1   Explaining contingency judgments with a computational model of instance-based memory

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Abstract

The purpose of this thesis was to develop and test an account of human contingency judgments based on a computational model of human memory. Specifically, we used MINERVA 2, a simulation model of episodic memory (Hintzman, 1986). MINERVA 2 is a model of encoding and retrieval. It assumes that individual experiences are encoded as memory traces, and that traces are retrieved as a function of their similarity to an environmental stimulus. I tested the model by attempting to simulate a study done by Crump, Hannah, Allan, and Hord (2007), that illustrated two major phenomena in the contingency judgment literature, such as the $\triangle$P and outcome density effects. I show that the model can successfully account for both normative contingency judgment behavior and the outcome density bias, and discuss implications for future research.

Keywords: memory, contingency judgments, MINERVA II, instance theory

Word count: 2180

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Introduction

Imagine driving down a highway at a speed of sixty miles per hour. Suddenly, you hear on the radio that a car accident has occurred on the same highway that you are driving on. Given your previous experience with driving on this highway, you anticipate a longer commute time. Why did you predict a longer commute? How was this anticipation formed? Is the anticipation accurate or biased? These types of questions are asked by researchers when studying contingency learning and judgments, which include the cognitive processes enabling people to become sensitive to whether particular stimulus predict particular outcomes. The purpose of studying human contingency judgments is to gain a better understanding of the way that people learn about the causal relationships between events (Beckers, De Houwer, & Matute, 2007).

There are many different theories of how people become sensitive to contingency information. The purpose of this thesis was to propose a new memory-based explanation using a computational model based on MINERVA 2, which is a simulation model of episodic memory Hintzman (1988). Before explaining the model, I will first review the current literature on human contingency judgments. The review will describe tasks and phenomena in the domain, discuss current theories of contingency learning and explain how they differ from my proposal. Finally, I will describe and explain MINERVA 2 before applying the model to simulate results from Crump, Hannah, Allan, and Hord (2007).

The contingency judgment literature: tasks and phenomena

What is a contingency?   Contingency is formally defined as a statistical relationship between two dichotomous, binary variables. In this thesis, I will refer to two kinds of events cues and outcomes, both of which can occur or not occur. For example, the notice of an accident on the highway could be a cue; and, observing a slow-down in traffic

Table 1

A 2x2 contingency table for a single cue and outcome.

	Outcome	No Outcome
Cue	A	B
No Cue	C	D

48 following the cue could be an outcome. The contingency between a cue and outcome is

49 established over a history of event pairings. There are four possible cue-outcome pairings

50 that can be presented with varying frequencies to manipulate the cue-outcome relationship;

51 and, these are shown in Table 1.

52   Table 1 displays a 2x2 contingency table representing the four different cue-outcome

53 pairings. The letters represent the frequency of each event pair. First, a cue is presented and

54 an outcome occurs (A). Second, a cue is presented and an outcome does not occur (B). Third,

55 a cue is not presented and an outcome occurs (C). Fourth, a cue is not presented and an

56 outcome does not occur (D). Conventionally, the contingency between the cue-outcome pairs

57 over all events is defined by the $\triangle$P rule (see Allan, 1980). The formula to compute $\triangle$P is:

58 $$\Delta P = \frac{A}{A+B} - \frac{C}{C+D}$$

59   In other words, ΔP is a difference between two conditional probabilities:

60 $$\Delta P = P(Cue|Outcome) - P(Cue|NoOutcome)$$

61      $\triangle$P can range from 1 to -1, and behaves similarly to a correlation coefficient. When

62 $\triangle$P is 1, the presence of a cue perfectly predicts the occurrence of an outcome; and, the

63 absence of the cue perfectly predicts the absence of the outcome. When $\triangle$P is -1, the

64 presence of a cue predicts the absence of an outcome, and the absence of cue predicts the

65 presence of the outcome. When $\triangle$P approaches 0, the presence or absence of the cue does

66 not predict the presence or absence of the outcome.

⁶⁷   To return to the highway traffic example, a person may anticipate that traffic will slow

⁶⁸ down after hearing about an accident because they may be sensitive to a positive

⁶⁹ contingency relationship between accident events and the consequences for traffic flow. In a

⁷⁰ laboratory scenario, it is possible to measure peoples' abilities to judge contingencies

⁷¹ between events using a variety of contingency judgment tasks discussed next.

⁷²   **Assessing contingency judgment ability.**   The discrete-trial procedure is a

⁷³ common method used to measure contingency judgment ability. Allan (1993) describes

⁷⁴ several studies that use the discrete-trial procedure. For example, on each trial, Allan and

⁷⁵ Jenkins (1980) presented subjects with an empty lake scene in which they could respond by

⁷⁶ either moving or not moving a joystick. After performing this action, the scene would either

⁷⁷ change to a picture of the lake with the Loch Ness Monster in it, or it would remain the

⁷⁸ same. Allan also describes a study by Shanks (1985), in which participants were shown a

⁷⁹ tank moving across the screen and passing a gun sight. Participants chose either to fire or

⁸⁰ not fire at the tank, and then observed whether or not the tank was destroyed. In general, in

⁸¹ these tasks, people are presented with pairs (ABCD frequency information) of cue-outcome

⁸² events over a series of trials, and then asked to make judgments of contingency between the

⁸³ cue and the outcome at the end of the series of trials. For example, in the Loch Ness

⁸⁴ monster example, after viewing a series of events, participants could be asked to rate the

⁸⁵ contingency between their response and the appearance of the Loch Ness Monster.

