter plot, we can see that the data points with a high mean value are primarily of low variance. Thus, to achieve a high mean value, the network needs consistency.

**Impact of Bias:** Interestingly, the usage of the bias had a relatively significant impact on the performance of the network in my experiments. Without bias, the networks seem to achieve better scores overall. All plots in Figure 2.5 illustrate the results without bias. For comparison, Figure 2.6 shows the results of a network with two hidden layers with the same configurations as before, but this time including bias. As we can see, the networks with bias connections generally have a much lower score. The number of networks that can consistently solve the task also decreases significantly. In the paper, the authors noted that the probability mass of top-performers generally increases when all tested environments drop the bias connections. Thus, this is not an isolated observation. However, they did not investigate this behavior further as it was not the focus of their paper.

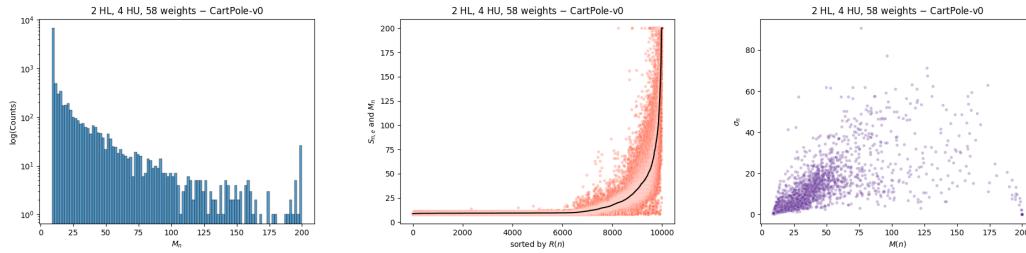


FIGURE 2.6: **Impact of bias.** The figures show the performance of a network with two hidden layers with the same settings as before, but here we include bias. We can see that the network with bias connections performs much worse than those without bias connections.

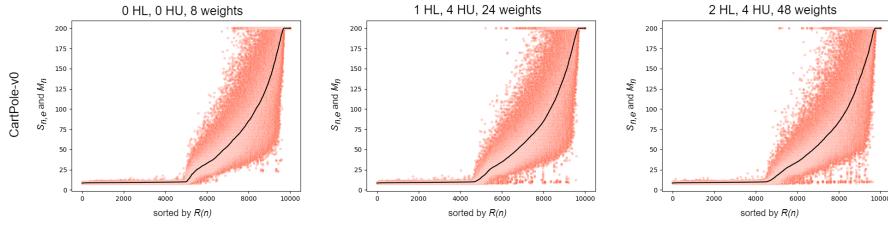
One possible explanation could be that guessing additional weights might be fatal for achieving a good score. In other words: the more possibilities we have to guess a weight falsely, the higher the probability of failing. To test this hypothesis, we can alter the number of weights and compare the results. With an increased number of weights, we would expect the networks to perform worse than before. However, it could also be that the number of weights is not as impactful as the complexity of the model. Randomly guessing the parameters of a simple model has a higher chance of resulting in a good (simple) model than guessing the parameters of a more complex model. The complexity of the network architecture gives an upper bound for the function that we can approximate. A network with high complexity maps into a larger search space with more complex functions. Since the size of the search space increases, there are also more possible samples that fail to solve the task. The paper showed that a simple model is sufficient to solve these environments. A complex model is oversized for our purpose here. Thus, randomly guessing a simple model can yield a model with good performance with enough attempts. However, it is unlikely to randomly guess a complex model that performs well without any training involved. To test this hypothesis, we can increase the complexity of a network by varying the number of hidden layers or the number of neurons in a hidden layer. With increased complexity, we would expect the networks to perform worse than before.

Another interesting aspect would be to inspect the role of the bias in connection with the environments. A simple model is sufficient to solve a simple task. Nevertheless, what if the environment is more difficult to solve? In that case, the undersized complexity would limit us from finding an appropriate solution as the search space is not large enough for this scenario. Therefore, including bias should improve the results.

Next to the analysis of the models, Chapter 3 also contains the experiments following this thought process.

### 2.2.2 Visualization

For the visualization of the results, I am using similar plots as Oller, Glasmachers, and Cuccu, 2020. Figure 2.5 already presented the type of plots used for the visualization in Chapter 3. However, I only use the scatter plots in the middle image because it transparently illustrates a high amount of information. Figure 2.7 shows an example of such a plot. The different architectures of the model are aligned in the columns, and the row represents the environment. In this case, I am using three neural network architectures and the CartPole environment. In each image, the samples



**FIGURE 2.7: Visualization of results.** The image shows an example of the visualization of the conducted experiments. The scatter plot represents the individual sample scores  $S_{n,e}$  for episode  $e$  over the rank  $R(n)$  of the sample  $n$ . The overlaid line plot consists of each sample's mean scores  $M_n$ .

are sorted according to their mean score for better analysis. The position of a sample  $n$  within the sorted list is referred to as its rank  $R(n)$ . In the plot, the samples are aligned according to their rank on the x-axis. Each red dot represents the individual sample score  $S_{n,e}$  for sample  $n$  in episode  $e$ . The overlaid line plot represents the mean score  $M_n$  of sample  $n$  over all episodes. Thus, the line plot is a naturally increasing curve of the mean scores. The scatter plot indicates the variety of scores for one sample. The title of the plot informs us which model and how many weights were used. In addition, the corresponding OpenAI gym environment is declared.

## 2.3 Alternative Models

This section describes the models I used in the experiments. Since we are using black-box optimization techniques, we have no constraints on the model. Therefore, we can use any function approximator. Note that we are in the field of direct policy search, where we directly search in the search space of policies for a suited approximation.

### 2.3.1 Polynomials

In mathematics, a polynomial is the sum of powers in one or more variables multiplied by constant coefficients. Given a field  $F$ , a polynomial in variable  $x$  with coefficients in  $F$  is a formal expression denoted by

$$f(x) = \sum_{i=0}^n a_i x^i \in F[x], \quad a_0, \dots, a_n \in F, \quad i \in \mathbb{N},$$

where  $F[x]$  represents the set of all such polynomials (Fischer, 2014, p. 61). The above formula shows the one dimensional case. For the multi-dimensional case,  $x$  and the coefficients are vectors instead of scalars. Thus, a polynomial  $p(\mathbf{x})$  with  $\mathbf{x} = [x_0, \dots, x_m]^T$  being a vector and of degree  $n$  can be represented by

$$p(\mathbf{x}) = \sum_{i=0}^n \mathbf{w}_i^T (x_k^i)_{k \in I} \in F^n[\mathbf{x}], \quad \mathbf{w}_0, \dots, \mathbf{w}_n \in F^n, \quad I = \{0, \dots, m\}.$$

Polynomials are relatively simple mathematical expressions and offer some significant advantages: their derivative and indefinite integral are easy to determine and are also polynomial. Due to their simple structure, polynomials can be valuable to analyze more complex functions using polynomial approximations. *Taylor's theorem* tells us that we can locally approximate any  $k$ -times differentiable function by a polynomial of degree  $k$ . We call this approximation *Taylor polynomial*. Furthermore, the

Weierstrass approximation theorem says that we can uniformly approximate every continuous function defined on a closed interval by a polynomial. Other applications of polynomials are *polynomial interpolation* and *polynomial splines*. Polynomial interpolation describes the problem of constructing a polynomial that passes through all given data points. Polynomial splines are piecewise polynomial functions that can be used for spline interpolation.

**Construction of the model:** For the experiments with the polynomial model in Chapter 3, I constructed the model  $P_1$ . It consists of one polynomial for each possible action in a discrete action space. The input of the model is the observation of the environment. The dimension of the weight vectors is according to the dimension of the input vector. For example, for the environment CartPole with the discrete action space  $\{0, 1\}$  and observation  $\mathbf{x} = [x_0, x_1, x_2, x_3]^T$ , this means that  $P_1$  consists of two polynomials:

$$\begin{aligned} p_0(\mathbf{x}) &= \sum_{i=0}^n \mathbf{w}_i^T (\mathbf{x}_k^i)_{k \in I} \in \mathbb{R}, & \mathbf{w}_0, \dots, \mathbf{w}_3, \mathbf{x} \in \mathbb{R}^4, & I = \{0, 1, 2, 3\} \\ p_1(\mathbf{x}) &= \sum_{i=0}^n \hat{\mathbf{w}}_i^T (\mathbf{x}_k^i)_{k \in I} \in \mathbb{R}, & \hat{\mathbf{w}}_0, \dots, \hat{\mathbf{w}}_3, \mathbf{x} \in \mathbb{R}^4, & I = \{0, 1, 2, 3\} \end{aligned}$$

In the formulas,  $n$  denotes the degree of the polynomial. For the experiments, I used polynomials of degrees one, two, and three. However, the output of the polynomials has no appropriate upper and lower limit, as illustrated in Figure 2.8. That makes it harder to interpret the results reasonably. So, I scaled the outputs with the logistic

