

Qwerty: A Basis-Oriented Quantum Programming Language

QCE '25

Austin J. Adams^{*}, Sharjeel Khan^{*}, Arjun S. Bhamra^{*}, Ryan R. Abusaada^{*},
Travis S. Humble[†], Jeffrey S. Young^{*}, Thomas M. Conte^{*}

September 5th, 2025

^{*}Georgia Tech and [†]ORNL

Two leaps:

Two leaps:

① Problem \rightarrow Algorithm

Two leaps:

① Problem \rightarrow Algorithm

② Algorithm \rightarrow Implementation

Two leaps:

① Problem \rightarrow Algorithm



② Algorithm \rightarrow Implementation

Quantum Programming is Tough for Newcomers

Two leaps:

① Problem \rightarrow Algorithm



② Algorithm \rightarrow Implementation

Quantum programming languages

Example operation:

$$|+\rangle|+\rangle|+\rangle|+\rangle \mapsto -|+\rangle|+\rangle|+\rangle|+\rangle$$

Example operation:

$$|+\rangle|+\rangle|+\rangle|+\rangle \mapsto -|+\rangle|+\rangle|+\rangle|+\rangle$$

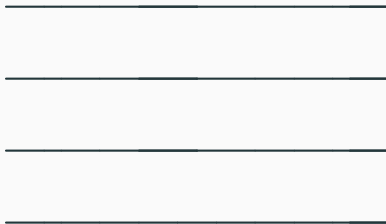
How does this look in most quantum programming languages?

Step 1: Do the math

$$U = -|++++\rangle\langle++++| \\ + (I^{\otimes 4} - |++++\rangle\langle++++|)$$

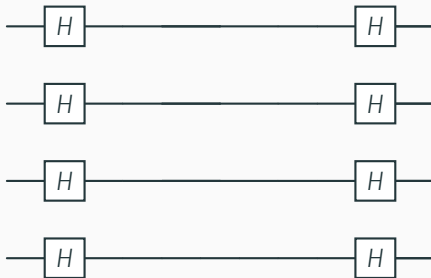
Step 2: Design a circuit

$$U = -|++++\rangle\langle++++| \\ + (I^{\otimes 4} - |++++\rangle\langle++++|)$$



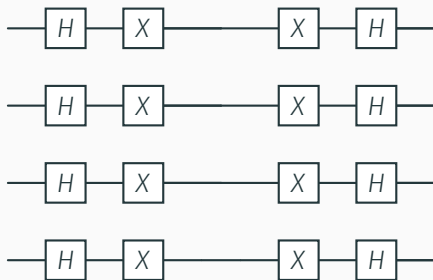
Step 2: Design a circuit

$$U = H^{\otimes 4}(-|0000\rangle\langle 0000| + (I^{\otimes 4} - |0000\rangle\langle 0000|))H^{\otimes 4}$$



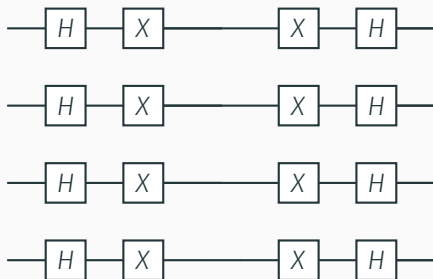
Step 2: Design a circuit

$$U = H^{\otimes 4} X^{\otimes 4} (-|1111\rangle\langle 1111| + (I^{\otimes 4} - |1111\rangle\langle 1111|)) X^{\otimes 4} H^{\otimes 4}$$



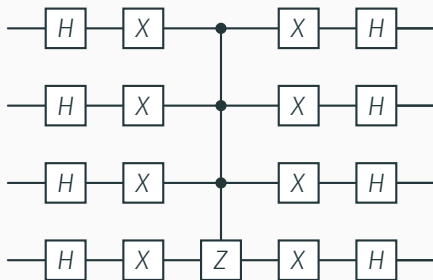
Step 2: Design a circuit

$$U = H^{\otimes 4} X^{\otimes 4} (-|1111\rangle\langle 1111| + (I^{\otimes 4} - |1111\rangle\langle 1111|)) X^{\otimes 4} H^{\otimes 4}$$