⁸⁶   This thesis provides a memory-based explanation for contingency judgment phenomena

⁸⁷ from Crump, Hannah, Allan, and Hord (2007), who used the streamed trials procedure. The

⁸⁸ streamed-trials procedure is essentially a faster version of the discrete-trials procedure. For

⁸⁹ example, in the streamed-trials procedure, cues and outcomes for each event are presented

⁹⁰ for 100 ms intervals, separated by another short intertrial interval of 100ms intervals. That

⁹¹ study used abstract cues and outcomes. For example, cues were displayed as a blue square

⁹² that appeared or or did not appear, and outcomes were displayed as a red circle that

⁹³ appeared or did not appear. A streamed-trial consisted of 60 events presented to participant,

followed by a question to rate the contingency between the cue and outcome. A benefit of the streamed-trials procedure was that individual participants could be presented with many streamed trials over the course of an experiment. As a result, it was possible to conduct within-subject repeated measures designs, and measure how a single subject's ratings changed across different levels of ΔP programmed into each streamed trial.Although there are many contingency judgment tasks to choose from, the modelling efforts in this thesis were focused on the design and results from Crump, Hannah, Allan, and Hord (2007). The important aspect was not the streamed trial nature of that design. Instead, the design conveniently demonstrated two important contingency judgment that are found across tasks. Thus, the major aim of the modelling effort was to determine if the model could successfully explain hallmark findings in the literature.

Classical contingency judgment phenomena.   The modelling effort focused on explaining two major phenomena defined as the contingency effect, and the outcome density effect. Both are explained in turn.The contingency effect is the basic finding that humans are capable of making contingency judgments that appear to track ΔP. For example, when the programmed ΔP value in a series of cue-outcome events is positive, participants judgements of contingency tend to be positive, and when the ΔP value in a series of cue-outcome events is negative, participants judgements of contingency tend to be negative. In other words, participants judgments of contingency tends to vary as a function of the actual ΔP between the events they are judging.

The outcome density effect is an example of a bias in contingency judgment. A bias occurs when people make contingency judgments that are different from the actual ΔP value between the events. The outcome density effect is the finding that ratings of contingency are also biased by the overall proportion of outcomes, which can be statistically independent from ΔP. Specifically, participants rate cues as more predictive of outcomes that are more frequent than less frequent. In summary, this thesis identified the contingency effect and outcome density bias as a useful starting point to test a new memory model of contingency

₁₂₁ judgments. Before elaborating on the new model, I now review a few of the existing theories.

Theoretical process accounts of Contingency judgments

₁₂₃ What psychological mechanisms are involved in making contingency judgments?
₁₂₄ Several theories can be used to explain the way in which contingency judgments work. Next,
₁₂₅ I review rule-based, associative, and signal detection accounts of contingency judgments
₁₂₆ before describing the memory-based approach taken in this thesis.

₁₂₇ **Rule-based accounts.** Rule-based theories explain contingency judgments in terms
₁₂₈ of the application of formulas, much like the way a statistician would compute contingency
₁₂₉ values. (Allan, 1993). For example, Allan discusses the $\triangle$P rule, which is defined as the
₁₃₀ difference between two independent conditional probabilities. It is possible that people act
₁₃₁ like "calculators," and use the $\triangle$P formula when making contingency judgments. A
₁₃₂ relatively recent development in ordering packages from Amazon is a real-world example we
₁₃₃ can use to illustrate this idea. The company now posts a photo online of the package on the
₁₃₄ doorstep to show that an order was delivered; however, this is not always the case. Four
₁₃₅ possible cue-outcome events can occur. First, an order is delivered (cue) and a picture
₁₃₆ (outcome) is posted (A). Second, an order is delivered and a picture is not posted. Third, an
₁₃₇ order is not delivered and a picture is posted. Fourth, an order is not delivered and a picture
₁₃₈ is not posted. Anyone familiar with Amazon delivery could be asked to rate the contingency
₁₃₉ between package delivery and photo evidence of the delivery. . According to rule-based
₁₄₀ theory, a person could do this by explicitly using the $\triangle$P formula.

₁₄₁ Explicit rule-based contingency judgments would involve a set of additional processes.
₁₄₂ First, a person would need to estimate or obtain the frequencies of each of the A, B, C, and
₁₄₃ D cue-outcome events. These frequencies could be estimated in some unknown fashion
₁₄₄ through some memory process, or a person could potentially review the history of Amazon
₁₄₅ orders to obtain accurate frequencies. Then, to make the contingency judgment, they would
₁₄₆ need to understand that the $\triangle$P formula is an appropriate measure of contingency judgment,

147 and they would need to enter the frequency values into the formula, calculate the $\triangle$P value,

148 and report the value as their contingency judgment.