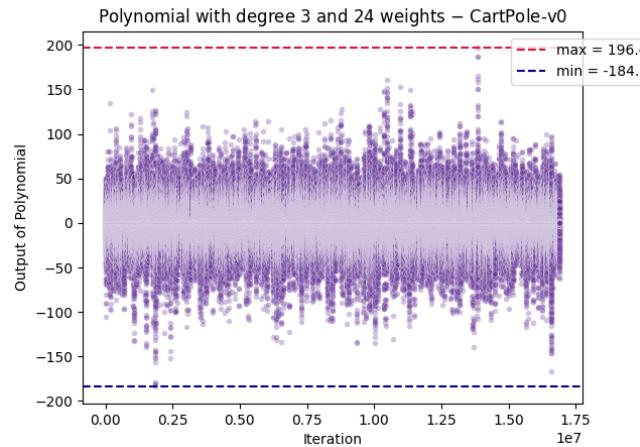


FIGURE 2.8: **Upper and lower bound.** The figure shows each output of the polynomial functions described above. As we can see, the functions are not well bound, and there are quite a few outliers. That makes it hard to interpret the output sensibly.

sigmoid function. We can interpret the output space of the sigmoid function as a probability, as discussed in Section 1.2.1. In this case, we search for the probability that a specific action is the reasonable one given an observation  $\mathbf{x}$ . Thus, for our example with the CartPole environment, we can interpret  $\text{sig}(p_0)$  as the probability that action 0 is the correct one and  $\text{sig}(p_1)$  as the probability that action 1 is the correct one. Putting this thought into a formula for  $P_1$  and the CartPole environment, we get:

$$P_1(\mathbf{x}) = \begin{cases} 1 & \text{if } \text{sig}(p_1(\mathbf{x})) > \text{sig}(p_0(\mathbf{x})), \\ 0 & \text{otherwise.} \end{cases}$$

### 2.3.2 Binary Trees

A binary tree is an abstract data type in computer science with a relatively simple design. A binary tree represents a hierarchical tree structure consisting of a set of connected nodes. A node is a structure that can hold data and connections to other nodes, also called links or edges. Nodes in a binary tree can have at most two child nodes. A node's child nodes are those connected to it and situated beneath it in the tree. The node is then called the parent node in the perspective of its child nodes. Since there are at most two child nodes, we typically refer to them as left child and right child, depending on their position. Figure 2.9 illustrates an example of a binary tree with seven nodes. The topmost node is called the root node. By design, there

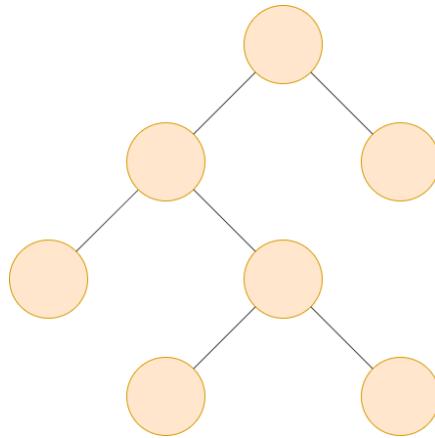


FIGURE 2.9: **Example of a binary tree.** The figure shows an example of a binary tree consisting of seven nodes.

exists a path from the root node to every other node of the tree. The nodes which do not have child nodes are commonly called leaf nodes.

There are many applications for binary trees. One important example is the binary search tree. Each node of the binary search tree contains a key. The nodes are positioned according to their key in the hierarchy of the tree. For this reason, binary search trees are very efficient for searching. In addition, we can use binary trees as decision trees for machine learning problems. Decision trees represent a predictive model to come to conclusions about a set of observations. For each node, we assess a feature, and a child node is selected accordingly. This process starts at the root node, and we repeat it until reaching a leaf node. The leaf node then holds the value of the target variable. In classification problems, the target variable takes on a value inside a discrete set of values. The leaves then represent the class labels. The target variable can also take on a constant value inside a specified range for regression problems.

The approach to decision trees is simple and intuitive when comparing them with other algorithms in machine learning. Decision trees split the input space into multiple partitions. When the dataset is splittable, it works well with fast predictions. Tree-based models like C4.5 which is an extension of the earlier ID3 algorithm and CART became quite popular (Quinlan, 2014; Breiman et al., 2017). C4.5 was even ranked one of the top ten algorithms in data mining (Wu et al., 2008). Another advantage of decision trees is their interpretability since they implement rules humans can easily analyze. There were also approaches combining neural networks and

decision trees. Yang, Morillo, and Hospedales, 2018 introduced a neural network-based tree model DNDT which performed better for some datasets than neural networks while providing an interpretable decision process. Despite the success of decision trees in supervised learning problems, decision trees are not commonly used in reinforcement learning. Reinforcement learning introduces many challenges, including that the model has to constantly adapt as the interaction between the agent and the environment continues. Silva et al., 2020 approached this challenge by allowing for a gradient update over the entire tree.

**Construction of the model:** Since this study aims to experiment with models that will be optimized with black-box optimization techniques, we can use a simple binary tree model without any constraints on it. Each node holds three properties: `values`, `left`, `right`. The variables `left` and `right` contain the two child nodes. If they are empty, the node is a leaf. The variable `values` holds the model's output if the node is a leaf. Otherwise, it contains the weights used for the decision strategy. Figure 2.10 illustrates the decision process of the binary tree model for the environment CartPole. The illustrated model has one node for the decision finding and two

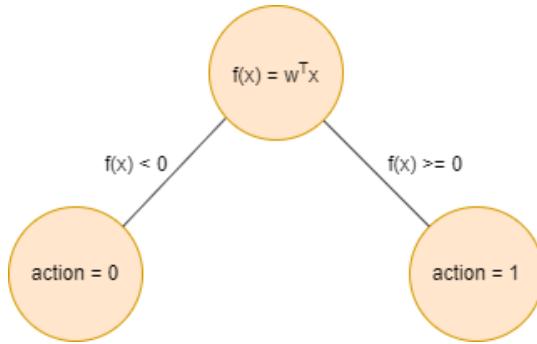


FIGURE 2.10: **Illustration of binary tree model.** The figure illustrates the binary tree model with the environment CartPole.

leaf nodes containing the two possible discrete actions, 0 and 1. The dot product defines the decision-making process. For the environment CartPole with observations  $\mathbf{x} = [x_0, x_1, x_2, x_3]^T$ , this results in the following formula:

$$f(\mathbf{x}) = \mathbf{w}^T \mathbf{x} = w_0 x_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$$

The variable  $w$  represents the weights of the model. The actions of the leaves are fixed, whereas an optimization technique optimizes the weights of the nodes involved in the decision-making process.

The implementation is straightforward for environments like CartPole where we only have two possible discrete actions. However, for an action space with more than two actions possible, the binary tree still has the same structure, but the decision is fixed between the minimum action and the maximum action. For example, for the environment Acrobot, the minimum action is to apply -1 torque to the joint, and the maximum action is to apply 1 torque to the joint. In this case, we discard the action of applying 0 torque to the joint. Similarly, for environments with a continuous action space, we again take the minimum and the maximum of the value of the action. For the environment Pendulum, this would be -2.0 and +2.0.

The only hyperparameter we have with this model is the number of nodes for the decision process. That makes hyperparameter tuning much more straightforward than it would be for other models like neural networks. Note that the leaves do not

count as nodes for the model's architecture since each path in the tree has to result in a final decision between two actions. The leaf nodes are mandatory in that sense and thus cannot be altered in their number.

## Chapter 3

# Experiments

This chapter aims to answer the research questions formulated in Section 1.7. First, I describe which experiments I conducted. Next, I show the results of these experiments. Finally, I discuss my findings further in the last section.

### 3.1 Experiments

This section describes the experiments I used in order to investigate and answer the formulated research questions. We can break down the experiments into three categories: reproduction of the results of the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing*, comparison of the alternative models described in Section 2.3 to neural networks, and an analysis of the impact of bias in these settings.

#### 3.1.1 Reproduction of the RWG-Paper

For a starting point, I reproduced the results of the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* discussed in Section 2.2. I used the same methodology as the authors of the paper did. In specific, I used three network architectures: a network without any hidden layers (0 HL, 0 HU), a network with a single hidden layer of 4 units (1 HL, 4 HU), and a network with two hidden layers of 4 units each (2 HL, 4 HU). I used the same procedure as explained in pseudocode in Algorithm 3 using RWG. I tested the three neural networks for all five classic control environments provided by the OpenAI Gym interface: CartPole, Acrobot, Pendulum, MountainCar, and MountainCarContinuous. For the next experiments, the reproduced results serve as a guideline to judge the effectiveness of the alternative models in comparison with neural networks.

#### 3.1.2 Comparison of Alternative Models to Neural Networks

This experiment aims to provide a comparison of alternative models to the commonly used neural network model. I selected a few promising models and analyzed them with the help of the classic control environments. Analogous to the neural networks, I used three architectures for each model with increasing complexity. I conducted the following experiments:

- (a) In the first experiment, I took the polynomial model  $P_1$  and tested it on the discrete classic control environments. I used polynomials of degrees 1, 2, and 3.
- (b) In the second experiment, I tested the binary tree model on all five classic control environments. I used binary trees with 1, 4, and 8 nodes.

