Pre - Calculus

Complex Numbers (Lesson 1)

i = imaginary unit = backbone of complex number system

$i^2 = -1$ $\quad$ $i = \sqrt{-1}$ $\quad\quad$ $i = \sqrt{-1}$

$i^0 = 1$ $\quad\quad$ $i^2 = -1$

$i^1 = i$

$i^2 = -1$

$i^3 = i^2 \times i = -1 \cdot i = -i$

$i^4 = i^3 \cdot i = -i \cdot i = (-1)(i)(i) = 1$

$i^5 = i^4 \cdot i = 1 \cdot i = i$

$i^6 = i^5 \cdot i = i \cdot i = -1$

- pure imaginary numbers is when one takes multiple of the imaginary unit i

  $3i$, $i\sqrt{5}$, and $-12i$

  $(3i)^2 = 3^2 \cdot i^2 = 9 \cdot (-1) = \boxed{-9}$

  $(4i)^2 = 4^2 \cdot i^2 = 16(-1) = -16$

- pure imaginary numbers are the square roots of negative numbers

  $\sqrt{-9} = 3i$ (3 imaginary units)

  $\sqrt{-5} = i\sqrt{5}$

  $-\sqrt{-144} = -12i$ (12 imaginary units)

  for $a > 0$, $\sqrt{-a} = i\sqrt{a}$

$\sqrt{9} = 3$

$\sqrt{-18} = i\sqrt{18} = i\sqrt{9 \cdot 2} = i\sqrt{9} \cdot \sqrt{2} = i \cdot 3 \cdot \sqrt{2} = \boxed{3i\sqrt{2}}$

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ when $a, b \geq 0$

9 is a perfect square factor of 18

25

$5 \cdot 5$

$\sqrt{-25} = a > 0, \sqrt{-a} = i\sqrt{a}$

$= \sqrt{-1} \cdot \sqrt{25} = i\sqrt{25} = i \cdot 5 = 5i$

$\sqrt{-10} = i\sqrt{10} = i\sqrt{5 \cdot 2} = i\sqrt{5} \cdot \sqrt{2}$

$\sqrt{-24} = i\sqrt{24} = i\sqrt{6 \cdot 4} = i \cdot \sqrt{6} \cdot 2$

$2i\sqrt{6}$

$\sqrt{-21} = i\sqrt{14} =$

7 2

Simplifying roots of negative numbers

$\sqrt{-52} = i\sqrt{52} = i\sqrt{4 \cdot 13} = i\sqrt{4} \cdot \sqrt{13} = 2i\sqrt{13}$

2 26

2 13

$\sqrt{-80} = i\sqrt{80}$ $= i\sqrt{1\ldots}$

20 4

$\sqrt{-70} = \sqrt{(-1)(70)}$

70 14 5

35 2 7 2

$\sqrt{-37} = i\sqrt{37}$

100

10 10

5 2 5 2

$2i \cdot \sqrt{\ldots}$

$2i \cdot 2$

$4i$

$\sqrt{-70} = i\sqrt{70} = i\sqrt{10} \cdot \sqrt{7}$

$= i\sqrt{35} \cdot \sqrt{2}$

7 5

$\sqrt{-100} = i\sqrt{100}$

85 19

5 5

$-64 = i\sqrt{64}$

$\sqrt{-9} = i\sqrt{9} = i \cdot 3 = 3i$

$\sqrt{-44} = i\sqrt{44}$

11 4

$\sqrt{90} = i\sqrt{90}$ $i\sqrt{10} \cdot \sqrt{9}$

10 9

3 3

$i\sqrt{10} \cdot 3 = 3i\sqrt{10}$

$i\sqrt{11}\sqrt{4}$

$2i\sqrt{11}$

$i\sqrt{25 \cdot 4} = i\sqrt{25} \cdot \sqrt{4}$ $i \cdot$

$\cancel{8i\sqrt{3}}$ 2

Powers of imaginary unit

$i^3 = i^2 \cdot i = (-1)i = \boxed{-i}$

$i^4 = i^3 \cdot i = -i(i) =$

$= i^2 \cdot i^2 = (-1)(-1) = \boxed{1}$

$i^5 = i^4 \cdot i = \boxed{i}$

$i^6 = i^4 \cdot i^2 = (1)(-1) = \boxed{-1}$

$i^7 = i^4 \cdot i^3 = (1)(-i) = \boxed{-i}$

$i^8 = i^4 \cdot i^4 = (1)(1) = \boxed{1}$

i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8
i	-1	$-i$	1	i	-1	$-i$	1

$i \to -1 \to -i \to 1 \to \ldots$

$(i^4)^{75}$

$i^{301} = i^{300} \cdot i^1$

$= (1)(i)$

$i^{31} = i^{28} \cdot i^3$

$1 \cdot (-i) = -i$

i raised to a multiple of 4 is $\boxed{1}$

i^{17}

$i^{2\ldots} =$

$i^{227} = i^{224} \cdot i^3$

$(1) = \boxed{-i}$

i^{2016}

$i^{-1} = \frac{1}{i}$

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i as the principal root of -1

$i = \sqrt{-1}$ a and b cannot both be negative

$-1 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} \neq \sqrt{-1 \cdot -1} = \sqrt{1} = 1$

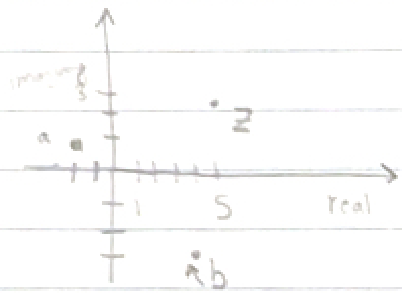
Intro to Complex Numbers

Real Numbers: $0, 1, 0.\overline{3}, \pi, e$

Imaginary Number: $i, -i, \pi i, ei$ $\qquad z = \underbrace{5 + 3i}_{\text{complex number}}$

$(i, -1, -i, 1)$

Complex Number: Real + Imaginary #



$a = -2 + 1i$

$b = 4 - 3i$

$$\underset{\substack{\uparrow \\ \text{real} \\ \text{part}}}{a} + \underset{\substack{\uparrow \\ \text{imaginary} \\ \text{part}}}{b}\, i$$

- There can be complex #'s that are neither real nor imaginary
  ie: $4 - 2i$
- 0 is a complex number $(0 + 0i)$
  real
  imaginary

$(-2 + 3i)$

$10.2 + 0i$

$0 + -17i$

$z = 77 - 27.8i$

Classifying Complex Numbers

$i, -1, -i, 1$

$$2 + 3i + 7i^2 + 5i^3 + 9i^4$$

$2 + 3i + (-7) + 5(-i) + 9$

$2 + 3i - 7 - 5i + 9$

$\boxed{4 - 2i}$

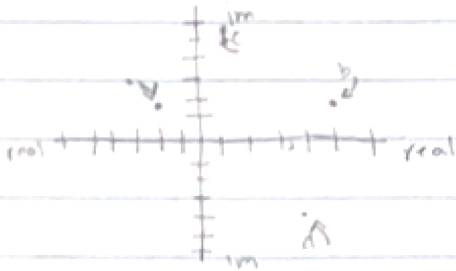
real $\quad$ imaginary

complex number

- any real number is a complex number
- any imaginary number is a complex number
- a complex number can also be a complex number

i, -1, -i, 1

Plotting Numbers on the Complex Plane



$\sqrt{2} = i\sqrt{2}$

$\boxed{\text{Unit test}}$

$\sqrt{-16} = i\sqrt{16} = i \cdot 4$

$4i$

$\sqrt{-27} \quad i \cdot \sqrt{27} = i\sqrt{3} \cdot \sqrt{9} = i\sqrt{3} \cdot 3$

$3 \quad 9 \qquad 3i\sqrt{3} \qquad = 3i\sqrt{3}$

Adding Complex Numbers

$(5 + 2i) + (3 - 7i)$ can only add like w/ like

$5 + 3 + 2i - 7i$ (real vs real i) (imaginary w/ imaginary)

