MATHEMATICAL PROOF

LECTURE 1 JAN 7

1A

Mathematical Proofs (Question)

In mathematics, a proof is
- A. Similar to a scientific experiment. Hypothesis Data to support
- B. A preponderance of evidence for the truth of a statement.
- C. A plausible argument that a statement is true.
- D. An evaluation of a boolean-valued expression. {could be
- E. None of the above.

Plausibility isn't enough for a statement to be true

Writing Traditional Proofs (Question)

To learn how to write traditional proofs, you should
- A. Read proofs given in mathematics articles and textbooks.
- B. Write proofs of facts you already know.
- C. Translate formal proofs into traditional proofs.
- D. Do exercises that ask for a proof.

Mathematical Proof ? argument showing that a conclusion follows from a set of premises

Theorum statement with proof

Conjecture could be true, no proof to supportge it

- Proofs are unique to math, theyre means of establishing truth. □.
  - ↳ communicate ideas, vehicles
  - ↳ certify a result is correct
  - ↳ Discovery through proofs
  - ↳ 'univerality' true no matter what

Other fields establish truth through other means i.e experiments

Styles of Mathematical Proof

- → description, describe the deduction
- → Prescription, Prescribe how to produce the proof; reasoning involved
- → two-col, left statements (intermediate), right reasons (justifications)
- → computation
- → construction
- → geometrically, rules of geometry (visual proof)
- → 'classical', non constructive reasoning, proof it exists w/o product
- → 'constructive' uses only constructive reasoning must have product

Traditional Proof Style:

- natural language (expression, ambiguous notation)
- may contain gaps (reader fills these)
- key ideas are understandable
- low-level details are computed or left for reader

good for communication
↳ organization
↳ discovery / beauty

- found in txtbooks, research papers

Formal Proof Style:

- clear rules, constrained
- described derivation or prescribed to create derivation
- Software can be implemented
- details must be correct
- not effective at communication but likely its correct

Proving a Conjunction (Question)

Which is not a valid way to prove $A \wedge B$?

A. Prove A and B separately.
B. Prove A and B together.
C. Prove A and then prove B assuming A.
D. Prove A assuming B and prove B assuming A.

C. is okay since we use A which is true to prove B

D. is invalid since your basing your proof on an unproven assumption.

METHODS OF PROOF FOR PROPOSITIONAL FORMULA

implication	$A \Rightarrow B$	to prove. We assume A to be true and use that to prove B
negation	$\neg A$	Assume A, derive a contridiction
conjunction	$A \wedge B$	Prove seperately or together, Prove A, Assuming A Prove B
disjunction	$A \vee B$	Prove one or the other, Assume $\neg A$ then prove B, vice versa
iff	$A \Leftrightarrow B$	Prove $A \Rightarrow B$, $B \Rightarrow A$

METHOD OF PROOF FOR QUANTIFIED FORMULAS

Universal stmnt $\forall x \in S \cdot A$

- Assume $x \in S$, Prove A
- $\exists x \in S \cdot \neg A$, derive a contradiction
- if S is finite, subs each member and prove A holds for each
- Induction

Existential stmnt $\exists x \in S \cdot A$

- for some 'a' $\in S$, $A[x := a]$
- Assume $\forall x \in S \cdot \neg A$, derive a contradiction

THEOREM TERMINOLOGY

theorem	statement result of proof, fundamental importance
axiom	theorem with assumed truth
lemma	theorem to help prove other theorems
corollary	theorem of fundamental importance
proposition	theorem that's obviously true

PROOF TERMINOLOGY

iff	logical equivalence $a \Leftrightarrow b$
obvious	no thinking
clearly	result is verified w/ little effort
trivial	extremely simple
straight-forward	every step is obvious
similar	same structure / techique

'brute force'	verify each case individually
'symmetric argument'	structure perserving transformation from another argument
'well-defined'	definition is fully given
'TFAE'	list of $\equiv$ statements
Without loss of generality	'WLOG', Prove special case is OK
'QED'	$\square$, $\blacksquare$ end of proof

DISCUSSION SESSION

deductive argument	if when A is true ⇒ B can be true
Mathematical Proofs depend on assumptions?	Some assumptions may not be true, but help us lead to truth
What does a m.proof say about the world?	We may apply a proof to the real world, and thus learn about what is true irl
Why gaps in trad. proofs?	B/c we don't want reader to get lost in details. We want to communicate idea of proof
Beautiful Proof.	Easier to remember, happier. Usually Better