I described the models in more detail in Section 2.1. For the experiments, I used the same procedure as in *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* with only slight adaption. Thus, we can directly compare the results of the alternative models to those of neural networks. Another important aspect of this procedure is that there is no learning involved. For their paper, the authors were interested in the complexity of the environment, whereas I aim to find out more about the nature of the models. The classic control environments are relatively easy to solve, as explained in Section 2.2. Therefore, we expect some controllers to solve the task even without any learning involved.

For the experiments, first, I initialized the environment. Second, I initialized the respective model. Then, I drew the weights of the model from the standard normal distribution  $\mathcal{N}(0, 1)$  or the uniform distribution  $U(a, b)$  depending on the model and environment used. Each of these instances of the model represents a sample. In total, I used 10'000 samples ( $N_{samples}$ ). Finally, I ran 20 episodes ( $N_{episodes}$ ) with each sample for an environment and stored the respective score as an entry of the score tensor  $S$ . Algorithm 4 shows an overview of the described procedure. As we can see, the procedure is very similar to the one shown in Algorithm 3.

---

**Algorithm 4** Procedure for alternative models using RWG

---

```

1: Initialize environment
2: Initialize model
3: Create array  $S$  of size  $N_{samples} \times N_{episodes}$ 
4: for  $n = 1, 2, \dots, N_{samples}$  do
5:   Sample model weights randomly from  $\mathcal{N}(0, 1)$  or  $U(a, b)$ 
6:   for  $e = 1, 2, \dots, N_{episodes}$  do
7:     Reset the environment
8:     Run episode with model
9:     Store accrued episode reward in  $S_{n,e}$ 
```

---

### 3.1.3 Analysis of the Impact of Bias

The authors of the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* indicate that using bias worsens the performance of neural networks in their experimental setting. This experiment serves to analyze this behavior. I used the same procedure as explained in the paper with the same three neural network architectures: a network without any hidden layers (0 HL, 0 HU), a network with a single hidden layer of 4 units (1 HL, 4 HU), and a network with two hidden layers of 4 units each (2 HL, 4 HU). First, I reproduced the results for the networks with bias to confirm their findings. Then, I varied the number of (a) weights, (b) hidden layers, and (c) neurons for a neural network. To get insights into whether this observation is specific to neural networks, I additionally used the polynomial model for the experiment concerning the variation of the weights of the model. In more detail, the experiments I conducted in this matter are the following:

- (a) In the first experiment, I only changed the number of weights for each of the three networks and the polynomial model  $P_1$  described in Section 2.3. I tripled the number of weights for all models. To achieve this, I constructed one weight  $\mathbf{w}_i$  out of three weights by addition:

$$\text{Triple number of weights: } \mathbf{w}_i = \mathbf{w}_{i1} + \mathbf{w}_{i2} + \mathbf{w}_{i3}$$

- (b) In the second experiment, I tested the neural network models with different numbers of hidden layers. Each layer still has the same number of hidden neurons as before. Thus, the networks have four hidden units for each layer. I analyzed a network with 4 hidden layers, one with 6, and one with 8.
- (c) In the last experiment concerning this subject, I varied the number of hidden neurons for a network. Here, I only used a network with two hidden layers. For the number of hidden neurons, I chose 5, 8, and 10.

## 3.2 Results

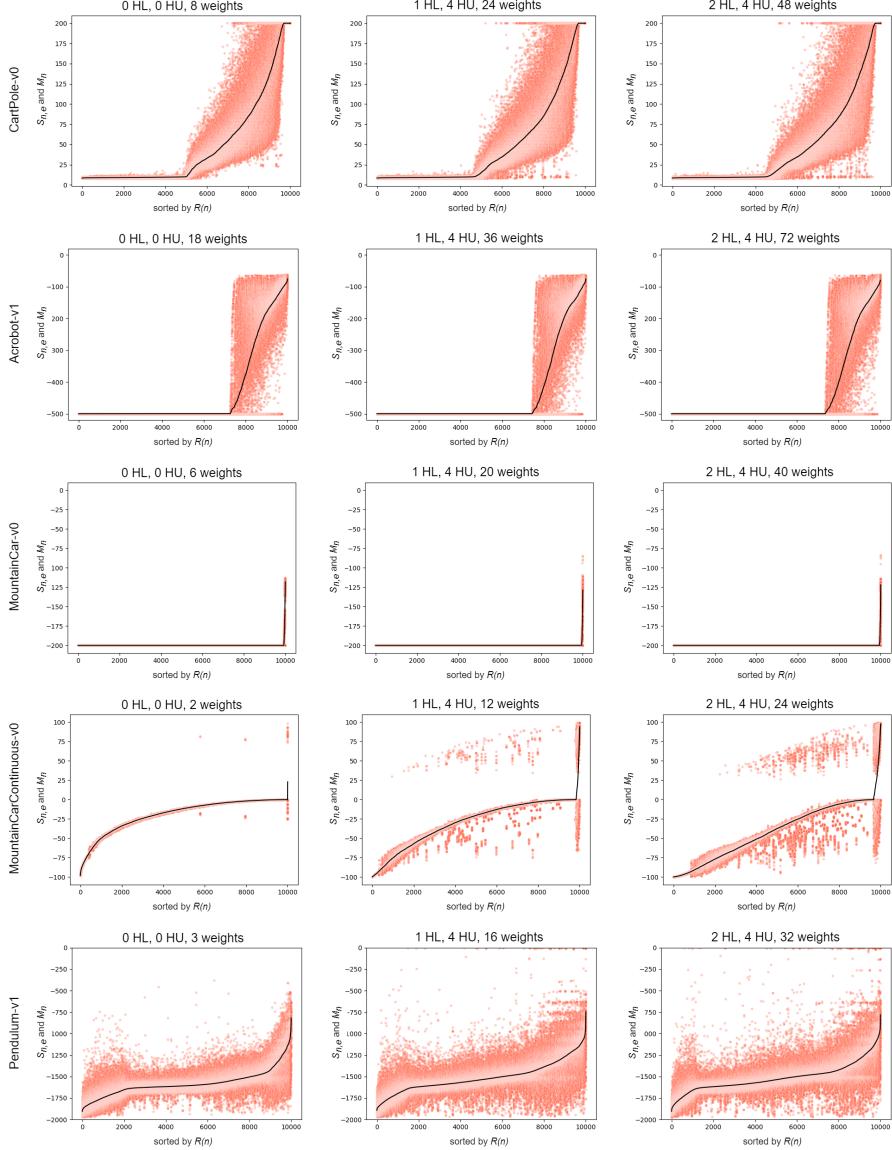
### 3.2.1 Reproduction of RWG-Paper

Figure 3.1 shows the results for all five classic control environments using neural networks without bias. For each plot, the samples are ranked according to their mean score and aligned on the  $x$ -axis according to their rank. The scatter plots show all scores of the samples, whereas the lineplot illustrates the mean of each sample over all episodes. Each column shows a different network architecture. The first column shows the results of the networks without hidden layers, the second column the networks with one hidden layer, and the last column the networks with two hidden layers. The rows are dedicated to the different classic control environments. These results are consistent with the ones from the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing*. In the subsequent sections, I will refer to these plots to compare the effectiveness of other models or configurations to these neural network architectures.

### 3.2.2 Polynomial Model

Figure 3.2 shows the results for the discrete classic control environments using the polynomial model  $P_1$  without bias. The visualization is identical to the one I used for the neural networks in Figure 3.1. The rows show the results for the different environments, and the columns show the polynomial with degrees 1, 2, and 3. Looking at the first column, we can see a striking resemblance to the performance of the neural network without hidden layers shown in Section 3.1 for all three environments. This observation is not entirely a surprise since the neural network without hidden layers represents a linear controller. However, the second and last columns show different results for the environments CartPole and Acrobot.