$\boxed{8 - 5i}$

Subtracting Complex Numbers

$(2 - 3i) - (6 - 18i)$

$2 - 3i - 6 + 18i$

$-6 + 2 - 3i + 18$

$\boxed{-4 + 15i = 15i - 4}$

$(8 - i) - (-82 + 2i)$

$8 - i + 82 - 2i$

$90 - 3i$

$(9 + 52i) - (1 + 0i)$

$9 + 52i - 1 \cancel{- 0i}$

$8 + 52i$

$(-3 + 7i) + (9 + 0i)$

$-3 + 7i + 9 + \cancel{0i}$

$-3 + 9 + 7i$

$6 + 7i$

$(-13 + 16i) - (3 + 4i)$

$-13 + 16i - 3 - 4i$

$-16 + 12i$

$(14 + 60i) + (-30 + 2i)$

$14 + 60i - 30 + 2i$

$-30 + 14 + 62i$

$-16 + 62i$

Multiply Complex Numbers $\quad i, -1, -i, 1$

$-4(13+5i)$
$-52+(-20i)$
$-52-20i$

$2i(3-8i)$
$6i-16i^2$
$6i-16(-1)$
$\boxed{6i+16} \cdot \boxed{16+6i}$

$3(-2+10i)$
$-6+30i$

$-6i(5+7i)$
$-30i-42i^2$
$-30i+42$

Multiplying 2 complex numbers

$(1+4i)(5+i)$
$5+i+20i+4i^2$
$5+21i-4$
$1+21i$

$(1+2i)(3+i)$
$3+i+6i+2i^2$
$3+7i-2$
$1+7i$

$(4+i)(7-3i)$
$28-12i+7i+3i^2$
$28-5i+3$
$31-5i$

$(2-i)(2+i)$
$4+\cancel{2i}-\cancel{2i}-i^2$
$4+1=5$

$(-1)(i2)$
$(-1)(-1)$

$(1+i)(1+i)$
$1+i+i+i^2$
$\cancel{1}+2i+\cancel{(-1)}$

$(a-bi)(a+bi)$
$a^2 \cancel{+abi-abi} -bi^2$
a^2+bi^2
a^2+b^2

$-5i(5i-5)$
$-25i^2+25i$
$25+25i$

$(1+3i)^2 \cdot (2+i)$
$(1+3i)(1+3i)\cdot(2+i)$
$1+3i+3i+9i^2\cdot(2+i)$
$1+6i-9\cdot(2+i)$
$(-8+6i)(2+i)$
$-16-8i+12i+6i^2$
$-16+4i-6$
$-22+4i$

$-9i(6i+8)$
$-54i^2-72i$
$54-72i$

$-10(13i-9)$
$-130i+90$

$10(-6-9i)$
$-60-90i$

$(2-2i)(4-4i)$

$8-8i-8i+8i^2$

$8-16i+8i^2$

$\cancel{8}-16i\cancel{-8}$

$(-3+3i)(3-2i)$

$-9+6i+9i-6i^2$

$-9+15i+6$

$\boxed{-3+15i}$

$-6i(6-2i)$

$-36i+12i^2$

$-36i-12$

$(2-i)(-3+2i)$

$-6+4i+3i-2i^2$

$-6+7i+2$

$\boxed{-4+7i}$

$(1-2i)(4+i)$

$4+i-8i-2i^2$

$4-7i+2$

$\boxed{6-7i}$

$(1+i)(3-5i)$

$3-5i+3i-5i^2$

$3-2i+5$

$8-2i$

$(-2-3i)(-5-2$

$10+4i+15i+6i^2$

$10+19i-6$

$4+19i$

$(7-10i)-(3+30i)$

$7-10i-3-30i$

$4-40i$

$8(11i+2)$

$=88i+16$

Complex Roots (Solving quadratics)

$2x^2+5=6x$

$2x^2-6x+5=0$

$$\frac{-6\pm\sqrt{(-6)^2-4(2)(5)}}{2(2)}$$

$ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\frac{-6\pm\sqrt{36-40}}{2(2)}=\frac{-6\pm\sqrt{-4}}{4}=\frac{-6\pm i\sqrt{4}}{4}=\frac{-6\pm 2i}{4}$$

$$\frac{6+2i}{4} \qquad \frac{6-2i}{4}$$

$$\frac{6}{4}+\frac{2i}{4} \qquad \frac{6}{4}-\frac{2i}{4}$$

$(3+i)(3+i)$

$9+3i+3i+i^2$

$9+6i-1$ $8+6i$

$$\boxed{\frac{3\pm i}{2}}$$

$$\frac{\cancel{2}(3+i)}{\cancel{4}_2} \quad \boxed{\frac{(3+i)}{2}} \quad \boxed{\frac{(3-i)}{2}}$$

$$2\left(\frac{3+i}{2}\right)^2+5=\overset{3}{\cancel{6}}\left(\frac{3+i}{\cancel{2}_1}\right)$$

$$\frac{\overset{1}{\cancel{2}}\,(\overset{4}{\cancel{8}}+\overset{3}{\cancel{6}}i)}{\cancel{2}\,\cancel{4}}+\frac{5}{1}=9+3i$$

$4-3i+5=9-3i$

$$\boxed{\begin{array}{l} ax^2 + bx + c = 0 \\ \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{array}}$$

$-5x^2 + x + 2 = 0$

$a = -5$

$b = 1$

$c = 2$

$$\frac{-1 \pm \sqrt{(1)^2 - 4(-5)(2)}}{2(-5)} = \frac{-1 \pm \sqrt{1+40}}{-10} = \frac{-1 \pm \sqrt{41}}{-10} \quad \frac{-1}{-10} \pm \frac{\sqrt{41}}{10}$$

$$-5\left(\frac{-1 + \sqrt{41}}{-10}\right)^2 + \left(\frac{-1 + \sqrt{41}}{-10}\right) + 2 = 0$$

$14x^2 - 11x + 10 = 0$

$a = 14$

$b = -11$

$c = 10$

$$\frac{+11 \pm \sqrt{121 - 4(14)(10)}}{2(14)} = \frac{11 \pm \sqrt{121 - 560}}{28} = \frac{11 \pm \sqrt{-439}}{28}$$

$$\frac{11}{28} \pm \frac{i\sqrt{439}}{28}$$

$x^2 - 5x - 10 = 0$

$a = 1$

$b = -5$

$c = -10$

$$\frac{5 \pm \sqrt{25 - 4(1)(-10)}}{2(1)} = \frac{5 \pm \sqrt{25+40}}{2} = \frac{5 \pm \sqrt{65}}{2}$$

$-x^2 + 7x - 14 = 0$

$a = -1$

$b = 7$

$c = -14$

$$\frac{-7 \pm \sqrt{49 - 4(-1)(-14)}}{2(-1)} = \frac{-7 \pm \sqrt{49 - 56}}{-2} = \frac{-7 \pm \sqrt{-7}}{-2}$$

$$= \frac{\cancel{-}7 \pm i\sqrt{7}}{\cancel{-}2}$$

Unit Test

$15(-4i+2)$ $(-8)-(6+30i)$ $(-8+0i)-(6+30i)$

$-60i+30$ $-2+30i$ $-8-6$

$-8+0i-6+30i$

$-14+30i$

$11x^2-6x-2=0$

$a=11$
$b=-6$
$c=-2$

$$\frac{-(-6)\pm\sqrt{36-4(11)(-2)}}{2(11)}=\frac{6\pm\sqrt{36-88}}{22}=\frac{\overset{3}{\cancel{6}}\pm i\sqrt{52}}{\underset{11}{\cancel{22}}}$$

$(3-91i)+(67+0i)$

$70-91i$

$11i(-8+10i)$ $i,-1,-i,1$

$-88i+120i^2$

$-88i+120(-1)$ $8(11i+2)$ $9i(-4-7i)$

$-88i-120$ $88i+16$ $-36i+63\cancel{i^2}$

$15i(-i-1)$ $63-36i$

$4x^2+10x+13=0$ $-15\cancel{i^2}-15i$ $15(-4i+2)$

$-60i+30$

$a=4$
$b=10$
$c=13$

$$\frac{-10\pm\sqrt{100-(4(4)(13))}}{2(4)}=\frac{-10\pm\sqrt{100-208}}{8}=\frac{-10\sqrt{-108}}{8}$$

$$=\frac{\overset{5}{\cancel{-10}}\pm i\sqrt{108}}{\underset{4}{\cancel{8}}}=\frac{-5\pm i\sqrt{}}{4}$$

$(1+5i)\cdot(-3-i)$ $\pm\sqrt{-38}=i\sqrt{38}$

$-3-i-15i+5\cancel{i^2}$

$-3-16i+5=2-16i$

$\sqrt{108}$
$3 \quad 36$

$$=\frac{-5\pm i\sqrt{3}\cdot\sqrt{}}{}$$

$$\frac{\overset{5}{\cancel{-10}}\pm\overset{3}{\cancel{6}}i\sqrt{3}}{\underset{4}{\cancel{8}}}$$