149   Although it is clear that people may have the ability to use algebra and apply

150 mathematical rules to transform frequency information into a contingency judgment, it is

151 much less clear that people use this approach in everyday life. For example, people seem able

152 to sense contingency information on an intuitive level without explicitly applying rules. And,

153 if people were using the $\triangle$P rule alone, then it is unclear why biases in contingency

154 judgment would appear such as the outcome density effect.

155   **Associative accounts.**   Associative theory explains that contingency judgments

156 depend on associative knowledge formed automatically through classical conditioning

157 processes. According to this line of reasoning, contingency knowledge, like associative bonds,

158 is learned through the repeated presentation of stimuli. And, people rely upon their existing

159 associative knowledge when making judgments. In other words, a contingency judgment

160 should roughly reflect a read-out of the strength of association between a cue and outcome.

161 One implication of the associative view, is that phenomena that influence how people learn

162 associations should also influence contingency judgments.

163   Consider the Rescorla-Wagner model of learning (Rescorla & Wagner, 1972)explains

164 that when a CS is frequently paired with a US and is consistent in eliciting a CR, the CS has

165 acquired associative strength. Under these circumstances, participants will easily come to

166 associate the CS with the US, and respond accordingly. This aspect of the model can

167 account both for the standard contingency effect and the outcome density effect in human

168 contingency judgments. For example, frequent exposure to cue-outcome events (CS-US

169 events) will increase the associative bond between the cue and outcome and support

170 increasingly more positive contingency judgments.

171   Additionally, the Rescorla-Wagner model suggests that the amount of learning

172 diminishes as a conditioned stimulus becomes more familiar. model explains that

173 Furthermore, Rescorla and Wagner state, "changes in the strength of a stimulus depend

174 upon the total associative strength of the compound in which that stimulus appears."

175 Rescorla and Wagner support their assertion that all stimuli present when the US occurs are

176 important to consider, by demonstrating that their model can explain the blocking effect

177 (Kamin, 1969), which occurs when a new association is unable to be properly formed due to

178 a previous association with the US. Similarly, blocking phenomena have been demonstrated

179 in human contingency judgment tasks (Shanks, 1985), which further supports the notion

180 that associative processes may be involved in human contingency judgment.

181 **Signal-detection accounts.** Signal detection theory (Green & Swets, 1966) is a

182 measurement framework that can separate one's ability to differentiate between actual

183 information (sensitivity) and biases that can emerge during a judgment task.For example,

184 contingency judgments are formed based on one's ability to report on their internal

185 sensitivity to contingency informationSiegel, Allan, Hannah, & Crump (2009). The signal

186 detection approach does not specify how people become sensitive to contingency information,

187 and it is compatible with other models like the associative account which could explain

188 sensitivity in terms of association bond formation. Several factors may influence one's

189 judgment. A minimum amount of change is necessary for one to be able to notice whether

190 something is different from what they have previously experienced. There is also a minimum

191 amount of stimulation required in order for someone to be aware that something is

192 happening. If a significant amount of time is elapsed between two events, one may be less

193 likely to predict that one event caused another. For example, if you eat spoiled food but do

194 not get sick until three weeks later, you may be less likely to predict that the food caused

195 the illness than if you had become sick immediately after consumption.Further, noise

196 interference also plays a role. This can be anything that distracts a participant while they

197 are trying to focus on the contingency task, such as thoughts, sounds, or other objects in

198 sight. These factors can influence, positively or negatively, one's memory of the contingency

199 task. A major contribution of the signal detection approach is to determine whether

contingency judgment phenomena potentially reflect changes in sensitivity or bias. For

example, Allan, Hannah, Crump, and Siegel (2008) demonstrated the outcome density effect

reflects a bias in reporting the strength of contingency rather than a more fundamental

change to the process responsible for acquiring associative knowledge.

Memory accounts.   Although memory for previous experience is clearly relevant to

the task of making judgments about contingency between cues and outcomes that appeared

over time, the role of memory processes in contingency judgments has not been broadly

discussed (but see, Arkes & Rothbart, 1985) or formalized. This thesis examines whether a

computational model of human memory, MINERVA II, could provide a useful memory-based

explanation of contingency judgment phenomena.

MINERVA II is a multiple-trace theory of memory. It is assumes that individual

experiences are encoded as traces into memory. Furthermore, it assumes that repeated

exposure to the same information creates multiple copies rather than strengthening the same

memory. A major assumption is that representations of an individual experience are roughly

wholistic, that is they include the collection of details present in the experience within the

representation for that trace. In other words, a single trace potentially contains all of the

contextual information embedded within a specific experience. MINERVA II can behave

similarly to an associative account, however it does not directly learn or form associative

bonds between cues and outcomes. Instead, the whole history of cue-outcome pairings is

stored within separate traces in memory. Minerva II also assumes that retrieval operates in a

cue and similarity driven fashion. A cue or pattern in the environment is compared to traces

in memory and activates similar memory traces in parallel, which are then used at retrieval.

In principle, MINVERA II should be able to perform a contingency judgment task. For

example, it would be possible to provide the memory with examples of cue-outcome events

just as participants are in the discrete trials procedure. Then, the memory could be probed

with a cue pattern or the absence of a cue pattern. In both cases, similar to the process of

cued-recall, the memory would retrieve different amounts of contextual information that were paired with the cue within the memory traces. In other words, MINERVA II should retrieve expectations for the outcome given a cue. The question I set out to answer in this thesis was whether the MINERVA II process would also produce contingency effects and the outcome density bias in the same way that people do. Before moving on to the simulations, I describe MINERVA II in more detail.