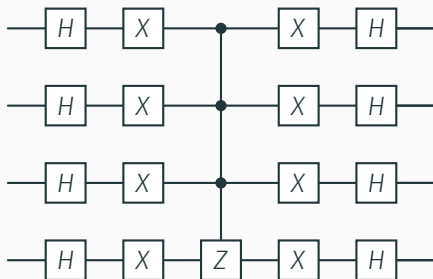
Step 2: Design a circuit

$$U = H^{\otimes 4} X^{\otimes 4} (CCCZ) X^{\otimes 4} H^{\otimes 4}$$



Step 2: Design a circuit

$$U = -|++++\rangle\langle++++| + (I^{\otimes 4} - |++++\rangle\langle++++|) \quad \checkmark$$



Step 3: Write synthesis code

QCL (2000)

```
1 operator diffuse(quireg q) {  
2     H(q);  
3     Not(q);  
4     CPhase(pi,q);  
5     !Not(q);  
6     !H(q);  
7 }
```


Step 3: Write synthesis code

QCL (2000)

```
1 operator diffuse(qreg q) {  
2   H(q);  
3   Not(q);  
4   CPhase(pi,q);  
5   !Not(q);  
6   !H(q);  
7 }
```

Q# (2025)

```
1 operation Diffuse(q : Qubit[])  
2     : Unit {  
3     for qi in q {  
4         H(qi);  
5         X(qi);  
6     }  
7     Controlled Z(Most(q),  
8                   Tail(q));  
9     for qi in q {  
10        X(qi);  
11        H(qi);  
12    }  
13 }
```

Step 3: Write synthesis code

QCL (2000)

```
1 operator diffuse(qreg q) {  
2   H(q);  
3   Not(q);  
4   CPhase(pi,q);  
5   !Not(q);  
6   !H(q);  
7 }
```

Q# (2025)

```
1 operation Diffuse(q : Qubit[])  
2     : Unit {  
3     within {  
4         for qi in q {  
5             H(qi);  
6             X(qi);  
7         }  
8     } apply {  
9         Controlled Z(Most(q),  
10             Tail(q));  
11     }  
12 }
```

Step 3: Write synthesis code

QCL (2000)

```
1 operator diffuse(qreg q) {  
2   H(q);  
3   Not(q);  
4   CPhase(pi,q);  
5   !Not(q);  
6   !H(q);  
7 }
```

Q# (2025)

```
1 operation Diffuse(q : Qubit[])  
2     : Unit {  
3     within {  
4         ApplyToEachA(H, q);  
5         ApplyToEachA(X, q);  
6     } apply {  
7         Controlled Z(Most(q),  
8                       Tail(q));  
9     }  
10 }
```

Step 3: Write synthesis code

QCL (2000)

```
1 operator diffuse(qureg q) {  
2   H(q);  
3   Not(q);  
4   CPhase(pi,q);  
5   !Not(q);  
6   !H(q);  
7 }
```

Q# (2025)

```
1 operation Diffuse(q : Qubit[])  
2     : Unit {  
3     within {  
4       ApplyToEachA(H, q);  
5       ApplyToEachA(X, q);  
6     } apply {  
7       Controlled Z(Most(q),  
8                     Tail(q));  
9     }  
10 }
```

Most Quantum PLs today require circuit synthesis expertise

Goal:

$$|+\rangle|+\rangle|+\rangle|+\rangle \mapsto -|+\rangle|+\rangle|+\rangle|+\rangle$$

Goal:

$$|+\rangle|+\rangle|+\rangle|+\rangle \mapsto -|+\rangle|+\rangle|+\rangle|+\rangle$$

Qwerty syntax:

'pppp' >> - 'pppp'

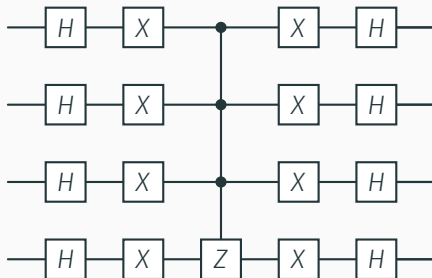
Goal:

$$|+\rangle|+\rangle|+\rangle|+\rangle \mapsto -|+\rangle|+\rangle|+\rangle|+\rangle$$

Qwerty syntax:

'pppp' >> - 'pppp'

Circuit synthesized by ASDf:



Our Contributions

1. Qwerty programs expressed with **basis translations** and **qubit literals**
2. Programmers can **interpret** the behavior of Qwerty programs **without circuit synthesis experience**
3. Qwerty integrates with a popular classical PL (**Python**)

Hello World in Qwerty

```
1 from qwerty import *  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18 print(grover())
```

Hello World in Qwerty

```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7
8
9
10
11
12
13
14
15
16
17
18 print(grover())
```