$(-7-10i)-(3+30i)$

$-10-40i$

$$=\frac{-5\pm3i\sqrt{3}}{4}$$

$(1+2i)(1-4i)$

$1-4i+2i+8i^2$

$1-2i+8 = 9-2i$

$i, -1, -i, 1$

$-20(2i-7)$

$-40i+140$

$\sqrt{-18}$ $\quad i\sqrt{18} = i\sqrt{2}\cdot\sqrt{9}$

$9\ 2 \quad 3i\sqrt{2}$

$(-21+65i)+(-30+12i)$

$-21+65i-30+12i$

$-51+77i$

$3x^2-2x+7=0$

$a=3$

$b=-2$

$c=7$

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-2)\pm\sqrt{4-(4(3)(7))}}{2(3)} = \frac{2\pm\sqrt{4-84}}{6}$$

$$= \frac{2\pm\sqrt{-80}}{6} = \frac{2\pm i\sqrt{80}}{6}$$

$$= \frac{2\pm i\sqrt{5}\cdot\sqrt{4}}{6}$$

$$= \frac{2\pm i\sqrt{5}\cdot 2}{6} = \frac{\cancel{2}^{1}\pm \cancel{2}^{1}i\sqrt{5}}{{}_{3}\cancel{6}}$$

$$= \frac{1\pm i\sqrt{5}}{3}$$

80

$10\ 2$

$5\ 2$

$2x^2+3x-11=0$

$a=2$

$b=3$

$c=-11$

$$\frac{-3\pm\sqrt{9-(4(2)(-11))}}{2(2)}$$

$$\frac{-3\pm\sqrt{9+88}}{4} = \frac{-3\pm\sqrt{97}}{4}$$

Adding Polynomials

$(5x^2 + 8x - 3) + (2x^2 - 7x + 13x)$

$\cancel{5x^2} + \cancel{8x} - 3 + \cancel{2x^2} - \cancel{7x} + \cancel{13x}$

$\boxed{7x^2 + 14x - 3}$

$(5h^3 - 2h) + (-2h^3 - h^2 - 2h)$

$5h^3 - 8h \quad \cancel{2h^3}\ \cancel{h^2} - 2h$

$3h^3 - h^2 - 10h$

$(-5n^4 + 8) + (5n^4 + 8n^3 + 3n^2)$

$\cancel{-5n^4} + 8 + \cancel{5n^4} + 8n^3 + 3n^2$

$0n^4 + \boxed{8n^3 + 3n^2 + 8}$

$(-4b^3 + b - 1) + (6b - 6)$

$\cancel{4b^3}\ \cancel{6b} - 1 + \cancel{6b} - 6$

$\boxed{-4b^3 + 7b - 7}$

$(7a^2 - 2a - 2) + (-5a + 3)$

$\cancel{7a^2} - 2a - 2 - 5a + 3$

$\boxed{7a^2 - 7a + 1}$

Subtract Polynomials

$(6x + 14) - (3x^2 + x - 9)$

$(6x + 4) + -1(3x^2 + x - 9)$

$\cancel{6x} + 4 - 3x^2 - x - 9$

$\boxed{-3x^2 + 15x + 23}$

$(d^2 + 6d + 9) - (d^2 + 6d + 9)$

$d^2 + \cancel{6d} + 9 - d^2 - \cancel{6d} - 9$

$\boxed{d^2 + d^2}$

(continued)

$(5b^3 + 9b + 4) - (9b - 4)$

$5b^3 + \cancel{9b} + 4 - \cancel{9b} + 4$

$\boxed{5b^3 + 8}$

$(3x^3 + 4x^2) - (2x^3 + 3x^2 - x)$

$\cancel{3x^3} + \cancel{4x^2} - \cancel{2x^3} - \cancel{3x^2} + x$

$x^3 + x^2 + x$

$(w^3 + 8w^2 - 3w) - (4w^2 + 5w - 7)$

$w^3 + \cancel{8w^2} - \cancel{3w} - \cancel{4w^2} - \cancel{5w} + 7$

$\boxed{-w^3 + 4w^2 - 8w + 7}$

Polynomial Subtract

Subtract $2x^2 + 4x - 1$ from $6x^2 + 3x - 9$

$(6x^2 + 3x - 9) - (2x^2 + 4x - 1)$

$\cancel{6x^2}\ \cancel{3x} - 9 + \cancel{2x^2} - 4x + 1$

$\boxed{8x^2 - x - 8}$

$P = -4b^2 + 6b - 9 \quad (P - Q)$

$Q = 7b^2 - 2b - 5$

$(-4b^2 + 6b - 9) - (7b^2 - 2b - 5)$

$4b^2 + \cancel{6b} - 9 - 7b^2 + 2b + 5$

$\boxed{-11b^2 + 8b - 4}$

$(b^2 + 8b - 9) - (11b^2 - 4b + 7)$

$b^2 + \cancel{8b} - 9 - \cancel{11b^2} + \cancel{4b} - 7$

$-10b^2 + 12b - 16$

(D - C)