MINERVA II

MINERVA II is a computational instance theory of human memory (Hintzman, 1984, 1986, 1988). It is conceptually similar to other global-similarity models of memory (Eich, 1982; Murdock, 1993). MINERVA II and related models have been applied to explain many kinds of cognitive phenomena and processes such as recognition memory (Arndt & Hirshman, 1998), probability judgment and decision-making (Dougherty, Gettys, & Ogden, 1999), artificial grammar learning (Jamieson & Mewhort, 2009a), serial reaction time task performance (Jamieson & Mewhort, 2009b), associative learning phenomena (Jamieson, Crump, & Hannah, 2012), and computational accounts of semantic knowledge (Jamieson, Avery, Johns, & Jones, 2018).

In MINERVA 2, memory is a matrix M. Each row represents a memory trace, and the columns represent features of the trace.

How does encoding work? Individual events are represented as feature vectors E, and new events are stored to the next row in the memory matrix M. Individual features are stored with probability L, representing quality of encoding. For example, in the present simulations I used a feature vector with 30 elements. The first 10 elements coded the presence or absence of the cue. When the cue was present, the elements were set to 1. When the cue was absent, the elements were set to 0. The 11th to 20th elements coded the outcome, again setting the elements to 1 or 0 depending on whether the outcome was present or absent.

251 The elements 21 to 30 coded the background context, and were set to 1 for all traces.

252 How does retrieval work? A probe (feature vector for a current event in the

253 environment) is submitted to memory, and causes traces to activate in proportion to their

254 similarity to the probe. Similarity between each trace and the probe is computed with a

255 cosine:

256 $$S_i = cos(\theta) = \frac{A\dot{B}}{||A||||B||}$$

257 $$S_i = \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2}\sqrt{\sum_{i=1}^{n} B_i^2}}$$

258 Where A is a probe and B is a memory trace in M.

259 Activation is a function of similarity raised to a power of three, and it is possible to

260 further raise the exponent The exponent, sometimes also called resonance, acts like a tuning

261 parameter causing only the most similar traces to be viable for retrieval as resonance is

262 increased.

263 $$A_i = S_i^3$$

264 Each trace is then weighted by its activation (cubed similarity) to the probe, and

265 summed to produce an echo.

266 $$C_j = \sum_{i=1}^{m} A_i \times M_{ij}$$

267 How is a contingency judgment computed? We take the raw values in the outcome

268 portion of the echo as measures of expectation for the outcome given the cue. More

269 specifically, we ask the model to derive a contingency judgment in two steps. In the first

270 step, the model is probed with a cue-present feature vector that contains no outcome. This

271 probe will tend to be more similar to traces that contain a cue-present feature, causing those

272 traces to be more highly represented in the retrieved echo. To the extent that those traces

273 include pairings with the outcome, the outcome features will also be present in the echo. To

274 measure the extent to which the cue retrieves an expectation for the outcome, we simply

275 recorded the raw value in the echo from the first element of the outcome field (the 11th unit).

276 In the second step, we probe the memory with a cue-absent feature vector and record the

277 expectation for the outcome. These two values are similar to the conditional probability of

278 receiving an outcome given a cue, and receiving an outcome given no cue. The model-based

279 contingency judgment was thus taken as a difference between the expectations.

<center>**Simulation 1**</center>

281 Our experiment is based on a research study performed by Crump, Hannah, Allan, and

282 Hord (2007). While this study involved presenting humans with a contingency task, our

283 computer model attempts to replicate the findings of this study, and expand upon it. The

284 Crump, Hannah, Allan, and Hord (2007) study tested for the contingency effect as well as

285 the outcome density effect by manipulating the number of cues and outcomes presented in

286 each condition. Specifically, they used a 2x2 design manipulating both contingency and

287 outcome density. The contingency manipulation involved streamed-trials that include zero

288 contingency or a medium positive contingency. The outcome density manipulation involved

289 streamed-trials that included a low or high proportion of outcomes. These levels were

290 crossed to produce the four conditions shown in Figure 1.

291 After viewing a streamed-trial participants were asked at random to complete a

292 contingency rating judgments or a frequency estimate judgments. The contingency rating

293 judgments were collected using a sliding scale, where participants could choose between -100

294 and +100. In order to take in frequency estimate judgments, participants were presented

295 with four images, each representing one of the four cue-outcome events. One field was left

296 empty, in which participants were told to write in an estimate of the frequency of occurrence

297 for each circumstance.

298 The original results from Crump, Hannah, Allan, and Hord (2007) are shown in Figure

Figure 1. Original 2x2 contingencies tables reprinted from Crump, Hanah, Allan, & Hord, (2007). The top two matrices outline the frequency of the four cue-outcome pairings in a (left) low outcome density noncontingent stream, $\triangle P = 0$, $P(O) = .2$, and a (right) high outcome density noncontingent stream, $\triangle P = 0$, $P(O) = .8$. The bottom two matrices define a (left) low outcome density contingent stream, $\triangle P = .467$, $P(O) = .33$, and (right) high outcome density contingent stream, $\triangle P = .467$, $P(O) = .67$.