Hello World in Qwerty

```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7 @qpu
8 def grover_iter(q):
9     return q | oracle.sign | 'pppp' >> -'pppp'
10
11
12
13
14
15
16
17
18 print(grover())
```

Hello World in Qwerty

```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7 @qpu
8 def grover_iter(q):
9     return q | oracle.sign | 'pppp' >> -'pppp'
10
11 @qpu
12 def grover():
13     return ('pppp' | grover_iter
14             | grover_iter
15             | grover_iter
16             | measure**4)
17
18 print(grover())
```

Qubit Initialization

Qubit Literals: String Analogy

- Standard basis: '0' and '1'

Qubit Literals: String Analogy

- Standard basis: '0' and '1'
- In typical classical PLs,
"yee" + "haw" == "yeehaw"

Qubit Literals: String Analogy

- Standard basis: `'0'` and `'1'`
- In typical classical PLs,
`"yee" + "haw" == "yeehaw"`
- In Qwerty,
`'0' * '1' == '01'`

- Phase $e^{i\pi/4} |1\rangle$ represented as *tilt*: '1' @ 45
- Syntax evokes '1' ↻45

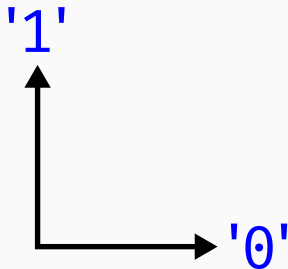
Qubit Literals: Tilt

- Phase $e^{i\pi/4} |1\rangle$ represented as *tilt*: '1' @ 45
- Syntax evokes '1' ↻45
- Fun to draw as ↗

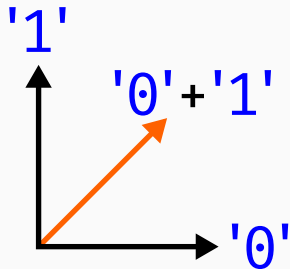
Qubit Literals: Tilt

- Phase $e^{i\pi/4} |1\rangle$ represented as *tilt*: '1' @ 45
- Syntax evokes '1' ↻45
- Fun to draw as ↗
- - '1' is syntactic sugar for '1'@180

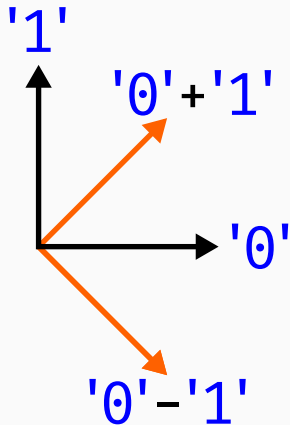
Qubit Literals: Superposition



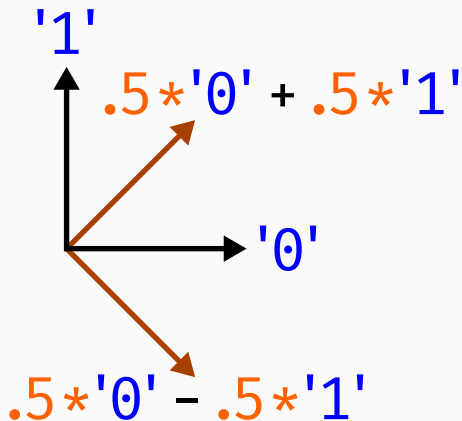
Qubit Literals: Superposition



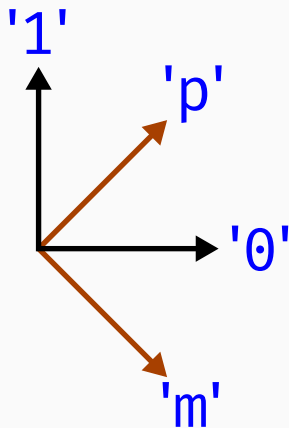
Qubit Literals: Superposition



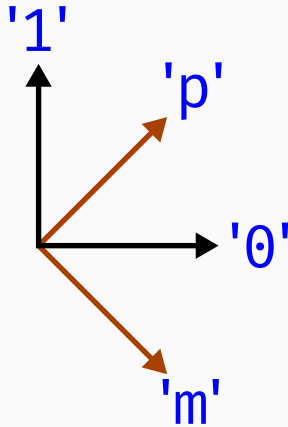
Qubit Literals: Superposition



Qubit Literals: Superposition



Qubit Literals: Superposition



Why **p** and **m** instead of **+** and **-**?