$D = -7y^2 + 3y - 6$

$C = 3y^2 + 4y + 4$

$(-7y^2 + 3y - 6) - (3y^2 + 4y + 4)$

$-\cancel{7y^2} + \cancel{3y} - 6 - \cancel{3y^2} - \cancel{4y} - 4$

$-10y^2 - y - 10$

$3x^2+7x-4$ from $8x^2-6x+2$

$(8x^2-6x+2)-(3x^2+7x-4)$

$\cancel{8x^2}-6x+2-\cancel{3x^2}-7x+4$

$5x^2-13x+6$

$\cancel{7w^2}+\cancel{4w}-6+\cancel{w^2}-\cancel{11w}+13$

$8w^2-7w+7$

$\cancel{2y^2}+7y+11-\cancel{8y^2}+5y-7$

$-6y^2+12y+4$

$(6c^2-2c-1)+(-4c^2+7c+5)$

$\cancel{6c^2}-\cancel{2c}-1-\cancel{4c^2}+\cancel{7c}+5$

$\boxed{2c^2+5c+4}$

$(6x^2+3x-9)-(-2x^2+4x-1)$

$\cancel{6x^2}+\cancel{3x}-9+\cancel{2x^2}-\cancel{4x}+1$

$8x^2-x-8$

$(-3y^2-5y-2)+(-7y^2+5y+2)$

$-\cancel{3y^2}-\cancel{5y}-\cancel{2}-\cancel{7y^2}+\cancel{5y}+\cancel{2}$

$-10y^2$

$(f^3-5f+25)-(4f^2-12f+9)$

$\cancel{f^3}-5f+25-\cancel{4f^2}+12f-9$

$f^3-4f^2+7f+16$

$\cancel{3t^2}-\cancel{5t}+6-\cancel{8t^2}-\cancel{7t}-9$

$\boxed{-5t^2+2t-3}$

$(p^2-5p+4)-(4p^2-9p+11)$

$\cancel{p^2}-\cancel{5p}+4-\cancel{4p^2}+\cancel{9p}-11$

$-3p^2+4p-7$

$-\cancel{8r^2}+\cancel{11r}-6-\cancel{7r^2}-\cancel{9r}+14$

$-15r^2+2r+8$

$\cancel{5a^2}-\cancel{6a}-4+\cancel{7a^2}-\cancel{3a}+9$

$\boxed{12a^2-9a+5}$

$-\cancel{3a^2}+\cancel{2a}-5+\cancel{2a^2}-\cancel{a}-6$

$\boxed{-a^2+a-11}$

$N=-3a^2+2a-5$

$T=-2a^2+a+6$

$N-T=-3a^2+2a-5-(-2a^2+a+6)$

$-\cancel{3a^2}+\cancel{2a}-5+\cancel{2a^2}-\cancel{a}-6$

$\boxed{-1a^2+a-11}$

$(6y^2+2y+5)-(5y^2-6y-11)$

$\cancel{6y^2}+\cancel{2y}+5-\cancel{5y^2}+\cancel{6y}+11$

$\boxed{y^2+8y+16}$

$A=7x^2-3x+10$

$B=-4x^2+6x-4$

$A-B=(7x^2-3x+10)-(-4x^2+6x-4)$

$\cancel{7x^2}-\cancel{3x}+10+\cancel{4x^2}-\cancel{6x}+4$

$\boxed{11x^2-9x+14}$

$(10a^2+3a+25)-(9a^2-6a+5)$

$\cancel{10a^2}+\cancel{3a}+25-\cancel{9a^2}+\cancel{6a}-5$

$\boxed{a^2+9a+20}$

$(2y^2+7y+11)-(8y^2-5y+7)$

$\cancel{2y^2}+\cancel{7y}+11-\cancel{8y^2}+\cancel{5y}-7$

$\boxed{-6y^2+12y+4}$

$E=6c^2-2c-1$

$F=-4c^2+7c+5$

$E+F=\cancel{6c^2}-\cancel{2c}-\cancel{1}-\cancel{4c^2}+\cancel{7c}+\cancel{5}$

$2c^2+5c+4$

Multiplying Binomials by Polynomials

$(y^2 - 6y)(3y^2 - 2y + 1)$

$3y^4 - 2y^3 + y^2 - 18y^3 + 12y^2 - 6y$

$\boxed{3y^4 - 20y^3 + 13y^2 - 6y}$

$(x+4)(x^2+3x+2)$

$x^3 + 3x^2 + 2x + 4x^2 + 12x + 8$

$\boxed{x^3 + 7x^2 + 14x + 8}$

$(w^2+2)(w^2+3w+9)$

$w^4 + 3w^3 + 9w^2 + 2w^2 + 6w + 18$

$\boxed{w^4 + 3w^3 + 11w^2 + 6w + 18}$

$(2y^3+5)(y^3+6y)$

$2y^6 + 12y^4 + 5y^3 + 30y$

$\boxed{2y^6 + 12y^4 + 5y^3 + 30y}$

$(2y^3+5)(y^3+6y)$

$2y^6 + 12y^4 + 5y^3 + 30y$

$(b^3+b^2)(b^2+7b+4)$

$b^5 + 7b^4 + 4b^3 + 1b^4 + 7b^3 + 4b^2$

$b^5 + 8b^4 + 15b^3$

$b^5 + 7b^4 + 4b^3 + b^4 + 7b^3 + 4b^2$

$b^5 + 8b^4 + 11b^3 + 4b^2$

$(2t+1)(t^2+7t+6)$

$2t^3 + 14t^2 + 12t + t^2 + 7t + 6$

$2t^3 + 15t^2 + 19t + 6$

$(3p^2+1)(p^3+4)$

$3p^5 + 12p^2 + p^3 + 4$

$3p^5 + p^3 + 12p^2 + 4$

$(a^2+3)(a^2+2a+5)$

$a^4 + 2a^3 + 5a^2 + 3a^2 + 6a + 15$

$\boxed{a^4 + 2a^3 + 8a^2 + 6a + 15}$

$(n^3+5n^2)(n^3+4n^2+3n)$

$n^6 + 4n^5 + 3n^4 + 5n^5 + 20n^4 + 15n^3$

$\boxed{n^6 + 9n^5 + 23n^4 + 15n^3}$

$(10a-3)(5a^2+7a-1)$

$50a^3 + 70a^2 - 10a - 15a^2 - 21a + 3$

$\boxed{50a^3 + 55a^2 - 21a + 3}$

$(2+3w)(w^2+6w+9)$

$2w^2 + 12w + 18 + 3w^3 + 18w^2 + 27w$

$\boxed{3w^3 + 20w^2 + 39w + 18}$

$(x^2+4)(x^2-6)$

$x^4 - 6x^2 + 4x^2 - 24$

$\boxed{x^4 - 2x^2 - 24}$

$(q^2-5)(q^2-2q+5)$

$q^4 - 2q^3 + 5q^2 - 5q^2 + 10q - 25$

$\boxed{q^4 - 2q^3 + 10q - 25}$

$(1+x)(x^2-5x-6)$

$x^2 - 5x - 6 + x^3 - 5x^2 - 6x$

$\boxed{-4x^2 + x^3 - 11x - 6}$

$(c^2-6)(2c^2+3c-1)$

$2c^4 + 3c^3 - c^2 - 12c^2 - 18c + 6$

$2c^4 + 3c^3 - 13c^2 - 18c + 6$

$a^2 + 2ab + b^2$

Polynomial Special Products: difference of squares

$(x+y)(x-y) = (x-y)^2$

$x^2 - xy + xy - y^2$

$= \boxed{x^2 - y^2}$

$(3+5x^4)(3-5x^4)$

$\boxed{9 - 25x^8}$

$(3y^2+2y^3)(3y^2-2y^3)$

$\boxed{= 9y^4 - 4y^6}$

$(8n^2+3n^7)(8n^2-3n^7)$

$64n^4 - 9n^{14}$

$(4b^2+3)(4b^2-3)$

$16b^4 - 9$

$(8-n^7)(8+n^7)$

$64 - n^{14}$

$(a+b)^2 = (a+b)(a+b) =$

$a^2 + ba + ba + b^2$

$\boxed{= a^2 + 2ba + b^2}$

$(5x^6+4)^2 = (5x^6+4)(5x^6+4)$

$= 25x^{12} + 20x^6 + 20x^6 + 16$

$= 25x^{12} + 40x^6 + 16$

$25x^{12} + 2(5x^6)(4) + 16$

$25x^2 + 20x^6 + 16$

$25x^2 + 40x^6 + 16$

$(3t^2 - 7t^6)^2$

$= 9t^4 - 49t^{12}$

$(3t^2 + 7t^6)^2$

$= 9t^4 + 2(3t^2 \cdot 7t^6) + 49t^{12}$

$= 9t^4 + 2(21t^8) + 49t^{12}$

$(3t^2 - 7t^6)^2$

$9t^2 + 2(3t^2 \cdot -7t^6) + (7t^6)^2$

$9t^2 + 2(-21t^8) + 49t^{12}$

$9t^2 - 42t^8 + 49t^{12}$

$(4d^2 - 2d^7)^2$

$= 16d^4 + 2(4d^2 \cdot (-2d^7)) + (-2d^7)^2$

$16d^4 + 2(-8d^9) + 4d^{14}$

$16d^4 - 16d^9 + 4d^{14}$

$(4d^2 - 2d^7)(4d^2 + 2d^7)$

$16d^4 - 8d^9 - 8d^9 - 4d^{14}$

$16d^4 - 16d^9 - 4d^{14}$

$(2a^6 - 6a^3)^2$

$4a^{12} + 2(2a^6 \cdot (-6a^3)) + (-6a^3)^2$

$4a^{12} + 2(-12a^9) + 36a^6$

$4a^{12} - 24a^9 + 36a^6$

$(9n^7 - 1)^2$

$81n^{14} + 2(9n^7 \cdot -1) + (-1)^2$

$81n^{14} - 2(-9n^7) + 1$

$81n^{14} - 18n^7 + 1$

$(8x^4 + 1)^2$

$64x^8 + 2(8x^4 \cdot 1) + 1^2$

$64x^8 + 2(8x^4) + 1$

$64x^8 + 16x^4 + 1$

$(3z^5 + 7z^2)^2$

$9z^{10} + 2(3z^5 \cdot 7z^2) + (7z^2)^2$

$9z^{10} + 2(21z^7) + 49z^4$

$a^2 + 2ab + b^2$

Polynomial Special Products : difference of squares

$(x+y)(x-y) \cdot (x-y)^2$

$x^2 - xy + xy - y^2$

$= \boxed{x^2 - y^2}$

$(3+5x^4)(3-5x^4)$

$\boxed{9 - 25x^8}$

$(3y^2 + 2y^3)(3y^2 - 2y^5)$