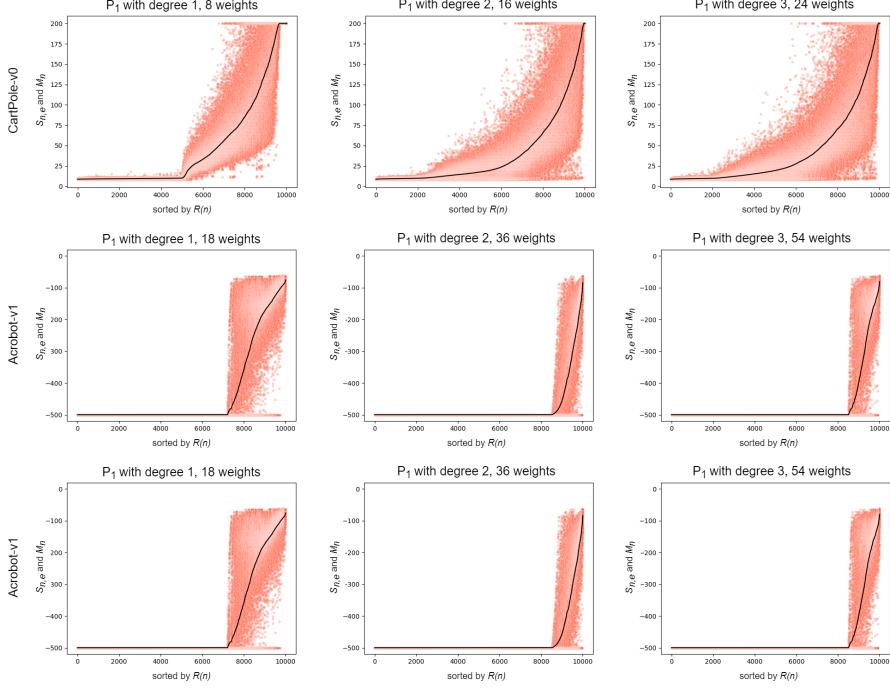
**CartPole:** For the environment CartPole, we can see that the curve of the mean score stays low until around 5'000 but then goes up relatively steeply. That means the linear model fails around 50% to guess the action correctly. However, after that, we have a high probability to achieve a good score or even solve the task entirely during multiple episodes. There are also quite a few samples that could solve the task each time, as indicated by the short straight black line at the top of the plot. Looking at the polynomials with degree 2 for CartPole, we can see that the scores increase already at around 2'000, but the slope is less steep than for the linear model. Therefore, we start earlier to get meaningful results, but the scores increase only slowly. This aspect is significant for a learning algorithm. In addition, there are fewer samples that could solve the task for each episode than there are for the linear model. Furthermore, the variance is higher for the polynomials with a higher degree compared to the linear model. If we look at the results of the polynomials with



**FIGURE 3.1: Results for all classic control environments using neural networks without bias.** Each row shows the results of one environment, whereas the columns represent the different network architectures. These plots are a reproduction of the results of the RWG-paper. My results are consistent with those shown in the paper.

degree 3, we can see that there is only a tiny boost compared to the polynomials of degree 2. The scores are overall slightly higher, but the slope is similar to before. The considerable difference lies between the polynomials with degree 1 and polynomials with degree 2, respectively, degree 3.

**Acrobot:** For the environment Acrobot, the polynomial model with degree one delivers a score distribution indistinguishable from the one of the neural network without hidden layers. The lineplot stays at -500 until around rank 7'500. Then it goes up steeply. Thus, we have a large plateau of minimum scores before we reach scores over -500. That opposes a higher difficulty level for the learning algorithm than CartPole. But with RWG, we can still reach relatively high scores, and the



**FIGURE 3.2: Results for the discrete classic control environments using the polynomial model  $P_1$  without bias.** Each row shows the results of one environment. The columns represent the different degrees of the polynomials. In these plots, the model  $P_1$  was used. The results aligned in the first column are almost indistinguishable from the ones of the neural networks without hidden layers. Nonetheless, the environments CartPole and Acrobot deliver different results for a higher degree of the polynomial.

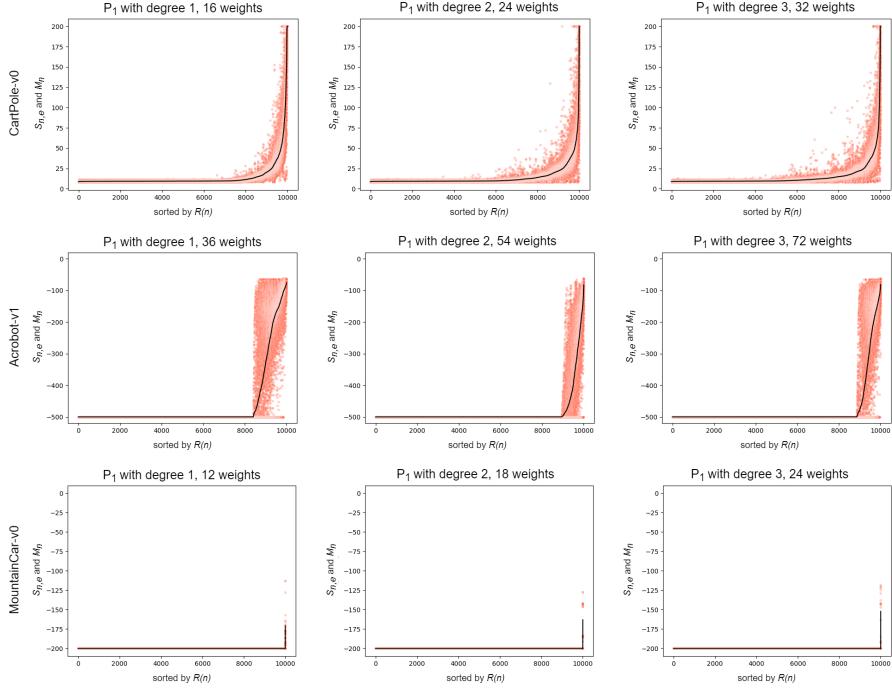
lineplot is continuous. Interestingly, the polynomial models with a higher degree performed noticeably worse. We cannot observe this behavior with neural networks.

**MountainCar:** The environment MountainCar is more challenging to solve than the other environments, as we can see by the results of both the neural networks and the polynomial model. Both models show very similar findings. There is a large fitness plateau, and only a few samples can reach a score higher than the minimum score of -200. That applies to both the linear models and the ones with higher complexity.

**Bias:** Figure 3.3 shows the results of using the polynomial model  $P_1$  to solve the discrete classic control environments using bias. Comparing these results with those without using bias in Figure 3.2 shows a significant difference. The bias influences the performance of the model negatively. The individual and mean scores are visibly lower for all three environments for all three configurations of the model. Thus, the observation about the bias the authors made in the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* is not specific to neural networks but seems to be more of a general factor.

### 3.2.3 Binary Tree Model

For the binary trees, I tested three models, one with 1 node, one with 4 nodes, and one with 8 nodes. Since the previous experiments suggest that using bias has a

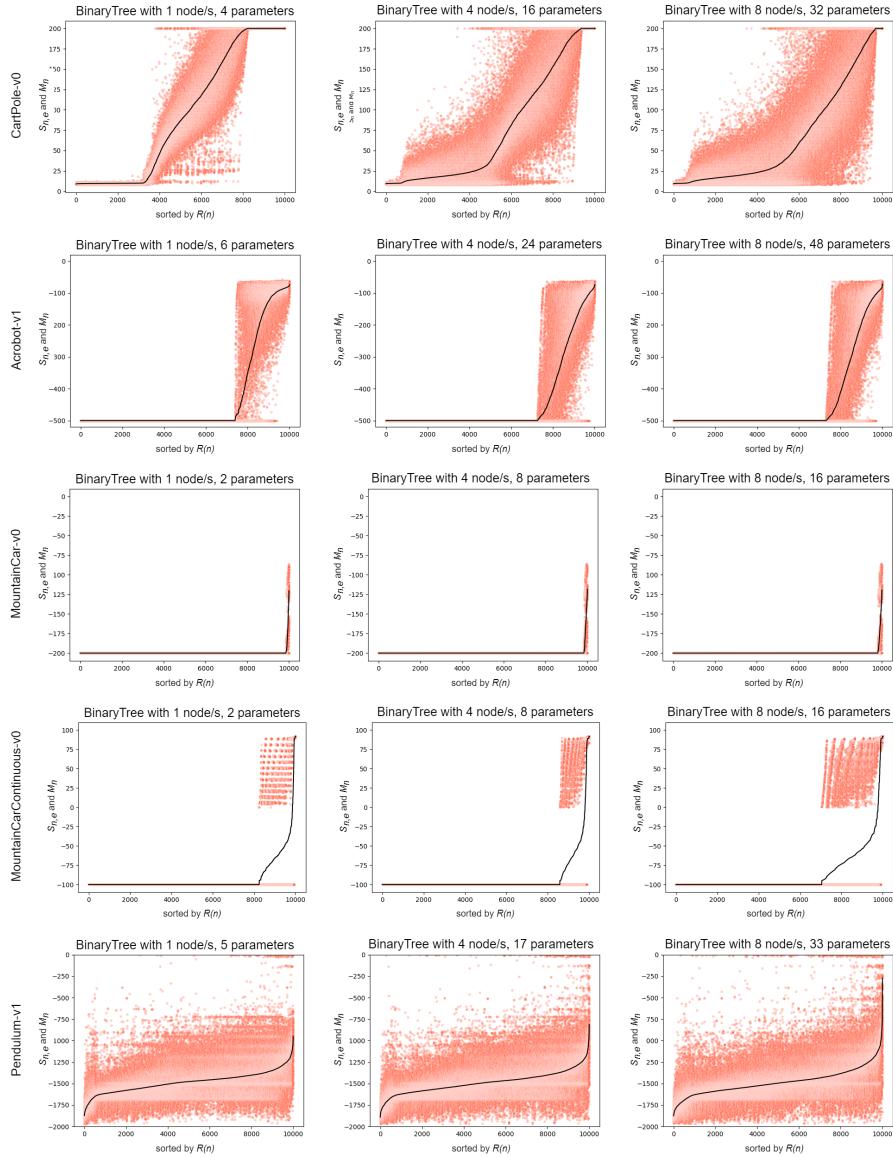


**FIGURE 3.3: Results for the discrete classic control environments using the polynomial model  $P_1$  with bias.** Each row shows the results of one environment. The columns represent the different degrees of the polynomials. For these plots, the polynomial model  $P_1$  was used. Overall, the model performs noticeably worse with bias as opposed to without bias.

negative effect, the following experiments do not include bias. Figure 3.4 shows the results for all five classic control environments with the binary tree model.