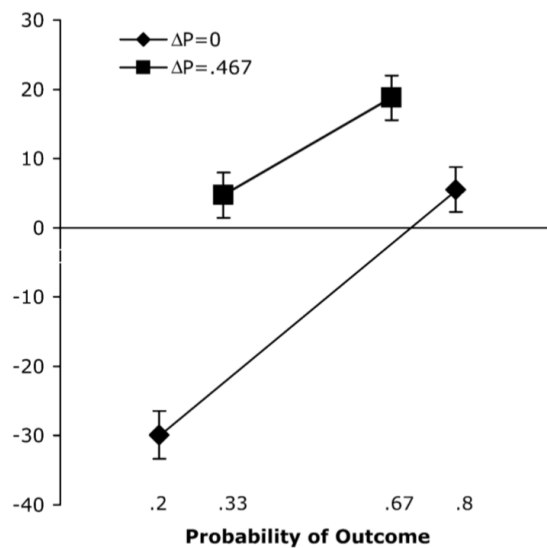*Figure 2*. Original results reprinted from Crump, Hanah, Allan, & Hord, (2007).

2. The figure shows that, for non-contingent conditions ($\triangle$P=0, diamond shape),

contingency ratings were lower for both low and high outcome density conditions.

Participants' contingency ratings were highest overall during contingent conditions

($\triangle$P=.467, diamond shape). This trend indicates that the $\triangle$P effect is present. However,

regardless of stream condition, contingency ratings were always higher when outcome density

was larger. This shows that the outcome density effect is present.

The purpose of simulation 1 was to deliver the same set of conditions as in Crump,

Hannah, Allan, and Hord (2007) to MINERVA II, and then determine whether the model

makes similar patterns of contingency judgments. In particular, we were interested in

whether the model would show both the standard contingency effect as well as the outcome

density effect.

Methods

We used RStudio to create a model of memory. Our model was presented with two

types of streams, non-contingent and contingent. Non-contingent refers to trials where $\triangle$P is

0. This means there is no relationship between the cues and outcomes shown, regardless of

outcome density. Cues do not predict outcomes, and outcomes do not predict cues.

Contingent refers to trials where $\triangle$P is .467, where the presence of a cue does foreshadow

the presence of an outcome. Each type of stream contained two conditions, low outcome

density and high outcome density. Low outcome density refers to a trial in which fewer

outcomes were presented than cues. Four types of trials were presented to the model. The

model was presented with either, a cue and an outcome, a cue and no outcome, no cue and

an outcome, or no cue and no outcome. After being shown these combinations many times,

our model was asked to predict whether an outcome would occur given that cues were

presented first with no outcomes.

As mentioned earlier, MINERVA II is a multiple trace model, and so it assumes that

each experience leaves an individual memory trace. With repeated exposure to the same information, it creates multiple copies rather than strengthening the same memory. MINERVA II is mostly focused on long-term memory, however, there is assumed to be a temporary buffer (short-term memory) that relay information to long-term memory. The model was programmed in R and the code is presented in Appendix 1.

The original experiment performed by Crump, Hannah, Allan, and Hord (2007) involved a blue square being presented as the cue and a red circle being presented as the outcome. Our model presents cues and outcomes to the model as sets of 0s and 1s. 0 being not present, 1 being present. If a cue was presented first (1), it may have either been followed by an outcome (1), or no outcome (0). If no cue was presented first (0), it was either followed by no outcome, or an outcome. In theory, the more cues and outcomes presented, the more accurate the model will be at predicting the presence or absence of each.

I used the following open-source software for the model code, analyses, and to create this thesis as a reproducible report, R [Version 4.0.2; R Core Team (2020)] and the R-packages *ggplot2* [Version 3.3.2; Wickham et al. (2020)], *knitr* [Version 1.30; Xie (2020)], *lsa* [Version 0.73.2; Wild (2020)], *papaja* [Version 0.1.0.9997; Aust and Barth (2020)], and *rmarkdown* [Version 2.6; Allaire et al. (2020)].