Because this looks confusing: **+'+-'**

State Evolution

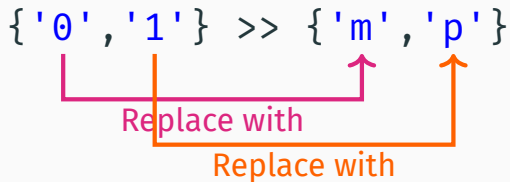
$\{ '0', '1' \} \gg \{ 'm', 'p' \}$

{ '0', '1' } >> { 'm', 'p' }



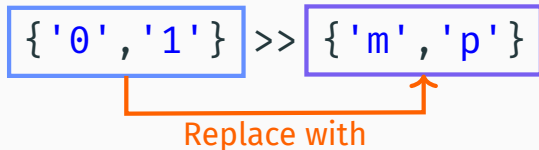
Replace with

Basis Translations

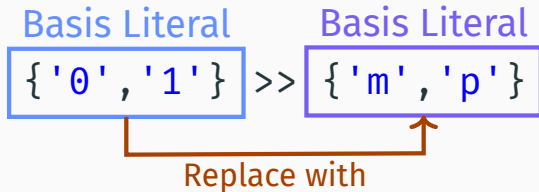


$\{ '0', '1' \} \gg \{ 'm', 'p' \}$

Basis Translations



Basis Translations



Basis Literals

- Ordered list of basis vectors
- Example: { '00', '01', '10', '11' } is two-qubit standard basis

- Ordered list of basis vectors
- Example: $\{ '00', '01', '10', '11' \}$ is two-qubit standard basis
- Could also write $\{ '0', '1' \} * \{ '0', '1' \}$

- Ordered list of basis vectors
- Example: $\{ '00', '01', '10', '11' \}$ is two-qubit standard basis
- Could also write $\{ '0', '1' \} * \{ '0', '1' \}$
- Or $\{ '0', '1' \} ** 2$

- Ordered list of basis vectors
- Example: $\{ '00', '01', '10', '11' \}$ is two-qubit standard basis
- Could also write $\{ '0', '1' \} * \{ '0', '1' \}$
- Or $\{ '0', '1' \} ** 2$
- Must be an orthonormal basis
 - Example: $\{ '00', '10', -'10' \}$ is invalid

Any $b_{\text{in}} \gg b_{\text{out}}$ requires $\text{span}(b_{\text{in}}) = \text{span}(b_{\text{out}})$.

Basis Translation Type Checking

Any $b_{\text{in}} \gg b_{\text{out}}$ requires $\text{span}(b_{\text{in}}) = \text{span}(b_{\text{out}})$.

✓ $\{'0', '1'\} \gg \{'0', '1'\} @ 90$

Basis Translation Type Checking

Any $b_{\text{in}} \gg b_{\text{out}}$ requires $\text{span}(b_{\text{in}}) = \text{span}(b_{\text{out}})$.

✓ $\{'0', '1'\} \gg \{'0', '1'\} @ 90$

✗ $\{'0'\} \gg \{'1'\}$

- **b.measure** measures in the basis **b**
- Measure in standard basis: **{ '0', '1' }.measure**

- **b.measure** measures in the basis **b**
- Measure in standard basis: **{ '0', '1' }.measure**
- Measure in Bell basis:
**{ '00' + '11',
 '00' - '11',
 '10' + '01',
 '01' - '10' }.measure**

Superdense Coding in Qwerty

```
1 message = bit[2](0b10)
2
3 @qpu
4 def superdense():
5     bit0, bit1 = message
6     alice, bob = '00' + '11'
7
8     sent_to_bob = (alice | ({'0','1'} >> {'1','0'}
9                             if bit0 else id)
10                    | ({'1'} >> {'-1'}
11                        if bit1 else id))
12
13     recovered_message = (sent_to_bob * bob
14                          | {'00'+ '11', '00'- '11',
15                            '10'+ '01', '01'- '10'}
16                          .measure)
17     return recovered_message
```

Superdense Coding in Qwerty

```
1 message = bit[2](0b10)
2
3 @qpu
4 def superdense():
5     bit0, bit1 = message
6     alice, bob = '00' + '11' Entangled pair
7
8     sent_to_bob = (alice | ({'0','1'} >> {'1','0'}
9                             if bit0 else id)
10                    | ({'1'} >> {'-1'}
11                        if bit1 else id))
12
13     recovered_message = (sent_to_bob * bob
14                           | {'00'+ '11', '00'-'11',
15                               '10'+ '01', '01'-'10'}
16                           .measure)
17     return recovered_message
```