**CartPole:** Comparing the results using binary trees with those using neural networks shown in Figure 3.1, we can see a few differences. For the environment CartPole, we can see that we generally reach better scores with the binary trees. Comparing the neural network without hidden layers with the binary tree with only one node, we have a similar curve, but the lineplot representing the mean scores sets off much earlier at around 3'500 for the binary tree than the one for the neural network, which sets off at around 5'000. In addition, the binary tree has a higher chance of reaching a maximal mean score, indicated by the straight black line at the top right. For the more complex architectures, it gets more interesting. The binary tree with four nodes can already raise the mean value under 1'000 samples. That tells us that the problem of the fitness plateau is reduced significantly. Thus, when using a learning algorithm, the binary tree has a better chance of solving the problem without getting stuck in a fitness plateau. The chance of reaching a maximal mean score decreases with adding more nodes to the binary tree. That could be caused by the increasing complexity of the binary tree, which makes it harder to guess the weights correctly. Furthermore, the scores are more spread out for the binary trees than for the neural networks, indicating a higher variance.

**Acrobot:** Comparing the results of the binary tree model with one node with the neural network without hidden layers for the environment Acrobot, the binary tree produces a better score distribution. Although the fitness plateau remains to the



**FIGURE 3.4: Results for the classic control environments using binary trees without bias.** Each row shows the results of one environment. The columns represent the different configurations for the binary tree models. Even with only one node, the results are comparable to those from neural networks or outperform them.

same extent, the individual scores are overall higher, and the slope of the mean scores has improved, demonstrating a steeper slope. The neural networks with higher complexity seem to slightly outperform the binary trees with four and eight nodes. The scores are slightly higher, and the slope of the mean scores is a bit steeper. However, the difference is not massive. Overall, we can say that both models reach similar scores and mean values.

**MountainCar:** Comparing the two models, binary trees and neural networks, for the environment MountainCar, we can see that they reach similar mean scores. However, the binary trees are more likely to reach more and higher individual scores. The risk of being stuck in a fitness plateau remains for the binary trees. In addition, the scores are spread wider for the binary tree than for the neural network.

**MountainCarContinuous:** The results for the environment MountainCarContinuous look very different for the binary trees than for the neural networks. Neural networks show a slowly increasing curve with a peak after 9'500-9'900 samples for the mean scores. The binary trees have a lot of low performers displayed as a plateau of minimum scores. We cannot see this bad behavior in the results of the neural network models. However, there are a few samples that can reach a positive score. The neural network without hidden layers could not achieve these high mean scores.

This behavior is likely caused by the binary tree outputting discrete values instead of continuous ones, as it is the case for neural networks. The actions are fixed with the current implementation of the binary tree model. The two possible actions are the minimum and maximum of the action defined by the environment. In the case of MountainCarContiuous, this results in either -1 or 1. However, experimenting with this action pair shows us some interesting results for the environment MountainCarContiuous. Fiugure 3.5 shows the results of using the action pair 0 and 1 instead of -1 and 1. There is a significant improvement in the model's performance.

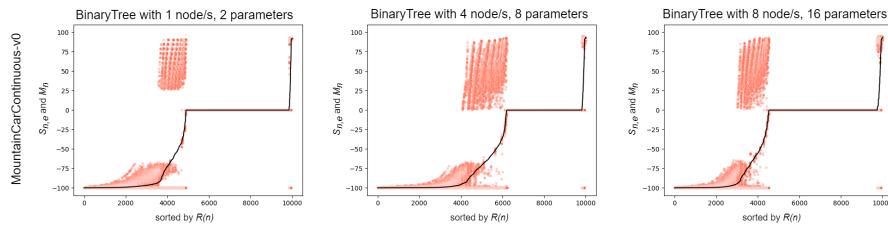


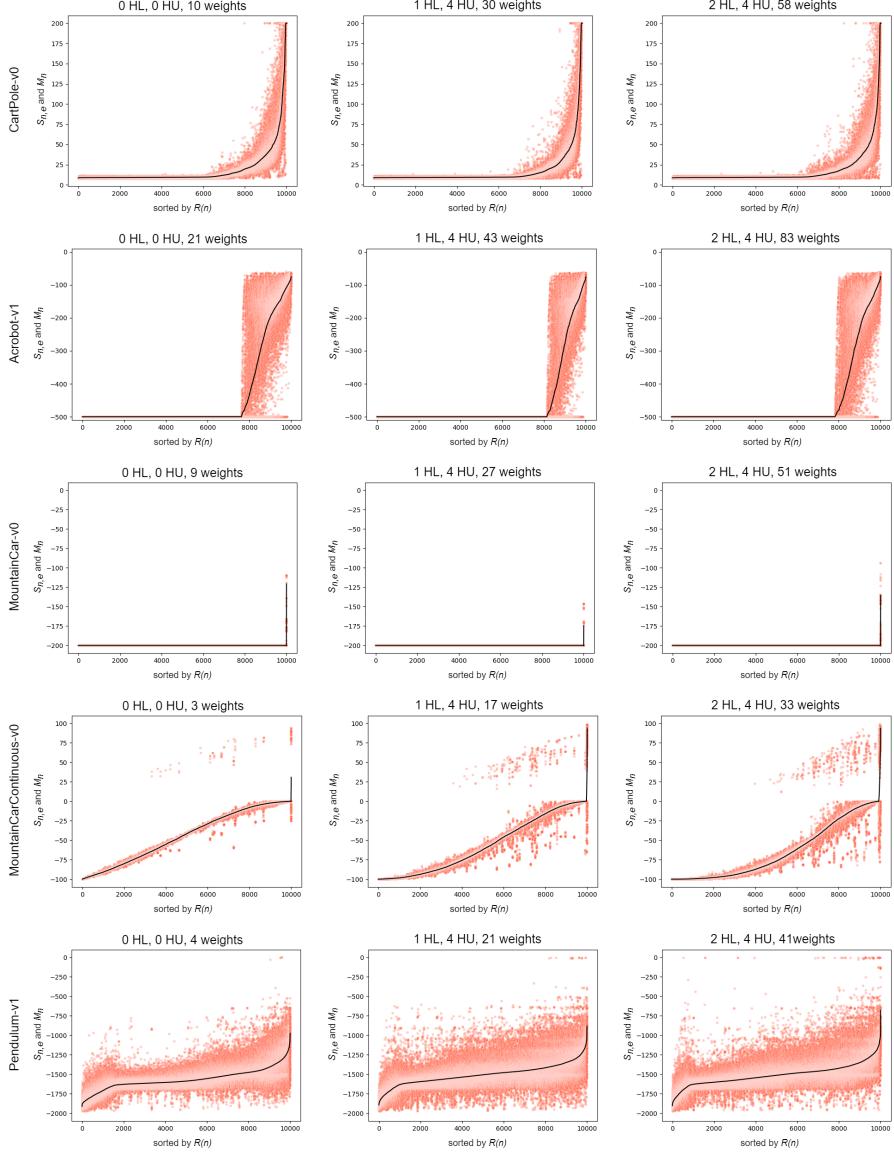
FIGURE 3.5: **MountainCarContiuous using the binary tree model with actions 0 and 1.** The figures show the results of the environment MountainCarContiuous using binary trees with fixed actions 0 and 1 as opposed to -1 and 1. The model's performance increased significantly compared to the results in Figure 3.4.

There are still some low performers, but the score distribution improved noticeably. However, many samples produce a zero score, resulting in a large fitness plateau.

**Pendulum:** For the Pendulum environment, we can see that the binary trees perform increasingly better with more nodes. The binary tree with one node reaches a mean score of around -1'500 in the large middle area. These scores are slightly higher than those of the neural network without hidden layers in the middle area. However, the neural network reached a higher maximum mean score than the binary tree. With an increased number of nodes, the binary tree model even outperforms the neural networks with remarkably higher mean scores. Even though the actions produced by the binary trees are discrete instead of continuous, the curve of the mean scores looks very similar to the one of neural networks.

### 3.2.4 Bias Investigation

In their paper, Oller, Glasmachers, and Cuccu (2020) mentioned that using bias negatively affects the score distribution leading to fewer top performers and generally lower scores. Figure 3.6 shows my results for all classic control environments with neural networks without using bias. Comparing these plots with the ones without using bias in Figure 3.1, we can see the negative impact the bias has on the effectiveness of the model. For the environment CartPole, the scores are overall lower and get slightly worse with the increased complexity of the model. Interestingly, for



**FIGURE 3.6: Results for all classic control environments using neural networks with bias.** Each row shows the results of one environment, whereas the columns represent the different network architectures. For most environments, we can see that introducing a bias has a negative effect on the score distribution.

the environment Acrobot, the network without hidden layers is resistant to the negative impact of the bias. However, the network with one hidden layer results in lower scores with bias. The network with two hidden layers has slightly better results than the one with one hidden layer but still performs worse than the one without bias. The same observation can be made for the environment MountainCar. For the environment MountainCarContinuous, we can see a negative effect on the score distribution for all three network architectures. The environment Pendulum does not seem to suffer much when introducing bias. For all three architectures, there is not much difference visible. The scores are generally lower and produce less top performers when using bias. Therefore, we can confirm the interesting observation of the authors, namely, that using bias is counterproductive in this experimental setting.