Results

Did our MINERVA model produce a similar $\triangle$P effect and outcome density effect to those found in the Crump, Hannah, Allan, and Hord (2007) study? The results of the model simulations are shown in Figure 3. For both contingent ($\triangle$P=.467) and non-contingent ($\triangle$P=0) streams of data, contingency ratings (Outcome Activation Strength in Echo) were lower when less outcomes were presented (low outcome density, lower Probability of Outcome). Just like the human participants in the original study, our computer model also had higher contingency ratings when more outcomes were presented than cues (high outcome
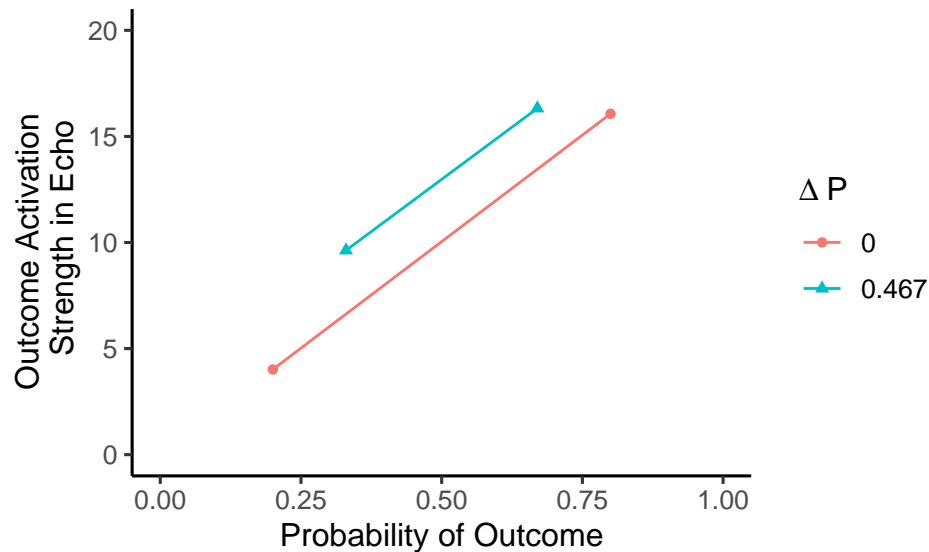
Figure 3. Mean Contingency Ratings Based on Outcome Density.

density, greater Probability of Outcome). In contingent conditions, contingency ratings were much higher overall than in non-contingent conditions. Overall, the results paralleled those of the original study, providing some initial support for a memory-based view of contingency judgment.

Simulation 2

I noted briefly earlier that Crump, Hannah, Allan, and Hord (2007) manipulated the judgment that participants gave following each trial. Specifically, participants either received a contingency rating task, or a frequency estimation task. For the frequency judgment task, participants were asked to estimate the frequency of each of the four cue-outcome event pairs that were presented in the trial. With this measure, it was possible to derive a contingency value from the frequency estimates. The results from the frequency judgments in the original study are presented in figure 4.

The interesting finding was that the outcome density bias was no longer present for contingency ratings derived from frequency estimates. The purpose of Simulation 2 was to determine whether MINERVA II would show a similar pattern if it was asked to make
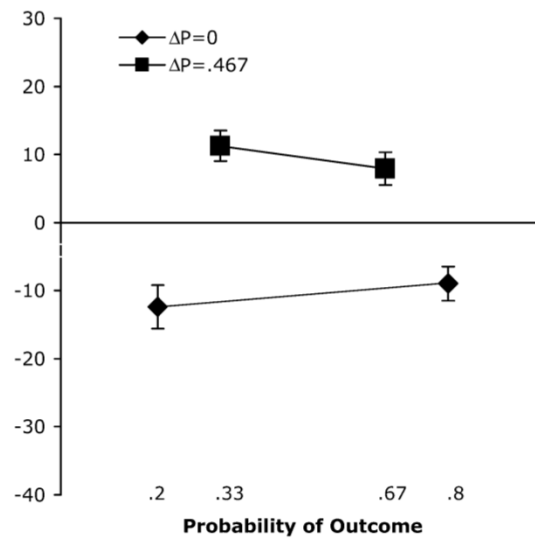
Figure 4. Mean transformed △P scores derived from participants frequency estimates (with standard error) as a function of △P (0 vs. .467) and outcome density. Reprinted from Crump et al. (2007)

364 frequency judgments. Notably, MINERVA II was developed originally in part to explain

365 frequency judgments (Hintzman, 1988), so it can naturally be applied to frequency

366 judgments in the contingency judgment task.

Methods

368   Simulation 2 was nearly identical to Simulation 1 with the following exceptions. After

369 each streamed trial, the model was probed with one of the four cue-outcome event pairs, and

370 following Hintzman (1988), the sum of the similarity weighted activations was recorded as

371 the model response. The four responses were then entered directly into the ΔP formula to

372 compute a frequency derived contingency value. Additionally, the model was evaluated for

373 different levels of resonance, which is the exponent used to raise the similarities. The

374 simulation code is reported in Appendix 2.

Results

376   Did our second simulation produce similar results to that of Crump, Hannah, Allan,

377 and Hord (2007)? The results of Simulation 2 are reported in figure 5. The figure shows the

³⁷⁸ model evaluated using six different levels of resonance. As the resonance level increased

³⁷⁹ (generalization decreased, exponent value increased), the model became more precise at

³⁸⁰ making estimates of frequency judgements. In other words, when the exponent value was

³⁸¹ low, the model was being asked to remember more generally. Under these circumstances

³⁸² (most often when resonance was 1), the model over-generalized, neglecting to discriminate

³⁸³ between positive contingencies and zero contingencies. Activation intensity was similar

³⁸⁴ regardless of which cue was used as a probe. The model gave similar frequency estimates for

³⁸⁵ all cue-outcome situations. This resulted in estimated delta P values of 0.

³⁸⁶   As it became greater, the model was being asked to remember more specific instances.

³⁸⁷ The outcome paralleled that of the original study, with the model not being biased by

³⁸⁸ outcome density. As expected, this contrasts the results of Simulation 1. For noncontingent

³⁸⁹ streams ($\triangle P = 0$), frequency estimates (Simulated $\triangle P$) tended to be closer to 0, and largely

³⁹⁰ did not differ as a function of Probability of Outcome or in conditions with greater resonance.