Superdense Coding in Qwerty

```
1 message = bit[2](0b10)
2
3 @qpu
4 def superdense():
5     bit0, bit1 = message
6     alice, bob = '00' + '11' Entangled pair
7
8     sent_to_bob = (alice | ({'0','1'} >> {'1','0'}
9                             if bit0 else id)
10                    | ({'1'} >> {'-1'}
11                      if bit1 else id))
12
13     recovered_message = (sent_to_bob * bob
14                          | {'00'+ '11', '00' - '11',
15                             '10'+ '01', '01' - '10'})
16                          .measure)
17     return recovered_message
```

Metaprogramming

- Tedious:

```
{'0', '1'}.measure
```

```
* {'0', '1'}.measure
```

```
* {'0', '1'}.measure
```

- Tedious:

```
{'0', '1'}.measure
```

```
* {'0', '1'}.measure
```

```
* {'0', '1'}.measure
```

- What about `{'0', '1'}.measure**3?`

- Tedious:
 { '0', '1' }.measure
 * { '0', '1' }.measure
 * { '0', '1' }.measure
- What about { '0', '1' }.measure**3?
- Or **std.measure**3**?

- Tedious:
 `{'0', '1'}.measure`
 `* {'0', '1'}.measure`
 `* {'0', '1'}.measure`
- What about `{'0', '1'}.measure**3`?
- Or `std.measure**3`?
- Or even `measure**3`?

- Tedious:
 `{'0', '1'}.measure`
 `* {'0', '1'}.measure`
 `* {'0', '1'}.measure`
- What about `{'0', '1'}.measure**3`?
- Or `std.measure**3`?
- Or even `measure**3`?
- *metaQwerty* expands to Qwerty

```
'0'.sym = __SYM_STD0__()  
'1'.sym = __SYM_STD1__()
```

```
'0'.sym = __SYM_STD0__()  
'1'.sym = __SYM_STD1__()  
'p'.sym = '0' + '1'  
  
'm'.sym = '0' + '1'@180
```

metaQwerty Prelude: Qubit Symbols

```
'0'.sym = __SYM_STD0__()  
'1'.sym = __SYM_STD1__()  
'p'.sym = '0' + '1'  
'i'.sym = '0' + '1'@90  
'm'.sym = '0' + '1'@180  
'j'.sym = '0' + '1'@270
```

```
std = {'0', '1'}  
pm  = {'p', 'm'}  
ij  = {'i', 'j'}  
bell = {'00' + '11',  
        '00' - '11',  
        '10' + '01',  
        '01' - '10'}
```

```
std = {'0', '1'}  
pm  = {'p', 'm'}  
ij  = {'i', 'j'}  
bell = {'00'+'11',  
        '00'-'11',  
        '10'+'01',  
        '01'-'10'}
```

```
fourier[1] = pm  
fourier[N] = fourier[N-1] // std.revolve
```

```
id = {'0', '1'} >> {'0', '1'}  
flip = {'0', '1'} >> {'1', '0'}  
measure = std.measure
```


metaQwerty Example: N-Qubit Grover's

```
1 def grover2(oracle, num_iter):
2     @qpu[[N]]
3     def grover_iter(q):
4         return (q | oracle.sign
5                 | 'p'**N >> -'p'**N)
6
7     @qpu[[N]]
8     def kernel():
9         return ('p'**N
10                | (grover_iter
11                   for i in range(num_iter))
12                | measure**N)
13
14     return kernel()
```

metaQwerty Example: N-Qubit Grover's

```
1 def grover2(oracle, num_iter):
2     @qpu[[N]] ← Polymorphism
3     def grover_iter(q):
4         return (q | oracle.sign
5                 | 'p'**N >> -'p'**N)
6
7     @qpu[[N]] ←
8     def kernel():
9         return ('p'**N
10                | (grover_iter
11                   for i in range(num_iter))
12                | measure**N)
13
14     return kernel()
```

metaQwerty Example: N-Qubit Grover's

```
1 def grover2(oracle, num_iter):
2     @qpu[[N]] ← Polymorphism
3     def grover_iter(q):
4         return (q | oracle.sign
5                 | 'p'**N >> -'p'**N)
6
7     @qpu[[N]] ←
8     def kernel():
9         return ('p'**N
10                | (grover_iter
11                  for i in range(num_iter))
12                | measure**N)
13
14     return kernel()
```