$\boxed{= 9y^4 - 4y^{10}}$

$(8n^2 + 3n^7)(8n^2 - 3n^7)$

$64n^4 - 9n^{14}$

$(4b^2 + 3)(4b^2 - 3)$

$16b^4 - 9$

$(8 - n^7)(8 + n^7)$

$64 - n^{14}$

$(a+b)^2 = (a+b)(a+b) =$

$a^2 + ba + ba + b^2$

$\boxed{= a^2 + 2ba + b^2}$

$(5x^6 + 4)^2 = (5x^6 + 4)(5x^6 + 4)$

$= 25x^{12} + 20x^6 - 20x^6 + 16$

$= 25x^{12} + 40x^6 + 16$

$25x^{12} + 2(5x^6)(4) + 16$

$25x^2 + 20x^6 + 16$

$25x^2 + 40x^6 + 16$

$(3t^2 - 7t^6)^2$

$= 9t^4 - 49t^{12}$

$(3t^2 + 7t^6)^2$

$= 9t^4 + 2(3t^2 \cdot 7t^6) + 49t^{12}$

$= 9t^4 + 2(21t^8) + 49t^{12}$

$(3t^2 - 7t^6)^2$

$9t^2 + 2(3t^2 \cdot -7t^6) + (7t^6)^2$

$9t^2 + 2(-21t^8) + 49t^{12}$

$9t^2 - 42t^8 + 49t^{12}$

$(4d^2 - 2d^7)^2$

$= 16d^4 + 2(4d^2 \cdot (-2d^7)) + (-2d^7)^2$

$16d^4 + 2(-8d^9) + 4d^{14}$

$16d^4 - 16d^9 + 4d^{14}$

$(4d^2 - 2d^7)(4d^2 + 2d^7)$

$16d^4 - 8d^9 - 8d^9 - 4d^{14}$

$16d^4 - 16d^9 - 4d^{14}$

$(2a^6 - 6a^3)^2$

$4a^{12} + 2(2a^6 \cdot (-6a^3)) + (-6a^3)^2$

$4a^{12} + 2(-12a^9) + 36a^6$

$4a^{12} - 24a^9 + 36a^6$

$(9n^7 - 1)^2$

$81n^{14} + 2(9n^7 \cdot -1) + (-1)^2$

$81n^{14} - 2(-9n^7) + 1$

$81n^{14} - 18n^7 + 1$

$(8x^4 + 1)^2$

$64x^8 + 2(8x^4 \cdot 1) + 1^2$

$64x^8 + 2(8x^4) + 1$

$64x^8 + 16x^4 + 1$

$(3z^5 + 7z^2)^2$

$9z^{10} + 2(3z^5 \cdot 7z^2) + (7z^2)^2$

$9z^{10} + 2(21z^7) + 49z^4$

$(9n^7-1)^2$

$81n^{14}+2(9n^7\cdot(-1))+(-1)^2$

$81n^{14}+2(-9n^7)+(1)$

$81n^{14}-18n^7+1$

$(5a^3-6a^2)(5a^3+6a^2)$

$\boxed{25a^6-36a^4}$

$25a^6+\cancel{30a^5}\cancel{-30a^5}-36a^4$

$(y^2+6)(-2y^2+7)$

$-2y^4+7y^2-12y^2+42$

$\boxed{-2y^4-5y^2+42}$

$(7m^6+6)(7m^6-6)$

$49m^{12}-36$

$(8x^4+1)^2$

$64x^8+2(8x^4\cdot 1)+1^2$

$64x^8+2(8x^4)+1$

$64x^8+16x^4+1$

Factoring Polynomials

$(6x^2+9x)(x^2-4x+4)$

$3x(2x+3)(x^2-4x+4)$

$3x(2x+3)(x-2)(x+2)$

$x^3-4x^2+6x-24$

$x(x^2-4x+6-24)$

$x(x^2-4x-18)$

$\boxed{x(x-6)(x+3)}$

$x^3-4x^2+6x-24$

$x^2(x-4)+6(x-4)$

$\boxed{(x-4)(x^2+6)}$

$16x^3+24x^2+9x$

$4x^2(4x+6)+9x$

$x(16x^2+24x+9)$

$x\ (4x+3)(4x+3)$

$\boxed{x(4x+3)^2}$

$(x^2-4)(x^2+6x+9)$

$(x-2)(x+2)(x+3)(x+3)$

$(x-2)^2(x+3)^2$

x^5-x^4+3x-3

$x^4(x-1)+3(x-1)$

$(x-1)(x^4+3)$

$(x+3)(x-2)$

$x^2-2x+3x-6$

x^2+x-6

x^2-4x+4

$(x-2)(x-2)$

$x^2-2x-2x+4$

x^2-4x+4

$5x^4-20x^3+20x^2$

$5x^2(x^2-4x+4)$

$(x^2+x-6)(2x^2+4x)$

$(x+3)(x-2)\,2x(x+2)$

$(x-2)^2$

$x^2-2(x\cdot(-2))+2^2$

$x^2-2(-2x)+4$

x^2-4x+4

$x^4+5x^3+4x+20$

$x^3(x+5)+4(x+5)$

$(x+5)(x^3+4)$

Identifying Quadratic Patterns

$36x^4 - (y+3)^2$

$36x^4 - y^2 + 2(y)(3) + 9$

$36x^4 - y^2 + 6y + 9$

$(6x^2 - 1)(y+3)^2$

$u^2 \cdot v^2 \quad u = 6x^2$

$36x^4 - (y+3 \quad v = y+3$

$x^4 + 9$

$(x^2+3)^2$

$3x^2 - 16y^5$

$x^4 + 3x^2y + 9y^2$

$x^2 \qquad 3y$

$(u+v)^2$

$(x^2+3y)(x^2+3y)$

$x^4 + 3x^2y + 3x^2y + 9y^2$

$x^4 + 6x^2y + 9y^2$

$(u-v)^2$

$(x^2-3y)(x^2-3y)$

$x^4 - 3x^2y - 3x^2y + 9y^2$

$x^4 - 6x^2y + 9y^2$

$(x^2-3y)(x^2-3y)$

$x^4 + 3x^2y - 3$

$x^4 - 14x^2 + 49$

$(x^2-7)(x^2-7)$

$x^4 - 7x^2 - 7x^2 + 49$

$u = (x+1)$

$(x+1)^2 - 4y^2 \quad v = -2y$

$(x+7)(x+7)$

$x^2 + 7x + 7x + 49$

$(x+7)^2 + 2y^2(x+7) + y^4$

$(x+7)^2 + 2y^2x + 14y^2 + y^4$

$\boxed{x^2 + 14x + 49 + 2y^2x + 14y^2 + y^4}$

$(u+v)^2$

$u = \boxed{(x+7)} \qquad = u^2 + 2(u \cdot v) + v^2$

$v = \boxed{y^2} \qquad 2((x+7) \cdot y^2)$

$= 2(y^2x + 7y^2)$

$= 2y^2x + 14y^2$

$2y^2(x+7)$

$(x+7)^2 + 2((x+7) \cdot y^2) + y^4$

$(x+7)^2 + 2y^2(x+7) + y^4$

$\boxed{x^2 + 14x + 49 + 2y^2x + 14y^2 + y^4}$

$(4x^2 - 9y^6)$

$(u+v)(u-v) = u^2 - v^2$

$u = 2x \quad \boxed{(2x+3y^3)(2x-3y^3)}$

$v = 3y^3$

$(u+v)^2 = u^2 + 2uv + v^2$

$(x-5)^2 + 2y^3(x-5) + y^6$

$u = (x-5) \qquad ((x-5) + y^3)^2$

$v = y^3$

$(u-v)^2 = u^2 + 2uv + v^2$

$16x^2 - 16xy^3 + 4y^6$

$u = 4x$

$v = 2y^3$

$i, -1, -i, 1$

$(u+v)(u-v) = u^2 + v^2$

$4x^4 - 9y^4 =$

$u = 2x^2$

$y = 3y^2$

$9x^4 - 6x^2y^4 + y^8$

$(u-v)^2 = (u-v)(u-v) = u^2 + 2uv + v^2$

$u = 3x^2$ $\quad (3x^2)^2 + 2(3x^2 \cdot -y^4) + (y^4)^2$

$v = y^4$ $\quad 9x^4 + 2(-3x^2y^4) + y^8$

$\boxed{9x^4 - 6x^2y^4 + y^8}$

$9x^2 - 4y^6$

$(u-v)(u-v) = \boxed{u^2 - v^2}$

$u = 3x$

$v = 2y^3$

$25x^4 - 30x^2 + 9$

$5x^2(5x^2 - 6) + 9$ $\quad 2(u \cdot v)$

$2(5x \cdot 3)$

$(5x)^2 \quad (3)$ $\quad 2(15x^2) = 30x^2$

$\boxed{(5x^2 - 3)(5x^2 - 3)}$

$x^2 - 49y^2$ $\quad (u+v)(u-v) = u^2 - v^2$

$u = x$ $\quad x^2 - 49y$

$v = 7y$ $\quad (x+7y)(x-7y)$

$36c^2 - 121d^2 = (u+v)(u-v)$

$u = 6c$ $\quad (6c+11d)(6c-11d)$

$v = 11d$

$9m^2 + 30mn + 25n^2$

$(u+v)^2 = u^2 + 2uv + v^2$

$u = 3m$

$v = 5n$ $\quad (3m + 5n)^2$

$9m^2 + 2(3m \cdot 5n) + 25n^2$

$9m^2 + 2(15mn) + 25n^2$

$\boxed{9m^2 + 30mn + 25n^2}$

$64y^6 - 48y^3 + 9$

$(u-v)^2 = u^2 + 2uv + v^2$

$8y^3 = u$ $\quad 64y^6 + 2(-24y^3) + 9$

$-3 = v$ $\quad \boxed{64y^6 - 48y^3 + 9}$

$16x^6 - 81$ $\quad (u+v)(u-v)$

$u = 4x^3$ $\quad (4x^3 + 9)(4x^3 - 9)$

$v = 9$

Zero's of polynomial Introduction

$p(x) = (x-1)(x+2)(x-3)(x+4)$

$p(x) = 0?$ $(x-1)(x+2)(x-3)(x+4) = 0$

$\boxed{1, -2, 3, -4}$ will give us 0

$p(1) = 0$

$p(-2) = 0$

$p(3) = 0$

$p(-4) = 0$

this lets us know at what points our function crosses the x-axis

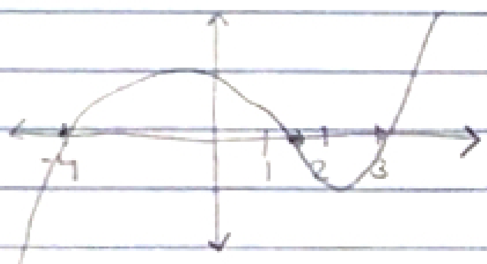
Shows us what to expect

Zero Polynomials: plotting zero

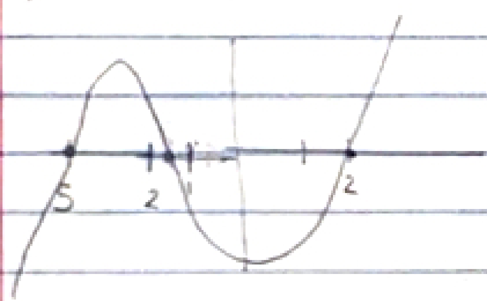
- gives us clarity where or what points where the x-axis is crossed