**Altering the number of weights:** To further investigate this counterintuitive behavior, Figure 3.8 shows the results of the number of weights for the polynomial model, and Figure 3.8 shows the results of tripling the number of weights for each network architecture. Comparing the results of the polynomial models, we cannot

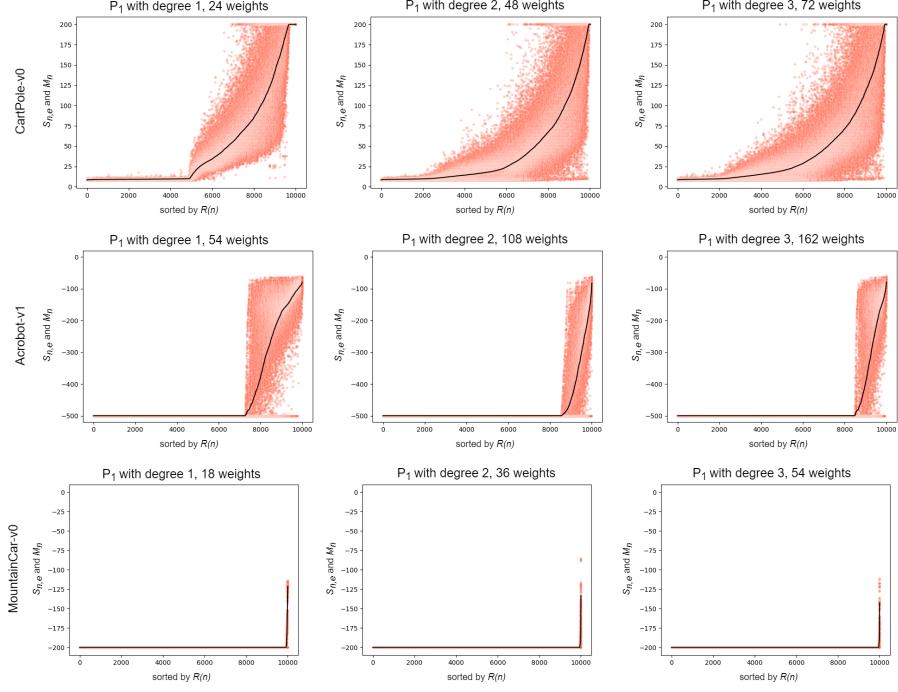
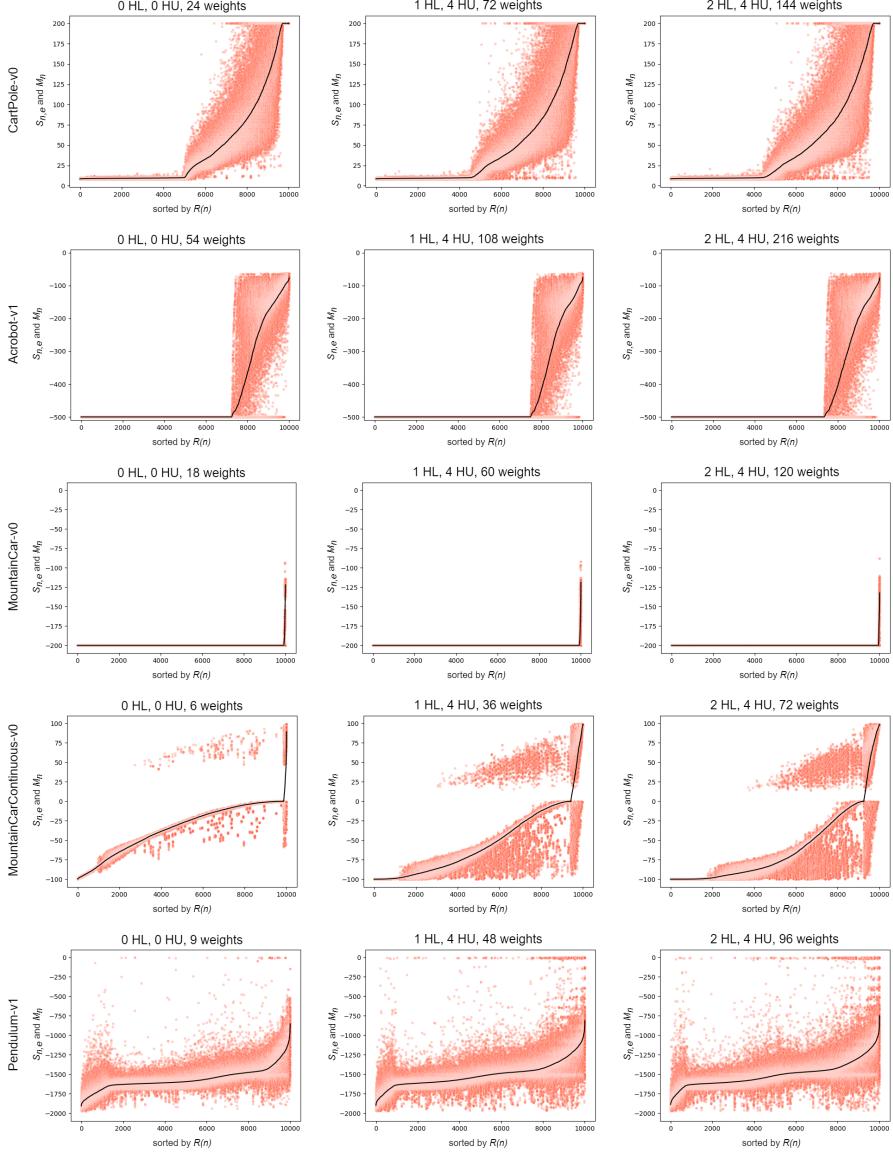


FIGURE 3.7: **Results of tripling the number of weights for the polynomial model.** The figure shows the results of tripling the number of weights for the polynomial model for the three discrete classic control environments. Despite the increased number of weights, there are only slight differences visible in the plots of the environment MountainCar compared to the ones shown in Figure 3.2. Acrobot and CartPole look indistinguishable from the other results.

see a difference for the environments CartPole and Acrobot comparing these results with those shown in Figure 3.2. The environment MountainCar shows only slight differences.

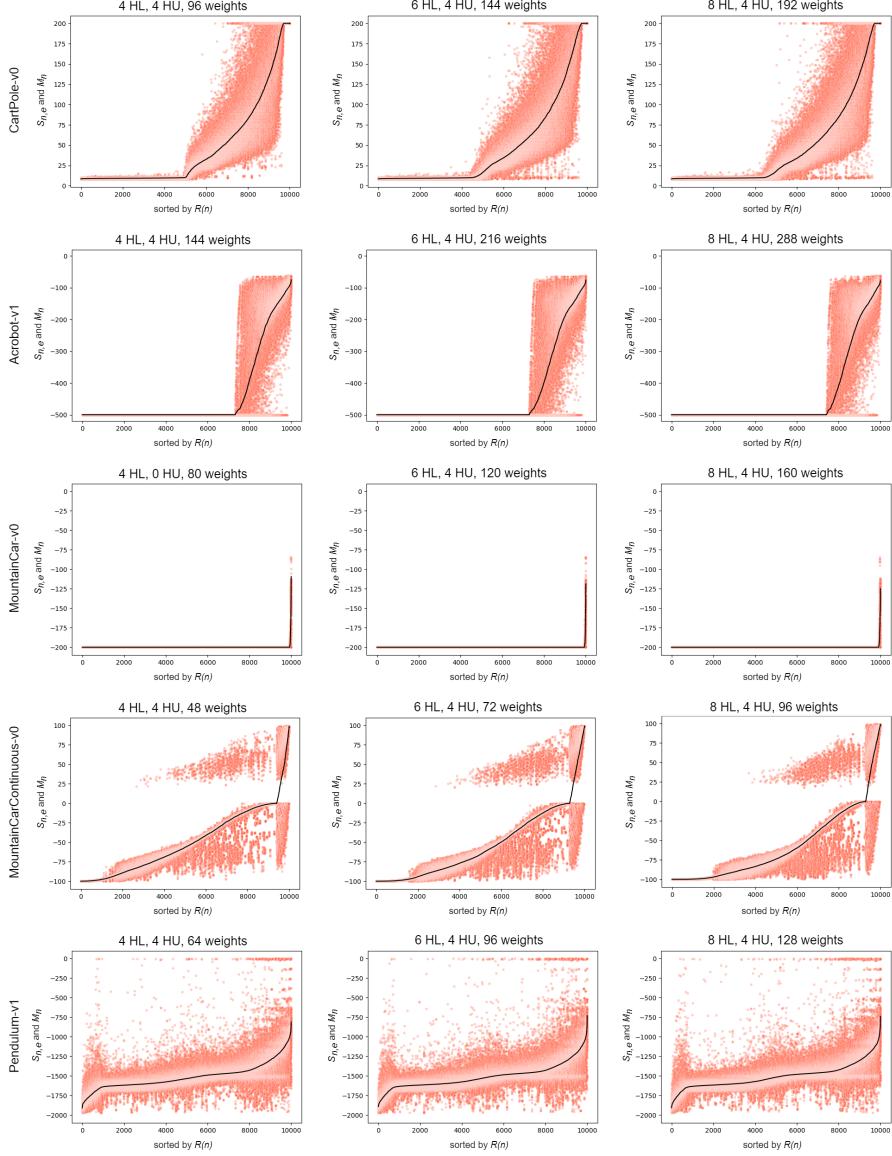
Comparing the plots in Figure 3.8 to those in Figure 3.1, we cannot see a huge difference. The score distribution of the environments CartPole and Acrobot look indistinguishable from those presented in Figure 3.1. We can see slight differences in the plots for the environment MountainCar. Increasing the number of weights for the network without hidden layers results in a few better individual scores without much difference in the mean score distribution. We see slightly better mean scores for the network with one hidden layer, whereas the network with two exhibits slightly worse mean scores. For the environment MountainCarContinuous, the mean scores are lower compared to the networks with more weights. However, there are more top performers. This is especially visible for the network without hidden layers. In addition, the scores are more spread out. There are a few more samples that reach a higher score for the environment Pendulum with the network without hidden layers. However, the networks with one and two hidden layers deliver similar results to those shown in Figure 3.1 for the environment Pendulum. To summarize, altering the number of weights for a network can produce a slight difference in the