Polymorphism

Macros

Predication

Flip right qubit if left is '1':

```
flip if '1_' else id
```

Predication Example

Flip right qubit if left is '1':

```
flip if '1_' else id
```

Basis pattern

Predication Example

Flip right qubit if left is '1':

flip if '1_' else id
Basis pattern

Performs:

'00' \mapsto '00'

'01' \mapsto '01'

'10' \mapsto '11'

'11' \mapsto '10'

Another Predication Example

```
pm >> std if {'p_p', 'm_m'} else id
```

Performs:

'**p****p****p**' \mapsto '**p**0**p**'

'**p****m****p**' \mapsto '**p**1**p**'

'mpp' \mapsto 'mpp'

'mmp' \mapsto 'mmp'

'ppm' \mapsto 'ppm'

'pmm' \mapsto 'pmm'

'**m****p****m**' \mapsto '**m**0**m**'

'**m****m****m**' \mapsto '**m**1**m**'

Full Example: Bernstein–Vazirani

Bernstein-Vazirani Algorithm

Input: Black box for $f(x) = \underbrace{x_1 s_1}_{\text{AND}} \underbrace{\oplus}_{\text{XOR}} x_2 s_2 \oplus \cdots \oplus x_n s_n$

Output: Secret bitstring s used to build oracle

Bernstein-Vazirani

Bernstein-Vazirani Algorithm

Input: Black box for $f(x) = \underbrace{x_1 s_1}_{\text{AND}} \underbrace{\oplus}_{\text{XOR}} x_2 s_2 \oplus \cdots \oplus x_n s_n$

Output: Secret bitstring s used to build oracle

- Any classical algorithm needs to run the black box n times:

$$s_1 = f(1000 \cdots 00)$$

$$s_2 = f(0100 \cdots 00)$$

$$\vdots$$

$$s_n = f(0000 \cdots 01)$$

Bernstein-Vazirani Algorithm

Input: Black box for $f(x) = \underbrace{x_1 s_1}_{\text{AND}} \underbrace{\oplus}_{\text{XOR}} x_2 s_2 \oplus \cdots \oplus x_n s_n$

Output: Secret bitstring s used to build oracle

- Any classical algorithm needs to run the black box n times:

$$s_1 = f(1000 \cdots 00)$$

$$s_2 = f(0100 \cdots 00)$$

$$\vdots$$

$$s_n = f(0000 \cdots 01)$$

- Quantum algorithm only needs 1 query!

Bernstein–Vazirani in Qwerty

```
1 from qwerty import *
2
3 def bv(secret_string):
4
5
6
7
8
9
10
11
12
13
14     return kernel()
15
16 secret_string = bit[4](0b1101)
17 print(bv(secret_string))
```

Bernstein–Vazirani in Qwerty

```
1 from qwerty import *
2
3 def bv(secret_string):
4     @classical
5     def f(x):
6         return (secret_string & x).xor_reduce()
7
8
9
10
11
12
13
14     return kernel()
15
16 secret_string = bit[4](0b1101)
17 print(bv(secret_string))
```

Bernstein–Vazirani in Qwerty

```
1 from qwerty import *
2
3 def bv(secret_string):
4     @classical
5     def f(x):
6         return (secret_string & x).xor_reduce()
7
8     @qpu[[N]]
9     def kernel():
10         return ('p'**N | f.sign
11                 | pm**N >> std**N
12                 | measure**N)
13
14     return kernel()
15
16 secret_string = bit[4](0b1101)
17 print(bv(secret_string))
```

Bernstein–Vazirani in Qwerty

```
1 from qwerty import *
2
3 def bv(secret_string):
4     @classical
5     def f(x):
6         return (secret_string & x).xor_reduce()
7
8     @qpu[[N]]
9     def kernel():
10        return ('p'**N | f.sign
11                | pm**N >> std**N
12                | measure**N)
13
14    return kernel()
15
16 secret_string = bit[4](0b1101)
17 print(bv(secret_string))
```


Bernstein–Vazirani in Qwerty

```
1 from qwerty import *
2
3 def bv(secret_string):
4     @classical
5     def f(x):
6         return (secret_string & x).xor_reduce()
7
8     @qpu[[N]]
9     def kernel():
10        return ('p'**N | f.sign
11                | pm**N >> std**N
12                | measure**N)
13
14    return kernel()
15
16 secret_string = bit[4](0b████)
17 print(bv(secret_string))
```

Tracing Bernstein–Vazirani in Qwerty

```
( '0' + '1' ) * ( '0' + '1' )  
* ( '0' + '1' ) * ( '0' + '1' )
```