³⁹¹ For contingent streams ($\triangle P = .467$), frequency estimates tended to be close to .467 in

³⁹² conditions where resonance was greater than 3. They also largely did not vary with regard to

³⁹³ Probability of Outcome. This again brings up the possibility that the type of judgement

³⁹⁴ requested may be the cause of bias. Notably, however, in conditions where resonance was

³⁹⁵ low, there is an inversion of the outcome density effect for frequency estimates close to zero.

³⁹⁶ Instead of estimating an effect when more outcomes were present, the model estimates a

³⁹⁷ greater simulated delta P value when less outcomes were present. This contrasting the

³⁹⁸ outcome density effect found in humans during the Crump et al. (2007) study.

General Discussion

⁴⁰⁰   The purpose of this experiment was to create a simulated version of the study

⁴⁰¹ performed by Crump, Hannah, Allan, and Hord (2007). In general, our model was able to

⁴⁰² replicate several attributes of the in-person study, such as the $\triangle P$ conditions and the

⁴⁰³ outcome densities associated with them. This suggests that contingency judgments can be
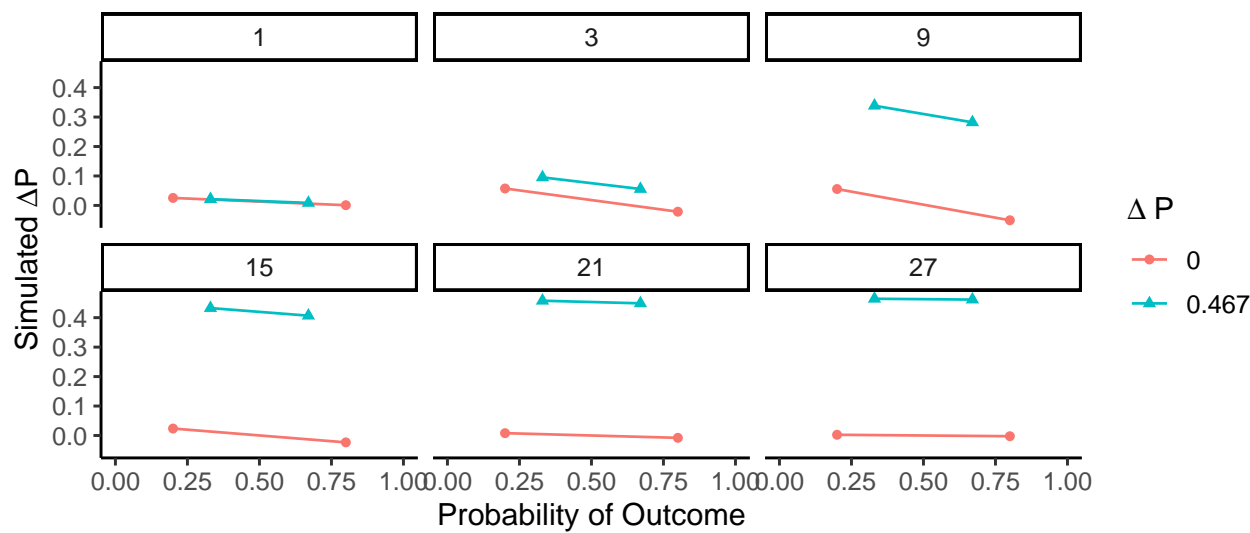
Figure 5. Mean model contingency ratings derived from model estimates of frequency judgments.

explained in terms of memory processes.

Limitations

Our model contains several key differences when compared with the original study done by Crump, Hannah, Allan, and Hord (2007). One major difference between our model and the in-person study is that our simulation did not produce any negative ratings. Specifically, the outcome density effect was not present. Several factors may explain this result, such as the fact that no human participants were included in our study. In the low outcome density condition ($\triangle P=0$) of the original study, human beings gave negative ratings. This was likely due to the outcome density effect. This phenomenon was not present in our simulation data. Another factor that may explain this result is the possibility that we overlooked particular variables when creating our model. It could be the case that we neglected to implement code for some aspect of attention or memory.

In order to create a simulated version of memory, our model makes several assumptions. For instance, our model assumes that each memory has its own trace (multiple-trace theory). In order to operationalize cue and outcome events, we use binary

₄₁₉ values to represent the presence or absence of an occurrence. To operationalize similarity

₄₂₀ between events, we use the cosine function. We also use resonance to operationalize a way to

₄₂₁ amplify the relative differences in our data.

₄₂₂   By studying contingency judgments, we can gain a better understanding of factors that

₄₂₃ influence learning, memory, and eventually decision making. Our results indicate that there

₄₂₄ is a relationship between the number of times an outcome is shown, and one's prediction of

₄₂₅ whether or not an outcome will occur based on a cue. This general principle may have

₄₂₆ real-world implications, such as in regard to interpreting data. One may falsely attribute one

₄₂₇ variable to causing another based on often experiencing both variables together. This relates

₄₂₈ to the statistical principle "correlation does not imply causation."