**FIGURE 3.8: Results of tripling the number of weights for neural networks.** The figure shows the results of tripling the number of weights for the three neural network architectures for the classic control environments. Despite the increased number of weights, there are only slight differences visible in the plots compared to the ones shown in Figure 3.1 for all environments with the exception of MountainCarContinuous.

results for some environments. But these differences are not as dramatic as we have seen for the networks without using bias.

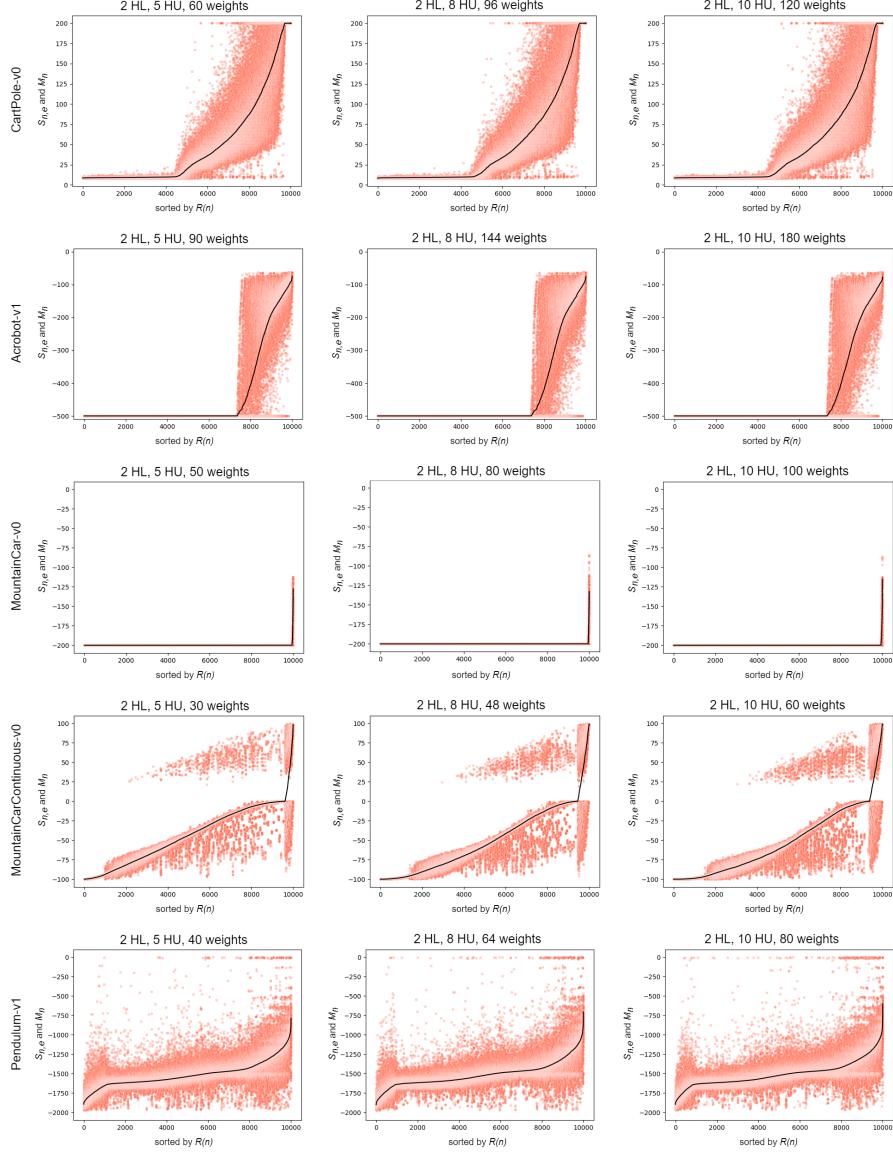
**Altering the number of layers:** Figure 3.9 shows the results of altering the number of layers in a network. Each layer still has the same amount of neurons, namely four neurons. For the environment CartPole, the lineplot of the mean scores seems to start earlier to rise above the minimum score, but the slope is also slightly less steep with an increasing number of hidden layers. The environment Acrobot does not seem to be affected anyhow by the change in the number of layers. The plots look indistinguishable. The mean scores for the environment MountainCar seem to



**FIGURE 3.9: Results of altering the number of layers for neural networks.** The plots show the results of increasing the number of hidden layers in a neural network gradually. The score distribution changes with an increased number of layers for some environments. However, the changes are less extreme compared to the results of the networks that used bias.

be best for the network with four hidden layers. The scores tend to get lower with an increased number of layers. For the environment MountainCarContinuous, the slope of the mean scores changes. The scores get generally slightly lower with more layers. Finally, we cannot see a big difference in the plots for the environment Pendulum by adding more layers. There may be a few more top performers with more hidden layers, but the difference is not significant and is not reflected in the mean score distribution.

**Altering the number of neurons:** Figure 3.10 shows the results of increasing the number of neurons in each hidden layer for a network with two hidden layers. We cannot see much difference in the plots for the environments CartPole and Acrobot.



**FIGURE 3.10: Results of altering the number of neurons for neural networks.** The plots show the result of increasing the number of neurons for each layer in a network with two hidden layers. For some environments, there is a change in the mean scores. However, the changes are not extreme.

For both, the score distribution seems to be unaffected by the increased number of neurons. The mean scores for the environment MountainCar first decrease with five and eight neurons. Then with ten neurons, we achieve higher scores again. For the environment MountainCarContinuous, the plots look similar to the ones in Figure 3.9 with an increased number of layers. We can see a slight decrease in the scores with increased neurons which is less extreme than when we increased the number of layers. With the environment Pendulum, we can reach higher mean scores with an increased number of neurons. Otherwise, the slope of the mean scores and the overall score distribution looks very similar.

### 3.3 Discussion

This chapter delivered a direct comparison between neural networks and alternative models like polynomials and binary trees. In addition, I investigated the impact of bias in these settings. The subsequent paragraphs aim to discuss the results to prepare for the conclusions and proposed future work in the next chapter.

**Comparison of polynomial model with neural networks:** As we have seen in the last section, the polynomial model can deliver comparable results to neural networks for discrete environments. The results of the environment `MountainCar` with the polynomial model look almost indistinguishable from those of the neural network model. For the environment `Acrobot` the polynomial model with degree delivers very similar results as the neural network without hidden layers. However, with the higher complexity of the model, neural networks outperform the polynomial model. Similarly, the polynomial of degree one results in the same score distribution as the neural network without hidden layers. Nonetheless, the score distribution differs noticeably from neural networks with higher complexity of the model. The mean scores show the same trend for all neural network architectures, which cannot be applied to the polynomial model. With the linear polynomial model, we have a 50% chance of failing, but the probability of actually solving the task for all episodes is higher than for the polynomials with a higher degree. That means with the linear model or neural networks, we have a larger fraction of samples that can solve the environment independently of the initialization conditions. At first glance, we could assume that this is an important aspect for such a model and choose the polynomial with degree 1 over one with a higher degree. However, we should remind ourselves that these experiments are rather unusual for an application since there is no learning involved and the number of samples is huge. In a standard application, we would use some kind of training and want the model to sequentially improve its performance. Considering this aspect, when using a learning algorithm, we would not prefer the polynomials with degree one or the neural networks as we might get stuck in a fitness plateau when the algorithm has no method of dealing with this behavior.