Tracing Bernstein–Vazirani in Qwerty

```
( '0' + '1' ) * ( '0' + '1' )  
* ( '0' + '1' ) * ( '0' + '1' )
```

① ↓ f.sign


```
f.sign == f_1.sign  
        * f_2.sign  
        * f_3.sign  
        * f_4.sign
```

where

$$f_i(x_i) = \underbrace{x_i s_i}_{\text{AND}}$$

Tracing Bernstein–Vazirani in Qwerty

$('0' + '1') * ('0' + '1')$
 $* ('0' + '1') * ('0' + '1')$

①  `f.sign`

$('0' + -'1') * ('0' + -'1')$
 $* ('0' + '1') * ('0' + -'1')$

Tracing Bernstein–Vazirani in Qwerty

$('0' + '1') * ('0' + '1')$
 $* ('0' + '1') * ('0' + '1')$

①  f.sign


$* \quad 'm' \quad * \quad 'm'$
 $* \quad 'p' \quad * \quad 'm'$

Tracing Bernstein–Vazirani in Qwerty

```
( '0' + '1' ) * ( '0' + '1' )  
* ( '0' + '1' ) * ( '0' + '1' )
```

①  f.sign

```
*      'm'      *      'm'  
*      'p'      *      'm'
```

②  pm**4 >> std**4


```
*      '1'      *      '1'  
*      '0'      *      '1'
```


Tracing Bernstein–Vazirani in Qwerty


```
( '0' + '1' ) * ( '0' + '1' )  
* ( '0' + '1' ) * ( '0' + '1' )
```

①  f.sign

```
*      'm'      *      'm'  
      'p'      *      'm'
```

②  pm**4 >> std**4

```
*      '1'      *      '1'  
      '0'      *      '1'
```

③  measure**4

```
bit[4](0b1101)
```

- Teleportation
- Quantum phase estimation (with tracing)
- Period finding (with tracing)
- Shor's (order finding)

Conclusion

In this talk, I presented Qwerty, a *basis-oriented* quantum programming language that allows non-experts to reason about quantum computation without knowing bra-ket notation or circuit synthesis.

Conclusion

In this talk, I presented Qwerty, a *basis-oriented* quantum programming language that allows non-experts to reason about quantum computation without knowing bra-ket notation or circuit synthesis.

Compiler (ASDF) paper:



CGO '25

Source code (GitHub):



github.com/gt-tinker/qwerty

Backup Slides

Fourier Basis Details

Fourier Basis: Motivation

QFT:


```
std**N >> fourier[N]
```

QFT[†]:

```
fourier[N] >> std**N
```

Fourier Basis

`fourier[1]`



`{ ('0' + '1'),
('0' + '1') }`

Fourier Basis

fourier[1]

fourier[2]

$\{$ $('0' + '1') * ('0' + '1') ,$
 $('0' + '1') * ('0' + '1') ,$
 $('0' + '1') * ('0' + '1') ,$
 $('0' + '1') * ('0' + '1') \}$

Fourier Basis

fourier[1]

fourier[2]

fourier[3]

Diagram illustrating a sequence of expressions enclosed in a large orange border. The expressions are arranged in four rows of three. Each expression is a sum of two terms, each term being a product of two terms. The terms are either '0' or '1' with a blue dot. The expressions are grouped by a blue box on the left and a red box on the right. A red arrow points from the top of the red box to the bottom of the red box, indicating a sequence of operations.

Expressions (from top to bottom):

- Row 1: $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$
- Row 2: $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$
- Row 3: $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$
- Row 4: $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$, $(0 + 1) * (0 + 1) * (0 + 1)$

Fourier Basis

fourier[1]

fourier[2]

fourier[3]

{ ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1'),
 ('0' + '1') * ('0' + '1') * ('0' + '1') }

{ '0' , '1' }.revolve

metaQwerty Prelude: Fourier Basis

```
fourier[1] = pm  
fourier[N] = fourier[N-1] // std.revolve
```

metaQwerty Prelude: Fourier Basis

```
fourier[1] = pm  
fourier[N] = fourier[N-1] // std.revolve
```

Basis generator

metaQwerty Prelude: Fourier Basis

```
fourier[1] = pm
```

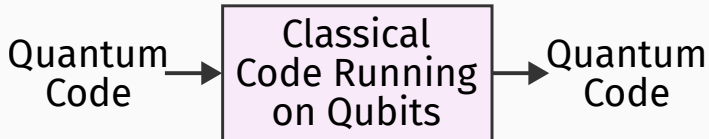
```
fourier[N] = fourier[N-1] // std.revolve
```