$p(x) = 2x(2x+3)(x-2)$ $\qquad 2x+3=0$
$\quad = (2x^2+6x)(x-2)$ $\qquad \frac{2x}{2} = \frac{-3}{2}$
$p(2) = 0$ $\qquad x = -\frac{3}{2}$
$p(0) = 0$
$p(-\frac{3}{2}) = 0$

Zeros of Polynomials: matching equation to graph



$p(x) = (x+4)(2x-3)(x-3)$



$p(x) = (x+5)(2x+3)(x-2)$

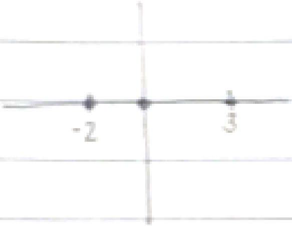
Zeros of Polynomials (w/ factoring): grouping

$p(x) = x^3 + x^2 - 9x - 9$
$x^2(x+1) - 9(x+1) = 0 \quad p(x) = 0, -1$
$(x+1)(x^2-9)$
$(x+1)(x-3)(x+3) = \boxed{p(x) = (-1, 3, -3)}$

Zeros of polynomials (w/ factoring): common factor

$p(x) = 5x^3 + 5x^2 - 30x$
$5x(x^2 + x - 6) \qquad p(x) = \boxed{-3, 0, 1}$
$5x(x+3)(x-2)$

$p(x) = 3x^3 - 3x^2 - 18x$



$3x(x^2 - x - 6) = 0$ Common number

$3x(x-3)(x+2) = 0$

$0 \quad 3 \quad -2$

$p(x) = (2x^2 + 7x + 5)(x-3)$

$(2x+5)(x+1)(x-3)$

factoring

$-\frac{5}{2} \quad -1 \quad 3$

$(u-v)(u+v)$

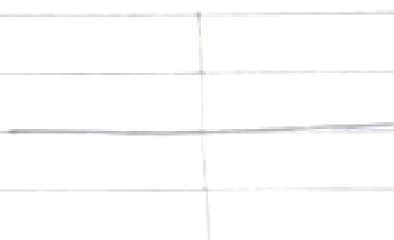
$u^2 - v^2$

$p(x) = (x^2 - 1)(x^2 - 5x + 6)$

$(x+1)(x-1)(x-2)(x-3)$

$-1, \ 1, \ 2, \ 3$

$p(x) = \underbrace{2x^3 + 5x^2}\ \underbrace{-2x - 5}$



grouping

$(2x^3 + 5x^2)(-2x - 5)$

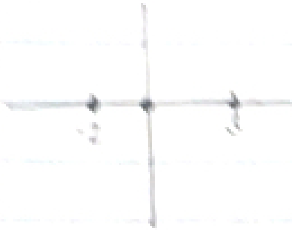
$x^2(2x+5)(-2x-5)$

$0, \frac{5}{2}, -\frac{5}{2} \quad u^2 - v^2 \quad (u-v)(u+v)$

$(2x+5)(x^2-1)$

$-\frac{5}{2} \quad (x+1)(x-1)$

$p(x) = 3x^3 - 3x^2 - 18x$



$3x(x^2 - x - 6) = 0$

$3x(x-3)(x+2) = 0$

$0 \quad 3 \quad -2$

(common number)

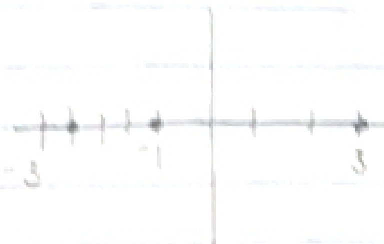
$p(x) = (2x^2 + 7x + 5)(x-3)$

$(2x+5)(x+1)(x-3)$

$-\frac{5}{2} \quad -1 \quad 3$

$(u-v)(u+v)$

$u^2 - v^2$

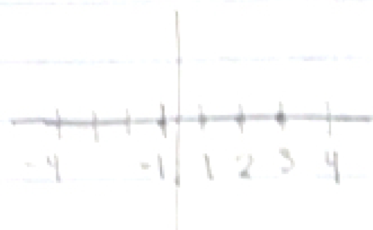


(factoring)

$p(x) = (x^2-1)(x^2-5x+6)$

$(x+1)(x-1)(x-2)(x-3)$

$-1, 1, 2, 3$



$p(x) = \underbrace{2x^3 + 5x^2}\ \underbrace{-2x-5}$

(grouping)

$(2x^3 + 5x^2)(-2x-5)$

$x^2(2x+5)(-2x-5)$

$0, \frac{5}{2}, -\frac{5}{2} \quad u^2 - v^2 \quad (u-v)(u+v)$

$(2x+5)(x^2-1)$

$-\frac{5}{2} \quad (x+1)(x-1)$

Polynomial Division

$$\frac{x^2+3x+2}{x+1} = \frac{(x+2)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$= \boxed{x+2} \quad \boxed{x \neq -1}$$

Polynomial Long Division?

$$\begin{array}{r} \boxed{x+2} \\ x+1 \,\big|\, x^2+3x+2 \\ -x^2 \;\; -x \downarrow \\ 0 \;\; 2x+2 \end{array}$$

$$\frac{x^2+5x+6}{x+2}$$

$$\frac{\cancel{(x+3)(x-2)}}{\cancel{x+2}}$$

$$\begin{array}{r} x+5 \\ x+2 \,\big|\, x^2+5x+6 \\ -(x^2+2x \downarrow) \\ 0 \;\; 3x+6 \\ -(5x \quad) \\ -2x \end{array}$$

$$\frac{x^4-2x^3+5x}{x} = \frac{\cancel{x}(x^3-2x^2+5)}{\cancel{x}}$$

$$\boxed{= x^3-2x^2+5} \neq 0$$

$$\frac{3x^5+x^4-4x^2}{x} = \cancel{x^2}\boxed{(3x^4+x^3-4x)}$$

$$x(3x^3+x^2-4)$$

$$x(x^3-2x^2+5)$$

$$\frac{3x^5+x^4-4x^2}{x}$$

$$\frac{\cancel{x}(3x^4+x^3-4x)}{\cancel{x}}$$

$$\frac{2x^4+4x^3-x^2}{x} = \frac{\cancel{x}(2x^3+4x^2-x)}{\cancel{x}}$$

$$2x^3+4x^2-x$$

$$\frac{x^5-3x^2+2x}{x} = \frac{\cancel{x(x^4-3x+2)}}{\cancel{x}}$$

$$\frac{x^4+5x^3+3x^2}{x} = \frac{\cancel{x(x^3+5x^2+3x)}}{\cancel{x}}$$

$$\frac{2x^4-3x}{x} = \frac{\cancel{x}(2x^3-3)}{\cancel{x}}$$

$$\frac{1}{x}(x^4-2x^3+5x)$$

$$\frac{x^4}{x} - \frac{2x^3}{x} + \frac{5x}{x}$$

$$x^3-2x^2+5$$

Divide Polynomials by x (w/ remainders)

$\frac{18x^4 - 3x^2 + 6x - 4}{6x}$

$\frac{3x(6x^3 - 1) + (6x - 4)}{6x}$

$\frac{6x^3 - x + 6x + 4}{2x} = \frac{2x(3x^2 + 3) + (6x + 1)}{2x}$

$3x^2 + 6x^4 + 7$

$\frac{18x^4}{6x} - \frac{3x^2}{6x} + \frac{6x}{6x} - \frac{4}{6x}$

$3x^3 - \frac{1}{2}x + 1 - \frac{2}{3} \cdot \frac{1}{x} = \frac{2}{3x} = \frac{2}{3}x^{-1}$

$\boxed{3x^3 - \frac{1}{2}x + 1 - \frac{2}{3x}}$

$\frac{6x^2 - 4x - 3}{x}$

$\frac{6x^2}{x} - \frac{4x}{1x} - \frac{3}{1x}$

$6x - 4 - \frac{3}{x}$

$\frac{x^4 + 2x^2 - 5}{x}$

$\frac{x^4}{x} + \frac{2x^2}{x} - \frac{5}{x} = x^3 + 2x - \frac{5}{x}x^{-1}$

$\frac{3x^3}{x} \frac{-x}{x} \frac{-2}{x} = 3x^2 - 1 - 2x^{-1}$

$\frac{6x^5 - 2x^4 - 1}{x} = \frac{6x^5}{x} - \frac{2x^4}{x} - \frac{1}{x}$

$= 6x^4 - 2x^3 - 1x^{-1}$

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Long Division

$\frac{2x + 4}{2}$

$\frac{2(x + 2)}{2} = \boxed{x + 2}$

$$\begin{array}{r} x + 2 \\ 2 \enclose{longdiv}{2x + 4} \\ 2x \downarrow \\ 4 \end{array}$$

$$\begin{array}{r} x + 2 \\ x + 1 \enclose{longdiv}{x^2 + 3x + 6} \\ -(x^2 + x) \downarrow \\ 2x + 6 \\ -(2x + 2) \\ 4 \end{array}$$

remainder

$\boxed{x + 2 + \frac{4}{x + 1}}$

$\frac{x^2 + 5x + 4}{x + 4}$

$\frac{(x + 4)\boxed{(x + 1)}}{(x + 4)} = x + 1$

$$\begin{array}{r} x + 1 \\ x + 4 \enclose{longdiv}{x^2 + 5x + 4} \\ -(x^2 + 4x) \downarrow \\ x + 4 \\ -4 \\ 0 \end{array}$$

$\frac{3x^3 - 2x^2 + 7x - 4}{x^2 + 1}$

$$\begin{array}{r} 3x - 2 \\ x^2 + 1 \enclose{longdiv}{3x^3 - 2x^2 + 7x - 4} \\ -(3x^3 \downarrow + 3x) \\ -2x^2 + 4x - 4 \\ -(-2x^2 \downarrow - 2) \\ 4x - 2 \end{array}$$

$3x - 2 + \frac{4x - 2}{x^2 + 1}$

Dividing by Polynomials by x (no remainders)

$$\frac{x^4-2x^3+5x}{x} = \frac{x^4}{x} - \frac{2x^3}{x} + \frac{5x}{x} = \boxed{x^3-2x^2+5}$$

$$\frac{2x^5+5x^3}{x} \quad \frac{2x^5}{x} + \frac{5x^3}{x} = 2x^4+5x^2$$

$$x^2(2x^2+5)$$

$$\frac{2x^4-3x}{x} = \frac{2x^4}{x} - \frac{3\cancel{x}}{\cancel{x}} = \boxed{2x^3-3}$$

$$\frac{2x^4+4x^3-x^2}{x} = \frac{2x^4}{x} + \frac{4x^3}{x} - \frac{x^2}{x} = \boxed{2x^3+4x^2-x}$$

$$\frac{x^5-3x^2+2x}{x} = \frac{x^5}{x} - \frac{3x^2}{x} + \frac{2x}{x} = \boxed{x^4-3x+2}$$

$$\frac{4x^3+x^2}{x} = \frac{4x^3}{x} + \frac{x^2}{x} = 4x^2+x$$

$$(u^2-v^2) = (u+v)(u-v)$$

$$\frac{(x^2-16)}{x+4} = \frac{(x-4)\cancel{(x+4)}}{\cancel{x+4}}$$

$$\frac{(x^2+7x+10)}{x+2} = \frac{(x+5)\cancel{(x+2)}}{\cancel{x+2}}$$

$$\frac{x^2+10x+25}{x-5} = \frac{(x+5)\cancel{(x+5)}}{\cancel{x+5}}$$

$$\frac{x^2-36}{x-6} = \frac{(x+6)\cancel{(x-6)}}{\cancel{x-6}}$$

Dividing Quadratics by linear expressions w/ remainders

- use in physical applications (qudratic = force or energy) (linear = time)
- relationship w/ something and time

$$\frac{x^2+5x+8}{x+2} = \frac{x^2+5x+6+2}{x+2}$$

$$= \frac{(x+3)\cancel{(x+2)}+2}{\cancel{x+2}}$$

$$= (x+3) + \frac{\boxed{2}}{x+2} \quad \boxed{x \neq -2,}$$ constrain the remainder

remainder

$$\begin{array}{r} \boxed{x+3\ r\ 2} \\ x+2\ \overline{)\ x^2+5x+8} \\ -(x^2+2x)\downarrow \\ 3x+8 \\ -3x\ 6 \\ \hline \boxed{2} \end{array}$$

Dividing quadratics by linear expressions w/ remainders : missing x-te

$$\frac{x^2+1}{x+2} = \frac{(u^2+v^2)}{x+2}$$

$$\begin{array}{r} \boxed{x-2}\ r\ 5 \\ x+2\ \overline{)\ x^2 \qquad 1} \\ -(x^2-2x) \\ \hline -2x+1 \\ -(+2x+4) \\ \hline 5 \end{array}$$

$x-2 \ r \frac{5}{x+2},\ x \neq -2$

$$\frac{x^2+4}{x-3} = \begin{array}{r} x+3\ r\ R \\ x-3\ \overline{)\ x^2 \qquad 3 \quad x\text{-}3} \\ -x^2+3x\downarrow \\ \hline +3x+3 \\ -3x+9 \\ \hline 12 \end{array}$$

$$\frac{x^2-5}{x-2}$$

$$\begin{array}{r} x+2 \\ x-2\ \overline{)\ x^2+0x-5} \\ -x^2+2x \\ \hline +2x-5 \\ -2x+4 \\ \hline -1 \end{array}$$

$$x+4 \overline{\smash{)}\,\cancel{x^2}+0x+1}^{\;x-4}$$
$$-\cancel{x^2}+4x$$
$$-4x+1$$
$$+4x+16$$
$$17$$

$$x+3 \overline{\smash{)}\,\cancel{x^2}+5x+5}^{\;x+2}$$
$$-\cancel{x^2}-3x$$
$$+\cancel{2x}+5$$
$$-\cancel{2x}-6$$
$$-1$$

$$\boxed{x+1,\ \frac{3}{x-1},\ x\neq 1}$$

$$x-1 \overline{\smash{)}\,\cancel{x^2}+0x+2}^{\;x+1}$$
$$-\cancel{x^2}+1x$$
$$\cancel{x}+2$$
$$-\cancel{x}+1$$
$$3$$

$$x-1 \overline{\smash{)}\,\cancel{x^2}+6x-4}^{\;x+7}\quad \boxed{\frac{3}{x-1}}$$
$$-\cancel{x^2}+x$$
$$7\cancel{x}-4$$
$$-7\cancel{x}+7$$
$$3$$

Dividing Polynomials by linear expressions

$$\frac{3x^3+4x^2-3x+7}{x+2}$$

$$\boxed{3x^2-2x+1 + \frac{5}{x+2}\ \ x\neq 2}$$

$$x+2 \overline{\smash{)}\,3\cancel{x^3}+4x^2-3x+7}$$
$$-\cancel{3x^3}-6x^2$$
$$-\cancel{2x^2}-3x$$
$$+\cancel{2x^2}+4x$$
$$x+7$$
$$-x-2$$
$$5$$

Dividing the Polynomials by linear expressions: missing term

$$\frac{2x^3-47x-15}{x-5}$$

$$\boxed{2x^2+10x+3} + \frac{0}{x-5}$$

$$x-5 \overline{\smash{)}\,2\cancel{x^3}+0x^2-47x-15}$$
$$-\cancel{2x^3}+10x^2$$
$$10x^2-47x$$
$$-10x^2+50x$$
$$3x-15$$
$$-3x+15$$
$$0$$

$\boxed{x\neq 5}$ constrain the domain

$$x-3\,\overline{\smash{)}\,x^3-0x^2-4x-15}$$
Quotient: x^2+3x+5

$-x^3+3x^2$

$3x^2-4x$

$-3x^2+9x$

$+5x-15$

$-5x+15$

0

$$x-5\,\overline{\smash{)}\,55x^3-22x^2-17x+11}$$
Quotient: $55x^2+253x+1282 \quad \frac{-6399}{x-5}$

$-55x^3+275x^2$

$253x^2-17x$

$-253x^2+1265x$

$1282x+11$

$1282x-6410$

$\boxed{-6399}$

$$x-5\,\overline{\smash{)}\,5x^3-22x^2-17x+11}$$
Quotient: $5x^2-12x-77$

$-5x^3+10x^2$

$-12x^2-17x$

$+12x^2-60x$

$-77x+11$

$+77x-385$

$$x-5\,\overline{\smash{)}\,2x^3-11x^2+0x+25}$$
Quotient: $2x^2-x-5$

$-2x^3+10x^2$

$-x^2+0x$

$+x^2-5x$

$-5x+25$

$+5x-25$

0

$$x+7\,\overline{\smash{)}\,x^3+5x^2+9x+30}$$
Quotient: $x^2-2x+23 \quad \frac{-131}{x+7} \quad x\neq-7$

$-x^3-7x^2$

$-2x^2+9x$

$-(+2x^2+14x)$

$23x+30$

$$x+1\,\overline{\smash{)}\,3x^3+10x^2+7x+0}$$
Quotient: $3x^2+7x$

$-3x^3+3x^2$

$7x^2+7x$

$-7x^2+7x$

$0 \quad 0 \quad 0$

$$x-6\,\overline{\smash{)}\,2x^3-13x^2+9x-16}$$
Quotient: $2x^2-x+3 \quad \frac{2}{x-6} \quad \boxed{x\neq6}$

$-2x^3+12x^2$

$-x^2+9x$

$+x^2-6x$

$3x-16$

$-3x+18$

2

$$x+1\,\overline{\smash{)}\,3x^3+0x^2+x-11}$$
Quotient: $3x^2-3x+4 \quad \frac{-15}{x+1} \quad x\neq-1$