To summarize, the results of the polynomial model are comparable to those of neural networks. However, this applies only for the three discrete environments since the polynomial model is not constructed to deal with a continuous action space.

**Comparison of binary tree model with neural networks:** Despite the simple structure of the binary tree model, it delivers comparable results to neural networks or even outperformed them. The binary tree model significantly outperforms neural networks for the environment `CartPole` for all three architectures. The binary tree with only one node shows a similar slope for the mean scores as the network without hidden layers, but it visibly reduces the plateau of minimum scores. The binary trees with higher complexity even manages to remarkably shrink the plateau, which none of the three architectures of the neural network did on that scale. The environment `MountainCar` yields similar mean scores for the binary trees and the neural networks. Thus, there is a large plateau of minimum scores for both models. However, when comparing the three configurations of the two models with each other, we can see that the binary tree model reaches more and higher individual scores than the neural network models. In addition, the results of the two models are similar for the two

models for the environment Acrobot. The binary tree with one node outperforms the neural network without hidden layers whereas the complexer neural networks outperform the binary trees with four and eight nodes.

Even though the binary tree model at its current implementation outputs only discrete action values, the results for the continuous environment Pendulum are comparable to those of neural networks. The binary tree with eight nodes even managed to outperform neural networks. For the environment MountainCarContinuous, neural networks delivered better results. Although the binary tree model with one node reached significantly higher mean scores than neural networks, it exhibits a large plateau of minimum scores that we do not see for the neural networks. The networks with higher complexity reached identical high mean scores. The considerable difference in the score distribution between the two models could be caused by the binary tree producing discrete actions instead of continuous ones, as neural networks did. For the binary tree model, the action is either -1 or 1, limiting the output space significantly. The environment MountainCarContinuous seems more sensitive to these discrete action outputs than the environment Pendulum. The experiments with the choice of actions resulted in a much better score distribution for the action pair 0 and 1.

All in all, the binary tree model produced a similar score distribution as neural networks did. For some environments, the binary trees even outperform neural networks. However, the environment MountainCarContinuous seems more difficult for the binary trees to learn than for neural networks.

**Impact of bias:** Including bias has a significant negative impact on almost all of the five classic control environments. For the environment Acrobot and Pendulum, the difference between using bias and not using bias seems less extreme than for the other environments depending on the number of layers used. To further investigate this behavior, I subsequently changed three aspects of the model's architecture. First, I increased the number of weights of the networks. Despite the remarkably increased number of weights, the results show only slight differences in the score distribution. The exception is the environment MountainCarContinuous, where the mean scores are remarkably lower. Except for the environment MountainCarContinuous, we can conclude that the increased number of weights does not impact the score distribution significantly. Second, I increased the number of hidden layers. Some environments show slightly different results like CartPole and MountainCar. However, the changes are less significant than in the experiments with bias connections. The exception is again MountainCarContinuous, where the difference is noticeably more extreme with an increased number of layers. Finally, I increased the number of neurons in each hidden layer. In the results of this experiment, some changes in the score distribution are visible. The mean scores of the environments Pendulum and MountainCar show slight differences. The environment MountainCarContinuous shows a similar picture as before.

In summary, no other aspect impacts the performance of the model as negative as using bias. The exceptions are the environment MountainCarContinuous, where we see even more extreme differences when changing the network architecture, and Pendulum, which already shows smaller differences than the other environments when using bias versus not using bias. It seems that this behavior is specific to the bias in the setting of RWG for the environments CartPole, Acrobot, and MountainCar. In addition, we have seen that the environment MountainCarContinuous generally reacts sensitive to a change in the model's architecture, showing a similar image as when using bias. In contrast, Pendulum does not seem to be significantly

affected by a change in the architecture. But as we have seen, the bias also affects it less.

## Chapter 4

# Conclusion

This work's driving question was whether we could find alternative models for reinforcement problems whose performance is comparable to the commonly used neural network model. To investigate how these models compare to neural networks and what advantages they have, two research questions were introduced in Section 1.7. In addition, one research question was formulated concerning the architecture of a model for additional analysis.

**RQ1: How do function approximators other than neural networks compare with the latter?** In this work, I directly compared the alternative models, the polynomial and binary tree models, to neural networks. The polynomial model produced results comparable to those of neural networks. For the environment `CartPole`, it delivered a more desirable score distribution for polynomials of degrees two and three. However, we would still prefer neural networks to tackle the problem of the `Acrobot` environment. The environment `MountainCar` seems to produce the same score distribution regardless of the model used. Thus, we cannot conclude that using the polynomial model robustly produces better results or results comparable to neural networks.

On the other hand, the binary tree model could deliver comparable results and even outperformed neural networks several times. The simplicity of the model did not limit the model's performance. The opposite is the case. It significantly outperformed neural networks for the environment `CartPole`. Since `CartPole` represents a relatively simple problem, the simple structure of binary trees is advantageous compared to the complex architecture of neural networks. Neural networks have proven that they are capable of solving many challenging problems. However, finding a good strategy with neural networks seems less efficient for a more straightforward task. The search space of the model can explain these results. A more complex model like neural networks with multiple hidden layers has a much larger search space in which they search for a good strategy for a given task. A large search space is a disadvantage when the task is easier to solve. Thus, we must carefully select our approach and adapt to the complexity of the task we want to solve.

The environment `MountainCar` again produces a similar score distribution. The mean values seem to be identical. There are, however, a few higher individual scores for the binary tree model. As we can see in the results, the problem implemented in `MountainCar` is hard to solve by RWG. All of the tested models showed similar findings. The large fitness plateau remains for all of them. To solve the problem, we need a learning algorithm high in exploration. For this use case, techniques in black-box optimization offer a good framework. Training a model with a learning algorithm prone to getting stuck in a local optimum will likely not result in a robust model that can solve the task. Also for the environment `Acrobot`, the results of the two models look very similar. Although there are a few differences, both models

seem equally suited to solve this task with the proper learning technique. Since a fitness plateau is present, an explorative learning algorithm should produce the best results.

Although the binary tree model only outputs discrete actions that are fixed, the results for the environment `Pendulum` can compare to those of neural networks. Again, both models seem to be suited to solve the task reliably with an appropriate learning algorithm. In this case, the danger of being stuck in a local optimum is less prevalent than for the environment `Acrobot`. Thus, the learning algorithm does not have to be as explorative. The environment `MountainCarContinuous` delivered counterintuitive results. The restriction of the binary tree model to only output the two actions -1 and 1 from the otherwise continuous action space negatively influenced the model's performance. However, changing the actions from -1 and 1 to 0 and 1 resulted in a significantly better score distribution. Since the action defines the directional force applied to the car, a value between 0 and 1 means that we apply no force at all or actively push the car to the right in the direction of the target. Thus, the results show that the model does not necessarily have to learn to accelerate the car to the left and right to solve the task.

To conclude these findings, the binary tree model looks promising for problems in reinforcement learning. Combined with a suited learning technique, it should robustly produce results comparable to those of neural networks.

**RQ2: What advantages and disadvantages can we see with other function approximators?** With neural networks, we have many hyperparameters that need fine-tuning. We can completely change the model architecture by changing the number of layers or the number of neurons. We have to set many other components, like the activation function. In both the polynomial model and the binary tree model, we have only one parameter, namely the degree of the polynomial and the number of nodes of the binary tree. Thus, finding a suitable configuration of the architecture of the alternative is straightforward and does not need much effort as opposed to neural network models. In addition, the binary tree model offers much more insight into decision-making than the neural network model. To comprehend how a neural network makes a decision is very hard. The binary tree model offers much more interpretability in that sense.

**RQ3: How does the bias influence the performance of the model?** Adding bias had a negative impact on all discrete models for the polynomial model and on almost all five classic control environments for the neural network model. In my experiments with the number of weights, the number of layers, and the number of neurons, none of these changes influenced the model's performance as much as the bias did. There seems to be something specific about the use of bias that changes the performance for these environments in the setting of RWG. Why this is the case needs further research. However, there are two exceptions. The environment `MountainCarContinuous` was very reactive to each change in the architecture or the number of weights. The environment `Pendulum` did not show a significant difference when using bias vs. when not using bias. A change in the architecture or the number of weights affected it similarly.

## 4.1 Future Work

There is much further research that can be done in this area. This work represents the first step in the direction of alternative models like the binary tree model. This thesis showed that the binary tree model looks promising and can produce results comparable to neural networks while maintaining a simple architecture and interpretability. However, it would be interesting how we can adapt the model to work better on specific problems. The current implementation of the model can undoubtedly be improved. For continuous action spaces, we can adapt the leaf nodes to be included in the learning algorithm or implement a function instead of outputting a fixed action.

So far, no practical learning technique has been involved. It would be interesting to see how the model performs with CMA-ES instead of RWG. Additionally, the model should be tested in more challenging benchmark problems like the environment `BipedalWalker` also included in the set of environments provided by the OpenAI Gym interface.

Finally, it would be interesting to see how other models perform in this setting. With techniques in black-box optimization, there are numerous possibilities. For example, we could use an adapted version of the Fourier transform.



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