Future Research

₄₃₀   In order to create a model that produces results that are more accurate to the original

₄₃₁ study, we could create a negative contingency condition. This condition would set P equal to

₄₃₂ -.467, meaning that the presence of a cue would predict the absence of an outcome. This has

₄₃₃ the potential to make the model more likely to give negative ratings of contingency.

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Appendix A

512 This appendix contains the R script for Simulation 1.

```r
### Model code for simulation 1

## create contingency task trials

create_trials <- function(A, B, C, D, vector_length, shuffle=FALSE){

  event_frequencies <- rep(c(1,2,3,4), times = c(A,B,C,D))

  event_pairs <- matrix(c(rep(c(1,1,1), each = vector_length),
rep(c(1,0,1), each = vector_length),
rep(c(0,1,1), each = vector_length),
rep(c(0,0,1), each = vector_length)),
ncol= vector_length*3,
byrow=TRUE)

  event_matrix <- event_pairs[event_frequencies,]

if(shuffle==FALSE) {return(event_matrix)
    }else{
return(event_matrix[sample(1:length(event_frequencies)),])
    }

}

## Run the model function

run_model <- function(trials,probe){
  memory <- trials

#probe memory with cue only

  similarities <- c()
for(i in 1:dim(memory)[1]){
    similarities[i] <- lsa::cosine(probe,memory[i,])
  }
  similarities[is.nan(similarities)]<-0

# generate echo
  weighted_memory <- memory*(similarities^3)
  echo <- colSums(weighted_memory)
```

```r
return(echo)
}

### DEFINE conditions

conditions <- list(list(outcome_density = .2,
delta_p = 0,
frequencies = c(6,24,6,24)),
list(outcome_density = .8,
delta_p = 0,
frequencies = c(24,6,24,6)),
list(outcome_density = .33,
delta_p = .467,
frequencies = c(17,13,3,27)),
list(outcome_density = .67,
delta_p = .467,
frequencies = c(27,3,13,17))
                    )

model_data<-data.frame() #initialize data frame

## Run the model across conditions

for(i in 1:4){
  trials <- create_trials(A=conditions[[i]]$frequencies[1],
B=conditions[[i]]$frequencies[2],
C=conditions[[i]]$frequencies[3],
D=conditions[[i]]$frequencies[4],
vector_length = 10,
shuffle=TRUE)
  model_output <- run_model(trials=trials,probe = rep(c(1,0,1),each=10))
  sim_data <- data.frame(outcome_density = conditions[[i]]$outcome_density,
delta_p = conditions[[i]]$delta_p,
expectation = model_output[11])
  model_data <- rbind(model_data,sim_data)
}

# print table
#knitr::kable(model_data)

# plot
library(ggplot2)

ggplot(model_data,aes(x=outcome_density,
```

```
y=expectation,
group=delta_p,
color=delta_p))+
geom_line()+
geom_point()
```

Appendix B

513 This appendix contains the R script for Simulation 2.

```r
## Simulation 2

run_model_activations <- function(trials,probe,resonance){
  memory <- trials

#probe memory with cue only

  similarities <- c()
for(i in 1:dim(memory)[1]){
    similarities[i] <- lsa::cosine(probe,memory[i,])
  }
  similarities[is.nan(similarities)] <- 0

# get activations
  activations <- sum(similarities^resonance)

return(activations)
}

# frequency judgment simulation
freq_model_data <- data.frame() #initialize data frame

## Run the model across conditions

for(tau in c(1,3,9,15,21,27)){
for(i in 1:4){
    trials <- create_trials(A=conditions[[i]]$frequencies[1],
B=conditions[[i]]$frequencies[2],
C=conditions[[i]]$frequencies[3],
D=conditions[[i]]$frequencies[4],
vector_length = 10,
shuffle=TRUE)

    freq_A <- run_model_activations(trials=trials,
probe = rep(c(1,1,1),each=10),
resonance = tau)
    freq_B <- run_model_activations(trials=trials,
probe = rep(c(1,0,1),each=10),
resonance = tau)
    freq_C <- run_model_activations(trials=trials,
```

```r
probe = rep(c(0,1,1),each=10),
resonance = tau)
    freq_D <- run_model_activations(trials=trials,
probe = rep(c(0,0,1),each=10),
resonance = tau)

    sim_data <- data.frame(outcome_density = conditions[[i]]$outcome_density,
delta_p = conditions[[i]]$delta_p,
expectation = (freq_A/(freq_A+freq_B)) - (freq_C/(freq_C+freq_D)),
resonance = tau)

    freq_model_data <- rbind(freq_model_data,sim_data)
  }
}
```

```r
# plot
library(latex2exp)
freq_model_data$delta_p <- as.factor(freq_model_data$delta_p)

ggplot(freq_model_data,aes(x=outcome_density,
y=expectation,
group=delta_p,
color=delta_p,
shape=delta_p))+
geom_line()+
geom_point()+
xlab("Probability of Outcome")+
ylab(TeX(r'(Simulated $\Delta$P)'))+
coord_cartesian(xlim=c(0,1))+
scale_color_discrete(name = expression(Delta~P))+
scale_shape_discrete(name = expression(Delta~P))+
theme_classic(base_size=12)+
facet_wrap(~resonance)
```