$-3x^3-3x^2$

$-3x^2+x$

$+3x^2+3x$

$4x-11$

$-4x-4$

-15

$$x+3\,\overline{\smash{)}\,2x^3-x^2-25x-12}$$
Quotient: $2x^2-7x-4$

$-2x^3-6x^2$

$-7x^2-25x$

$+7x^2+21x$

Factoring Using Polynomial Division

The polynomial $p(x) = 4x^3 + 19x^2 + 19x - 6$ has a known factor of $(x+2)$.
Re-write as a product of linear factors

$$x+2\ \overline{)\ 4x^3 + 19x^2 + 19x - 6}\quad \text{quotient: } 4x^2 + 11x - 3 \quad \frac{8}{x+2}$$

$-4x^3 - 8x^2$

$11x^2 + 19x$

$-11x^2 - 22x$

$-3x - 6$

$+3x + 6$

$\boxed{4x^2 + 11x - 3}$

linear · quadratic

$(x+2)(4x^2 + 11x - 3)$

needs to be broken down more

$p(x) = \boxed{(x+2)(4x-1)(x+3)}$

re-write as product of linear factors

$$x+6\ \overline{)\ x^3 + 9x^2 + 0x - 108}\quad \text{quotient: } x^2 + 3x - 18$$

$-x^3 - 6x^2$

$3x^2 + 0x$

$-3x^2 - 18x$

$-18x - 108$

$+18x + 108$

$p(x) = (x+6)(x^2 + 3x - 18)$

$p(x) = (x+6)(x+6)(x-3)$

$x+3\ \overline{)\ 2x^3 - x^2 - 25x - 12}$

$-2x^3 - 6x^2$

$-7x^2 - 25x$

$+7x^2 + 21x$

$-4x - 12$

$+4x + 12$

Product of linear factors

$$x+2\ \overline{)\ x^3 + 0x^2 - 19x - 30}\quad \text{quotient: } x^2 - x - 17 \quad \frac{4}{x+2}$$

$-x^3 - x^2$

$-x^2 - 19x$

$+x^2 + 2x$

$-17x - 30$

$+17x + 34$

4

$p(x) = (x+2)(x^2 - x - 17)\left(\frac{4}{x+2}\right)$

$(x+2)(\quad)(\quad)$

$$x+2\ \overline{)\ x^3 + 0x^2 - 19x - 30}\quad \text{quotient: } x^2 - 2x - 15$$

$-x^3 - 2x^2$

$-2x^2 - 19x$

$+2x^2 + 4x$

$-15x - 30$

$+15x + 30$

$p(x) = (x+2)(x^2 - 2x - 15)\,0$

$p(x) = (x+2)(x-5)(x+3)$

linear factors of the polynomial $x^3 - 19x -$

$$x+3\ \overline{)\ 2x^3 - x^2 - 25x - 12}\quad \text{quotient: } 2x^2 + 5x - 40$$

$2x^3 + 6x^2$

$5x^2 - 25x$

$-5x^2 - 15x$

$-40x - 12$

$-40x - 120$

$(x+3)(2x^2 - 7x - 4)$

$(x+3)(2x+1)(x-4)$

$$\begin{array}{r} x^2+4x-12 \\ x+3 \overline{)\,x^3+7x^2+0x-36} \\ -x^3-3x^2 \downarrow \\ \hline 4x^2+0x \\ -4x^2-12x \\ \hline -12x-36 \\ +12x+36 \\ \hline 0 \end{array}$$

$(x+3)(x+6)(x-2)$

$$\begin{array}{r} 5x^2-9x-2 \\ x-7 \overline{)\,5x^3-44x^2+61x+14} \\ -5x^3+35x^2 \\ \hline -9x^2+61x \\ +9x^2-63x \\ \hline -2x+14 \\ +2x-14 \\ \hline 0 \end{array}$$

$p(x)=(x-7)(5x^2-9x-2)$

$p(x)=(x-7)(5x+1)(x-2)$

$p(x)+\frac{k}{x+3}$

1.

$$\begin{array}{r} x-3 \quad \frac{2}{x+3} \quad x \neq -3 \\ x+3 \overline{)\,x^2+0x-7} \\ -x^2-3x \\ \hline -3x-7 \\ +3x+9 \\ \hline 2 \end{array}$$

$p(x)=x-3 \quad (x-3)+\frac{2}{x+3}$

$k=2$

2. product of linear factors $p(x)$

$$\begin{array}{r} x^2+4x+4 \\ x-1 \overline{)\,x^3+3x^2+0x-4} \\ -x^3+x^2 \\ \hline 4x^2+0x \\ -4x^2+4x \\ \hline 4x-4 \\ -4x+4 \\ \hline 0 \end{array}$$

$(x-1)(x^2+4x+4)=(x-1)(x+2)(x+2)$

$p(x)=(x-1)(x+2)(x+2)$

3. $p(x)+\frac{k}{x+1}$

$$\begin{array}{r} x-3 \quad -\frac{5}{x+1} \\ x+1 \overline{)\,x^2-2x-8} \\ -x^2-x \\ \hline -3x-8 \\ +3x+3 \\ \hline -5 \end{array}$$

$$\begin{array}{r} x-3 \\ x+1 \overline{)\,x^2-2x-8} \\ x^2+x \\ \hline -3x-8 \\ +3x+3 \\ \hline -5 \end{array}$$

6. plot on line

$p(x) = 2x^3 + 4x^2 - 6x$

$2x(x^2 + 2x - 3)$

$2x(x + 3)(x - 1)$

$2x(x+3)(x-1)$

$x = -3$

$x = 1$

7. expand + combine like terms

$u^2 - v^2$

$= 4x^8 - 9x^6$

$(2x^4 + 3x^3)(2x^4 - 3x^3)$

$4x^8 - 6x^7 + 6x^7 - 9x^6$

$4x^8 - 9x^6$

$x^6(4x^2 - 9)$

$x^6(2x + 3)(2x - 3)$

8. Factor Completely

$4p^2 - 25q^2$

$(2p + 5q)(2p - 5q)$

[illegible]

9. M - T

$M = -8r^2 + 11r - 6$

$T = -7r^2 - 9r + 14$

$-\cancel{8r^2} + \cancel{11r} - \cancel{6} - \cancel{7r^2} \cancel{9r} + \cancel{14}$

$\boxed{-15r^2 + 2r + 8}$

10. Combine like terms

$(5 - 2x^4)(5 + 2x^4) = u^2 - v^2$

$\boxed{25 - 4x^6}$

$25 + \cancel{10x^4} - \cancel{10x^4} - 4x^8$

$\boxed{25 - 4x^8}$

Unit 3

Shifting Functions Introduction

x^3-2x

$x(x^2-2)$ a^2-b^2

$x(x+\sqrt{2})(x-\sqrt{2})$

$0, -\sqrt{2}, \sqrt{2}$

4. product of linear factors

$x-4 \overline{)\,3x^3-20x^2+37x-20}$ = $3x^2-8x+5$

$-3x^3+12x^2$

$-8x^2+37x$

$+8x-32x$

$5x-20$

$-5x+20$

0

$(x-4)(3x^2-8x+5)$

$(x-4)(3x-5)(x-1)$

5. $x+4 \overline{)\,x^2+0x+1}$ = $x-4 \quad \frac{17}{x+4}$

$-x^2-4x$

$-4x+1$

$+4x+16$

17

$x+4 \overline{)\,x^2+0x+1}$ = $x-4$

$-x^2-4x$

$-4x+1$

$+4x+16$

17

$\boxed{\text{Unit Test}}$

1. $(x+3) \overline{)\,x^2+5x+5}$ = $x+2 \quad -\frac{1}{x+3} \quad x \neq -3$

$-x^2-3x$

$2x+5$

$-2x-6$

-1

2. $(3z^5+7z^2)^2$

$(3z^5+7z^2)(3z^5+7z^2)$

$9z^{10}+21z^7+21z^7+49z^4$

$9z^{10}+42z^7+49z^4$

$u^2+2uv+v^2$

$= 9z^{10}+2(3z^5\cdot 7z^2)+49z^4$

$9z^{10}+2(21z^7)+49z^4$

$9z^{10}+42z^7+49z^4$

3. linear factors

$\frac{x^3-6x^2+32}{x-4}$

$x-4 \overline{)\,x^3-6x^2+0x+32}$ = x^2-2x-8

$-x^3+4x^2$

$-2x^2+0x$

$+2x^2-8x$

$-8x+32$

$+8x-32$

0

$(x-4)(x^2-2x-8)$

$p(x)=(x-4)(x-4)(x+2)$

4. Subtract

$(p^2-5p+4)-(4p^2-9p+11)$

$p^2-5p+4-4p^2+9p-11$

$\boxed{-3p^2+4p-7}$

5. expand

$(2a^2+7a-10)(a-5)$

$2a^3+7a^2-10a-10a^2-35a+50$

$2a^3-3a^2-45a+50$