Basis generator

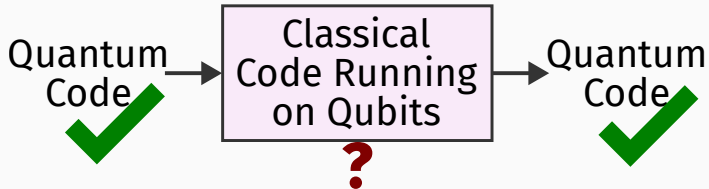
↑
Apply basis generator

Embedding Classical Functions

Plugging the Oracle Hole



Plugging the Oracle Hole



Revisiting Grover's in Qwerty: Classical Embeddings

```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7 @qpu
8 def grover_iter(q):
9     return q | oracle.sign | 'pppp' >> -'pppp'
10
11 @qpu
12 def grover():
13     return ('pppp' | grover_iter
14            | grover_iter
15            | grover_iter
16            | measure**4)
17
18 print(grover())
```

Revisiting Grover's in Qwerty: Classical Embeddings

```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7 @qpu
8 def grover_iter(q):
9     return q | oracle.sign | 'pppp' >> -'pppp'
10
11 @qpu
12 def grover():
13     return ('pppp' | grover_iter
14            | grover_iter
15            | grover_iter
16            | measure**4)
17
18 print(grover())
```

Classical func. def.

Revisiting Grover's in Qwerty: Classical Embeddings

```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7 @qpu
8 def grover_iter(q):
9     return q | oracle.sign | 'pppp' >> -'pppp'
10
11 @qpu
12 def grover():
13     return ('pppp' | grover_iter
14            | grover_iter
15            | grover_iter
16            | measure**4)
17
18 print(grover())
```

Classical func. def.

Embedding

Revisiting Grover's in Qwerty: Classical Embeddings

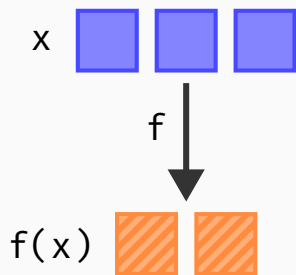
```
1 from qwerty import *
2
3 @classical
4 def oracle(x: bit[4]) -> bit:
5     return x[0] & ~x[1] & x[2] & ~x[3]
6
7 @qpu
8 def grover_iter(q):
9     return q | oracle.sign | 'pppp' >> -'pppp'
10
11 @qpu
12 def grover():
13     return ('pppp' | grover_iter
14            | grover_iter
15            | grover_iter
16            | measure**4)
17
18 print(grover())
```

Classical func. def.

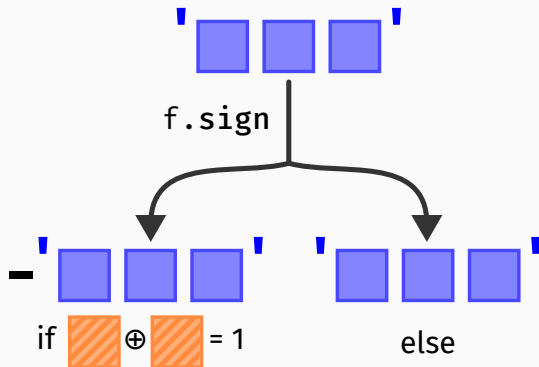
Embedding → 3 kinds

Sign Embedding

Classical Function:

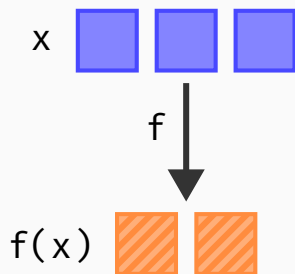


Sign Embedding:

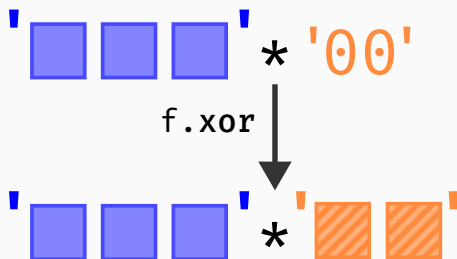


XOR Embedding

Classical Function:

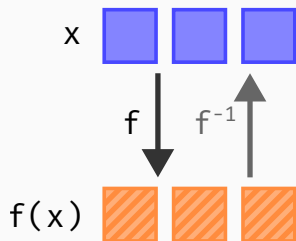


XOR Embedding:



In-Place Embedding

Classical Function:



In-Place Embedding:

