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MASTER THESIS

Non-Differentiable Function Approximation: Beyond Neural Networks

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Abstract

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Non-Differentiable Function Approximation: Beyond Neural Networks

Neural networks as generic function approximators can solve many challenging problems. However, they can only be applied successfully for a suited problem structure. In specific, neural networks require differentiability. But there are many areas where calculating an accurate gradient is non-trivial, including problems in Reinforcement Learning (RL). In contrast, Black-Box Optimization (BBO) techniques are less limiting. They presume no constraints on the problem structure, the model, or the solution. With this flexibility, we can study alternative models to neural networks that are yet unexplored in the context of RL. This thesis aims to achieve good results with a function approximator other than neural networks. I analyze promising models optimized with BBO methods.

Problem -> Solution -> Results
TODO: describe models and results

Keywords: Black-Box Optimization, Reinforcement Learning

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Chapter 1

Introduction

1.1 Reinforcement Learning Control Problems

Control problems appear in the context of both reinforcement learning and control theory. Thus, we often see vocabulary specific to control theory in the context of reinforcement learning. A control problem, generally explained, is a dynamic system described by state variables. The choice of controls or actions determines how the system will behave in the future. The goal of a control problem is to work out a strategy that causes the system to terminate in the desired target state. Reinforcement learning and approaches in control theory offer a framework to solve such control problems (Buşoniu et al., 2018). Even though the two fields embark on the same problem, the research communities remained mostly disjoint, leading to different approaches to the same problem. Reinforcement learning often makes model-free predictions from data, whereas control theory often defines well-specified models (Recht, 2018). In this thesis, I will focus on reinforcement learning frameworks to solve control problems.

Reinforcement learning is an area of machine learning that involves learning the optimal behavior to maximize a reward signal inside a dynamic environment. The learning process is marked by trial-and-error interactions with the environment. The learner does not know which actions lead to which rewards until he tries them (Kaelbling, Littman, and Moore, 1996).

Reinforcement learning differs from supervised learning, where learning is achieved through a training set with labeled examples. Each sample of the training data represents a description of the situation together with a label that marks how the system should react in that situation. The goal of the learning process is to generalize the optimal actions given a certain situation so that the system acts correctly in situations outside of the training set. In reinforcement learning, there are no labels available beforehand. In addition, we can also not classify reinforcement learning as unsupervised learning, where we typically try to find a hidden structure in a set of unlabeled data. With reinforcement learning, we do not try to find some structure in the data but solely try to maximize the reward signal. Therefore, alongside supervised learning, unsupervised learning, and maybe additional paradigms, reinforcement learning is considered to be a third machine learning paradigm. One additional challenge we do not see in the other branches of machine learning is the trade-off between *exploration* and *exploitation*. For a high reward, the reinforcement learning agent should take an action that it has tried in the past and was effective. It should therefore exploit the knowledge gained previously. However, to discover this beneficial action, it has to explore actions it has not taken before (Sutton and Barto, 2018).

We can represent many problems in machine learning, including reinforcement learning problems, as a function mapping an input space into an output space. The

output space or *search space* denotes the space of all feasible solutions. In reinforcement learning, the output space contains any action that could be taken given a certain situation. The underlying function is the output of the learning process of an algorithm and is often called the *target function*. Optimally, we would derive the formula of the function explicitly. However, even though we suspect that such a function exists, we usually do not have enough information to derive a formula directly. Especially in reinforcement learning, the available information is very sparse. Thus, we aim to approximate the target function with *function approximators* by using the available data. Hereafter, I will often talk about models to solve a reinforcement learning task. The models generally represent function approximators to solve a given problem.

To overcome the mentioned difficulties unique to reinforcement learning, we need to experiment with alternative models and strategies than those used for supervised or unsupervised learning.

Include MDP.

1.2 Classic Reinforcement Learning

In reinforcement learning, we have an agent and an environment. The agent can take on actions to interact with the environment. The environment, on the other hand, always holds a certain state that can be altered by the actions of the agent. The goal of the agent in classic reinforcement learning is to learn a policy π that drives the system into the desired state by taking the right actions. For example, the goal might be to solve skill games like Ball-in-a-Cup or Kendama (Kober, Mohler, and Peters, 2010). As mentioned before, we do not have labeled data beforehand as we do in supervised learning problems. We only get a scalar reward signal that indicates how well the agent performs. The goal of the agent is to maximize the cumulative rewards received by the environment. Therefore, the agent needs to learn optimal behavior only through trial-and-error interaction with the environment. This makes the problem structure much more challenging. Figure 1.1 illustrates the loop be-

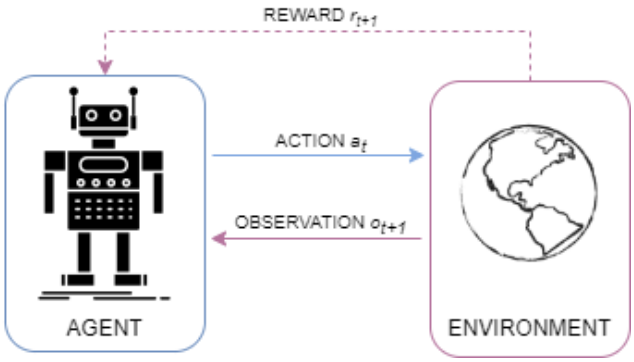


FIGURE 1.1: **Interaction loop between an RL agent and the environment.** At timestep t, the agent receives the observation o_t from the environment. The agent decides to select action a_t and interacts with the environment. The enviornment changes and returns the next observation o_{t+1} and the reward r_{t+1} to the agent.

tween the agent and the environment. The agent has received the observation o_t from the environment and therefore knows the environment's current state. The agent then performs some action a_t in the environment. The environment's state

changes according to the received action and returns an observation o_{t+1} indicating the current state of the environment and a reward r_{t+1}. One such interaction between the agent and the environment represents one *timestep*.

The rewards correspond to an underlying reward function specific to the environment. The reward function is a function that maps the current state of the environment, the current action, and the subsequent state into a scalar value. It can be deterministic or can contain a random component making it stochastic. Depending on the problem, the reward can be sparse or dense. Sparse reward means that the reward function returns zero in most of its domain and only returns a meaningful reward in a few rare settings or at the end of the experiment. That introduces a immense challenge since we cannot know which particular actions led to success or failure. Optimization problems with this setting are generally challenging to solve. Dense reward environments return a meaningful reward at almost every time step, making the optimization process much easier.

Learning a policy that maximizes cumulative rewards is generally non-trivial. The agent can only learn through trial and error. If the policy is not explorative enough, we might get stuck in a local optimum and never reach the goal. However, if there is too much exploration we receive less reward. Thus, there needs to be a balance between exploration and exploitation. In addition, there is often a time delay for the reward. The consequence of an action might not be immediately visible to the agent. This problem is known as the credit-assignment problem in the literature (Sutton and Barto, 2018).

We can solve reinforcement learning problems with methods based on value functions or methods based on policy search. The value function is a prediction of the future reward of landing in a specific state. It indicates how good or bad being in a certain state is. With methods based on value functions, the agent chooses the best action in the state. Methods based on policy search directly search for the optimal policy π^* and do not need to maintain a value function model (Arulkumaran et al., 2017).

As interest in neural networks and deep learning increased, deep reinforcement learning, which uses a neural network to estimate policies or value functions, attracted more attention.

Talk about sample efficiency? Include Q-Learning, reward shaping? show more the difference between this and dps

1.2.1 Neural Networks as Models

Neural networks are machine learning algorithms inspired by the functionalities of the human brain. They are applied successfully in many areas including in reinforcement learning problems. Neural networks consist of connected neurons or nodes that imitate the biological neurons of the brain sending signals to each other. A neuron receives one or multiple input values and calculates one or multiple output values. Figure [insert figure] illustrates one neuron. The neuron receives three input values and calculates one output value. For the caluclation of the ouptut, the neurons uses the assigned weights w. Thus, the output represents the sum of weighted inputs.

In a neural network, the neurons are arranged in connected layers. A network has at least an input and an output layer. We can further expand it with one or more hidden layers. Depending on the model, the connectivity between the layers differs. Figure 1.2 shows a sketch of a network with three hidden layers that are

fully connected. Each neuron holds an associated weight and threshold. It receives

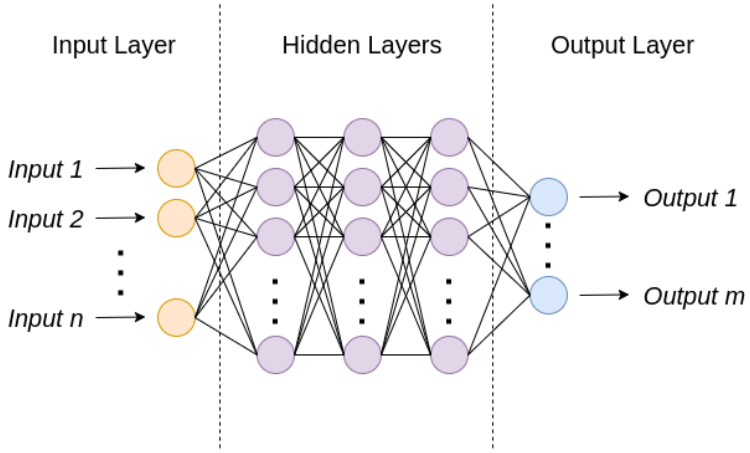


FIGURE 1.2: **Sketch of a neural network.** The image shows a sketch of a neural network with three fully connected hidden layers. The network expects n inputs and has m possible outputs.

an input and outputs the sum of weighted inputs. If this sum is greater than the associated threshold, the neuron gets activated.

A large part of the success of neural networks can be traced to the use of *backpropagation*. Backpropagation uses information from the previous epoch (i.e. iteration) to adjust the weights of the network using the error gradient.

A neural network with one or more hidden layers defines a non-linear function. It is theoretically able to represent any function. Thus, neural networks are *generic function approximators*. In addition, neural networks are highly expressive and flexible. In the context of reinforcement learning, they can be used to approximate a value function or a policy function. Deep reinforcement learning combines reinforcement learning with the methods from deep learning. It has been used in many applications including robotics, video games, and computer vision (François-Lavet et al., 2018).

A sigmoid function is a mathematical function that maps an arbitrary input space into an output space with a small range, for example, 0 and 1. The function has a characteristic S-shaped curve. We can interpret the output space of the sigmoid function as a probability. The formula and a plot of the logistic sigmoid function in 2D are shown in Figure 1.3.

The backpropagation algorithm's efficiency in calculating the gradient of an objective function is a key component of deep learning's success. However, there are certain limitations. For the backpropagation algorithm to be efficient, it relies on calculating accurate gradients. In fields like reinforcement learning, where we only have a reward signal that might be time delayed, this is a non-trivial task. In addition, there is a risk of getting stuck in a local optimum (Ha, 2017). These limits raise the question of which alternative models or techniques could be used that overcome these difficulties.

Integrate sigmoid function. Better connect text.

Should NN be explained in more detail? Write more about DRL, include recent results, advantages, disadvantages. Improve transition to DRL and Sigmoid.

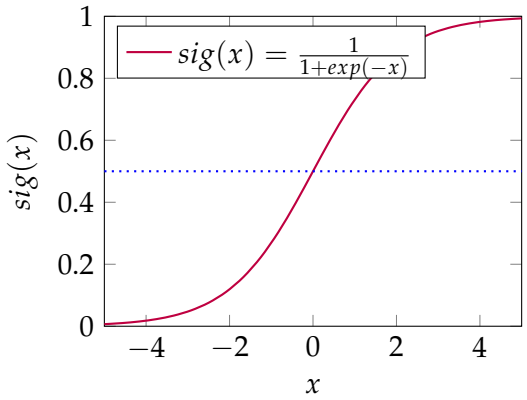


FIGURE 1.3: **Sigmoid function.** The figure shows a plot of the logistic sigmoid function. The function maps an arbitrary input space into the range between 0 and 1. The output of the function can be interpreted as a probability. It is useful to scale data into a meaningful value.

1.3 OpenAI Gym

The research community requires high-quality benchmarks to compare algorithms and models applied to problems in reinforcement learning. There were already a few released benchmarks like the Arcade Learning Environment (Bellemare et al., 2013) or the RLLab benchmark for continuous control (Duan et al., 2016). OpenAI Gym[1] combined the elements of these existing benchmarks in one toolkit. It provides a collection of tasks (called *environments*) with varying levels of difficulties and challenges. The interface is the same for all environments, which makes it convenient to use. The environments represent general reinforcement learning problems, including control problems, that are widely used in the literature to compare different approaches. To interact with an environment, we first have to initialize it. We can do this with the predefined function `make()`. The following Python code shows the initialization of the environment `CartPole`:

```
import gym
env = gym.make("CartPole-v0")
```

As we are in the setting of reinforcement learning, we assume that an agent is acting inside the dynamic environment. In each step, the agent can take some action and receives a reward and a new observation in return from the environment. Both the action and observation space are predefined by the environment. In practice, an agent returns an action `a0` with which we call the function `step()` of the environment. The next code snippet shows an example of this behavior:

```
observation, reward, done, info = env.step(a0)
```

The observation and reward returned by the environment applies to the current time step. OpenAI Gym focuses on episodic reinforcement learning, where the agent acts in a series of episodes with a fixed length. The initial state of the environment in each episode is sampled randomly. An episode ends when certain termination criteria are met. For example, when the task is solved or when we max out on timesteps (Brockman et al., 2016).

[1] https://www.gymlibrary.ml/

Explain more, compare with Section 2.2 to not repeat

1.4 Direct Policy Search

mention it as an alternative to Deep RL, mention it uses networks to approximate policy rather than value function

Although using value functions to solve reinforcement learning problems has worked well in many applications, there are limitations. If the state space is not entirely observable, convergence problems can arise (El-Fakdi, Carreras, and Palomeras, 2005). Additionally, in traditional value-based methods like Q-learning or temporal difference learning, the value function is computed iteratively with the help of bootstrapping. That often results in a bias in the quality assessment of state-action pairs for continuous state spaces. Another disadvantage of these methods is that the value functions are often discontinuous (Deisenroth, Neumann, Peters, et al., 2013).

Studies suggest that directly approximating a policy can be easier and delivers better results (Sutton et al., 1999; Anderson, 2000). This alternative method to methods based on value functions is called *direct policy search*. With direct policy search, the algorithm directly searches in the search space of policy representations discarding any value function (Wierstra, 2010). Instead of approximating a value function, we approximate the policy directly with an independent function approximator. Thus, we directly approximate the action-state mapping.

Explain more, go more into advantages, find more papers with comparison

1.5 Neuroevolution

special case of DPS: NN as policy approximator, train with evo

Neuroevolution is a method that we can use for direct policy search. Like deep reinforcement learning, we employ a neural network as our model. In deep reinforcement learning, we typically use gradient-based learning strategies, whereas, in neuroevolution, we employ evolutionary algorithms to optimize the network parameters. That reduces the risk of being unable to estimate accurate gradients or being stuck in a local optimum. Evolutionary algorithms only require a measure of the network's performance, making them directly applicable for reinforcement learning problems. Thus, we can use it more widely in a variety of problem classes. Such et al., 2017 showed that genetic algorithms, a subgroup of evolutionary algorithms, work well for the parameter search of neural networks. Even in hard reinforcement learning settings, including Atari and humanoid locomotion, they were able to achieve results at the scale of traditional deep reinforcement learning frameworks.

Explain more. more with direct policy search

1.6 Black-Box Optimization

but what is evo? here's a primer on black box optimization

In Black-Box Optimization (BBO), the problem structure, as well as the model, remains unknown. BBO methods optimize a parametrization without any constraints on the problem, model or solution needed. Thus, there is no constraint on differentiability or convexity. The optimization is done solely based on a score or cumulative reward, which makes these methods directly applicable to reinforcement learning problems.

With BBO, we can use any function approximator.

Explain more

1.6.1 Random Weight Guessing

Randomization and RWG (uniform distribution)

Random Weight Guessing (RWG) is the simplest BBO method. With RWG, we repeatedly randomly initialize the weights of a model until the resulting model classifies all training instances correctly. Then we test the model on a separate test set (Schmidhuber, Hochreiter, and Bengio, 2001). RWG is not a reasonable learning algorithm, but it can be used as an analysis method (Oller, Glasmachers, and Cuccu, 2020).

Explain more, include limitations (expensive algorithms)

1.6.2 Evolution Strategies

ES-(1+1) (normal, adapt mean)

Evolution Strategies (ES) belong to the class of evolutionary algorithms and are best suited for continuous optimization. They are directly inspired by the idea of evolution in nature and use the concept of mutation and selection. ES are implemented in a loop where one iteration is called a generation. In each generation new individuals (candidate solutions) are generated using the current parents. We continue with the next generation until a stopping criteria is met. ES are multi-agent algorithms which means that they generate multiple parametrizations that explore a different path or area in the optimization space independently. This exploration prevents us from landing in a local optimum.

(1 + 1)-ES: The population model (1 + 1) means that we maintain only one individual. For the next generation, we generate one new offspring as a variation of the parent, drawn from the normal distribution. For this, we adapt the mean of the distribution according to the parent. Then we keep either the parent or the child based on which performed had a better fitness score. Algorithm 1 shows this process in pseudocode.

Algorithm 1 (1 + 1)-ES in d dimensions

Parameter: $\sigma > 0$
Initialization: $x = x_0 \in \mathbb{R}^d$
while not done **do**
    Sample $x' \sim \mathcal{N}(x, \sigma^2 I)$
    if $f(x') \leq f(x)$ **then**
        $x \leftarrow x'$
Return x

Explain more

1.6.3 Covariance Matrix Adaptation Evolution Strategy

CMA-ES (which is a ES-(mu,lambda)) (normal, adapt mean, adapt covariance, large pop)

Covariance Matrix Adaptation Evolution Strategy (CMA-ES) is a method for non-linear, non-convex optimization problems in continuous domain. For each iteration (generation), a population of new individuals is generated by sampling a multivariate normal distribution. After each iteration, the mean and the covariance matrix is updated (Hansen, 2016).

Include formulas from tutorial

1.7 Research Questions

Start from the experiments. They are your answer. Which question do they answer? These are your research questions.
In the experiments section, mention explicitly which question each answers

The main goal of this thesis is to compare alternative models to neural networks. To analyze and compare the different models and their architectures, I formulated the following research questions:

1. How do function approximators other than neural networks compare with the latter?
2. What advantages and disadvantages can we see with other function approximators?
3. What impact has the bias on the performance of the model?

Work on questions.

Chapter 2

Method

2.1 Benchmarks in Reinforcement Learning

When developing a novel algorithm, it is important to compare our results with existing models. For this evaluation, we need standard benchmark problems. These are a set of standard optimization problems. OpenAI Gym is a toolkit created for exactly this scenario. As mentioned in Section 1.3, it contains a collection of benchmark problems with various levels of difficulty. However, not all benchmark problems are meaningful for the evaluation of an algorithm. If a problem is too trivial to solve, the results do not reflect the quality of the model adequately. We do not need to put a large amount of effort into the creation of a complex model for an easy-to-solve task.

In the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* (Oller, Glasmachers, and Cuccu, 2020), the authors analyze and visualize the complexity of standard RL benchmarks based on score distribution. They tested their approach on the five Classic Control benchmark problems from the OpenAI Gym interface: `CartPole`, `Acrobot`, `Pendulum`, `MountainCar`, and `MountainCarContinuous`. Given an RL environment, the authors conducted a fixed series of experiments. For these experiments, they used three neural network architectures ($N_{architectures} = 3$): a network without any hidden layers (0 HL, 0 HU), a network with a single hidden layer of 4 units (1 HL, 4 HU), and a network with two hidden layers of 4 units each (2 HL, 4 HU). With these, they cover a variety of network models that are suited to solve the given tasks. The evaluation of the benchmark problems should be as objective as possible and should not include bias in the data. To achieve this, the authors did not include any learning opportunities for the network models. Instead, they chose the network weights i.i.d. from the standard normal distribution $\mathcal{N}(0, 1)$ with Random Weight Guessing (RWG). This approach assures randomness and no directed learning. The goal of the paper was not to further analyze the network models but to investigate the benchmark problems themselves. With this in mind, they initialized 10^4 samples ($N_{samples} = 10^4$) with different random weights. The number of samples would be too large for a reasonable learning strategy. However, the large number of samples serves a different purpose than optimizing the results. Instead, the aim is to draw statistical conclusions. Each of these samples of a neural network represents a controller that maps observations to actions in the environment. Later in this thesis, I will explore function approximators other than neural networks representing the controller. In the paper, the authors tested the controllers for each environment during 20 independent episodes ($N_{episodes} = 20$). For each episode, they saved the score in the score tensor S. Algorithm 2 illustrates the procedure with pseudocode.

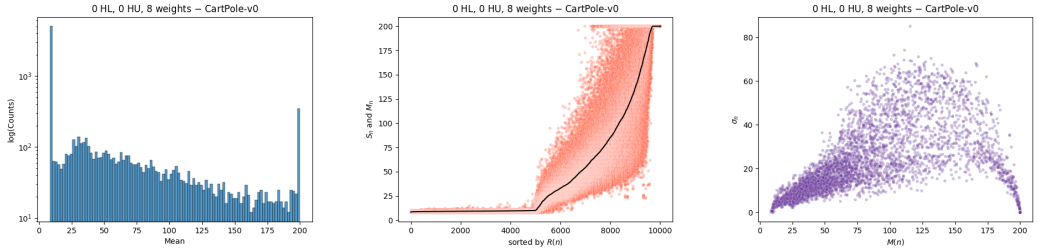
After the authors obtained the scores, they calculated the mean performance over all episodes from a sample and its variance. These statistics are significant insights. They can reveal how stable the network models are in completing a given task. A

Algorithm 2 Evaluation process taken from Oller, Glasmachers, and Cuccu, 2020

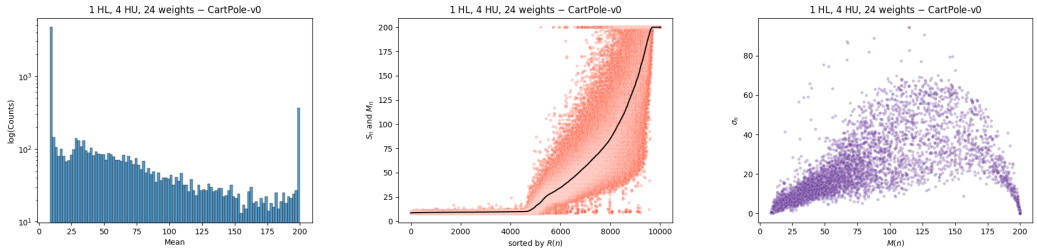
```
Initialize environment
Create array S of size N_architectures × N_samples × N_episodes
for n = 1, 2, ..., N_samples do
    Sample NN weights randomly from 𝒩(0, 1)
    for e = 1, 2, ..., N_episodes do
        Reset the environment
        Run episode with NN
        Store accured episode reward in S_{a,n,e}
```

low mean value suggests that, in general, the network cannot complete the task. The variance gives us further insight into the score distribution. It illustrates how spread out the scores are from their respective mean score. A high value means that we have high variability. A controller is valuable if it can solve a specific task reliably and stable. Therefore, we strive for a high value for the mean and a low value for the variance. However, training a network with random weight guessing should generally not result in a stable controller. If this is the case, we can assume that the task to solve was too trivial and is not valuable for evaluation measurements. In the illustrations of the paper, the authors ranked the samples according to their mean scores. They then visualized their results with three plots: a log-scale histogram of the mean scores, a scatter plot of the sample scores over their rank, a scatter plot of score variance over the mean score.

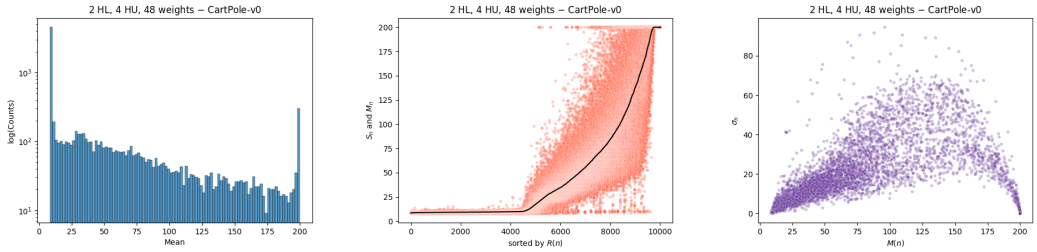
I reproduced the results of the authors following the mentioned methodology. My findings for the environment `CartPole` are displayed in Figure 2.1. The plots illustrate the results for each of the three network architectures. Each row shows the histogram of the mean score values in the left image, the scatter plot of all scores over their rank in the image in the middle, and the scatter plot of the score variance over the mean score in the right image for a specific network architecture. There are few differences, but overall all network architectures deliver similar insights. The histogram plots show that the majority of networks receive a low score. Since the weights of the networks were chosen with RWG, this is rather unsurprising. But there is still a significant amount of networks that were able to achieve a high mean value or even the maximum value of 200. With a score of 200, the network was able to solve the task each episode. Therefore, the network could reliably solve the task without any learning technique involved. This should not be the case for a complex task. Furthermore, in the scatter plot in the middle, we can see that the line plot of the mean scores is a continuous increasing line without any jumps. Thus, a suited RL algorithm should generally be able to learn the task incrementally without converging into a local optimum. At the top of the scatter plot, we can see quite a few data points with a score of 200 that have a relatively low mean score. This indicates that a network that generally performs poorly can still solve the task with the right initialization conditions. Lastly, in the scatter plot on the right, we can see the distribution of the variance according to the mean value. On the left side, we have low scores of variance corresponding with a low mean value. These networks were consistently unable to achieve a high score. Without any training involved, we can expect most networks to be in this area. However, in the middle of the plot, the data points are spread out. For a high variance, the scores of a network differ highly from the mean value. Thus, we might get lucky and receive a high score depending on initialization conditions, but we might as well get a low score. These networks are inconsistent and unstable. On the right side of the scatter plot, we can see that



(A) Results of network architecture without hidden layers



(B) Results of network architecture with one hidden layer



(C) Results of network architecture with two hidden layers

FIGURE 2.1: **Results of the benchmark evaluation.** The plots show a log-scale histogram of the mean scores (left images), a scatter plot of the sample scores over their rank (middle images), a scatter plot of score variance over the mean score (right image). As expected with RWG, most networks were not able to solve the given task. However, there is still a significant amount of samples achieving a mean score of 200. That suggests that the environment is trivial to solve.

the data points with a high mean value are mostly of low variance. Thus, to achieve a high mean value, the network needs consistency.

2.1.1 Impact of Bias

Interestingly, the usage of the bias had a relatively large impact on the performance of the network in my experiments. Without bias, the networks seem to achieve overall better scores. All plots in Figure 2.1 illustrate the results without bias. For comparison, Figure 2.2 shows the results of a network with two hidden layers with the same configurations as before but this time including bias. As we can see, the networks with bias connections have a much lower score in general. The number of networks that can consistently solve the task also decreases significantly. In the paper, the authors noted that the probability mass of top-performers generally increases when dropping the bias connections for all tested environments. Thus, this is not an isolated observation. However, they did not investigate this behavior further as it was not the focus of their paper. One possible explanation could be that guessing additional weights might be fatal for achieving a good score. Or in other words: the

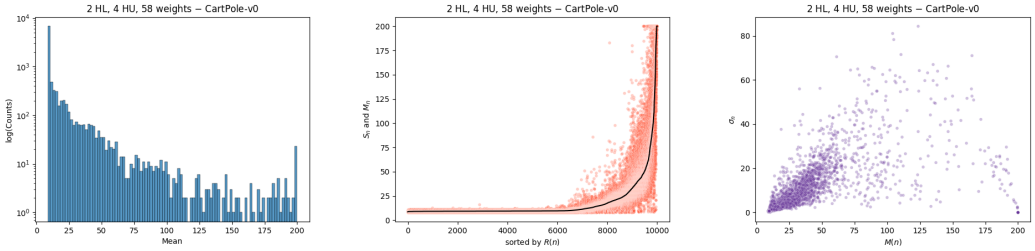


FIGURE 2.2: **Impact of bias.** The figures show the performance of a network with two hidden layers with the same settings as before but here we include bias. We can clearly see that the network with bias connection performs much worse than the one without.

more possibilities we have to falsely guess a weight, the higher the probability to fail. To test this hypothesis, we can alter the number of weights and compare the results. With an increased number of weights, we would expect the networks to perform worse than before. However, it could also be that the number of weights is not as impactful as the complexity of the model in general. Randomly guessing the parameters of a simple model has a higher chance to result in a good (simple) model than guessing the parameters of a more complex model. The complexity of the network architecture gives an upper bound for the function that can be approximated. A network with high complexity maps into a larger search space with more complex functions. Since the size of the search space increases, there are also more possible samples that fail to solve the task. As the paper showed, a simple model is sufficient to solve these environments. A complex model is oversized for our purpose here. Thus, randomly guessing a simple model can yield a model with good performance with enough attempts. However, it is unlikely to randomly guess a complex model that performs well without any training involved. To test this hypothesis, we can increase the complexity of a network by varying the number of hidden layers or the number of neurons in a hidden layer. With increased complexity, we would expect the networks to perform worse than before.

Another interesting aspect would be to inspect the role of the bias in connection with the environments. A simple model is sufficient to solve a simple task. But what if the environment is more difficult to solve? In that case, the undersized complexity would limit us from finding an appropriate solution as the search space is not large enough for this scenario. Therefore, including bias should improve the results.

The experiments following this thought process are described in Chapter 3.

How much should be included here and how much in experiments/visualization? Rename section name?

2.2 OpenAI Gym Environments

OpenAI Gym offers five classic control environments ready to use: `CartPole`, `Acrobot`, `MountainCar`, `MountainCarContiuous`, and `Pendulum`. The initial state of each environment is stochastic. Out of all environments OpenAI Gym provides, the classic control environments are considered the easier ones to solve. Each environment holds an observation and an action space. An agent performs some action from the action space and receives an observation describing how the environment's state changed.

2.2.1 CartPole

The `CartPole` environment corresponds to the pole-balancing problem formulated in *Neuronlike Adaptive Elements That Can Solve Difficult Learning Control Problems* (Barto, Sutton, and Anderson, 1983). A pole is attached to a cart that the agent can move along a one-dimensional track. Fig. 2.3 shows two possible states of this problem. In the image on the left, the pole is perfectly balanced and upright. The right image shows the pole tilted to the left. Thus, the cart should go to the left to correct the angle of the pole.

FIGURE 2.3: **Illustration of the environment CartPole.** The image shows two possible states of the `CartPole` environment. The left image shows a perfectly balanced pole. In the right image, the pole tilts to the left.

Action Space: The `CartPole` environment accepts one action per time step and has a discrete action space of {0, 1}. The action indicates the direction in which we push the cart by a fixed force. The actions are described in Table 2.1.

Action	Description
0	Push cart to the left
1	Push cart to the right

TABLE 2.1: **Action space of the environment CartPole.** Description of all possible actions for the `CartPole` environment.

Observation Space: For `CartPole`, we receive four observations that inform us about the position and velocity of the cart and the pole. Table 2.2 shows all observations with the range of the corresponding values.

Observation	Min	Max
Cart Position	-4.8	4.8
Cart Velocity	-Inf	Inf
Pole Angle	-0.418 rad (-24°)	0.418 rad (24°)
Pole Angular Velocity	-Inf	Inf

TABLE 2.2: **Observation space of the environment CartPole.** Description of all observations for the `CartPole` environment.

Reward: The goal of this task is to keep the pole upright as long as possible. Thus, we get a positive reward of +1 for each step that does not meet the termination

criteria. An episode terminates when the pole angle is $\pm 12°$, the cart position is ± 2.4, or the episode length is higher than 200 (500 for v1).

2.2.2 Acrobot

The environment `Acrobot` is based on the paper *Generalization in Reinforcement Learning: Successful Examples Using Sparse Coarse Coding* (Sutton, 1995) and the book *Reinforcement Learning: An Introduction* (Montague, 1999). The system contains a chain with two links, with one end of the chain fixed. The goal is to swing the free end of the chain upwards until reaching a certain height. Fig 2.4 shows in the left image one possible state for this problem with the fixed height indicated by a horizontal line. The right image of the figure illustrates one possible state that reached the target height.

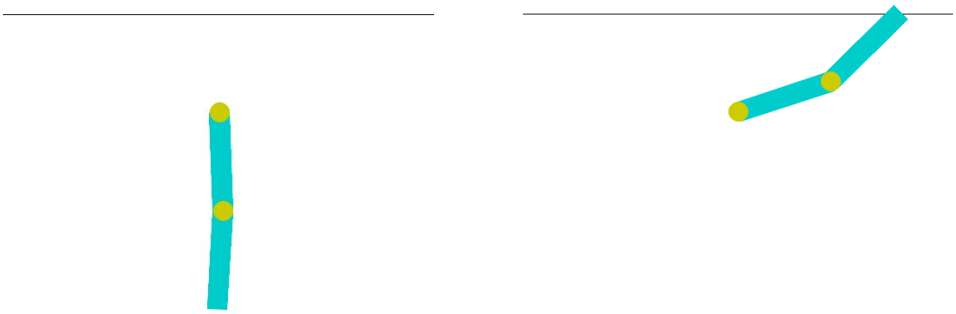


FIGURE 2.4: **Illustration of the environment Acrobot.** The image on the left shows one possible state of the environment `Acrobot`. The image on the right shows one possibility of reaching the target height.

Action Space: For `Acrobot`, the action space is discrete and deterministic, representing the torque applied to the joint between the two links, including the loose end. Table 2.3 shows a description of all possible actions.

Action	Description	Unit
0	apply -1 torque to the actuated joint	torque (N m)
1	apply 0 torque to the actuated joint	torque (N m)
2	apply 1 torque to the actuated joint	torque (N m)

TABLE 2.3: **Action space of the environment Acrobot.** Description of all possible actions for the `Acrobot` environment.

Observation Space: `Acrobot` returns six observations that inform us about the two rotational joint angles and their angular velocities. Table 2.4 contains a description for each observation. theta1 is the angle of the first joint, where an angle of 0 indicates that the first link is pointing directly downwards. theta2 is relative to the angle of the first link. An angle of 0 corresponds to having the same angle between the two links.

Observation	Min	Max
Cosine of theta1	-1	1
Sine of theta1	-1	1
Cosine of theta2	-1	1
Sine of theta2	-1	1
Angular velocity of theta1	-12.567 (-4 * π)	12.567 (4 * π)
Angular velocity of theta2	-28.274 (-9 * π)	28.274 (9 * π)

TABLE 2.4: **Observation space of the environment Acrobot.** Description of all observations for the `Acrobot` environment.

Reward: The goal of this task is to reach the target height with as few steps as possible. Thus, each step that does not attain the goal receives a negative reward of -1. Reaching the target height results in a reward of 0. The reward threshold is -100.

2.2.3 MountainCar and MountainCarContiuous

The environments `MountainCar` and `MountainCarContiuous` are two different versions of the same problem. The difference between the two is that `MountainCar` has a discrete action space, and `MountainCarContinuous` has a continuous action space. The goal of the task is to maneuver a car up a hill to its target. To achieve this, we need to strategically accelerate the car. An agent ought to learn how to swing from left to right to reach the target. This task is rather difficult to learn since we counterintuitively need to move in the opposite direction of the target to swing up. The problem first appeared in Andrew Moore's PhD thesis *Efficient memory-based learning for robot control* (Moore, 1990). Fig 2.5 shows one possible state in the left image and the target state of the two environments in the right image.

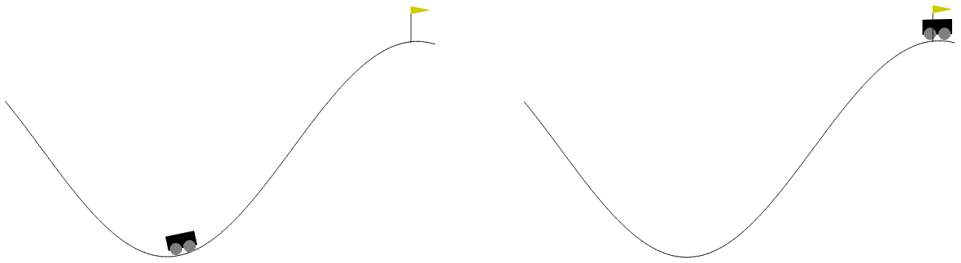


FIGURE 2.5: **Illustration of the environments MountainCar and MountainCarContinuous.** The image shows one possible state of the environments `MountainCar` and `MountainCarContiuous` on the left side. The image on the right shows the target state of the two environments.

Action Space: The action space for the environment `MountainCar` consists of three discrete actions. We can accelerate the car to the right or the left, or we do not interact with it. The actions are further described in Table 2.5. The environment `MountainCarContiuous`, on the other hand, accepts one continuous action per step. The action represents the directional force on the car and is clipped in the range of [-1, 1] and multiplied by a power of 0.0015.

Action	Description	Value	Unit
0	Accelerate to the left	Inf	position(m)
1	Don't accelerate	Inf	position(m)
2	Accelerate to the right	Inf	position(m)

TABLE 2.5: **Action space of the environment MountainCar.** Description of all possible actions for the `MountainCar` environment.

Observation Space: The environments `MountainCar` and `MountainCarContiuous` share the same observation space. They return two observations with information about the car's position and velocity. The observations are further described in Table 2.6.

Observation	Min	Max	Unit
Position of the car along the x-axis	-Inf	Inf	position(m)
Velocity of the car	-Inf	Inf	position(m)

TABLE 2.6: **Observation space of the environment MountainCar.** Description of all observations for the `MountainCar` and `MountainCarContiuous` environments.

Reward: To solve the task, the car has to reach the flag on the right hill as fast as possible. Each time step in which the agent cannot get tp the target, it receives a negative reward of -1 for the environment `MountainCar`. For the environment `MountainCarContiuous`, we receive a negative reward of $-0.1 * \text{action}^2$ for each step in which the car does not reach the target to penalize actions of large magnitude. When reaching the goal, we receive a positive reward of +100.

2.2.4 Pendulum

For the environment `Pendulum`, the system consists of a pendulum attached at one end to a fixed point and the other end being free. The goal is to apply torque on the free end to swing it to an upright position. This task is known as the inverted pendulum swing-up problem and is based on the classic problem in control theory. Figure 2.6 shows one possible state of the environment in the left image and the target state in the right one.

Action Space: The environment accepts one action per step. The action represents the torque applied to the free end of the pendulum.

Action	Description	Min	Max
0	Torque	-2.0	2.0

TABLE 2.7: **Action space of the environment Pendulum.** Description of all possible actions for the `Pendulum` environment.

Observation Space: The environment returns three observations. The first two observations represent the x- and y-coordinate of the pendulum's free end in a cartesian coordinate system. The variable theta defines the angle in radians. The third observation is the angular velocity of the free end of the pendulum.

FIGURE 2.6: **Illustration of the environment Pendulum.** The left image shows one possible state of the `Pendulum` environment. In the right image, the pendulum is in the upright position, which denotes the target state.

Observation	Min	Max
x = cos(theta)	-1.0	1.0
y = sin(angle)	-1.0	1.0
Angular Velocity	-8.0	8.0

TABLE 2.8: **Observation space of the environment Pendulum.** Description of all observations for the `Pendulum` environment.

Reward: The reward function is defined as r = -(theta^2 + 0.1 * theta_dt^2 + 0.001 * torque^2). Regardless of the received reward, an episode ends after 200 time steps.

2.3 Visualization

For the visualization of the results, I am using similar plots as Oller, Glasmachers, and Cuccu, 2020. The type of plots I will be using is already presented in Figure 2.1. However, I will mainly be using the scatter plots shown in the middle. Figure 2.7 shows an example of such a plot. For a better analysis, the samples are sorted according to their mean score. The position of a sample n within the sorted list is referred to as its rank $R(n)$. In the plot, the samples are aligned according to their rank on the x-axis. Each red dot represents the individual sample score $S_{n,e}$ for sample n in episode e. The overlaid line plot represents the mean score M_n of sample n over all episodes. Thus, the line plot is a naturally increasing curve of the mean scores. The scatter plot indicates the variety of scores for one sample. The title of the plot informs us which model and how many weights were used. In addition, the corresponding OpenAI gym environment is declared.

Rethink paper section, present plots here for the first time?

2.4 Alternative Models

This section describes the models I used in the experiments. Since we are using black-box optimization techniques, we have no constraints on the model. Therefore, we can use any function approximator. Note that we are in the field of direct

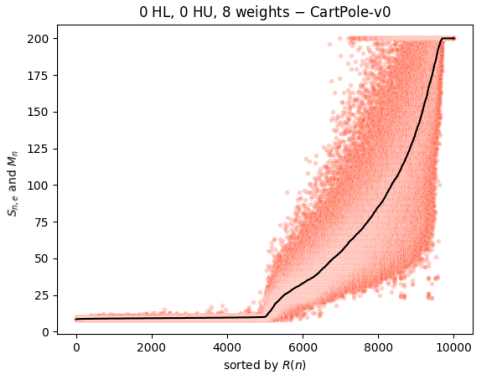


FIGURE 2.7: **Visualization of results.** The image shows an example of the visualization of the conducted experiments. The scatter plot represents the individual sample scores $S_{n,e}$ for episode e over the rank $R(n)$ of the sample n. The overlaid line plot consists of the mean scores M_n for each sample.

policy search, where we directly search in the search space of policies for a suited approximation.

2.4.1 Polynomials

In mathematics, a polynomial is the sum of powers in one or more variables multiplied by constant coefficients. Given a field F, a polynomial in variable x with coefficients in F is a formal expression denoted by

$$f(x) = \Sigma_{i=0}^{n} a_i x^i \in F[x], \qquad a_0, ..., a_n \in F, \;\; i \in \mathbb{N},$$

where $F[x]$ represents the set of all such polynomials (Fischer, 2014, p. 61). The above formula shows the one dimensional case. For the multi-dimensional case, x and the coefficients are vectors instead of scalars. Thus, a polynomial $p(\mathbf{x})$ with $\mathbf{x} = [x_0, ..., x_m]^T$ being a vector and of degree n can be represented by

$$p(\mathbf{x}) = \Sigma_{i=0}^{n} \mathbf{w_i}^T (x_k^i)_{k \in I} \in F^n[\mathbf{x}], \qquad \mathbf{w_0}, ...\mathbf{w_n} \in F^n, \;\; I = \{0, ..., m\}.$$

Polynomials are relatively simple mathematical expressions and offer some significant advantages: their derivative and indefinite integral are easy to determine and are also polynomial. Due to their simple structure, polynomials can be valuable to analyze more complex functions using polynomial approximations. *Taylor's theorem* tells us that we can locally approximate any k-times differentiable function by a polynomial of degree k. We call this approximation *Taylor polynomial*. Furthermore, the *Weierstrass approximation theorem* says that we can uniformly approximate every continuous function defined on a closed interval by a polynomial. Other applications of polynomials are *polynomial interpolation* and *polynomial splines*. Polynomial interpolation describes the problem of constructing a polynomial that passes through all given data points. Polynomial splines are piecewise polynomial functions that can be used for spline interpolation.

For the experiments with the polynomial model in Chapter 3, I constructed the model P_1. It consists of one polynomial for each possible action in a discrete action space. The input of the model is the observation of the environment. The dimension of the weight vectors is according to the dimension of the input vector. For example,

for the environment `CartPole` with the discrete action space $\{0, 1\}$ and observation $\mathbf{x} = [x_0, x_1, x_2, x_3]^T$, this means that P_1 consists of two polynomials:

$$p_0(\mathbf{x}) = \Sigma_{i=0}^{n} \mathbf{w_i}^T (x_k^i)_{k \in I} \in \mathbb{R}, \qquad \mathbf{w_0}, ..., \mathbf{w_3}, \mathbf{x} \in \mathbb{R}^4, \ I = \{0, 1, 2, 3\}$$
$$p_1(\mathbf{x}) = \Sigma_{i=0}^{n} \hat{\mathbf{w}}_\mathbf{i}^T (x_k^i)_{k \in I} \in \mathbb{R}, \qquad \hat{\mathbf{w}}_\mathbf{0}, ..., \hat{\mathbf{w}}_\mathbf{3}, \mathbf{x} \in \mathbb{R}^4, \ I = \{0, 1, 2, 3\}$$

In the formulas, n denotes the degree of the polynomial. For the experiments, I used polynomials of degrees one, twp, and three. However, the output of the polynomials has no appropriate upper and lower limit, as illustrated in Figure 2.8. That makes it harder to interpret the results reasonably. So, I scaled the outputs with the logistic

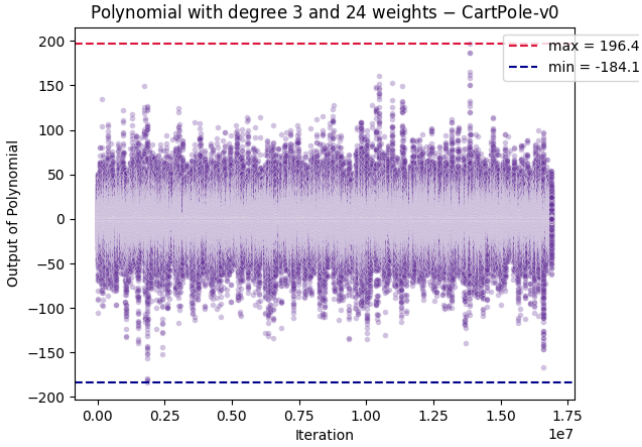


FIGURE 2.8: **Upper and lower bound.** The figure shows each output of the polynomial functions described above. As we can see, the functions are not well bound, and there are quite a few outliers. That makes it hard to interpret the output sensibly.

sigmoid function. We can interpret the output space of the sigmoid function as a probability, as discussed in Section 1.2.1. In this case, we search for the probability that a specific action is the reasonable one given an observation $\mathbf{x}$. Thus, for our example with the `CartPole` environment, we can interpret $sig(p_0)$ as the probability that action 0 is the correct one and $sig(p_1)$ as the probability that action 1 is the correct one. Putting this thought into a formula for P_1 and the `CartPole` environment, we get:

$$P_1(\mathbf{x}) = \begin{cases} 1 & \text{if } sig(p_1(\mathbf{x})) > sig(p_0(\mathbf{x})) \,, \\ 0 & \text{otherwise} \,. \end{cases}$$

Include theorems (Taylor, Weierstass)
Explain difference between approximation and interpolation
Does F^n make sense?

2.4.2 Splines

General splines, adjustments

Splines can also be used as function approximators. However, with splines we do not aim to approximate the whole function but rather return a piecewise approximation.

First was sind splines, dann herleitung in this case.

2.4.3 Binary Trees

A binary trees are data structures in computer science with a rather simple structure. The tree consists of nodes that each have at most two children, referred to as left child and right child.

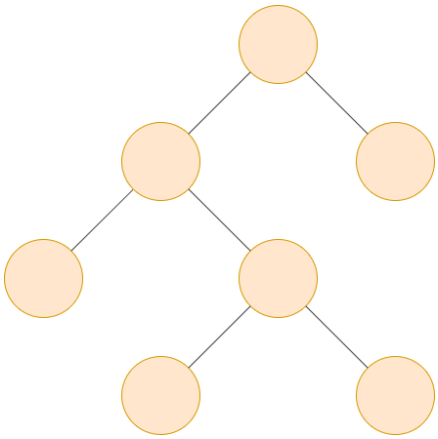


FIGURE 2.9: **Example of a binary tree.** The figure shows an example of a binary tree with seven nodes.

There exist already some tree-based machine learning algorithms. However, they are mostly used in supervised learning.

Herleitung von splines erklären -> vielleicht in vorheriger subsection? Explain continuous envs.

Chapter 3

Experiments

This chapter aims to answer the research questions formulated in Section 1.7. First, I describe which experiments I conducted. Next, I show the results of these experiments. Finally, I discuss my findings further in the last section.

3.1 Experiments

This section describes the experiments I used in order to investigate and answer the formulated research questions. We can break down the experiments into three categories: reproduction of the results of the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing*, comparison of the alternative models described in Section 2.4 to neural networks, and an analysis of the impact of bias in these settings.

3.1.1 Reproduction of RWG-Paper

For a starting point, I reproduced the results of the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* discussed in Section 2.1. I used the same methodology as the authors of the paper did. In specific, I used three network architectures: a network without any hidden layers (0 HL, 0 HU), a network with a single hidden layer of 4 units (1 HL, 4 HU), and a network with two hidden layers of 4 units each (2 HL, 4 HU). I used the same procedure as explained in pseudocode in Algorithm 2 using RWG. I tested the three neural networks for all five classic control environments provided by the OpenAI Gym interface: `CartPole`, `Acrobot`, `Pendulum`, `MountainCar`, and `MountainCarContinuous`. For the next experiments, the reproduced results serve as a guideline to judge the effectiveness of the alternative models in comparison with neural networks.

3.1.2 Comparison of Alternative Models to Neural Networks

This experiment aims to provide a comparison of alternative models to the commonly used neural network model. I selected a few promising models and analyzed them with the help of the classic control environments. Analogous to the neural networks, I used three architectures for each model with increasing complexity. I conducted the following experiments:

(a) In the first experiment, I took the polynomial model P_1 and P_2 and tested them on the discrete classic control environments. I used polynomials of degrees 1, 2, and 3.

(b) In the second experiment, I tested the binary tree model on all five classic control environments. I used binary trees with 1, 4, and 8 nodes.

I described the models in more detail in Section 2.2. For the experiments, I used the same procedure as in *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* with only slight adaption. Thus, we can directly compare the results of the alternative models to those of neural networks. Another important aspect of this procedure is that there is no learning involved. For their paper, the authors were interested in the complexity of the environment, whereas I aim to find out more about the nature of the models. The classic control environments are relatively easy to solve, as explained in Section 2.1. Therefore, we expect some controllers to solve the task even without any learning involved.

For the experiments, first, I initialized the environment. Second, I initialized the respective model. Then, I drew the weights of the model from the standard normal distribution $\mathcal{N}(0,1)$ or the uniform distribution $U(a,b)$ depending on the model and environment used. Each of these instances of the model represents a sample. In total, I used 10′000 samples ($N_{samples}$). Finally, I ran 20 episodes ($N_{episodes}$) with each sample for an environment and stored the respective score as an entry of the score tensor S. Algorithm 3 shows an overview of the described procedure. As we can see, the procedure is very similar to the one shown in Algorithm 2.

Algorithm 3 Procedure for alternative models using RWG

```
Initialize environment
Initialize model
Create array S of size N_samples × N_episodes
for n = 1, 2, ..., N_samples do
    Sample model weights randomly from 𝒩(0,1) or U(a,b)
    for e = 1, 2, ..., N_episodes do
        Reset the environment
        Run episode with model
        Store accured episode reward in S_{n,e}
```

Include splines or not?

3.1.3 Analysis of the Impact of Bias

The authors of the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* indicate that using bias worsens the performance of neural networks in their experimental setting. This experiment serves to analyze this behavior. I used the same procedure as explained in the paper with the same three neural network architectures: a network without any hidden layers (0 HL, 0 HU), a network with a single hidden layer of 4 units (1 HL, 4 HU), and a network with two hidden layers of 4 units each (2 HL, 4 HU). First, I reproduced the results for the networks with bias to confirm their findings. Then, I varied the number of (a) weights, (b) hidden layers, and (c) neurons for a neural network. To get insights into whether this observation is specific to neural networks, I additionally used the polynomial model for the experiment concerning the variation of the weights of the model. In more detail, the experiments I conducted in this matter are the following:

(a) In the first experiment, I only changed the number of weights for each of the three networks and the polynomial model P_1 described in Section 2.4. I tripled the number of weights for all models. To achieve this, I constructed one weight

$\mathbf{w_i}$ out of three weights by addition:

$$\text{Triple number of weights:} \qquad \mathbf{w_i} = \mathbf{w_{i1}} + \mathbf{w_{i2}} + \mathbf{w_{i3}}$$

(b) In the second experiment, I tested the neural network models with different numbers of hidden layers. Each layer still has the same number of hidden neurons as before. Thus, the networks have four hidden units for each layer. I analyzed a network with 4 hidden layers, one with 6, and one with 8.

(c) In the last experiment concerning this subject, I varied the number of hidden neurons for a network. Here, I only used a network with two hidden layers. For the number of hidden neurons, I chose 5, 8, and 10.

3.2 Results

3.2.1 Reproduction of RWG-Paper

Figure 3.1 shows the results for all five classic control environments using neural networks without bias. For each plot, the samples are ranked according to their mean score and aligned on the $x-$axis according to their rank. The scatter plots show all scores of the samples, whereas the lineplot illustrates the mean of each sample over all episodes. Each column shows a different network architecture. The first column shows the results of the networks without hidden layers, the second column the networks with one hidden layer, and the last column the networks with two hidden layers. The rows are dedicated to the different classic control environments. These results are consistent with the ones from the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing*. In the subsequent sections, I will refer to these plots to compare the effectiveness of other models or configurations to these neural network architectures.

3.2.2 Polynomial Model

Figure 3.2 shows the results for the discrete classic control environments using the polynomial model P_1 without bias. The visualization is identical to the one I used for the neural networks in Figure 3.1. The rows show the results for the different environments, and the columns show the polynomial with degrees 1, 2, and 3. Looking at the first column, we can see a striking resemblance to the performance of the neural network without hidden layers shown in Section 3.1 for all three environments. This observation is not entirely a surprise since the neural network without hidden layers represents a linear controller. However, the second and last columns show different results for the environments `CartPole` and `Acrobot`.

CartPole: For the environment `CartPole`, we can see that the curve of the mean score stays low until around 5′000 but then goes up relatively steeply. That means the linear model fails around 50% to guess the action correctly. However, after that, we have a high probability to achieve a good score or even solve the task entirely during multiple episodes. There are also quite a few samples that could solve the task each time, as indicated by the short straight black line at the top of the plot. Looking at the polynomials with degree 2 for `CartPole`, we can see that the scores increase already at around 2′000, but the slope is less steep than for the linear model. Therefore, we start earlier to get meaningful results, but the scores increase only

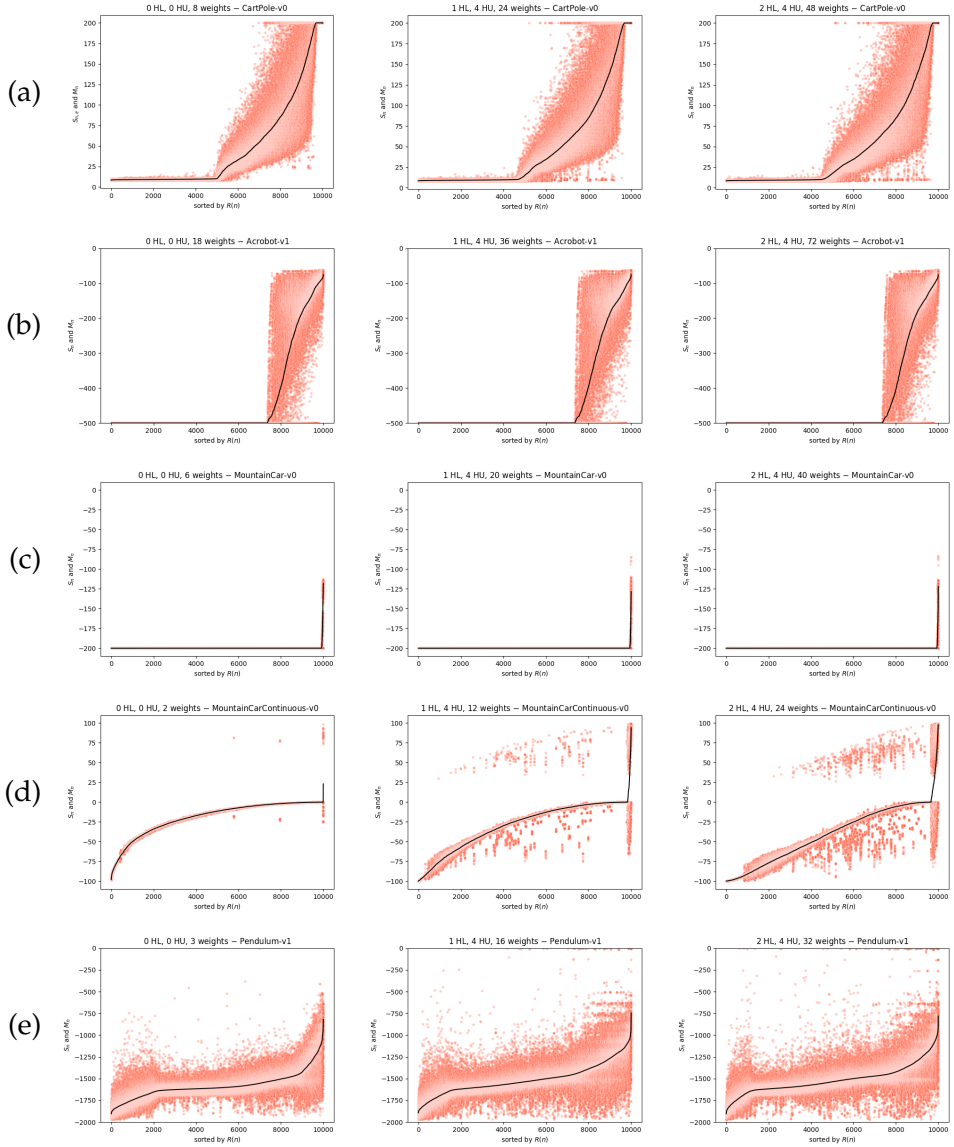


FIGURE 3.1: **Results for all classic control environments using neural networks without bias.** Each row shows the results of one environment, whereas the columns represent the different network architectures. These plots are a reproduction of the results of the RWG-paper. My results are consistent with those shown in the paper.

slowly. This aspect is significant for a learning algorithm. In addition, there are fewer samples that could solve the task for each episode than there are for the linear model. Furthermore, the variance is higher for the polynomials with a higher degree compared to the linear model. If we look at the results of the polynomials with degree 3, we can see that there is only a tiny boost compared to the polynomials of degree 2. The scores are overall slightly higher, but the slope is similar to before. The considerable difference lies between the polynomials with degree 1 and polynomials with degree 2, respectively, degree 3.

Acrobot: For the environment `Acrobot`, the polynomial model with degree one delivers a score distribution indistinguishable from the one of the neural network

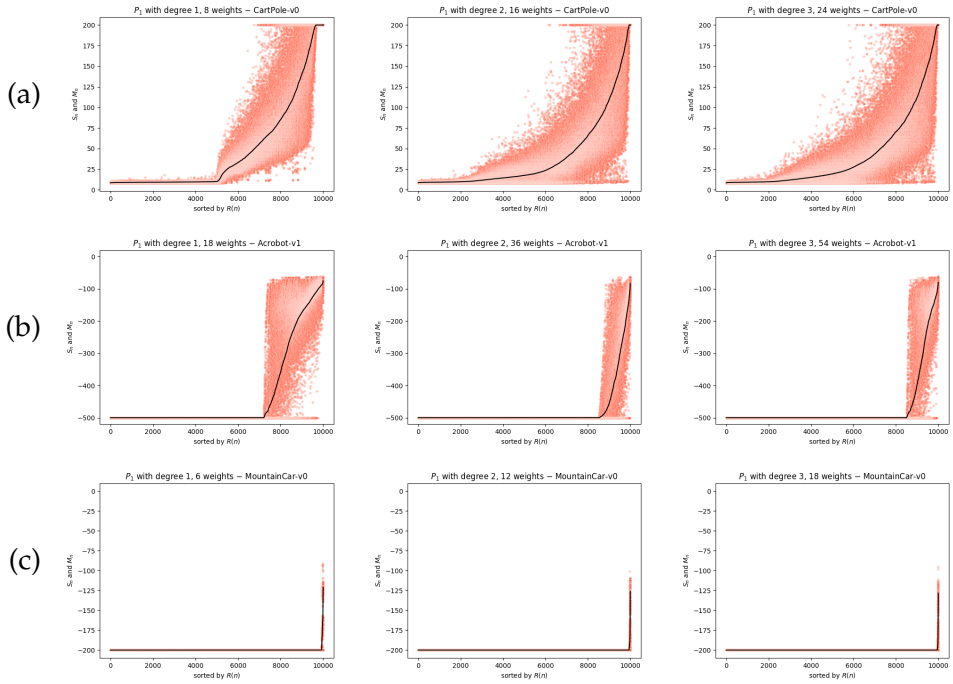


FIGURE 3.2: **Results for the discrete classic control environments using the polynomial model P_1 without bias.** Each row shows the results of one environment. The columns represent the different degrees of the polynomials. In these plots, the model P_1 was used. The results aligned in the first column are almost indistinguishable from the ones of the neural networks without hidden layers. Nonetheless, the environments `CartPole` and `Acrobot` deliver different results for a higher degree of the polynomial.

without hidden layers. The lineplot stays at -500 until around rank 7'500. Then it goes up steeply. Thus, we have a large plateau of minimum scores before we reach scores over -500. That opposes a higher difficulty level for the learning algorithm than `CartPole`. But with RWG, we can still reach relatively high scores, and the lineplot is continuous. Interestingly, the polynomial models with a higher degree performed noticeably worse. We cannot observe this behavior with neural networks.

MountainCar: The environment `MountainCar` is more challenging to solve than the other environments, as we can see by the results of both the neural networks and the polynomial model. Both models show very similar findings. There is a large fitness plateau, and only a few samples can reach a score higher than the minimum score of -200. That applies to both the linear models and the ones with higher complexity.

Bias: Figure 3.3 shows the results of using the polynomial model P_1 to solve the discrete classic control environments using bias. Comparing these results with those without using bias in Figure 3.2 shows a significant difference. The bias influences the performance of the model negatively. The individual and mean scores are visibly lower for all three environments for all three configurations of the model. Thus, the observation about the bias the authors made in the paper *Analyzing Reinforcement Learning Benchmarks with Random Weight Guessing* is not specific to neural networks but seems to be more of a general factor.

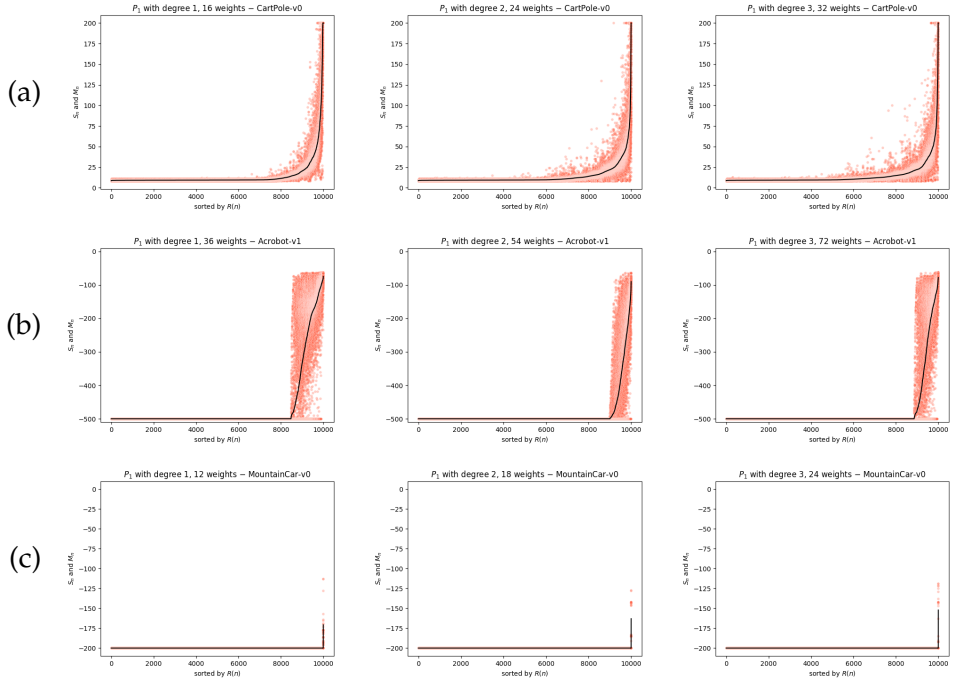


FIGURE 3.3: **Results for the discrete classic control environments using the polynomial model P_1 with bias.** Each row shows the results of one environment. The columns represent the different degrees of the polynomials. For these plots, the polynomial model P_1 was used. Overall, the model performs noticeably worse with bias as opposed to without bias.

Mention P_2 (failed for MountainCar otherwise similar results).

3.2.3 Binary Tree Model

For the binary trees, I tested three models, one with 1 node, one with 4 nodes, and one with 8 nodes. Since the previous experiments suggest that using bias has a negative effect, the following experiments do not include bias. Figure 3.4 shows the results for all five classic control environments with the binary tree model.

CartPole: Comparing the results using binary trees with those using neural networks shown in Figure 3.1, we can see a few differences. For the environment `CartPole`, we can see that we generally reach better scores with the binary trees. Comparing the neural network without hidden layers with the binary tree with only one node, we have a similar curve, but the lineplot representing the mean scores sets off much earlier at around 3'500 for the binary tree than the one for the neural network, which sets off at around 5'000. In addition, the binary tree has a higher chance of reaching a maximal mean score, indicated by the straight black line at the top right. For the more complex architectures, it gets more interesting. The binary tree with four nodes can already raise the mean value under 1'000 samples. That tells us that the problem of the fitness plateau is reduced significantly. Thus, when using a learning algorithm, the binary tree has a better chance of solving the problem without getting stuck in a fitness plateau. The chance of reaching a maximal mean score decreases with adding more nodes to the binary tree. That could be caused

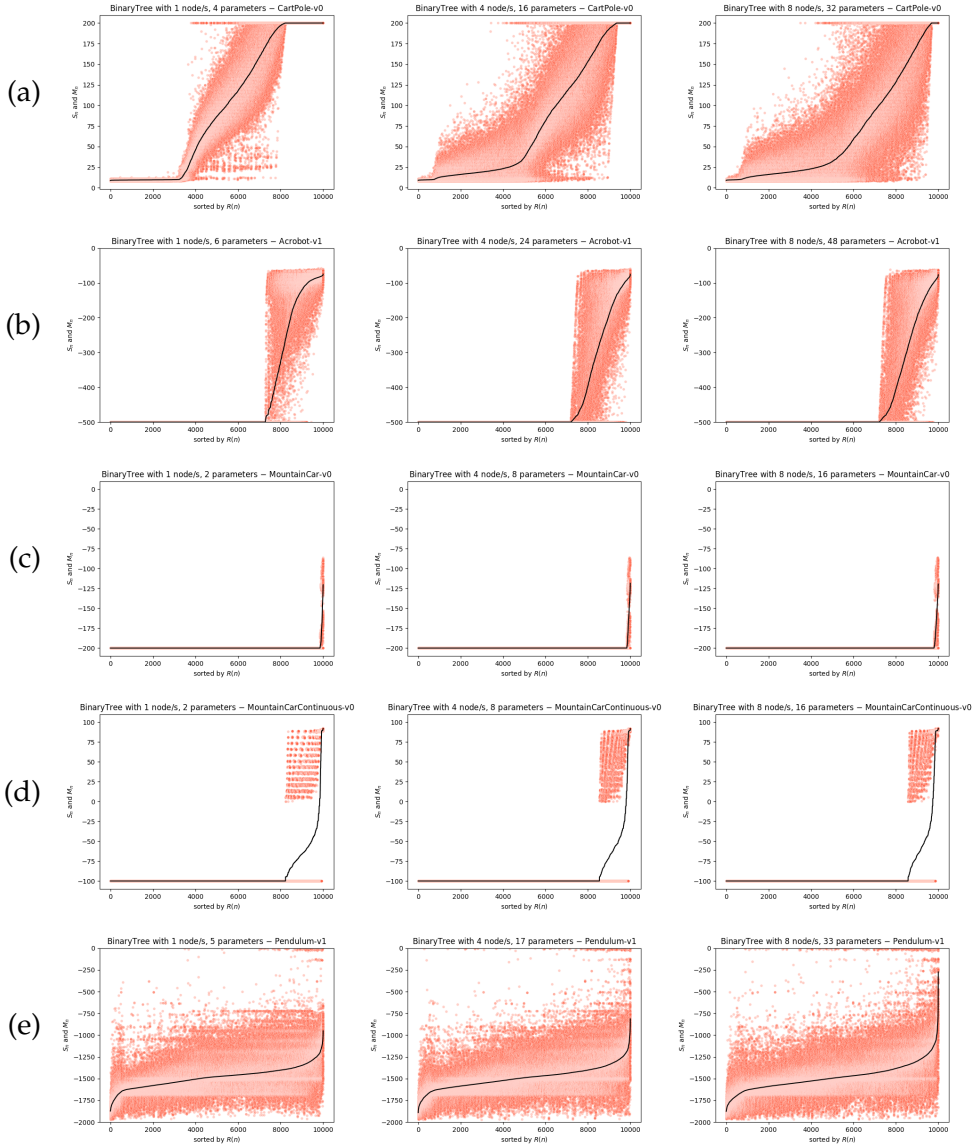


FIGURE 3.4: **Results for the classic control environments using binary trees without bias.** Each row shows the results of one environment. The columns represent the different configurations for the binary tree models. Even with only one node, the results are comparable to those from neural networks or outperform them.

by the increasing complexity of the binary tree, which makes it harder to guess the weights correctly. Furthermore, the scores are more spread out for the binary trees than for the neural networks, indicating a higher variance.

Acrobot: Comparing the results of the binary tree model with one node with the neural network without hidden layers for the environment `Acrobot`, the binary tree produces a better score distribution. Although the fitness plateau remains to the same extent, the individual scores are overall higher, and the slope of the mean scores has improved, demonstrating a steeper slope. The neural networks with higher complexity seem to slightly outperform the binary trees with four and eight nodes. The scores are slightly higher, and the slope of the mean scores is a bit steeper.

However, the difference is not massive. Overall, we can say that both models reach similar scores and mean values.

MountainCar: Comparing the two models, binary trees and neural networks, for the environment `MountainCar`, we can see that they reach similar mean scores. However, the binary trees are more likely to reach more and higher individual scores. The risk of being stuck in a fitness plateau remains for the binary trees. In addition, the scores are spread wider for the binary tree than for the neural network.

MountainCarContinuous: The results for the environment `MountainCarContinuous` look very different for the binary trees than for the neural networks. Neural networks show a slowly increasing curve with a peak after 9'500-9'900 samples for the mean scores. The binary trees have a lot of low performers displayed as a plateau of minimum scores. We cannot see this bad behavior in the results of the neural network models. However, there are a few samples that are able to reach a positive score. The neural network without hidden layers could not achieve these high mean scores.

Pendulum: For the `Pendulum` environment, we can see that the binary trees perform increasingly better with more nodes. The binary tree with one node reaches a mean score of around -1'500 in the large middle area. These scores are slightly higher than those of the neural network without hidden layers in the middle area. However, the neural network reached a higher maximum mean score than the binary tree. With an increased number of nodes, the binary tree model even outperforms the neural networks with remarkably higher mean scores. Even though the actions produced by the binary trees are discrete instead of continuous, the curve of the mean scores looks very similar to the one of neural networks.

Explain how model handeled continuous environments in models section.
Show MountainCarContiuous with better results?

3.2.4 Bias Investigation

In their paper, Oller, Glasmachers, and Cuccu (2020) mentioned that using bias negatively affects the score distribution leading to fewer top performers and generally lower scores. Figure 3.5 shows my results for all classic control environments with neural networks without using bias. Comparing these plots with the ones without using bias in Figure 3.1, we can see the negative impact the bias has on the effectiveness of the model. For the environment `CartPole`, the scores are overall lower and get slighly worse with the increased complexity of the model. Interestingly, for the environment `Acrobot`, the network without hidden layers is resistent to the negative impact of the bias. However, the network with one hidden layer results in lower scores with bias. The network with two hidden layers with has slightly better results than the one with one hidden layer but still performs worse than the one without bias. The same observation can be made for the environment `MountainCar`. For the environment `MountainCarContinuous`, we can see a negative effect on the score distribution for all three network architectures. The environment `Pendulum` does not seem to suffer much when introducing bias. For all three architectures, there is not much difference visible. The scores are generally lower and produce less

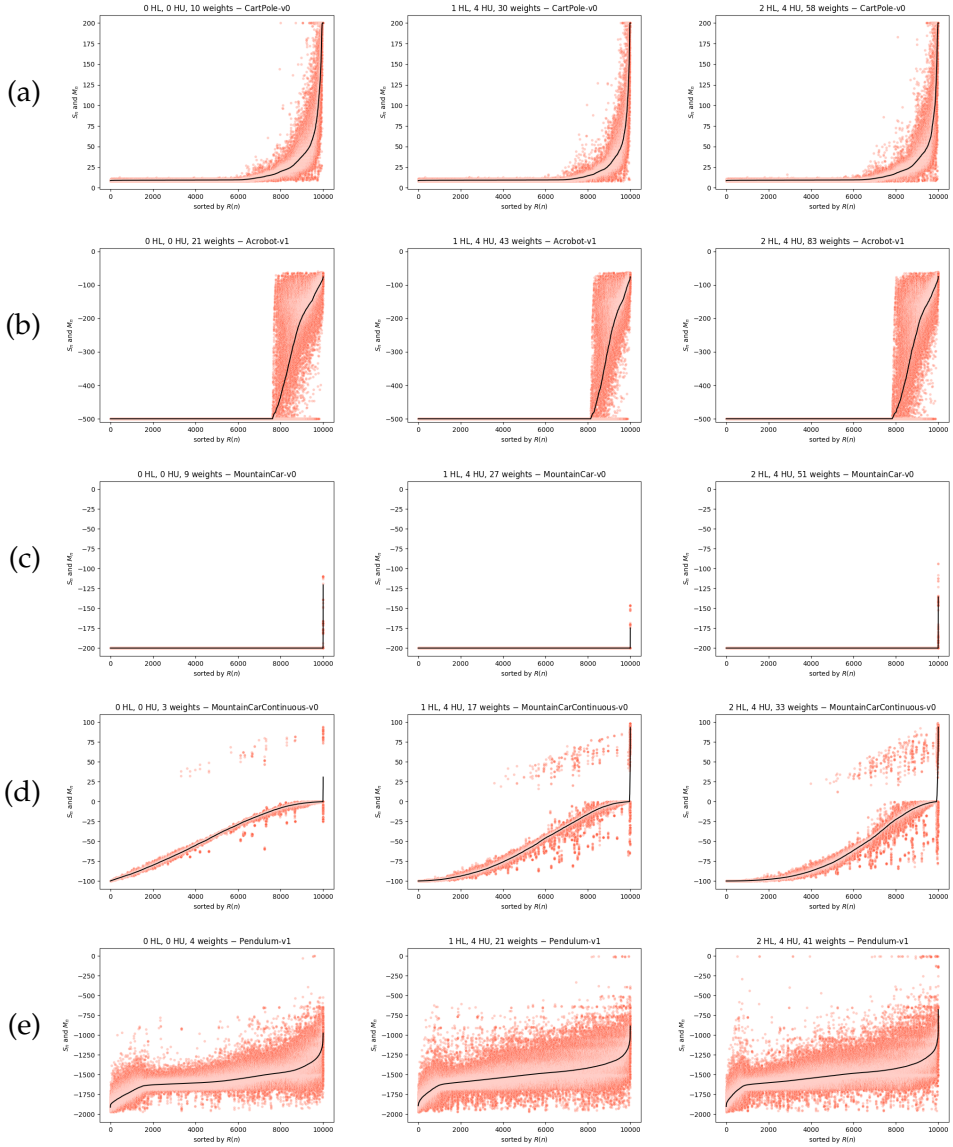


FIGURE 3.5: **Results for all classic control environments using neural networks with bias.** Each row shows the results of one environment, whereas the columns represent the different network architectures. For most environments, we can see that introducing a bias has a negative effect on the score distribution.

top performers when using bias. Therefore, we can confirm the interesting observation of the authors, namely, that using bias is counterproductive in this experimental setting.

Altering the number of weights: To further investigate this counterintuitive behavior, Figure 3.6 shows the results of tripling the number of weights for each network architecture for all classic control environments. Comparing these plots to those in Figure 3.1, we cannot see a huge difference. The score distribution of the environments `CartPole` and `Acrobot` look indistinguishable from those presented in Figure 3.1. We can see slight differences in the plots for the environment `MountainCar`. Increasing the number of weights for the network without hidden layers results in

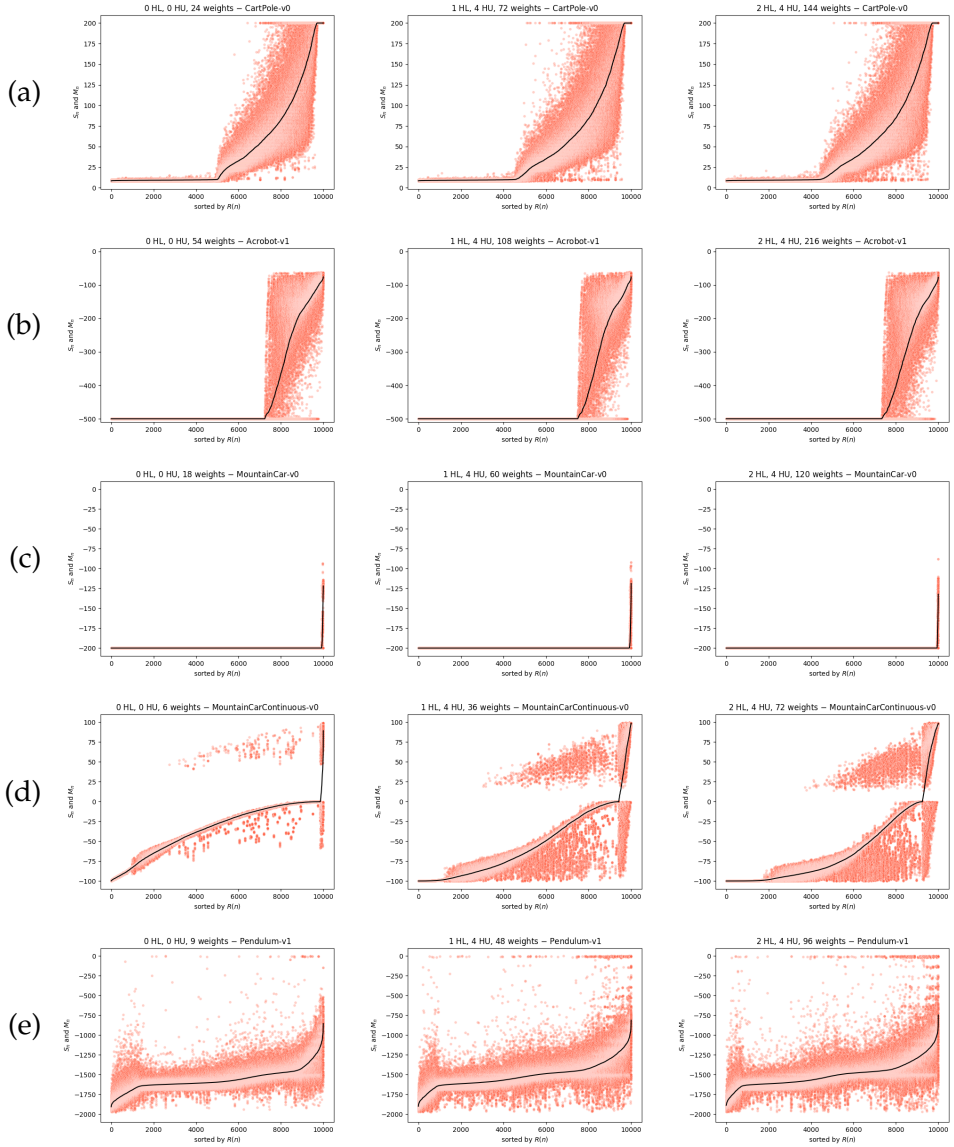


FIGURE 3.6: **Results of tripling the number of weights for neural networks.** The figure shows the results of tripling the number of weights for the three neural network architectures for the classic control environments. Despite the increased number of weights, there are only slight differences visible in the plots compared to the ones shown in Figure 3.1 for all environments with the exception of `MountainCarContinuous`.

a few better individual scores without much difference in the mean score distribution. We see slightly better mean scores for the network with one hidden layer, whereas the network with two exhibits slightly worse mean scores. For the environment `MountainCarContinuous`, the mean scores are lower compared to the networks with more weights. However, there are more top performers. This is especially visible for the network without hidden layers. In addition, the scores are more spread out. There are a few more samples that reach a higher score for the environment `Pendulum` with the network without hidden layers. However, the networks with one and two hidden layers deliver similar results to those shown in Figure 3.1 for the

environment `Pendulum`. To summarize, altering the number of weights for a network can produce a slight difference in the results for some environments. But these differences are not as dramatic as we have seen for the networks without using bias.

Altering the number of layers: Figure 3.7 shows the results of altering the number of layers in a network. Each layer still has the same amount of neurons, namely four neurons. For the environment `CartPole`, the lineplot of the mean scores seems

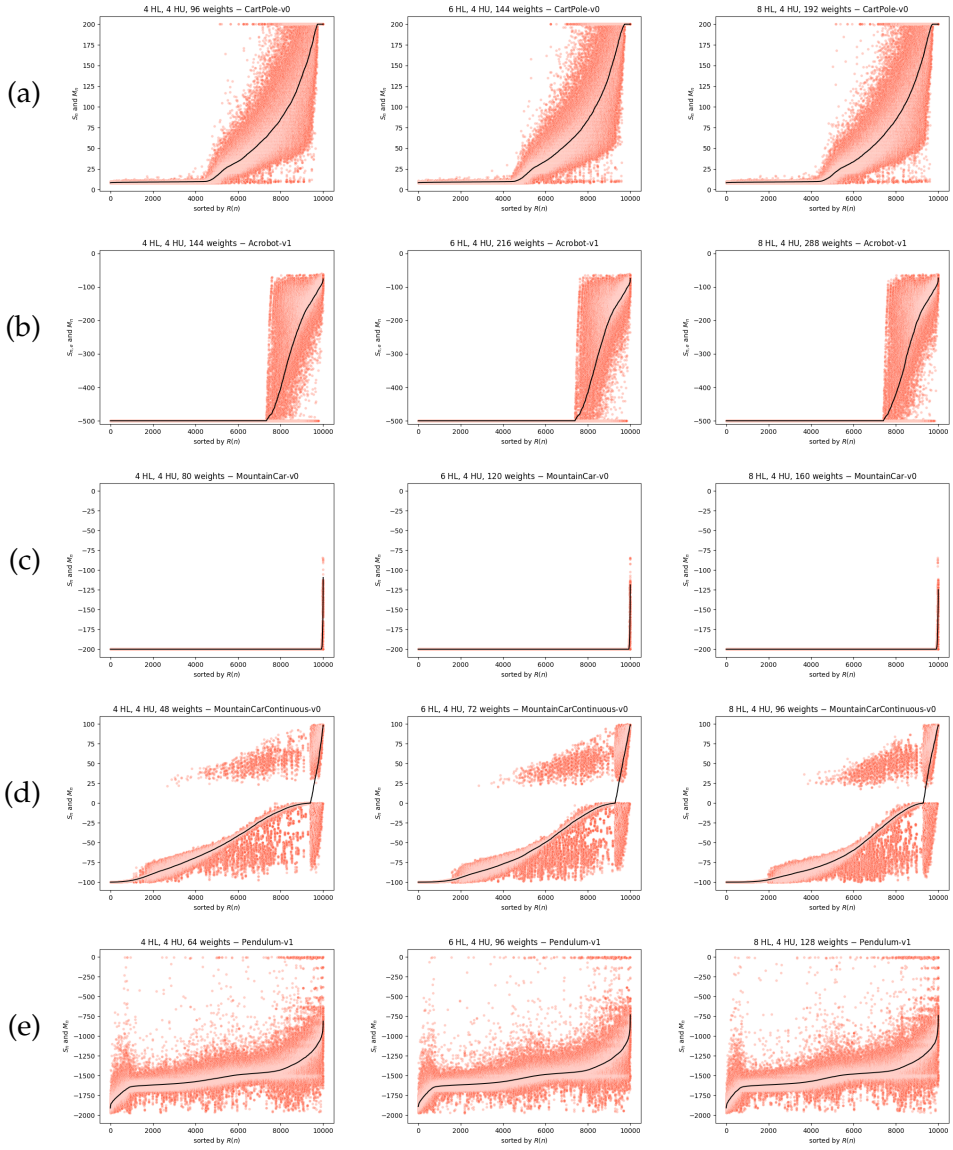


FIGURE 3.7: **Results of altering the number of layers for neural networks.** The plots show the results of increasing the number of hidden layers in a neural network gradually. The score distribution changes with an increased number of layers for some environments. However, the changes are less extreme compared to the results of the networks that used bias.

to start earlier to rise above the minimum score, but the slope is also slightly less steep with an increasing number of hidden layers. The environment `Acrobot` does not seem to be affected anyhow by the change in the number of layers. The plots

look indistinguishable. The mean scores for the environment `MountainCar` seem to be best for the network with four hidden layers. The scores tend to get lower with an increased number of layers. For the environment `MountainCarContinuous`, the slope of the mean scores changes. The scores get generally slightly lower with more layers. Finally, we cannot see a big difference in the plots for the environment `Pendulum` by adding more layers. There may be a few more top performers with more hidden layers, but the difference is not significant and is not reflected in the mean score distribution.

Altering the number of neurons: Figure 3.8 shows the results of increasing the number of neurons in each hidden layer for a network with two hidden layers. We

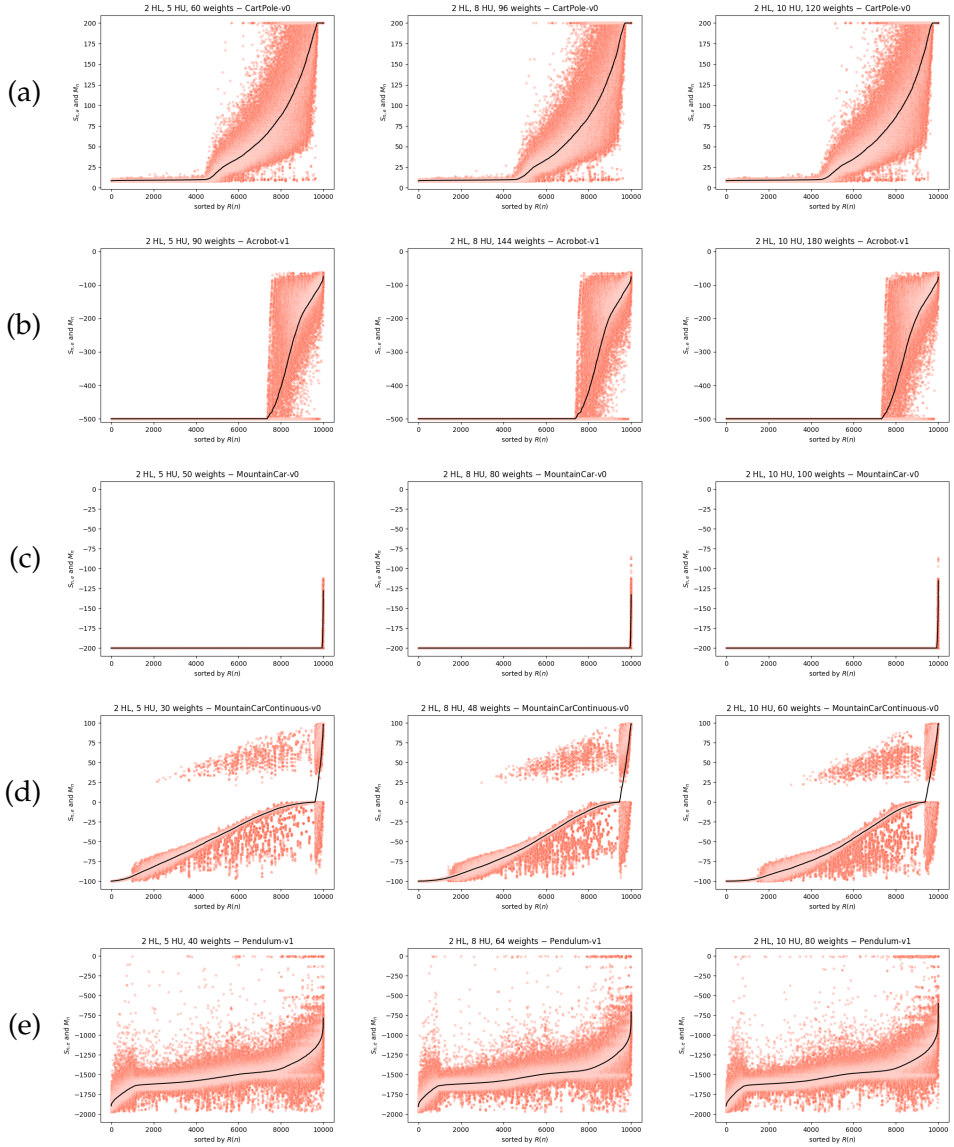


FIGURE 3.8: **Results of altering the number of neurons for neural networks.** The plots show the result of increasing the number of neurons for each layer in a network with two hidden layers. For some environments, there is a change in the mean scores. However, the changes are not extreme.

cannot see much difference in the plots for the environments `CartPole` and `Acrobot`. For both, the score distribution seems to be unaffected by the increased number of neurons. The mean scores for the environment `MountainCar` first decrease with five and eight neurons. Then with ten neurons, we achieve higher scores again. For the environment `MountainCarContinuous`, the plots look similar to the ones in Figure 3.7 with an increased number of layers. We can see a slight decrease in the scores with increased neurons which is less extreme than when we increased the number of layers. With the environment `Pendulum`, we can reach higher mean scores with an increased number of neurons. Otherwise, the slope of the mean scores and the overall score distribution looks very similar.

Mention P_1 for weight experiment.

3.3 Discussion

This chapter delivered a direct comparison between neural networks and alternative models like polynomials and binary trees. In addition, I investigated the impact of bias in these settings. The subsequent paragraphs aim to discuss the results to prepare for the conclusions and proposed future work in the next chapter.

Comparison of polynomial model with neural networks: As we have seen in the last section, the polynomial model can deliver comparable results to neural networks for discrete environments. The results of the environment `MountainCar` with the polynomial model look almost indistinguishable from those of the neural network model. For the environment `Acrobot` the polynomial model with degree delivers very similar results as the neural network without hidden layers. However, with the higher complexity of the model, neural networks outperform the polynomial model. Similarly, the polynomial of degree one results in the same score distribution as the neural network without hidden layers. Nonetheless, the score distribution differs noticeably from neural networks with higher complexity of the model. The mean scores show the same trend for all neural network architectures, which cannot be applied to the polynomial model. With the linear polynomial model, we have a 50% chance of failing, but the probability of actually solving the task for all episodes is higher than for the polynomials with a higher degree. That means with the linear model or neural networks, we have a larger fraction of samples that can solve the environment independently of the initialization conditions. At first glance, we could assume that this is an important aspect for such a model and choose the polynomial with degree 1 over one with a higher degree. However, we should remind ourselves that these experiments are rather unusual for an application since there is no learning involved and the number of samples is huge. In a standard application, we would use some kind of training and want the model to sequentially improve its performance. Considering this aspect, when using a learning algorithm, we would not prefer the polynomials with degree one or the neural networks as we might get stuck in a fitness plateau when the algorithm has no method of dealing with this behavior.

To summarize, the results of the polynomial model are comparable to those of neural networks. However, this applies only for the three discrete environments since the polynomial model is not constructed to deal with a continuous action space.

Comparison of binary tree model with neural networks: Despite the simple structure of the binary tree model, it delivers comparable results to neural networks or even outperformed them. The binary tree model significantly outperforms neural networks for the environment `CartPole` for all three architectures. The binary tree with only one node shows a similar slope for the mean scores as the network without hidden layers, but it visibly reduces the plateau of minimum scores. The binary trees with higher complexity even manages to remarkably shrink the plateau, which none of the three architectures of the neural network did on that scale. The environment `MountainCar` yields similar mean scores for the binary trees and the neural networks. Thus, there is a large plateau of minmum scores for both models. However, when comparing the three configurations of the two models with each other, we can see that the binary tree model reaches more and higher individual scores than the neural network models. In addition, the results of the two models are similar for the two models for the environment `Acrobot`. The binary tree with one node outperforms the neural network without hidden layers whereas the complexer neural networks outperform the binary trees with four and eight nodes. Even though the binary tree model at its current implementation outputs only discrete action values, the results for the continuous environment `Pendulum` are comparable to those of neural networks. The binary tree with eight nodes even managed to outperform neural networks. For the environment `MountainCarContinuous`, neural networks delivered better results. Although the binary tree model with one node reached significantly higher mean scores than neural networks, it exhibits a large plateau of minimum scores that we do not see for the neural networks. The networks with higher complexity then managed to reach the same high mean scores. The considerable difference in the score distribution between the two models could be caused by the binary tree producing discrete actions instead of continuous ones, as neural networks did. For the binary tree model, the action is either -1 or 1, limiting the output space significantly. It seems that the environment `MountainCarContiuous` reacts more sensible to these discrete action outputs than the environment `Pendulum`.

All in all, the binary tree model produced a similar score distribution as neural networks did. For some environments, the binary trees even outperform neural networks. However, the environment `MountainCarContinuous` seems more difficult for the binary trees to learn than for neural networks.

Impact of bias: Including bias has a significant negative impact on almost all of the five classic control environments. For the environment `Acrobot` and `Pendulum`, the difference between using bias and not using bias seems less extreme than for the other environments depending on the number of layers used. To further investigate this behavior, I subsequently changed three aspects of the model's architecture. First, I increased the number of weights of the networks. Despite the remarkably increased number of weights, the results show only slight differences in the score distribution. The exception is the environment `MountainCarContinuous`, where the mean scores are remarkably lower. Except for the environment `MountainCarContinuous`, we can conclude that the increased number of weights does not impact the score distribution significantly. Second, I increased the number of hidden layers. Some environments show slightly different results like `CartPole` and `MontainCar`. However, the changes are less significant than in the experiments with bias connections. The exception is again `MountainCarContinuous`, where the difference is noticeably more extreme with an increased number of layers. Finally, I increased the number of neurons in each hidden layer. In the results of this experiment, some changes in the score distribution are visible. The mean scores of the environments `Pendulum` and

`MountainCar` show slight differences. The environment `MountainCarContinuous` shows a similar picture as before.

In summary, no other aspect impacts the performance of the model as negative as using bias. The exceptions are the environment `MountainCarContinuous`, where we see even more extreme differences when changing the network architecture, and `Pendulum`, which already shows smaller differences than the other environments when using bias versus not using bias. It seems that this behavior is specific to the bias in the setting of RWG for the environments `CartPole`, `Acrobot`, and `MountainCar`. In addition, we have seen that the environment `MountainCarContinuous` generally reacts sensitive to a change in the model's architecture, showing a similar image as when using bias. In contrast, `Pendulum` does not seem to be significantly affected by a change in the architecture. But as we have seen, the bias also affects it less.

Explain more. Present or past? Talk about search space for why some complexer models perform better or worse and scores were more distributet in Conculsion chapter. Did Acrobot significantly change? Incldue the better images of MountainCarContinuous

Chapter 4

Conclusion

4.1 Conclusion

- Summarize the thesis, this time with a focus on the results obtained
- Add what deductions you have from the discussion above

In this work, I delivered a direct comparison of neural networks to the polynomial and binary tree models. I was able to show that especially the binary tree model can deliver comparable results with a simpler structure and less needed parameters. The last aspect makes the process of parameter tuning straightforward.

Explain more. Answer research questions.

4.2 Future Work

- Future work
- Fourier? You should add your own ideas rather than mine :)
- Particularly, please do not mention Bezier yet :) that was in confidence
- Harder benchmarks
- Ways to complexify the function from the b-tree

The continuation of this work includes...

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Chapter 5

Appendix

5.1 Additional Plots